The Calculation of Mutual Coupling Between Two Antennas and its Application to the Reduction of Mutual Coupling Effects in a Pseudo-Random Phased Array

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Introduction

A relatively sparse, pseudo-random arrangement of 256 antenna stands is currently proposed for LWA stations and is described in [1]. This approach has the advantage that a large aperture can be formed with relatively few antenna elements while maintaining reasonable sidelobe levels. As discussed in [2], however, the pseudo-random geometry appears to have the disadvantage that antenna terminal currents vary significantly from one antenna to the next and for different beam pointing directions. This could severely complicate the task of system calibration. The results in [2] appear to indicate that a more compact and uniform arrangement of antenna stands provides significantly better uniformity in terminal currents across the array, and presumably, for different pointing directions. The disadvantage of this approach, however, is that a much greater number of antenna stands (perhaps by a factor of more than two) is required to fill the 100 m station footprint in order to meet the station beamwidth requirement. This would significantly increase the cost of a station. Particularly given cost considerations, it is highly desirable to reduce the effect of mutual coupling between antenna elements in the pseudo-random array layout in order to improve the calibratibility of this station design. This is the topic of the present work.

This memo first discusses the calculation of mutual coupling effects between two antennas. A "receive mode"-based definition of mutual coupling which involves exciting the two antennas by an incident plane wave, rather than a "transmit mode"-based definition which involves placing an excitation source at the feed of one of the antennas, is proposed as a indicator for the performance of a given antenna element design in the full array. It is shown that by proper selection of different antenna element design parameters, the receive mode mutual coupling between two antennas can be reduced significantly over a wide range of frequencies and scan angles. Different element designs are then simulated in both a sparse, pseudo-random array and a dense, uniform array. It is shown that antenna element designs which exhibit lower receive mode coupling also exhibit improved performance in terms of terminal current uniformity when operated in the pseudo-random array such that the array performance is closer to that of a dense, uniform array. This suggests that the receive mode coupling calculation would be useful in the design of improved antenna elements for use in a pseudo-random array. Since only two antennas elements need to be included in the calculation, this approach would offer a significant reduction in terms of computational complexity compared with a design approach that involves all or a significant portion of the antenna elements of a large array.

As in [2], analysis in this study is primarily focused on relatively narrowband wire inverted-V dipoles operating over an infinite PEC ground. This greatly simplifies simulating the response of an array consisting of hundreds of antenna elements. Some limited results are also presented for blade-like dipoles, however, in order to demonstrate that the analysis techniques discussed in this memo can also be used to reduce coupling effects between more complex and wideband antennas.

Calculation of Mutual Coupling Between Two Antennas

Two unique definitions of mutual coupling between a pair of antennas are described in [3]. In the "transmit mode" coupling definition, an excitation source is placed at the feed of one antenna, and the other antenna is terminated in a load. Coupling between the two antennas is then calculated by

$$C_{tr} = \frac{P_L}{P_D} \qquad (1)$$

where P_D is the power delivered (or radiated) by the excited antenna, and P_L is the power delivered to the load of the un-excited antenna. This is the typical notion of coupling between a pair of antennas, which can be easily calculated by simulation or measured. S-parameter measurements taken between two antennas by a network analyzer are in fact related to the transmit-mode coupling calculation by

$$\left|S_{21}\right|^2 = \left(1-\left|S_{11}\right|^2\right)C_{tr}\,. \qquad (2)$$

When considering the interaction between antennas in a receive-only phased array, this definition of coupling seems problematic since the excitation is generated by a transmitting antenna at a fixed orientation relative to the other antenna. Therefore, there is no way to evaluate the change in coupling between the two antennas when they are excited by an incident wave from different directions. It is also expected that different current modes are setup in the antennas when excited by a plane wave rather than by a source placed at the feed of one of the antennas.

An alternate, "receive mode" definition of mutual coupling between two antennas is also proposed in [3]. In this definition, excitation is provided to one antenna, but not to the other by an incident plane wave from a direction (θ,φ). The un-excited antenna is terminated in a load. Coupling between the two antennas is calculated using

$$C_{rec} = \frac{P_{L,un-ex.}}{P_{L,ex.}} \qquad (3)$$

where $P_{L,ex}$ is the power delivered to the load of the excited antenna, which is due to the incident wave and scattering between the antennas while $P_{L,un-ex}$ is the power delivered to the load of the un-excited antenna which is only due to scattering between the antennas. This definition of coupling has the advantage that both antennas are operating in a receive mode and the coupling response can be evaluated for different directions of the incident field. This definition is perhaps not as intuitive, however, and is much more complicated to measure as compared with the transmit mode definition. Receive mode coupling is straightforward to calculate through simulation, though, by simply enforcing the proper boundary conditions. For instance, in a method of moments code such as NEC, this can be accomplished by zeroing out all the entries in the excitation vector except for those corresponding to the basis functions on the excited antenna.

A version of NEC2 written in C++, nec2++, was modified as described above to enable the calculation of receive mode coupling. Transmit and receive mode coupling calculations as defined in Equations 1 and 3 were then performed with the modified code to verify that it gave identical results as those given in [3] for short dipoles operating in free space.

An initial study was then performed to determine the coupling properties of wire inverted-V dipoles operating over an infinite PEC ground. The antenna design parameters considered in this study include the antenna element length, L, the height of the antenna above the ground, H, the droop angle of the element, α, and the wire radius, r_w, as show in Figure 1.

For a given distance between the two antennas, d, both parallel and co-linear antenna orientations as shown in Figure 2 are considered when calculating mutual coupling. The original design considered was the compact array element design from [2] with dimensions L = 1.77 m, H = 1.77 m, $\alpha = 45^o$, and $r_w = 9.5$ mm. The load impedance was assumed to be $Z_L = 100\ \Omega$.

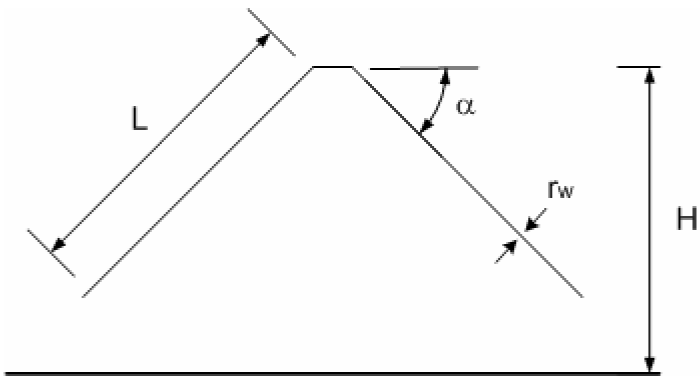

Figure 1. Geometry of inverted-V wire dipole antenna.

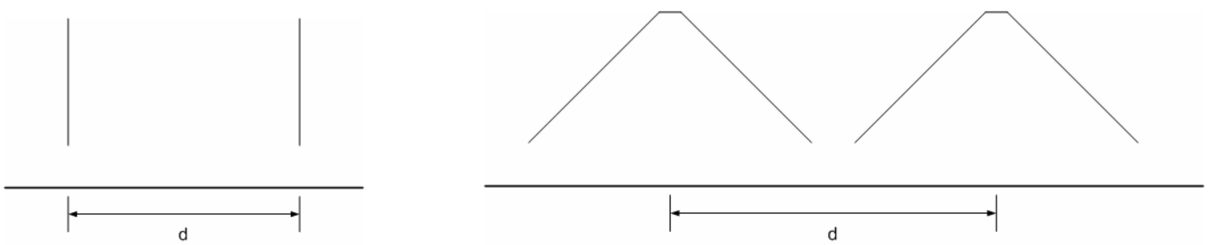

Figure 2. Parallel dipole orientation (left), co-linear dipole orientation (right).

Assuming d = 4 m, the transmit mode and receive mode coupling was calculated for the original inverted-V design for parallel and co-linear antenna orientations as a function of frequency, and the results are given in Figure 3. For both antenna orientations, receive coupling is calculated for zenith angles, $z = 0^o$, 22.5^o, 45^o, and 67.5^o in the E-plane of the antennas. Since the plane wave excites the dipole in a symmetric manner in the H-plane, receive coupling for $z \neq 0^o$ in that plane is identical to that for $z = 0^o$, and therefore these values are not shown in the plots. Except for the fact that both tend to peak up near the half-wave resonance of the dipole, which in this case is near 40 MHz, transmit and receive coupling calculation exhibit considerably different trends with frequency. Transmit coupling is much higher at low frequencies (below half-wave resonance) than receive coupling, while at higher frequencies, receive coupling tends to be higher than transmit coupling, especially for parallel dipoles. Also, while receive coupling is similar for different incidence zenith angles at lower frequencies, there can be significant variation with incidence angle at higher frequencies especially for co-linear dipoles where very high coupling values are exhibited. This behavior of course cannot be predicted by the transmit coupling calculation since excitation is only provided in one direction.

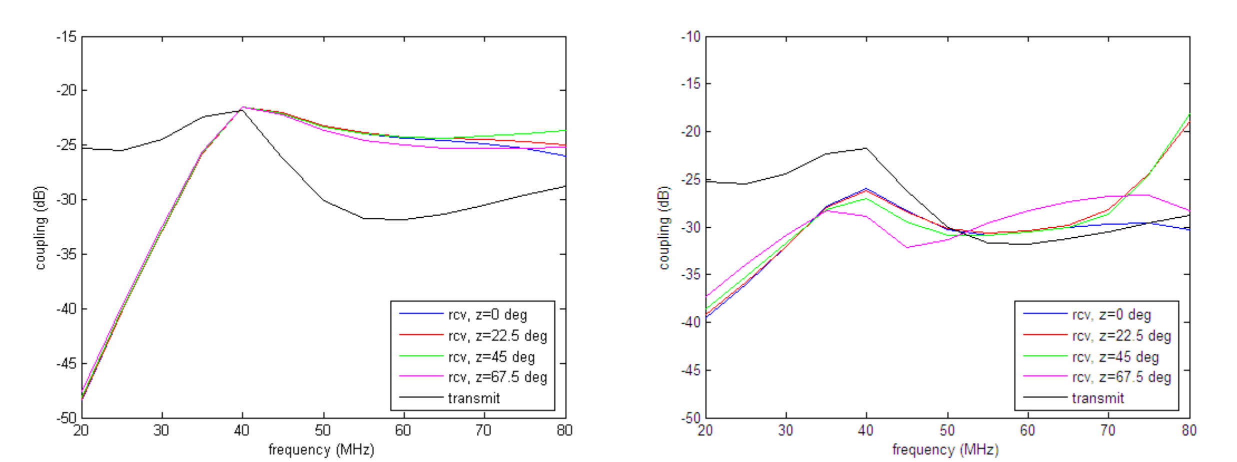

Figure 3. Comparison of transmit and receive mode mutual coupling calculations for the original wire inverted-V dipole design. Parallel dipoles (left), co-linear dipoles (right).

<u>Reduction of Mutual Coupling Between Two Wire Inverted-V Dipoles</u>

With the suspicion that it may serve as a useful indicator of receive array performance, an effort was made to reduce the receive mode coupling between two inverted-V dipoles as compared with the original design presented in the previous section. The distance between the dipoles, the length of a dipole arm, L, and the wire radius were maintained the same as in the original design (d = 4 m, L = 1.77 m, and r_w = 9.5 mm). The coupling between the dipoles was evaluated for different values of the design parameters, H, α, and Z_L. While H and α were allowed to vary over a relatively wide range of values, only a limited set of values of Z_L was considered: 100 Ω, 150 Ω, 200 Ω, 300 Ω, and 400 Ω.

It was found to be rather challenging to achieve a simultaneous improvement in coupling over all frequencies, both dipole orientations (parallel and co-linear), and all scan angles by manual adjustment of design parameters. This was due to the fact that what parameter changes tended to reduce coupling for the parallel orientation, in many cases also tended to increase coupling for the co-linear orientation. Additionally, parameter changes which tended to reduce coupling for lower frequencies (below half-wave resonance), in many cases also tended to increase coupling for higher frequencies (above half-wave resonance.) Luckily, parameter changes which tended to reduce coupling for a given scan angle, also tended to reduce coupling for other scan angles for the same dipole orientation and frequency, though some exceptions to this were noted. After some trial and error, a combination of parameters, H = 1.2 m, α = 30º, and Z_L = 200 Ω, was found which gave reduced receive coupling for both orientations, all frequencies, and all scan angles as shown in Figure 4. The new dipole design exhibits a reduction in receive coupling of up to 8 dB at 40 MHz, and a reduction of better than 3 dB over all frequencies above 30 MHz for both orientations and scan angles. It should be noted that if the load impedance were constrained to be a lower value such as 100 Ω and 150 Ω due to pre-amp design considerations, the dipole design would exhibit a smaller improvement in coupling compared with the original design below the first wave resonance (~ 40 MHz in this case). However, the improvement in coupling at higher frequencies would be increased somewhat.

The transmit mode coupling for the original and new inverted-V dipole designs are also compared in Figure 4. Note that this calculation does not predict a consistent improvement in coupling over all frequencies due to the new design as does the receive calculation. This agrees with the earlier observation that the transmit and receive mode coupling calculations exhibit notably different properties.

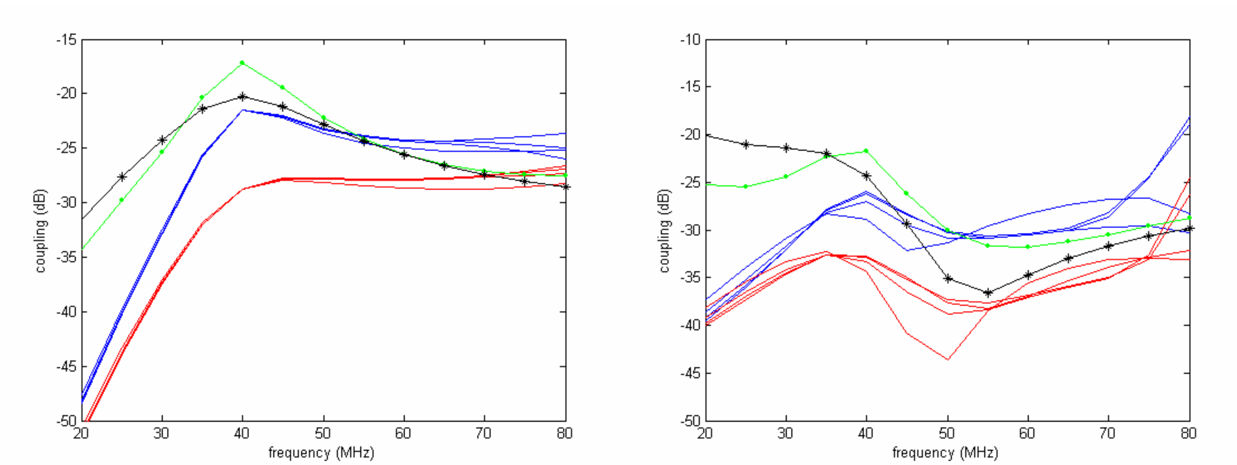

Figure 4. Comparison of mutual coupling calculations for the original and the new, reduced coupling wire inverted-V dipole designs. Parallel dipoles (left), co-linear dipoles (right). The blue traces are receive coupling for original dipole design for z^{inc} = [0°, 22.5°, 45°, 67.5°], the red traces are receive coupling for new dipole design for z^{inc} = [0°, 22.5°, 45°, 67.5°], the green trace is transmit coupling for the original design, and the black trace is transmit coupling for the new design.

The sky noise frequency responses of the two wire inverted-V dipole designs are given in Figure 5. As can be seen, the new, reduced coupling design exhibits increased sky noise compared with the original design at all frequencies except near 40 MHz where there is significant "excess" sky noise. Therefore, the reduction achieved in mutual coupling does not come at the expense of sensitivity.

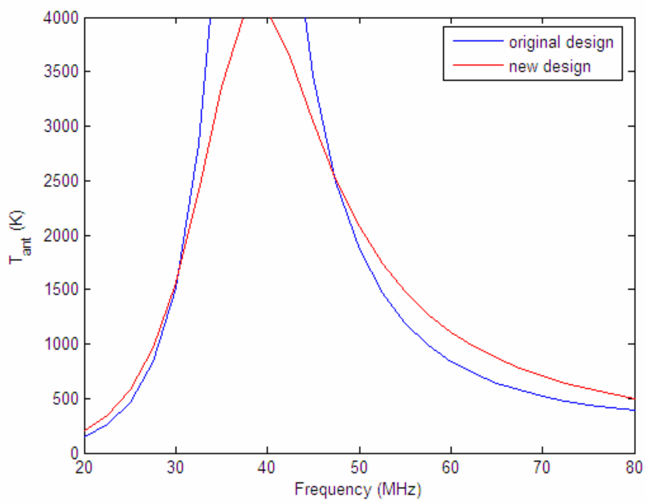

Figure 5. Sky noise frequency responses for original and new, reduced coupling wire inverted-V dipole designs.

Reduction of Mutual Coupling Effects in a Phased Array of Wire Inverted-V Dipoles

The performance of the wire inverted-V dipole designs considered above when operated in large phased arrays is now considered. An approach for evaluating different array designs using NEC is described in [2]. In this approach, multiple instances of a given antenna model are generated and placed according to a specified array layout. An impedance load is placed at the terminals of each antenna in order to simulate the effect of the pre-amp. The modeled array is then excited by a plane wave incident at different directions. Note that in this case, all antennas in the array are being excited by the plane wave, which differs from the receive mode coupling calculation in which only one of the two antennas is excited. The variation in terminal currents across different antennas in the array and for different array scan directions is then analyzed to determine the quality of the array design. As suggested in [2], an array which exhibits significant variation in terminal currents will be more difficult to calibrate with an accuracy equal to one which exhibits lower terminal current variation. Thus, it is desired to achieve an array design with low terminal current variation.

The results in [2] showed that a compact array consisting of a uniform arrangement of dipoles performed significantly better in terms of terminal current variation than a sparse array consisting of a pseudo-random (PR) arrangement of dipoles. Results were provided for only a single frequency, however, and coupling characteristics were not considered when selecting the wire inverted-V dipole designs used in that analysis. Therefore the array calculations were repeated for multiple frequencies between 20 to 80 MHz and for both the original and reduced coupling dipole designs.

The same two array designs considered in [2] are considered here - that is, a uniform array with element spacing in both x and y directions of 3.61 m, and the PR array with 4 m minimum element spacing described in [1]. However, in order make the array calculations in NEC more tractable, particularly when many frequencies are considered, the size of the arrays are reduced to include roughly 128 elements each. Since the number of elements included in the simulation is still very high, it is believed that the terminal current behavior exhibited in the smaller versions of the arrays is indicative of the general behavior of the larger versions of the arrays. In order to reduce the number of elements in the uniform array, its radius is decreased from 100 m to 23 m. All of the elements within a 33 m radius of the center of the original PR array are included in the smaller array. The layouts of the arrays used are shown in Figure 6.

The original wire inverted-V dipole design (which is the compact dipole design in [2]) is evaluated in both the uniform and PR arrays. The new, reduced coupling wire inverted-V design is evaluated only in the PR array. As is [2], the array is excited by a linearly polarized wave, and thus it is only necessary to include the co-polarized dipole of each antenna stand in the simulation.

The terminal current magnitudes and phases from all of the antennas for the original dipole design in both the uniform and PR arrays, and the new dipole design in the PR array are compared in Figures 7 and 8. Figure 7 provides results at 40 MHz for z = 0° (boresight) and Figure 8 provides results at 75 MHz for z = 45° in the E-plane of the array. The geometric component of the phase due to the spacing between antennas has been removed from these results so that only the perturbations due to mutual coupling remain. The results are sorted in ascending order as a function of distance from the center of the array. As can be seen, when the original dipole design is considered, the uniform array exhibits significantly lower variation in both terminal current magnitude and phase than the PR array for both frequency / pointing direction combinations shown. This is consistent with the results in [2]. However, the current variations are significantly lower for the PR array for both frequency / pointing direction combinations when the new dipole design is used as compared with the original dipole design so that the performance is more comparable to that of the uniform array. This is consistent with the receive coupling results given in the last section.

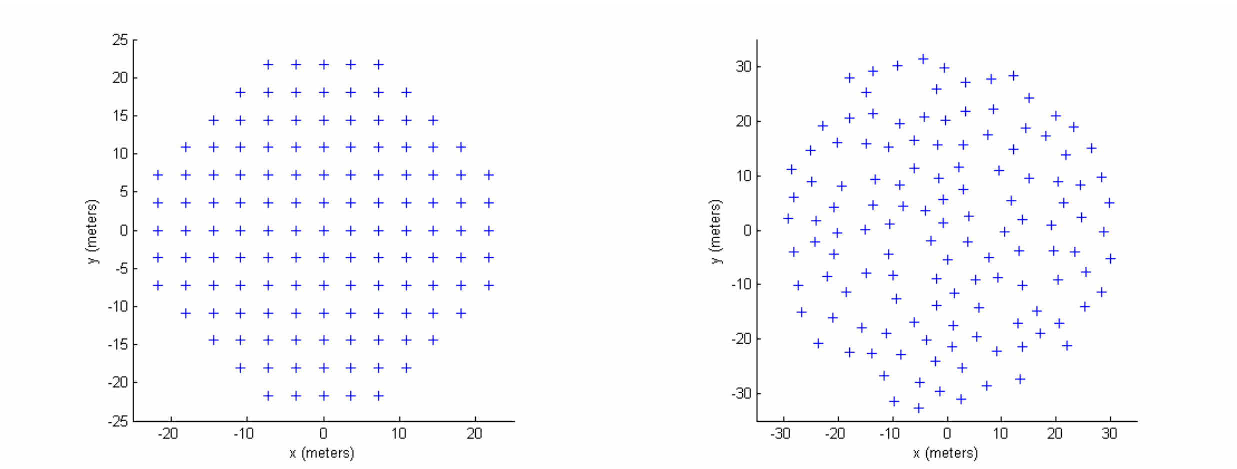

Figure 6. Array layouts considered. Compact uniform array (left), sparse pseudo-random array (right).

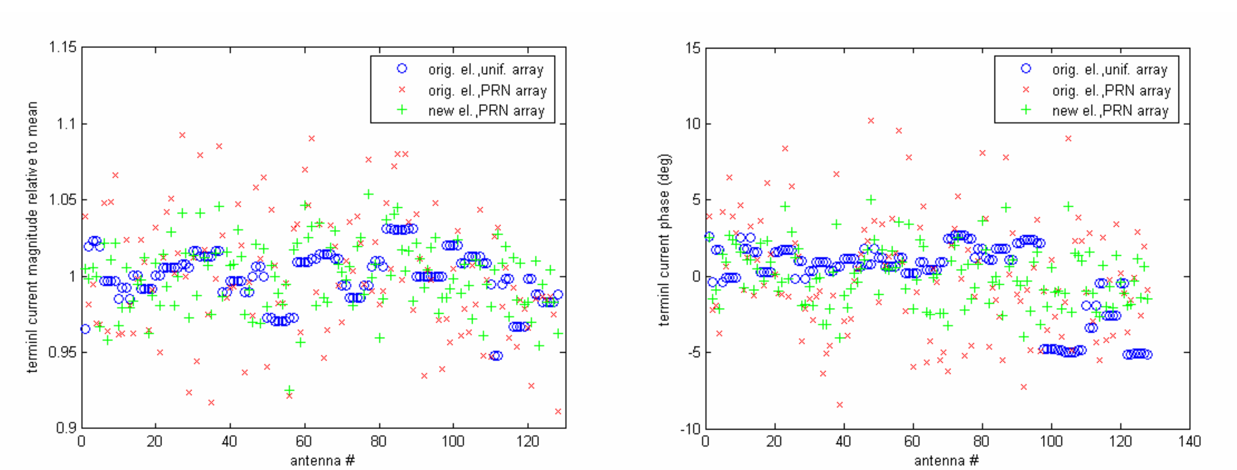

Figure 7. Comparison of terminal current magnitudes (left) and phases (right) for all three array / dipole combinations at 40 MHz, for co-polarized excitation at z = 0° (boresight).

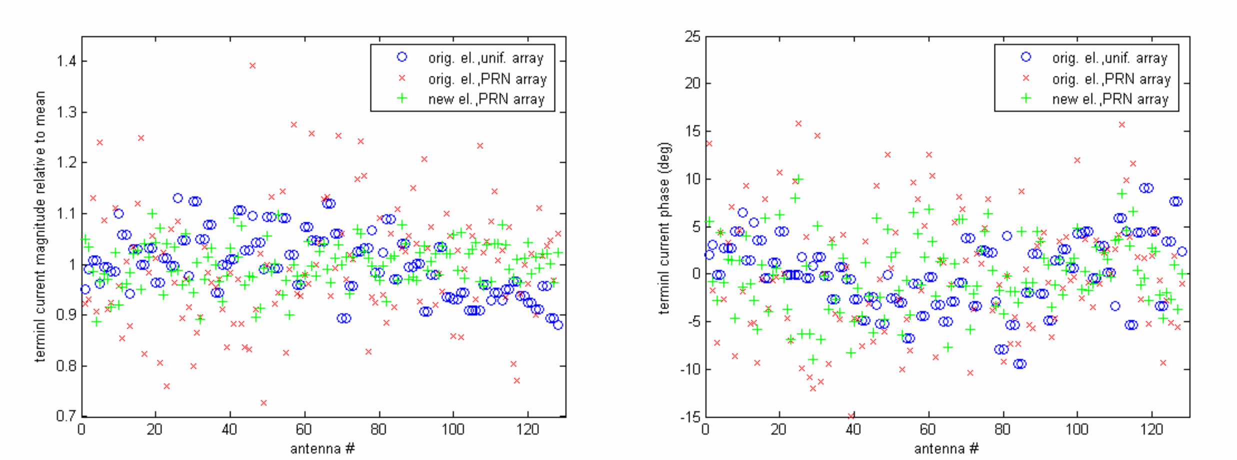

Figure 8. Comparison of terminal current magnitudes (left) and phases (right) for all three array / dipole combinations at 75 MHz, for co-polarized excitation at z = 45°, φ = 0° (E-plane).

In order to simplify analysis of array performance over a wide frequency range, statistics are calculated at each frequency over the terminal currents from all antennas. These include the standard deviation of current phase, and the standard deviation of current magnitude relative to mean magnitude; it is necessary to define the magnitude statistic this way since the mean value changes at each frequency. These statistics are plotted in Figure 9 for all three array design / dipole design combinations in 5 MHz steps between 20 to 80 MHz for two different pointing angles, z = 0° and z = 45° in the E-plane. Both magnitude and phase statistics are significantly better over most frequencies and pointing directions with the uniform array than with the PR array when the original dipole design is used, though this is not the case below 30 MHz or in the phase at 50 MHz. The statistics of the PR array are significantly improved (by a factor of 2 or more in some cases), however, by using the new, reduced coupling dipole design as compared with the original design for all frequencies and pointing directions. This is completely consistent with the receive coupling behavior of the two dipole designs described in the previous section. The performance of the PR array with the new dipole design is in fact comparable to that of the uniform array over the entire frequency range. It should be noted that all array design / dipole design combinations exhibit a significant increase in both current magnitude and phase variation at the high end of the operating band when scanning to lower angles. This behavior is also evident in the receive coupling calculations shown in Figure 4. These results appear to indicate that there is indeed a correlation between the receive coupling calculation between two dipoles of a common design and the performance of a phased array when that same dipole design is used in the array.

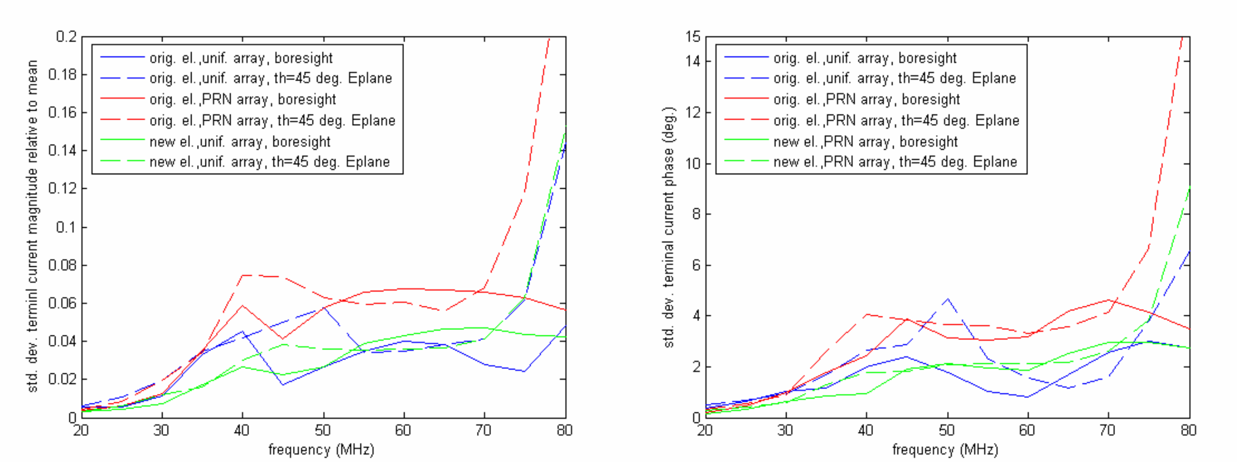

Figure 9. Standard deviations, calculated over all antennas, of terminal current magnitude (left), and phase (right) over LWA band for all three array / dipole combinations.

Initial Results for Broadband Dipoles

The analysis techniques described above are now applied to the blade antenna, which is a broadband dipole design currently being considered for use in the LWA. The "big blade" dipole design is first considered; the assumed dimensions of this antenna are: total element length, L = 1.72 m, total element width, W = 0.42 m, antenna height, H = 1.52 m, droop angle, $\alpha = 45^{o}$, and load impedance, $Z_L = 100\ \Omega$. The receive mode coupling for this design assuming a separation of 4 m between the dipoles was calculated and is given in Figure 10. The dimensions of the antenna were then adjusted in order to reduce mutual coupling. It was found that, as with wire inverted-V dipoles, a combination of lowering the height of the antenna (to 1.27 m), reducing the droop angle (to 35^{o}), and increasing the load impedance (to 200 Ω) could be used to reduce the coupling between blade dipoles over a wide range of frequencies. A further reduction in coupling at all frequencies can be achieved by reducing the width of the element to 0.28 m. As shown in Figure 10, the new blade dipole design exhibits a reasonable reduction in coupling as compared with the original design over all frequencies, both dipole orientations, and both pointing directions considered.

The sky noise frequency responses of the two blade designs are compared in Figure 11. Although the new design exhibits a slight decrease in sky noise at the low end of the operating band, high frequency performance is improved, and high sky noise levels are maintained at intermediate frequencies. Therefore as with wire inverted-V dipoles, a reduction in coupling between blade dipoles can be achieved without significant impact to the sensitivity.

In comparing the coupling calculations for the blade dipole designs in Figure 10 to those for the wire inverted-V dipole designs in Figure 4, it is evident that the two types of antennas exhibit similar coupling characteristics with frequency, though coupling is generally lower for wire inverted-V dipoles; the only exception is near the half-wave resonance, at roughly 40 MHz, of the original wire inverted-V dipole design. The improvement at low frequency is likely mostly due to the shorter electrical length of the wire inverted-V designs as compared with the blade designs, which has the effect of shifting the coupling response up in frequency. However, this does not explain the large difference between the two types of antennas at higher frequencies. For instance, the coupling for the wire inverted-V designs is roughly 10 dB better at 80 MHz for co-linear dipoles than the corresponding blade designs, despite the fact that the overall dimensions of the antennas are similar. A difference in sensitivity between the two types of antennas (the mismatch loss of blade dipole is generally lower than the wire dipole) could explain some of this difference, but not all of it. This suggests that, fundamentally, wire dipoles will couple less at higher frequencies than blade-like dipoles. This conclusion seems to be reinforced by the result discussed earlier that reducing the width of the blade dipole reduces coupling over all frequencies.

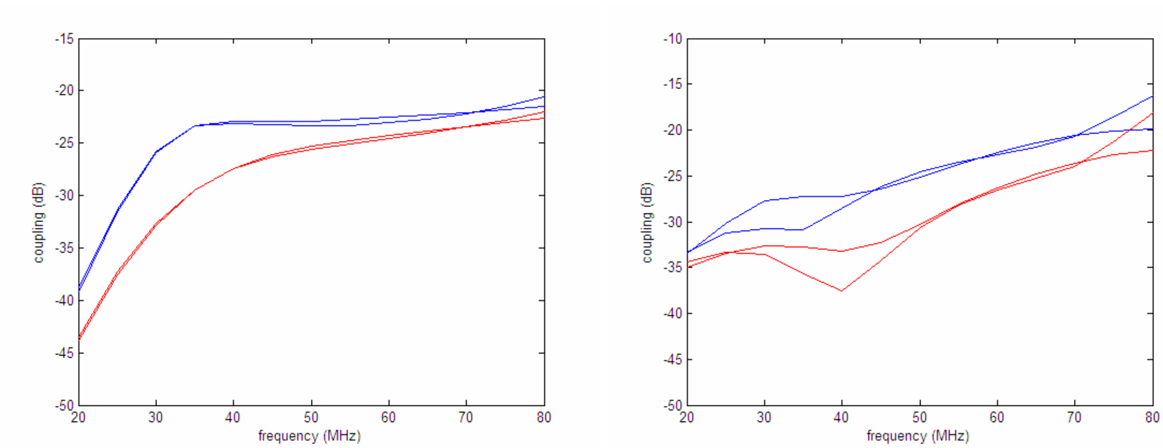

Figure 10. Comparison of receive coupling calculations for different blade-like dipole designs. Parallel dipoles (left), co-linear dipoles (right). Blue traces: original big blade design, red traces: reduced coupling blade design. Results provided for z^{inc} = 0° and 45° for each design.

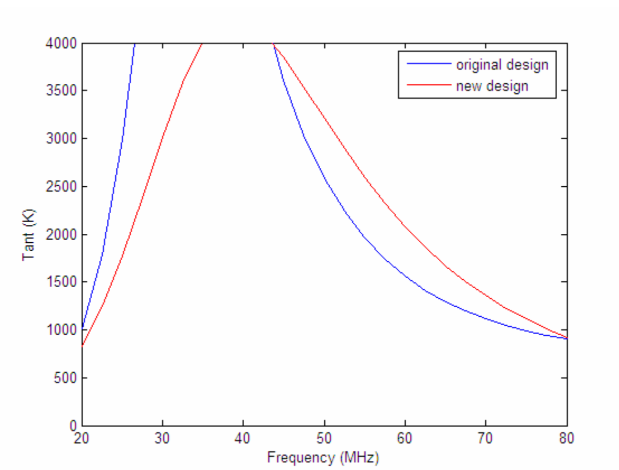

Figure 11. Sky noise frequency response of the original big blade and the reduced coupling blade dipole designs.

The performance of both the original and the new, reduced coupling blade designs when operated in a PR phased array are considered. Since many more unknowns are needed to accurately model a blade dipole than a wire inverted-V dipole in NEC, 200-300 versus 13, respectively for the results presented here, only a relatively small PR array could be simulated over many frequencies in a reasonable amount of time. Therefore, the 16 element array layout used for LWDA, which is a subset of the 256 element baseline PR layout for LWA, was used. Due to time constraints, each design was only evaluated for boresight illumination.

The standard deviations of current magnitude and phase as a function of frequency for both the original and the new, reduced coupling blade designs when operated in the 16 element PR array are given in Figure 12. The terminal current magnitude and phase variation using the new blade design are reduced at all frequencies compared with the original design, which agrees with the trend predicted by the receive coupling calculation. Comparing Figure 12 to Figure 9, it can be seen that the current variation at boresight at higher frequencies is higher for the blade dipoles than for the wire inverted-V dipoles, as was also predicted by the receive coupling calculation. For instance, the current magnitude standard deviation is up 2.5 times higher and the phase standard deviation up to 2.4 times higher for the blade than the wire inverted-V. These results further support the assertion made in the previous section that the calculations of receive coupling and the variation of currents in an array are related.

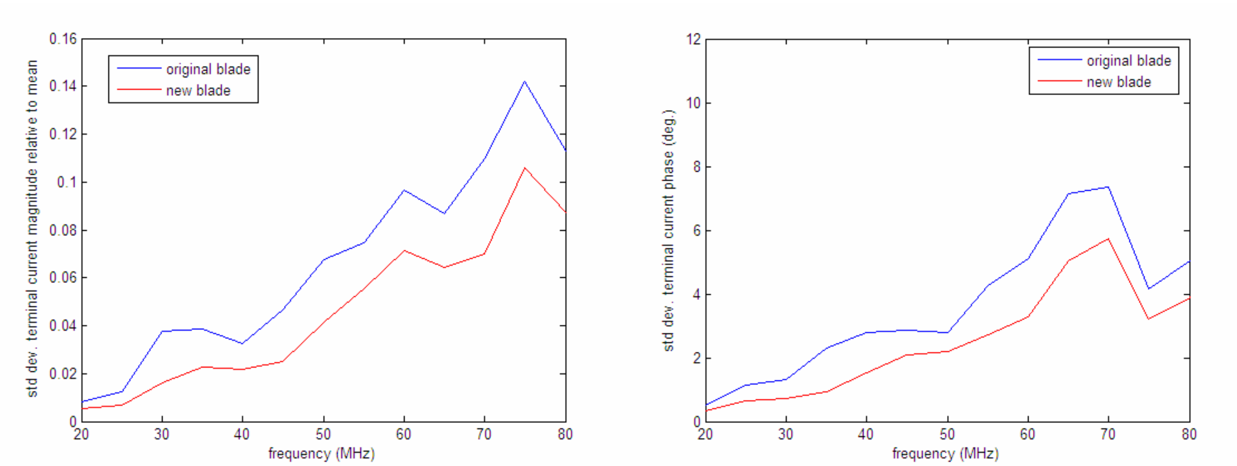

Figure 12. Standard deviations, calculated over all antennas, of terminal current magnitude (left), and phase (right) over the LWA band for original and new, reduced coupling blade designs for boresight illumination.

Conclusions

The results presented in this memo suggest that there is indeed a relationship between the two antenna receive mode coupling calculation and the amount of terminal current variation in a receive-based phased array. For both wire and blade-like dipoles, it was shown that the variation in terminal currents in a pseudo-random phased array could be reduced significantly *without changing the array layout* by designing the antenna elements so that they exhibit low receive coupling. Since only two antenna elements need to be considered, the problem of designing a pseudo-random array with reduced mutual coupling effects is greatly simplified compared to the typical design approach, which involves calculating the individual element responses for the entire or a large portion of the array.

Future effort will be focused on including mutual coupling as a design objective in genetic algorithm (GA) optimizations of antennas for use in LWA, which to date have only considered sky noise frequency response and radiation pattern quality as objectives [4]. Through this approach, it is planned to continue comparisons of different types of antennas, which would nominally include wire inverted-V, blade, and fork [5] dipoles.

Additionally, measurements should be performed to verify the accuracy of the NEC mutual coupling calculations. While it would be ideal to perform measurements of terminal current variation in a large array consisting of many 10's or 100's of antenna elements, a much smaller array consisting of, say 16 to 32 elements may be quite suitable for this purpose. Such a measurement effort seems to have much in common with the proposed Rapid Test Array (RTA) effort, particularly given that the RTA proposal calls for the construction of antenna elements, which could be re-arranged to evaluate different array layout designs [6]. Note that it would also be helpful if some of the antenna element dimensions (e.g. antenna height, element droop angle) could be adjusted so that the trends predicted in this memo could be checked. Although its layout and antenna elements are fixed, measurements with the LWDA, especially when paired with an outlier antenna (or small array) to improve signal to noise, would be useful to verify that the results given by NEC are at least reasonable.

References

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