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Coherent Subtraction of Narrowband Radio Frequency Interference

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Abstract: The ability to do radio astronomy at wavelengths > $1 \mathrm{m}$ is 7 frequently restricted by narrowband anthropogenic radio frequency interference (RFI). Much new science requires observations at frequencies where RFI 9 can not be avoided. There have been attempts to suppress RFI using sig-10 nal processing techniques, with performance which is found to be limited by 11 the interference-to-noise ratio (INR). This paper describes a generic sup-12 pression method applicable to all narrowband RFI signals, as well as a method 13 which is particularly effective against narrowband frequency modulated (NBFM) 14 signals. Both methods are based on a strategy of estimation followed by co-15 herent subtraction, and neither requires an additional reference antenna. The 16 generic method is shown to be somewhat "toxic" to underlying noise and as-17 tronomical signals but may nevertheless have applications; whereas the NBFM-18 specific method is shown to be relatively benign to astronomical signals but 19 also somewhat less effective in suppressing RFI. The performance of both 20 methods is demonstrated with a real-world data consisting of 4 narrowband 21 FM signals. 22

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1. Introduction

Anthropogenic radio frequency interference (RFI) poses severe challenges for radio 23 astronomy [*Thompson et al.*, 1991]. Whereas it is often possible to employ a strategy 24 of avoidance – e.g., observing in bands which are protected by regulation, or from 25 sites at which RFI is acceptably weak – much important science requires observations 26 at sites and in bands in which RFI cannot be effectively avoided. For example, a 27 topic which is currently of intense interest to the astronomical community is the 28 possibility of studying the Epoch of Reionization using the redshifted 21 cm emission 29 of hydrogen (e.g., [Furlanetto et al., 2004]), and is a prime motivator for a number of 30 new radio telescopes currently under construction including the Low Frequency Array 31 (LOFAR) [Butcher, 2004], the Mileura Widefield Array (MWA) [MWA website], and 32 the Square Kilometer Array (SKA) [SKA website]. This effort requires access to large 33 contiguous segments of quiet spectrum ranging from ~ 75 MHz to ~ 250 MHz. For 34 a variety of reasons the new instruments LOFAR and the Long Wavelength Array 35 LWA) [LWA website] intend to operate at frequencies as low as 20 MHz. 36

Although many forms of RFI exist in the range 20–250 MHz, a common and particularly onerous source is "narrowband" (3 kHz to 25 kHz bandwidth) voice communications using various amplitude, phase, and frequency modulation techniques. This category of signals can be found nearly everywhere, both because they are frequently used for mobile communications including aircraft, and because signals at these frequencies can propagate over extraordinary distances. These aspects limit the extent

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to which a strategy of avoidance can be successful, and motivate technical solutions
in which RFI-afflicted data might instead be cleaned, preferably in real time.

Although extensive literature exists on the problem of mitigation of RFI 45 (e.g., [Laster and Reed, 1997] and references therein) the vast majority of past work is 46 oriented toward communication applications, as opposed to radio science applications. 47 The resulting emphasis is on the reduction of very strong RFI – i.e., interference-to-48 noise ratios (INR) orders of magnitude greater than 1. RFI whose INR is less than or equal to 1 is typically no longer limiting to communication performance. In radio 50 science applications, in contrast, input INR ≤ 1 can be devastating; e.g., in spec-51 troscopy involving long integrations. Thus, we seek algorithms which are effective in 52 suppressing weak RFI to INR $\ll 1$. Additionally, we seek algorithms which do this 53 without distorting the underlying astronomy and, preferably, also not distorting the 54 underlying noise. 55

Notable previous attempts to mitigate narrowband RFI in a manner germane to 56 radio astronomy includes [Barnbaum and Bradley, 1998], which addressed the fea-57 sibility of using a time-domain adaptive canceler in this application. This approach 58 requires an auxiliary signal which has nominally a high-INR copy of the RFI and neg-59 ligible SNR for the astronomical signal of interest, and must therefore be obtained 60 from a separate "reference" antenna. Thus limitations of this approach are that (1)61 some a priori information about the location of the signal is required so as to prop-62 erly point the reference antenna(s), and (2) the technique is limited to suppression 63 which is proportional to the INR achieved in the reference channel. We refer to the 64

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⁶⁵ principle that this class of mitigation techniques tends to produce output INR equal ⁶⁶ to the inverse of the input INR as "power inversion". Power inversion tends to limit ⁶⁷ the usefulness of affected algorithms since RFI which appears weak over short time– ⁶⁸ frames can easily ruin observations made over longer time periods. In the Bradley ⁶⁹ and Barnbaum approach, the only alternative is to increase the gain of the reference ⁷⁰ antenna.

An interesting alternative approach can be found in the synthetic aperture radar 71 literature [Miller et al., 1997], in which the RFI is modeled as sinusoids of unknown 72 frequency, magnitude, and phase. The proposed algorithm then consists of dynami-73 cally estimating these parameters, synthesizing new (noise-free) versions of the signals 74 using the estimated parameters, and then coherently subtracting these from the orig-75 inal data. This technique turns out to be highly effective especially when the sample 76 rate is much greater than the Nyquist criterion for any given RFI signal, since in 77 this case, narrowband RFI signals are well-modeled as unmodulated carriers, even 78 over large numbers of samples. Furthermore, there is no need for a separate reference 79 antenna. To accommodate frequency-modulated signals having bandwidth too great 80 to model effectively as sinusoids, this approach adds an additional parameter – the 81 first derivative of frequency with respect to time – which in effect extends the signal 82 model to include "linear chirp" signals. The disadvantage of this approach is that 83 the algorithm has no means to distinguish between RFI and noise, and thus tends 84 to suppress both. This can be problematic for the detection of weak signals underly-85 ing the RFI. Ironically, the performance of this approach is also constrained by the 86

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power inversion principle; in fact severely so since, in the basic algorithm, there is 87 no reference antenna to increase the input INR. Detailed discussion of this issue can 88 be found in [*Ellingson*, 2002]. [*Ellingson and Hampson*, 2003] describes a specific 89 application of the sinusoidal estimation and subtraction approach to L-band astron-90 omy afflicted by ground based aviation radar, where the limitations of both power 91 inversion and detection sensitivity are observed. Although one might consider some 92 combination of the above two approaches to overcome this difficulty; that is, using 93 reference antenna to increase the input INR and thereby improving the estimation 94 of sinusoidal parameters, in the present work we wish to avoid the implementation 95 problems associated with a reference antenna. 96

[Ellingson et al., 2001] addressed this difficulty by employing a priori information 97 about the modulation of the RFI signal to improve the "effective" INR of the received 98 RFI. They demonstrate more than 20 dB of suppression of a GLONASS (direct se-99 quence spread spectrum satellite) signal received at an INR of -20 dB; i.e., suppres-100 sion at least 20 dB greater than that implied by the power inversion constraint. In 101 this case, the fact that the modulated bandwidth is orders of magnitude greater than 102 the message (pre-modulation) signal – i.e., large "processing gain" – is exploited to 103 convert the problem from that of estimating the parameters of the rapidly-changing 104 modulated RF waveform with low INR to that of estimating the parameters of the 105 slowly-varying message waveform with high INR. In terms of implementation, this 106 turns out to be a simple matter of demodulating the signal to retrieve the information 107

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signal, and then remodulating the signal to obtain a noise-free version of the original
 signal.

In this paper, we attempt to extend this "modulation-savvy" approach to improve the performance of RFI mitigation following the parametric estimation and subtraction strategy of [*Miller et al.*, 1997]. Specifically, we exploit the significant (but relatively modest) processing gain and "constant modulus" (magnitude) properties of narrowband FM (NBFM) signals to achieve the benefits of this approach with reduced distortion of the underlying signals of interest.

This paper is organized as follows. Section 2 presents a RFI signal model and its characteristics, the background theory of the suggested RFI mitigation method, and design parameters appropriate for mitigation of RFI of interest. In Section 3, we demonstrate the performance of the mitigation method with real-world data. Section 4 provides concluding remarks including recommendations for further improvement.

2. Theory

In this paper, we model the signal received by a radio telescope as

$$x(t) = \sum_{n=1}^{N} s_n(t) + s_a(t) + z(t)$$
(1)

where z(t) is additive white Gaussian noise, $s_a(t)$ is an astronomical signal of interest, and $s_n(t)$ are RFI signals having the form:

$$s_n(t) = a_n(t)e^{j\{\omega_n(t)t + \theta_n(t)\}}$$
(2)

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The meanings and behaviors of the parameters in (2) depend on the modulation of the 122 RFI signal. For amplitude-modulated signals including single-sideband (SSB), $a_n(t)$ 123 is the audio "message" signal; $\omega_n(t)$ is the carrier frequency which varies only slightly 124 and slowly due to a combination of the limited frequency stability of transmitting 125 equipment and Doppler effects; and $\theta_n(t)$ is an arbitrary phase offset which also varies 126 very slowly, primarily in response to propagation conditions. The parameters result 127 in occupied bandwidth of less than 5 kHz, which is only slightly larger than the 128 typical message bandwidth of 3 kHz. 129

For NBFM signals, the message is represented by $\theta_n(t)$, and $a_n(t)$ varies only 130 slightly and slowly, relative to the inverse message bandwidth, due primarily to multi-131 path fading [Jakes, 1974]. Because $a_n(t)$ is nominally constant, FM signals are said to 132 have "constant modulus", a property we exploit in this paper. Standard bandwidths 133 for NBFM signals are 25 kHz, 12.5 kHz, and (now rare but likely to become more 134 common) 6.25 kHz [TIA, 2004]. The relatively large occupied bandwidth compared 135 to the message bandwidth means NBFM signals contain a considerable degree of re-136 dundant information; this is essentially the "processing gain" referred to in Section 1. 137 This processing gain is exploited in NBFM systems to achieve improved audio qual-138 ity; however here we take advantage of it in much the same manner it is exploited 139 in *[Ellingson et al.*, 2001]; that is, to bypass the power inversion constraint and to 140 enhance discrimination between signal and noise. 141

The proposed algorithm is shown in Figure 1. It works by (1) finding the number of signals (i.e., "model order estimation") and making coarse estimates ω_n^c of their center

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frequencies, (2) estimating parameters, a_n , ω_n , and θ_n , for each signal identified, (3) 144 synthesizing estimates of the associated signals $s_n(t)$, and (4) subtracting these from 145 a delayed version of x(t), nominally yielding the desired signal $y(t) = s_a(t) + z(t)$. 146 Important to note in Figure 1 is that the "primary path" includes delay to account 147 for latency incurred in the estimation and synthesis of RFI signals in other paths. 148 The latency of concern is dominated by the delay incurred in finite impulse response 149 (FIR) filters, thus the primary path latency is deterministic and easily introduced in 150 the form of an all-pass FIR filter of appropriate length. 151

Many possible methods exist for model order estimation and coarse frequency es-152 timation [Stoica and Moses, 2005]. In our work, this is conveniently implemented 153 using a fast fourier transform (FFT). The length of the FFT, T_{FFT} , is chosen so as 154 to yield a resolution bandwidth small enough to clearly resolve adjacent signals. The 155 update rate T_1 for the estimates of model order and coarse frequency can be orders of 156 magnitude greater than $T_{\rm FFT}$, and thus the sensitivity of detection can be increased 157 by using the smallest possible T_{FFT} and averaging the resulting spectra over T_1 . The 158 criteria for detection is based on a threshold test. The value of the threshold β is 159 most conveniently expressed in units of standard deviations (σ) of the noise, z(t), 160 distribution above the mean of z(t), and the selected value is chosen as a tradeoff 161 between good sensitivity (tending to relatively low β) and low "false alarm" rate 162 (tending to relatively high β). The specific danger in having an excessive false alarm 163 rate is that the misguided attempts to estimate and subtract non-existent signals 164

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will result in increased distortion of z(t) and possibly also $s_a(t)$. In practice, β in the range 5σ to 10σ is known to be effective [*Ellingson and Hampson*, 2003].

Figure 2 shows a generic estimation procedure which is applicable to all narrowband modulations. In this procedure, RFI signal of interest is first isolated using a bandpass filter (BPF) centered at ω_n^c . Then, the parameters, a_n , ω_n , and θ_n , are estimated by assuming that RFI is sinusoidal over the period of T_2 since the last update of the parameters. With ω_n^c from coarse frequency search, the procedure is to find ω_n which maximizes the magnitude of (3) given by

$$A = \frac{1}{T_2} \int_0^{T_2} x_n(t) e^{-j\omega_n t} dt$$
 (3)

This is conveniently done as a binary search over the bandwidth of the BPF, starting at ω_n^c . For ω_n maximizing (3), A becomes

$$A = a_n e^{j\theta_n} \tag{4}$$

Then, a_n , ω_n , and θ_n are the optimal Maximum Likelihood estimates.

The precise implementation of the generic estimation procedure should be opti-168 mized to the type of signal being processed. The bandpass filter should be wide 169 enough to include the entire bandwidth of the RFI signal, plus any error in the ini-170 tial coarse frequency estimate; reasonable values are 5 kHz for SSB and 15 kHz for 171 NBFM. The update rate T_2 for sinusoidal parameter estimation is a tradeoff between 172 sensitivity (favoring relatively long T_2) and desire to accurately track parameter vari-173 ations resulting from modulation (favoring relatively short T_2). In practice, T_2 should 174 be approximately one-tenth of the inverse bandwidth of the signal; i.e., 20 μ s for SSB 175

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and 6.7 μ s for NBFM. To avoid jitter resulting from endpoint effects in the estimation of parameters, it is recommended to apply a Bartlett (triangular) window over the time-domain samples to be processed to obtain any given update.

Figure 3 shows an improved procedure which is applicable to NBFM signals. In this 179 case, $x_n(t)$ is demodulated by computing the time-derivative of $\theta_n(t)$, which yields 180 the message signal. This is then low-pass filtered to suppress extraneous noise. The 181 processing gain is obtained by the fact that the message signal is mostly preserved, 182 whereas any noise outside the low-pass bandwidth is suppressed. The message signal 183 is then remodulated by time-domain integration and addition of initial phase offset 184 to obtain the improved estimate of θ_n . Concerning LPF design, the passband must 185 include the entire message spectrum, and passband ripple must be minimized to 186 prevent error in the remodulated signal. Note that the principal advantage in this 187 procedure is that the new sinusoidal phase estimates now respond primarily to the 188 signal *alone*, as opposed to the combination of signal plus noise. Thus, this procedure 189 reduces the extent to which the algorithm distorts $s_a(t)+z(t)$, as will be demonstrated 190 in the next section. 191

¹⁹² A second modification for the NBFM-specific procedure of Figure 3 is that the ¹⁹³ constant modulus property of NBFM is exploited to improve the estimation of a_n . ¹⁹⁴ This is achieved by averaging the estimates over a period T_3 which is as long as ¹⁹⁵ possible, but much shorter than the "channel coherence time" which is the time over ¹⁹⁶ which the gain associated with propagation channel can be assumed to be relatively ¹⁹⁷ constant. The channel coherence time is approximately given by $T_C \approx 0.423/f_{Dm}$

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where f_{Dm} is the maximum Doppler shift [Rappaport, 1996]. In the experiment shown 198 in Section 3, for example, the carrier frequency is ~ 162 MHz. Therefore, assuming 199 the RFI transmitter is moving with the speed of 30 m/s, T_C becomes 26 ms; thus a 200 reasonable value for T_3 is 3.9 ms, i.e, much shorter than T_C and much longer than 201 noise correlation distance induced by the BPF. It should be noted that the advantage 202 obtained by enforcing constant modulus is similar to that obtained using the "demod-203 remod" technique to improve θ_n ; that is, that estimates now respond primarily to 204 the signal alone, as opposed to the combination of signal plus noise. 205

3. Experiment

To demonstrate the effectiveness of the proposed algorithm, we applied it to a real-206 world example of NBFM signals. The data consists of four separate weather radio 207 stations operated by the U.S. National Oceanic and Atmospheric Administration 208 located in Arizona (Southwest USA). This includes a station at 162.400 MHz located 209 at Flagstaff, a station at 162.425 MHz located at Payson (Mt. Ord), a station at 210 162.500 MHz located at Globe (Signal Peak), and a station at 162.550 MHz located at 211 Phoenix. The data provided to the algorithm is in complex baseband form sampled 212 at 256 kSPS, and the total record to be evaluated is ~ 1 seconds long. The average 213 spectrum is shown in Figure 4. It can be seen that the modulation is of the 12.5 kHz 214 variety, and that the INRs range from ~ 0.7 dB to ~ 24 dB within ~ 200 kHz-wide 215 passband. Note that the spectrum of each signal includes a strong narrow peak 216 which seems to be independent of the expected modulation. The explanation for this 217

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feature can be seen in Figure 5, which shows the dataset in the form of a spectrogram. The narrow peak corresponds to periods, visible in the spectrogram, over which the carrier is effectively unmodulated. These unmodulated periods occur because these stations continue to transmit even when there is no voice activity.

The algorithm parameters used to process this data are as follows: in model order detection and the coarse frequency search, $T_{\rm FFT} = 2 \text{ ms}$, $\beta = 10\sigma$, and detection is done just once over the duration of the dataset ($T_1 \approx 1 \text{ s}$).

First, we consider the results using the generic processing method of Figure 2 with $T_2 = 6.7 \ \mu s$. The result is shown in Figure 6. Note that the generic processing algorithm results in the deep suppression of all four NBFM signals. However, noise which occupies the same bandwidth as RFI signal is also suppressed. This is due to the algorithm's inability to discriminate RFI from noise.

²³⁰ Next, we consider the results using the NBFM-specific processing method of Fig-²³¹ ure 3 with $T_2 = 6.7 \ \mu s$ and $T_3 = 3.9 \ ms$. The bandwidth of the LPF following the ²³² differentiator is set at 4 kHz, which includes the entire message spectrum. ω_n is ²³³ estimated just once, i.e., $T_4 \sim 1$ s. The result is shown in Figure 7. Note in this case ²³⁴ noise spectrum is preserved, but the RFI is not as deeply suppressed.

Finally, we consider the potential toxicity of the RFI processing to weak astronomical signals, $s_a(t)$. To do this, we created simulated spectral lines in the form of sinusoids added to the data, with two cases of frequency offset, 1 and 5 kHz, above the center frequency of each of the 4 RFI signals, and with magnitude such that it has SNR of ~6 dB with respect to the resolution bandwidth of the spectrum shown in

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Figure 1. The spectrum of these signals by themselves is shown in Figure 8 for 1 kHz 240 offset. Figures 9–11 show the results for generic and NBFM-specific processing. In 241 Figure 9, it can be seen that the generic processing method suppresses all signals, 242 including the artificial spectral lines. Thus, the generic algorithm is not suitable 243 for spectroscopy. In Figures 10 and 11, the suppression results with NBFM–specific 244 processing are seen to depend on the frequency offsets. In case of 1 kHz offset, the 245 simulated spectral lines are suppressed by $\sim 50\%$. In case of 5 kHz offset, they are 246 preserved. The reason the lines with 5 kHz offset are preserved whereas the lines with 247 1 kHz offset are not is explained by the bandwidth of the LPF in the "demod-remod" 248 path in Figure 3, which is 4 kHz in this example. Thus, the choice of the bandwidth 249 of this LPF is a trade-off between a large value (e.g., 5 kHz), which provides the 250 most effective suppression, and a small value (e.g., 3 kHz), which protects largest 251 fraction of bandwidth against the toxic effect of the mitigation algorithm. 252

4. Concluding Remarks

This paper has described and demonstrated a method for mitigation for a broad class of narrowband signals which are common sources of interference to radio science observations in the range 20–250 MHz and beyond. The basic approach is to model RFI signals as sinusoids, and then to synthesize and subtract noise-free versions of these sinusoids from the original received signal. Two methods for estimating signal parameters were presented: A generic method which assumes only that RFI is well modeled as sinusoids, and a NBFM-specific method which exploits the constant

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²⁶⁰ modulus property and significant processing gain inherent in NBFM signals to im-²⁶¹ prove discrimination between RFI and noise. It was demonstrated that the former ²⁶² tends to distort components of the signal other than the RFI, whereas the latter is ²⁶³ considerably better in this regard, but less effective in suppression the RFI.

We should note that the generic method, despite its tendency to "eat" noise and 264 other signals underlying the RFI signals, is not completely without utility. For exam-265 ple, elimination of strong RFI – even if the underlying spectrum is rendered unusable 266 may nevertheless be desirable as it allows the data to be represented using a smaller 267 number of bits, which may have compelling advantages in some systems. Although 268 the same effect can be accomplished by, for example, computing spectra and then 269 blanking (zeroing) RFI-afflicted bins, this method is entirely coherent and does not 270 require transformation of the signal into the frequency domain, which might be prob-271 lematic or impossible in some systems. Another possible application of the generic 272 algorithm is pulsar processing, which is less sensitive to the observed "notching" 273 behavior than spectroscopy. 274

It should be pointed out that while we have not explicitly quantified the computational complexity of the proposed algorithms, we emphasize that it is well within the capabilities of existing conventional off-the-shelf real-time digital signal processing. In fact, this processing is similar or identical to processing performed by the communications equipment associated with these signals; e.g., detection, demodulation, and so on. Because these are typically commercial/commodity products, there has

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²⁸¹ been considerable previous effort invested in identifying minimum cost/complexity
 ²⁸² implementations; see for example [*Vassilevsky*, *V.L.*, 2007].

Finally, we should note that the algorithms described in Section 2 are applicable 283 to all forms of narrowband amplitude, phase, and frequency-modulated signals that 284 can be described in terms of the signal model of equation (2). This includes CW (i.e., 285 morse code), and the plethora of narrowband phase- and frequency-modulated digital 286 signals that exist. Moreover, it should be noted that digital signals have the additional 287 property of "finite alphabet" – that is, the parameters of the transmitted signal are 288 constrained to take on only a countable number of values, in contrast to analog 289 signals in which the parameters vary continuously. This is another form of a priori 290 waveform knowledge that can be exploited for additional performance improvement. 291 The possibility of exploiting the finite alphabet property of narrowband digital signals 292 is not considered here but is addressed in the context of a wideband digital signal in 293 Ellingson et al., 2001]. 294

Conversely, we should point out that there are a number of RFI signals in the 295 frequency range of interest for which the performance of the approach proposed here 296 is relatively poor. Prominent among these are broadcast FM signals ranging from 297 88 to 108 MHz in the U.S., which are relatively wideband (bandwidths on the order 298 of 200 kHz) and thus require update times (T_2) which are very short. In this case, 299 the linear chirp approach used in [Miller et al., 1997] might be helpful. In addition, 300 broadcast FM signals contain additional structure including a pilot signal, subcarri-301 ers for stereo, radio broadcast data system (RBDS), and subsidiary communications 302

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³⁰³ authorization (SCA). They must be taken into account to obtain the best perfor-³⁰⁴ mance. Performance will be limited for analog TV (NTSC) and digital TV (ATSC) ³⁰⁵ for similar reasons. However, we note that there is no reason that the concepts de-³⁰⁶ scribed in this paper and in [*Ellingson et al.*, 2001] could not be combined and/or ³⁰⁷ extended for these RFI signals.

Acknowledgments. The data used in Section 3 was provided by Michael Gray, who has made this and other datasets freely available via his website http://www.kd7lmo.net/ground_gnuradio_ota.html.

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Figure 1. Top-level block diagram of the algorithm.



Figure 2. Detail of generic estimation procedure.



Figure 3. Detail of improved estimation procedure for NBFM signals.



Figure 4. Power spectral density (PSD) of test dataset averaged over 0.967 s.

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Figure 5. Spectrogram (power spectral density vs. time and frequency) for the dataset.



Figure 6. Averaged power spectral density before (top) and after (bottom) application of the generic mitigation algorithm (Figure 2).



Figure 7. Averaged power spectral density before (top) and after (bottom) application of the NBFM–specific mitigation algorithm (Figure 3).



Figure 8. Averaged power spectral density of 4 sinusoids located at 1 kHz above the center frequencies of the RFI signals. The dash line is the mean of noise power spectral density.



Figure 9. Averaged power spectral density after application of the generic mitigation algorithm (Figure 2) with 4 additional sinusoid signals at 1 kHz offset.



Figure 10. Averaged power spectral density after application of the NBFM– specific mitigation algorithm (Figure 3) with 4 additional sinusoid signals at 1 kHz offset. Markers indicate the proper frequency and level of the additional signals.



Figure 11. Averaged power spectral density after application of the NBFM– specific mitigation algorithm (Figure 3) with 4 additional sinusoid signals at 5 kHz offset. Markers indicate the proper frequency and level of the additional signals.