Preliminary Analysis of the Effects of Front End Non-Linearity on LWA/LWDA Spectroscopy

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1 Introduction

The Long Wavelength Array (LWA) and Long Wavelength Development Array (LWDA) both require a broadband, low-noise gain stage immediately following the antenna. In addition to having sufficient gain and sufficiently-low noise temperature to fix the system temperature well below that of the Galactic background, this gain stage must also be robust to radio frequency interference (RFI). The latter is primarily a linearity requirement, and is typically expressed in terms of input 1-dB compression point ($P_{1\mathrm{dB}}$), input second-order intercept point (IIP2), and input third-order intercept point (IIP3). The primary concern in setting a linearity specification is that the intermodulation generated by the gain stage not obscure an unacceptably large fraction of the desired spectrum. These requirements are relatively easy to set if the spectrum is dominated by one or two strong tone-like sources of RFI [1]. However, the actual input spectrum is likely to be much more complex; thus it is a bit difficult to know quantatively how much intermodulation at levels deleterious to radio astronomy is generated based on the three linearity specifications alone.

This report attempts to shed some light on this issue through simulation. First, a mathematical model of the active balun designed by Brian Hicks of NRL (based on the Mini-Circuits GALI-74 MMIC amplifier) [2] is developed and shown to be a reasonable representation of the genuine article. Next, a rudimentary time-domain simulation of the expected input spectrum is developed. Finally, the input input spectrum is applied to the gain stage model and the resulting intermodulation spectrum is analyzed. This report concludes with a summary of results and some suggestions for next steps.

2 Gain Stage Modeling

The Hicks active balun has been found to have gain $G = +24.5$ dB ($G = +21.5$ with respect to each of the balanced inputs individually), $P_{1\mathrm{dB}} = -5$ dBm, IIP2= +19 dBm, and IIP3= +7.5 dBm [2]. A voltage transfer characteristic having approximately the same characteristics, developed from an initial educated guess and refined through trial and error, is the truncated Maclaurin series

$$y = 11.75x + 0.6x^2 - 9x^3 \tag{1}$$

where x and y are the input and output voltages in units of $\sqrt{\mathrm{mW}}$.

This transfer characteristic is illustrated in Figure 1. The model produces $P_{1\mathrm{dB}} = -8$ dBm, which is about 3 dB lower than measured. This was a tradeoff to improve the accuracy of the model's IIP2 and IIP3, as will be discussed below. Also note that the model accuracy is poor for input power $P_x > -10$ dBm; however this is not of much concern since it would be very poor engineering practice to operate in this region for any length of time.

The IIP2 and IIP3 of the model were determined by simulation. Equal-power tones at 31 MHz and 32 MHz were synthesized, and the model was applied to the resulting time-domain signal. The result was transformed into the frequency domain and the power of the resulting intermodulation products at 33 MHz and 63 MHz were measured. The former is a third-order intermodulation product (IM3), whereas the latter is a second-order intermodulation product (IM2). Repeating this procedure while varying the level of the input tones generates the result shown in Figure 2. The slope of the IM2 and IM3 curves are very close to the expected 2:1 (2.0001:1) and 3:1 (2.9709:1) respectively. Extrapolating these curves to their respective linear gain intercept points results in the following estimates: IIP2= +19.8 dBm and IIP3= +7.3 dBm. These are very close to the values of +19 dBm and = +7.5 dBm (respectively) measured for the Hicks active balun.

It is concluded that the polynomial model given by Equation 1 yields G, IIP2, and IIP3 which is very close to that of the Hicks active balun, subject to a few caveats. First, the model is not accurate for total power greater than -10 dBm or so. Second, the failure to consider non-linearities greater than third-order can be expected to lead to some inaccuracy in modeling intermodulation, although one expects the intermodulation from these higher-order linearities to weak relative to that associated with the lower-order terms. Third, it should be noted that this is a *memoryless* model; i.e., the model does not depend on inputs prior to the current instant. In practice, all amplifiers

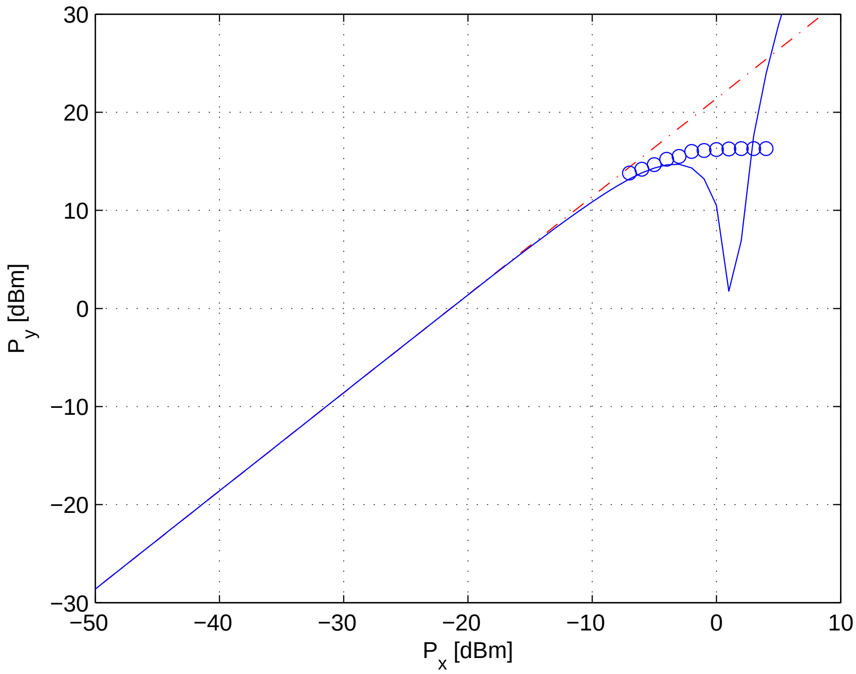

Figure 1: Voltage transfer characteristics, expressed in terms of power transfer: *Solid/Blue*: Maclaurin series model; '∘': Points read from measurement shown in [2]; *Dash-Dot/Red:* A perfectly linear amplifier with $G = 21.4$ dB. P_x and P_y are input and output power respectively.

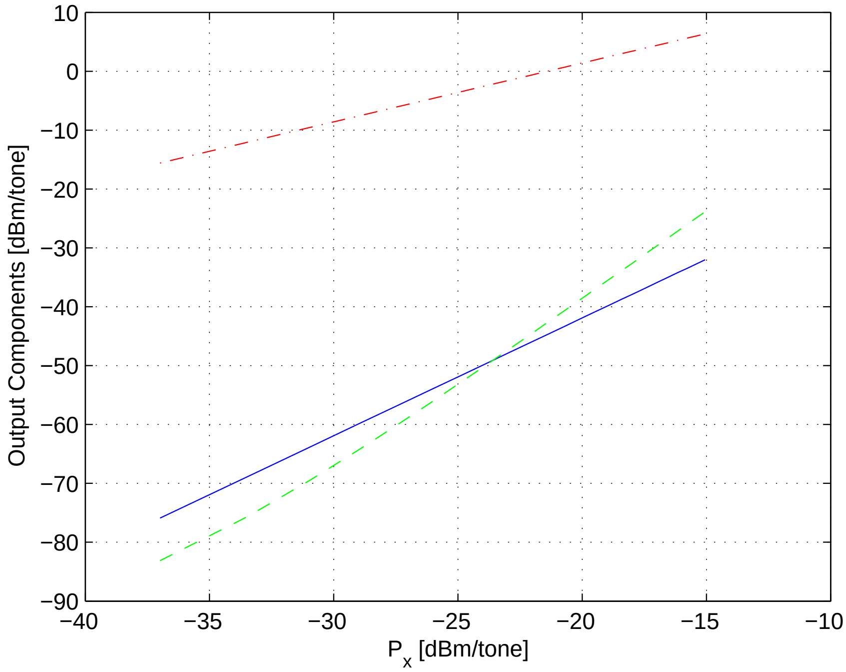

Figure 2: Levels of the 63 MHz (IM2, *Solid/Blue*) and 33 MHz (IM3, *Dash/Green*) intermodulation products in response to equal-strength input tones at 31 MHz and 32 MHz. *Dash-Dot/Red*: Linear gain response.

Freq [MHz]	Power [dBm]	Source
21.00	-31	Simulating an HF broadcast
42.00	-42	Simulating a Mobile Base Station
55.25	-55	TV Ch 2 Video
58.83	-60	TV Ch 2 Color
59.75	-55	TV Ch 2 Audio
67.25	-16	TV Ch 4 Video
70.83	-55	TV Ch 4 Color
71.25	-25	TV Ch 4 Audio
89.10	-39	FM Broadcast

Figure 3: Summary of the input spectrum model. See text for details.

have some memory; however, if the total signal level is maintained well below P_{1dB}, one expects the effects of memory to be relatively minor. A final caveat is that this model should not be taken as a alternative to field testing: Whereas this model is probably useful for understanding roughly what the behavior of the intermodulation is likely to be, it is impossible to ensure that all the relevant phenomenology has been reasonably captured. Thus, this analysis should not be considered to be a substitute for field testing.

3 Input Spectrum Modeling and Analysis

For this initial investigation, the input spectrum is modeled as the sum of sinusoids, where each sinusoid represents a narrowband (but not necessarily CW) carrier, as indicated in Figure 3. The frequencies and powers were chosen to correspond *roughly* to spectrum observed at a rural site, familiar to the author, and presented in [3]. Note that no attempt was made to include *all* the signals observed in [3] (only a few examples of the strongest are included), or to acheive any particular accuracy in frequencies or powers. The model is generated by simply adding up each of the sinusoids computed with a constant and randomly-determined phase (to reduce the potential for unrealistic spurious harmonic reinforcement). The time-average power in the resulting signal is −15.3 dBm, which is well below the −10 dBm criterion for model accuracy and −8 dBm P_{1dB} described above, and is as probably just about the largest input for which one should reasonably expect useable performance from the given amplifier. The model includes no noise; however the Galactic Background will be considered separately, below.

Figure 4 shows what this model looks like as a 400 MSPS × 16K- (16,384-) sample which is windowed using a Bartlett (triangle) window and transformed into the frequency domain using a 16K FFT. Immediately apparent is the fact that Bartlett windowing suppresses FFT edge effects (namely, the "skirt" around each sinusoid) to a level of only −80 dBm or so. Also shown in Figure 4 is the expected Galactic Background signal measured by a (mythical) perfectly-matched low-gain antenna, calculated as explained in [4]. Comparison of the level of the Galactic background to that of the FFT skirt artifacts makes it clear that an improved analysis procedure is required to observe intermodulation blocking levels comparable to the Galactic background. This example is included to illustrate one of the potential pitfalls in high-dynamic range analysis (be it for simulation or using actual data); it is very easy to mistake the FFT edge effects as *bona fide* signal power, when in fact they are simply processing artifacts.

This problem can be dealt with in at least two ways. The simplest are (1) to apply a more aggressive window function and (2) to extend the length of the FFT. For the sake of computational efficiency, it is chosen to apply a two-term Blackman window in place of the Bartlett window. The spectral sidelobes of the Blackman window are 10's of dB lower than those of the Bartlett window, with the tradeoff that Blackman this smears the power of the sinusoid over several spectral channels. Figure 5 shows the result with the Blackman window; note that this is adequate for a preliminary (at least) analysis.

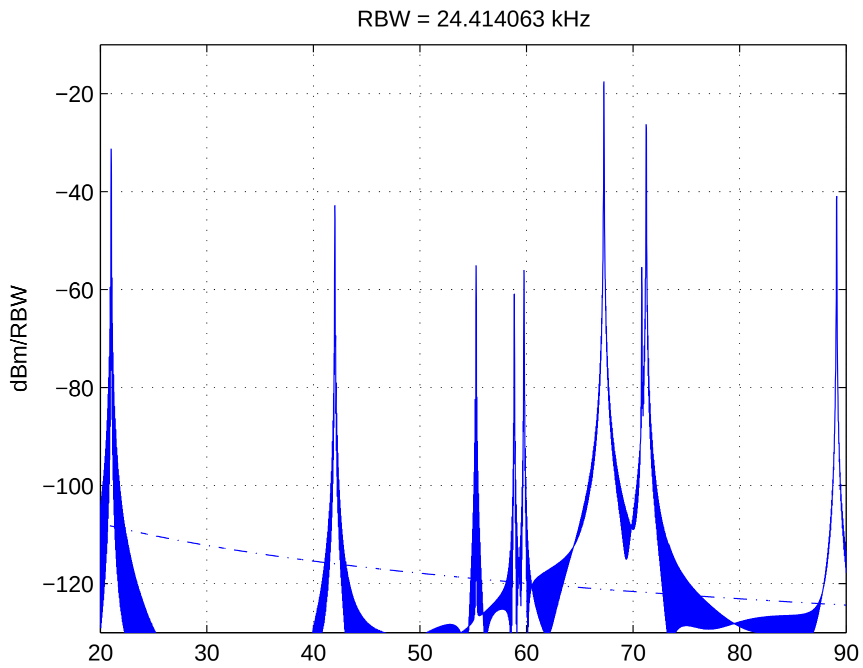

Figure 4: Input spectrum model *before* application of non-linear amplifier model. *Solid/Blue*: 16K FFT with Bartlett window. *Dash/Blue*: Galactic background.

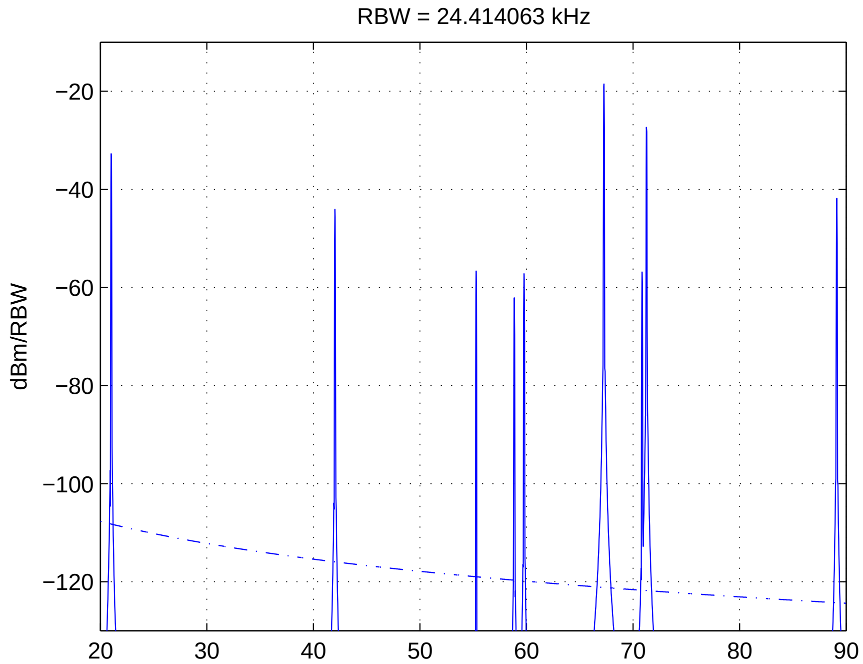

Figure 5: Input spectrum model *before* application of non-linear amplifier model. *Solid/Blue*: 16K FFT with two-term Blackman window. *Dash/Blue*: Galactic background.

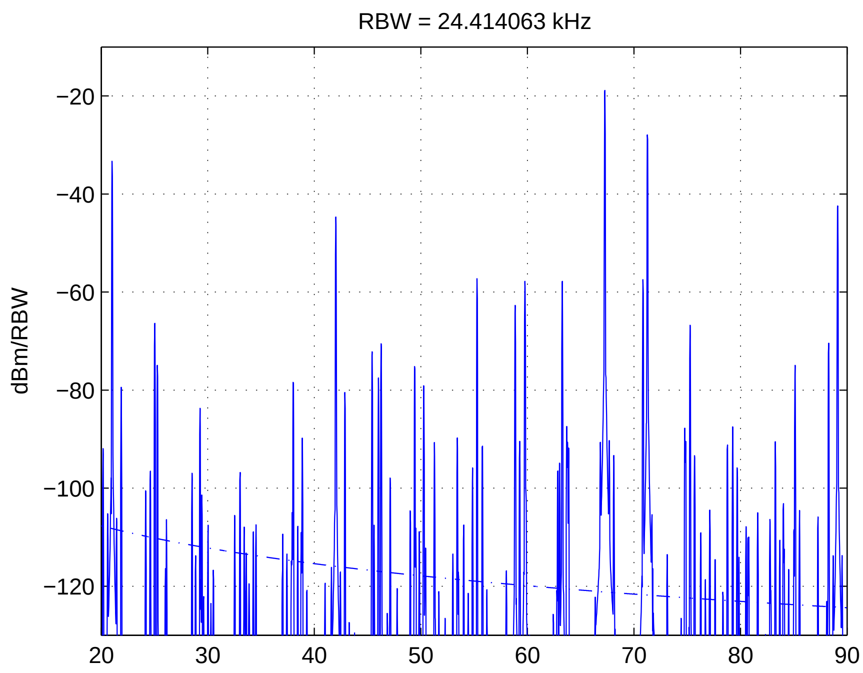

Figure 6: Input spectrum model *after* application of non-linear amplifier model. *Solid/Blue*: 16K FFT with two-term Blackman window, minus 21.5 dB such that display is referenced to the antenna terminals (for easy comparison with previous results). *Dash/Blue*: Galactic background.

4 Analysis of the Output Spectrum

To analyze the production of intermodulation by the first gain stage, the polynomial voltage transfer function shown in Equation 1 is applied to the time-domain model described in the previous section. Figure 6 shows the results when 16K samples of the resulting signal are windowed using the two-term Blackman window and the FFT is applied. As expected, there is copious generation of intermodulation throughout the spectrum, at levels blocking the Galactic background.

Recall that the signal model used to generate the above result has a total signal (RFI) input power (measured at the antenna terminals) of about -15.3 dBm. At many (hopefully, most) sites, the total RFI power could be much lower. Figure 7 shows the result when the total signal power is reduced by 10 dB (to -25.3 dBm) by simply reducing the power of each sinusoid in the signal model by 10 dB. As expected, the level of intermodulation is dramatically reduced.

5 Conclusions and Suggested Next Steps

This report demonstrates that it is reasonable to model the generation of intermodulation in the first gain stage of an LWA/LWDA receiver using a simple third-order polynomial, subject to the following caveats:

- The model is heuristic in the sense that it ignores the actual physics of the amplifier and instead strives only to produce the same specifications of G, IIP2, IIP3, and $P_{1\mathrm{dB}}$. This is fairly common practice in the RF engineering field, but it does not preclude the possibility that some important phenomenology has been neglected. In particular, it was noted that this is a *memoryless* nonlinearity model.
- The model is valid only for signal levels less than about -10 dBm.

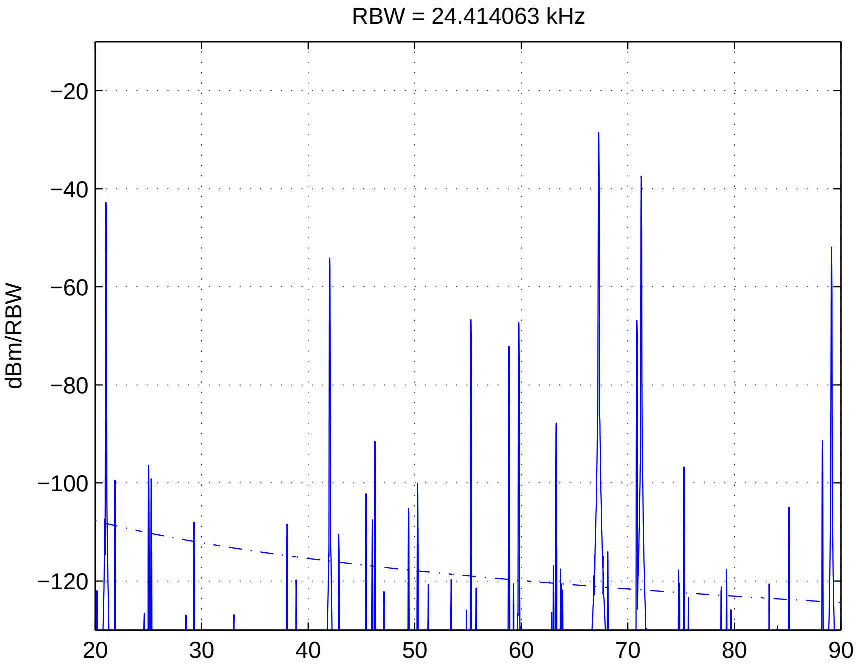

Figure 7: *Reduced-power* (by 10 dB) input spectrum model after application of non-linear amplifier model. *Solid/Blue*: 16K FFT with two-term Blackman window, minus 21.5 dB such that display is referenced to the antenna terminals (for easy comparison with previous results). *Dash/Blue*: Galactic background.

- Higher-order (4th and above) non-linearities are neglected. Although these are expected to be much weaker than the intermodulation associated with 2nd- and 3rd-order non-linearities, they may nevertheless become important at some point due to the extraordinary dynamic range which we are expecting from the receiver.

A simple input signal model was developed which very roughly approximates an expected spectrum, with a total input power about 8 dB below $P_{1\mathrm{dB}}$. Applying the amplifier model to this data resulted in the expected behavior – generation of copious amounts of intermodulation. It was shown if the input signal level is less by 10 dB (about 18 dB below $P_{1\mathrm{dB}}$), the situation is dramatically improved. However, this is a luxury that may not be possible at all LWA/LWDA sites. Furthermore, the input signal model was somewhat "easier" than the actual expected input, since it consisted only of a handful of sinusoidal signals with time-invariant parameters, as opposed to the scores of signals of varying bandwidths and time-varying behaviors that are seen in practice.

The following next steps are recommended:

1. The weakest point in this analysis is judged to be the fidelity of the input signal model. It would be very useful and not too difficult to refine this model such that, for example, Broadcast FM is represented as time-varying wideband frequency-modulated carriers, Broadcast TV signals are modeled as actual video plus wideband FM audio, a greater number of HF broadcast and low-band VHF two-way mobile radio signals, and so on. These simple changes could significantly change the nature of the results.

2. Perhaps the best way to use this analysis is in conjunction with field RFI measurements. The input signal model can be designed to conform to the results of these measurements, in which case the analysis should provide a useful characterization of the actual intermodulation scenario that can be expected.

3. A useful metric for analyzing the deleterious effects of intermodulation may be to compute the predicted time-frequency density of levels of intermodulation which block (exceed) the Galactic background for a given instrumental time-frequency resolution. This could be expressed anecdotally in the form of spectragrams, or stochastically in the form of a probability distribution function; both would likely provide some additional insight.

References

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