LARGE ARRAY SIGNAL PROCESSING FOR DSN APPLICATIONS: PART I

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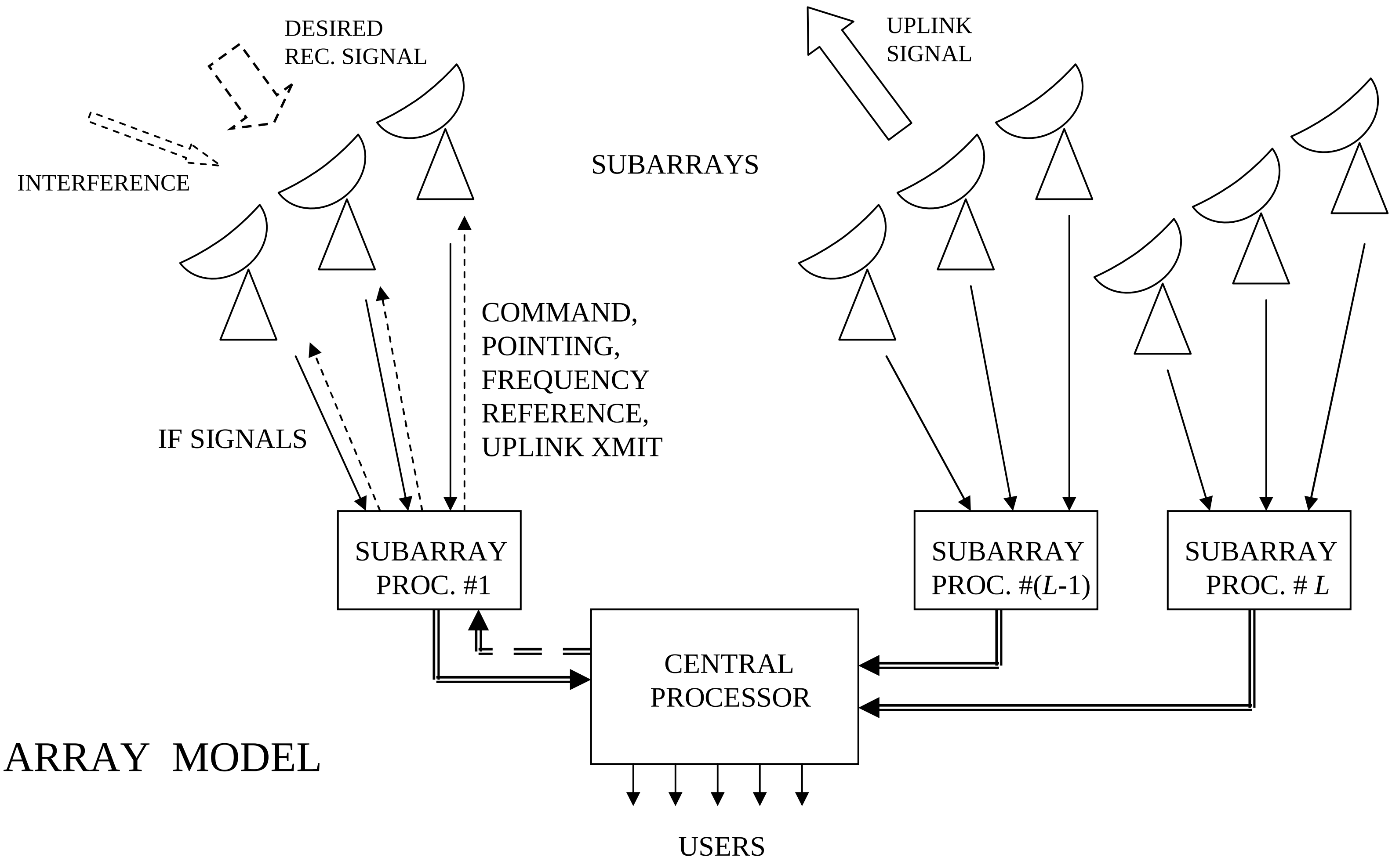
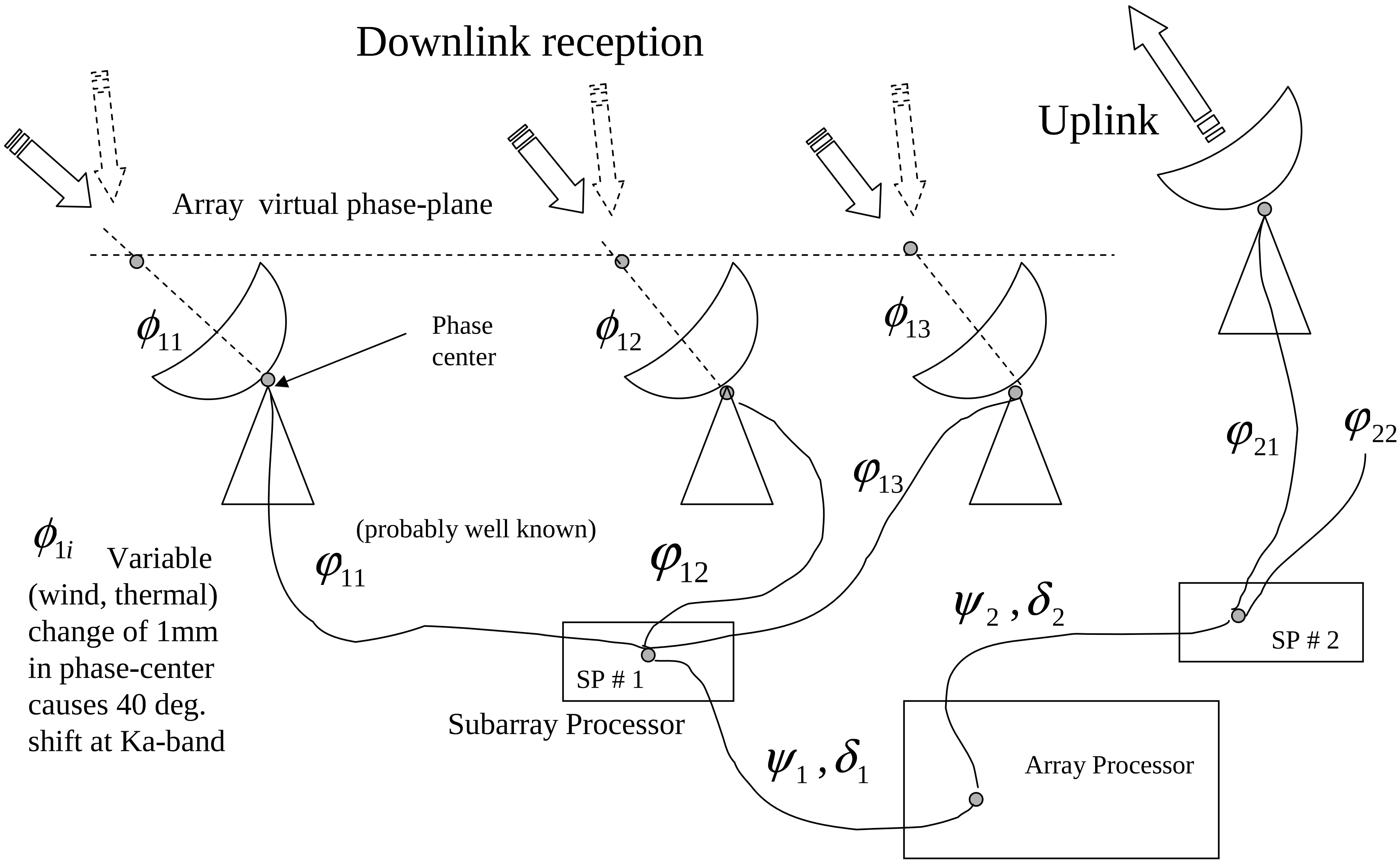
MAY 21, 2002

LARGE ARRAY SIGNAL PROCESSING

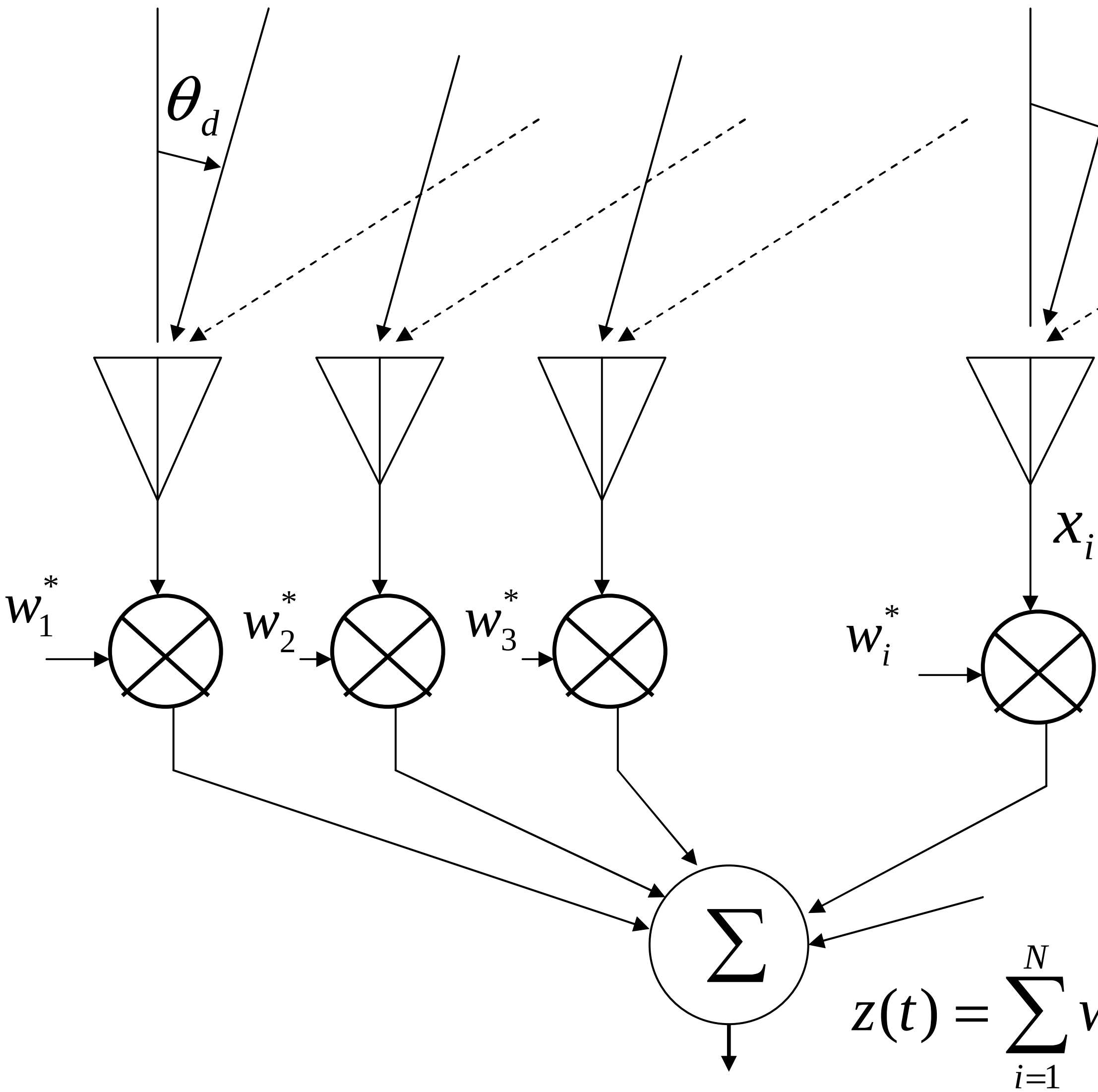


ARRAY MODEL

LARGE ARRAY SIGNAL PROCESSING



LARGE ARRAY SIGNAL PROCESSING



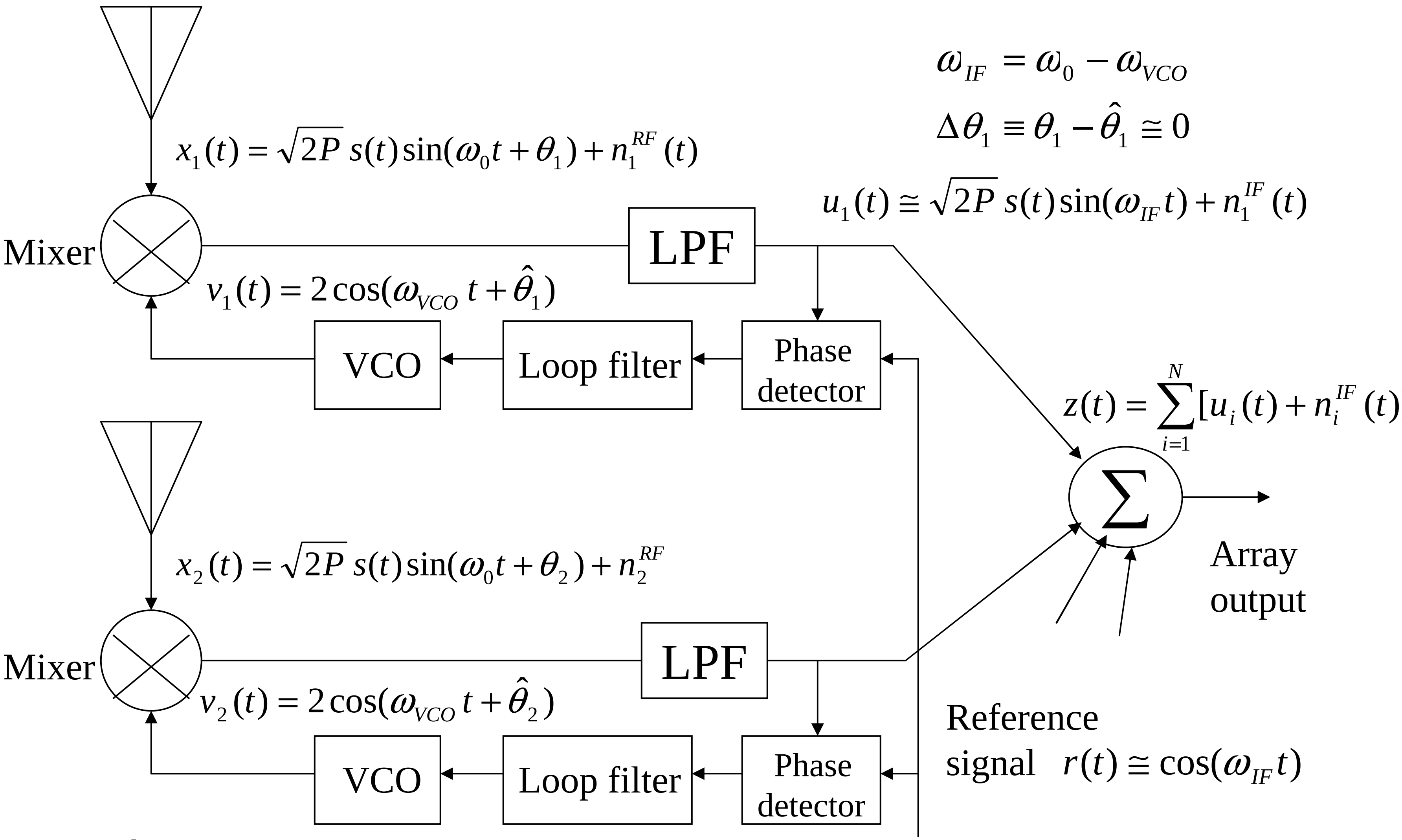
$$x_i(t) = d_i(t) + I_i(t) + n_i(t)$$

Reed, Mallett, Brennan (RMB) notation: $w_i^* x_i$

$$z(t) = \sum_{i=1}^{N} w_i^* \, x_i(t)$$

LARGE ARRAY SIGNAL PROCESSING

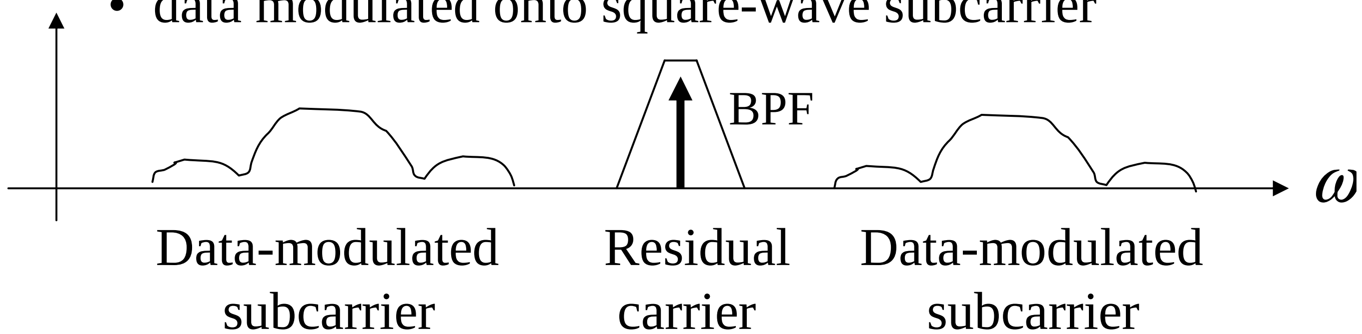
THE PHASE-LOCK LOOP ARRAY



LARGE ARRAY SIGNAL PROCESSING

- Reference Signal Requirements
  - must be **correlated** with the desired signal
  - must be **uncorrelated** with interference

- EXAMPLE: NASA Deep-Space Modulation Format
  - residual carrier usually present
  - data modulated onto square-wave subcarrier

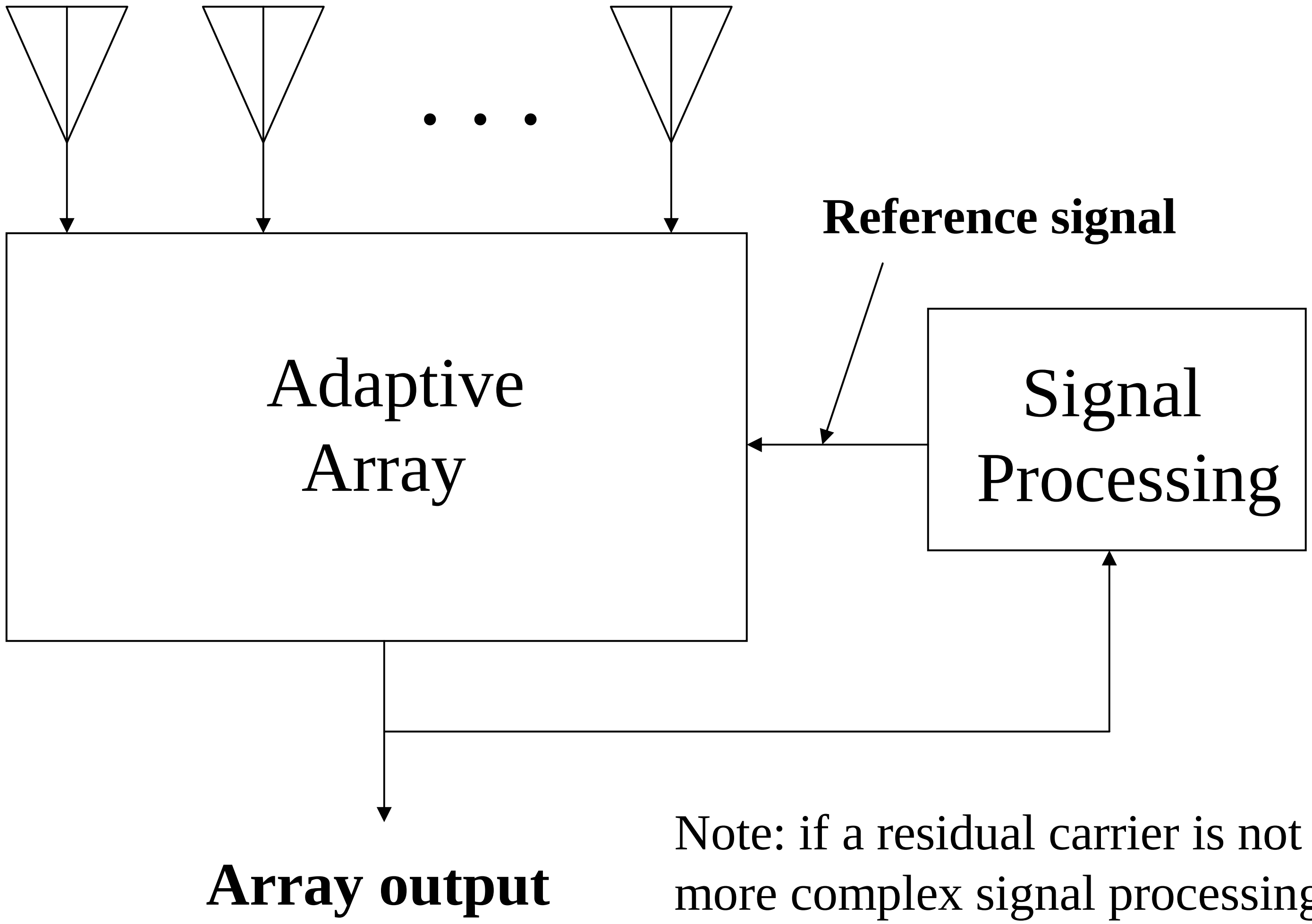


- *PLL* Array Output *SNR* (perfect reference):

$$SNR \equiv \frac{\left(N\sqrt{2P}s(t)\sin(\omega_{IF}(t))\right)^2}{N\sigma^2} = N\frac{P}{\sigma^2}$$

LARGE ARRAY SIGNAL PROCESSING

REFERENCE SIGNAL GENERATION



Note: if a residual carrier is not available, more complex signal processing often yields a useful reference signal.

LARGE ARRAY SIGNAL PROCESSING

COMPLEX VECTOR FORMULATION OF THE **LARGE ARRAY SIGNAL PROCESSING** PROBLEM

$$\mathbf{X} = [x_1(t), x_2(t), \cdots, x_N(t)]^T; \quad \mathbf{W} = [w_1, w_2, \cdots, w_N]^T$$

$$\mathbf{X} = \mathbf{X}_d + \mathbf{X}_I + \mathbf{X}_n$$

$$z(t) = \sum_{i=1}^{N} w_i^* \, x_i(t) = \mathbf{W}^{*T}\mathbf{X} \equiv z_d(t) + z_I(t) + z_n(t)$$

$$P_d = E\,|\,z_d(t)\,|^2; \quad P_I = E\,|\,z_I(t)\,|^2; \quad P_n = E\,|\,z_n(t)\,|^2$$

$$SINR = \frac{P_d}{P_I + P_n} = \frac{P_d}{P_u}; \quad \text{want } \mathbf{W}_{opt} \text{ that maximizes } SINR$$

LARGE ARRAY SIGNAL PROCESSING

NARROWBAND ASSUMPTION:

$$\mathbf{X}_d = s(t)\,\mathbf{U}_d \quad \text{signal times "source direction vector"}$$

$$= A_d(t)\{\exp[j(\psi_d + \varphi_d(t))]\}\,[1, e^{j\theta_{d2}}, \cdots, e^{j\theta_{dN}}]^T$$

DESIRED ARRAY OUTPUT SIGNAL AND POWER:

$$y_d(t) = \mathbf{W}^{*T}\mathbf{X}_d = s(t)\,\mathbf{W}^{*T}\,\mathbf{U}_d$$

$$P_d = E\,|\,s(t)|^2\,|\,\mathbf{W}^{*T}\,\mathbf{U}_d\,|^2 = E\,|\,s(t)|^2\,\mathbf{W}^{*T}\mathbf{U}_d\mathbf{U}_d^{*T}\mathbf{W}$$

$$= \mathbf{W}^{*T}E[\,s(t)\,\mathbf{U}_d\,s^*(t)\mathbf{U}_d^{*T}\,]\mathbf{W}$$

$$= \mathbf{W}^{*T}\,E[\mathbf{X}_d\,\mathbf{X}_d^{*T}\,]\mathbf{W}$$

$$= \mathbf{W}^{*T}\,\mathbf{\Phi}_d\,\mathbf{W}$$

LARGE ARRAY SIGNAL PROCESSING

THE "UNDESIRED" COMPONENTS:

$$\mathbf{\Phi}_u = \mathbf{\Phi}_I + \mathbf{\Phi}_n \equiv E[\mathbf{X}_I \mathbf{X}_I^{*T}] + E[\mathbf{X}_n \mathbf{X}_n^{*T}] = E[\mathbf{X}_u \mathbf{X}_u^{*T}]$$

$$P_u = P_I + P_n = \mathbf{W}^{*T} \mathbf{\Phi}_I \mathbf{W} + \mathbf{W}^{*T} \mathbf{\Phi}_n \mathbf{W} = \mathbf{W}^{*T} \mathbf{\Phi}_u \mathbf{W}$$

"SIGNAL TO INTERFERENCE PLUS NOISE RATIO"

$$SINR = \frac{P_d}{P_u} = \frac{\mathbf{W}^{*T} \mathbf{\Phi}_d \mathbf{W}}{\mathbf{W}^{*T} \mathbf{\Phi}_u \mathbf{W}} = E\,|\,s(t)\,|^2 \,\frac{|\,\mathbf{W}^{*T} \mathbf{U}_d\,|^2}{\mathbf{W}^{*T} \mathbf{\Phi}_u \mathbf{W}}$$

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MAXIMIZATION OF ***SINR*** (known look direction)

EXAMPLE 1: $\mathbf{\Phi}_I = \mathbf{0}, \quad \mathbf{\Phi}_u = \mathbf{\Phi}_n = \sigma^2 \mathbf{I}, \quad \mathbf{\Phi}_u^{-1} = \frac{1}{\sigma^2}\mathbf{I}$

$$SNR = E\,|\,s(t)\,|^2 \, \frac{|\,\mathbf{W}^{*T}\mathbf{U}_d\,|^2}{\mathbf{W}^{*T}\mathbf{\Phi}_u\,\mathbf{W}} = E\,|\,s(t)\,|^2 \, \frac{\left|\sum_{i=1}^{N} w_i^* \, u_{d,i}\right|^2}{\sigma^2 \sum_{i=1}^{N} |\,w_i\,|^2} \qquad (1)$$

Schwarz inequality: $\left|\sum_{i=1}^{N} w_i^* \, u_{d,\,i}\right|^2 \le \sum_{i=1}^{N} |\,w_i\,|^2 \, \sum_{i=1}^{N} |\,u_{d,\,i}\,|^2$

with equality iff $w_i = c\,u_i$.

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The optimum weights are $w_{opt,\,i} = u_i$. Letting $c = 1/\sigma^2$, the optimum weight vector can be expressed as

$$\mathbf{W}_{opt} = \mathbf{\Phi}_u^{-1}\mathbf{U}_d$$

and yields $$SNR = \frac{E\,|\,s(t)\,|^2}{\sigma^2}\sum_{i=1}^{N}|\,u_i\,|^2 = N\,\frac{P}{\sigma^2}$$

- Note that with **optimum weights**, the array output ***SNR*** is N times the elemental ***SNR***, as with a *PLL* Array
- Need to determine “source direction” separately
  - once source direction is determined, the optimum weights are also known

LARGE ARRAY SIGNAL PROCESSING

EXAMPLE 2: $\mathbf{\Phi}_I = \mathbf{0}, \quad \mathbf{\Phi}_u = \mathbf{\Phi}_n = diag[\sigma_1^2, \sigma_2^2, \cdots, \sigma_N^2]$

$$\mathbf{\Phi}_u = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_N^2 \end{bmatrix}; \quad \mathbf{\Phi}_u^{-1} = diag[\sigma_1^{-2}, \sigma_2^{-2}, \cdots, \sigma_N^{-2}]$$

$$SNR = E\,|\,s(t)\,|^2 \frac{|\,\mathbf{W}^{*T}\mathbf{U}_d\,|^2}{\mathbf{W}^{*T}\mathbf{\Phi}_u\,\mathbf{W}} = E\,|\,s(t)\,|^2 \frac{\left|\sum_{i=1}^{N} w_i^*\,u_{d,i}\right|^2}{\sum_{i=1}^{N} \sigma_i^2\,|\,w_i\,|^2} \qquad (2)$$

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First rewrite inner product, then apply Schwarz inequality:

$$\left| \sum_{i=1}^{N} w_i^* \, u_{d,i} \right|^2 = \left| \sum_{i=1}^{N} \sigma_i w_i^* \frac{u_{d,i}}{\sigma_i} \right|^2 \le \sum_{i=1}^{N} \sigma_i^2 \, | w_i |^2 \sum_{i=1}^{N} \frac{| u_{d,i} |^2}{\sigma_i^2}$$

$$\frac{SINR}{E\,| s(t) |^2} = \frac{\left| \sum_{i=1}^{N} \sigma_i w_i^* \frac{u_{d,i}}{\sigma_i} \right|^2}{\sum_{i=1}^{N} \sigma_i^2 \, | w_i |^2} \le \frac{\sum_{i=1}^{N} \sigma_i^2 \, | w_i |^2 \sum_{i=1}^{N} \frac{| u_{d,i} |^2}{\sigma_i^2}}{\sum_{i=1}^{N} \sigma_i^2 \, | w_i |^2} = \sum_{i=1}^{N} \frac{| u_{d,i} |^2}{\sigma_i^2} \qquad (3)$$

with equality iff $\sigma_i w_i = c \dfrac{u_{d,i}}{\sigma_i}; \quad \rightarrow \quad w_{opt,\,i} = \dfrac{u_{d,i}}{\sigma_i^2}$

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The optimum weight vector can again be expressed as

$$\boxed{\mathbf{W}_{opt} = \mathbf{\Phi}_u^{-1}\mathbf{U}_d} \qquad (4)$$

***SINR* of combined array output:**

From (3), the *SINR* of the array output with optimum weights is:

$$SINR = \frac{|\,\mathbf{W}_{opt}^{*T}\,\mathbf{U}_d\,|^2}{\mathbf{W}_{opt}^{*T}\,\mathbf{\Phi}_u\,\mathbf{W}_{opt}} = \sum_{i=1}^{N}\frac{E\,|\,s(t)\,|^2}{\sigma_i^2} = \sum_{i=1}^{N}\frac{P}{\sigma_i^2}$$

The maximum value of the ***SINR*** is achieved by the optimum weights $\mathbf{W}_{opt}$. When these weights are applied, the ***SINR*** of the output is equal to the sum of elemental ***SINR***-s.

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EXAMPLE 3: the general case: $\boldsymbol{\Phi}_u = \boldsymbol{\Phi}_I + \boldsymbol{\Phi}_n$ **(Applebaum)**

The covariance matrix of the general undesired component is Hermitian, therefore it can be diagonalized by a unitary transformation. Let $\mathbf{A}$ be a unitary matrix, so that

$$\mathbf{A}^{*T} = \mathbf{A}^{-1}; \qquad \mathbf{A}^{*T}\mathbf{A} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

and define the "transformed" vectors as

$$\mathbf{Y}_d = \mathbf{A}\,\mathbf{X}_d\,, \quad \mathbf{Y}_I = \mathbf{A}\,\mathbf{X}_I\,, \quad \mathbf{Y}_n = \mathbf{A}\,\mathbf{X}_n$$

These "transformed" vectors represent the original input vectors in a new, "rotated" coordinate system, and lead to a minor conceptual modification to the system block diagram.

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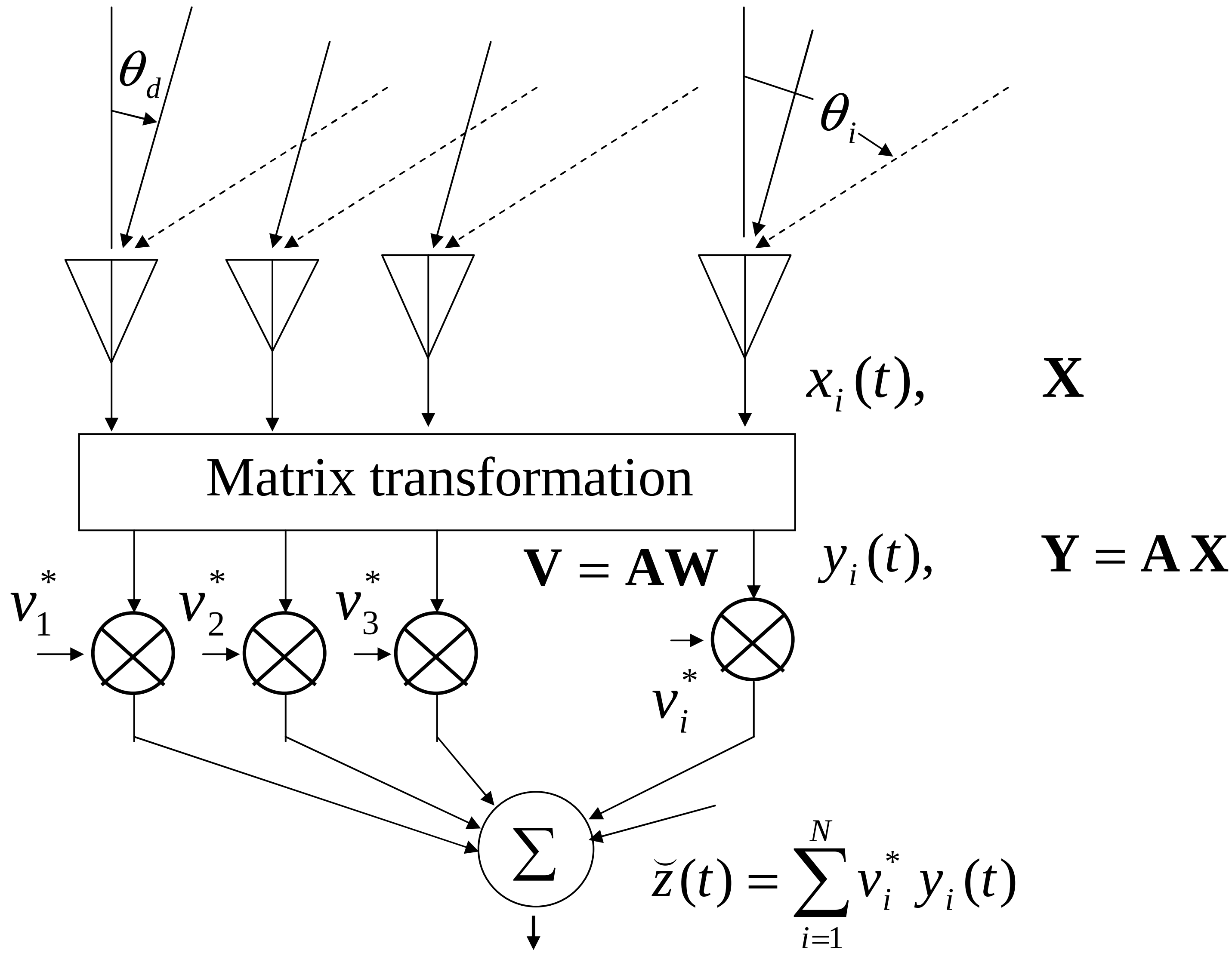

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Let the covariance matrix of the signals in the rotated coordinate system be designated by Ψ_u, defined as

$$\mathbf{\Psi}_u = E\left[\mathbf{Y}_u \mathbf{Y}_u^{*T}\right] = E\left[\mathbf{A}\mathbf{X}_u (\mathbf{A}\mathbf{X}_u)^{*T}\right] = E\left[\mathbf{A}\mathbf{X}_u \mathbf{X}_u^{*T}\mathbf{A}^{*T}\right]$$
$$= \mathbf{A}E\left[\mathbf{X}_u \mathbf{X}_u^{*T}\right]\mathbf{A}^{*T} = \mathbf{A}\mathbf{\Phi}_u\mathbf{A}^{*T}$$

If the columns of $\mathbf{A}$ correspond to the eigenvectors of $\mathbf{\Phi}_u$, then the transformation diagonalizes $\mathbf{\Phi}_u$, with the eigenvalues of $\mathbf{\Phi}_u$ occupying the diagonal:

$$\mathbf{\Psi}_u = \mathrm{diag}[\lambda_1, \lambda_2, \cdots, \lambda_N] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_N \end{bmatrix}$$

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Maximization of *SINR* for the general case:

The solution for the case of a diagonal covariance matrix has already been solved in Example 2: the optimum weights are proportional to the ratio of source-direction vector (in the rotated coordinates) to the total noise power. Defining the "rotated source-direction" as $\mathbf{Q} = \mathbf{A}\mathbf{U}_d$, the optimum weights can be obtained from equation (4) by inspection:

$$\mathbf{V}_{opt} = \mathbf{\Psi}_u^{-1}\mathbf{Q}_d$$

But $\mathbf{V} = \mathbf{A}\mathbf{W}$ implies that $\mathbf{W}_{opt} = \mathbf{A}^{-1}\mathbf{V}_{opt}$, so we can write

$$\begin{aligned}\mathbf{W}_{opt} &= \mathbf{A}^{-1}[\mathbf{\Psi}_u^{-1}\mathbf{A}\mathbf{U}_d] = [\mathbf{A}^{-1}\mathbf{\Psi}_u^{-1}\mathbf{A}]\mathbf{U}_d \\ &= [\mathbf{A}^{-1}\mathbf{\Psi}_u\mathbf{A}]^{-1}\mathbf{U}_d;\ \text{using}\,[\mathbf{ABC}]^{-1} = [\mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}] \qquad (5)\end{aligned}$$

LARGE ARRAY SIGNAL PROCESSING

Since $\mathbf{\Psi}_u = \mathbf{A}\,\mathbf{\Phi}_u\,\mathbf{A}^{*T}$, it follows that

$$\mathbf{A}^{-1}\,\mathbf{\Psi}_u\mathbf{A} = \mathbf{A}^{-1}[\mathbf{A}\,\mathbf{\Phi}_u\,\mathbf{A}^{*T}]\mathbf{A} = \mathbf{\Phi}_u$$

Substituting into (5) yields the optimum weight vector that maximizes ***SINR*** for the general case:

$$\mathbf{W}_{opt} = [\mathbf{A}^{-1}\mathbf{\Psi}_u\mathbf{A}]^{-1}\mathbf{U}_d = \mathbf{\Phi}_u^{-1}\,\mathbf{U}_d$$

Since a constant scale factor, μ, applied to the weight vector does not change the ***SINR***, we can also write:

$$\boxed{\mathbf{W}_{opt} = \mu\,\mathbf{\Phi}_u^{-1}\,\mathbf{U}_d}$$

LARGE ARRAY SIGNAL PROCESSING

PROCESSORS FAMILIAR FROM THE LITERATURE:

1. **Conventional Beamformer:** $\mathbf{W}_{opt} = (\text{constant})\mathbf{U}_d$

2. **NAME (noise-alone matrix inverse):**

$$\mathbf{W}_{opt} = \mathbf{\Phi}_u^{-1}\mathbf{U}_d$$

3. **SPNAMI (signal-plus-noise matrix inverse):**

$$\mathbf{W}_{opt} = \mathbf{\Phi}^{-1}\mathbf{U}_d$$

Both NAME and SPNAMI achieve the same *SINR*

LARGE ARRAY SIGNAL PROCESSING

THE EQUIVALENCE OF USING $\mathbf{\Phi}_u^{-1}\,\mathbf{U}_d^*$ **OR** $\mathbf{\Phi}^{-1}\,\mathbf{U}_d^*$ **TO MAXIMIZE *SINR*** (Applebaum-Compton notation: $w_i\ x_i$)

Recall that $\mathbf{\Phi} = \mathbf{\Phi}_d + \mathbf{\Phi}_u = E\,|\,s(t)\,|^2\,\mathbf{U}_d^*\,\mathbf{U}_d^T + \mathbf{\Phi}_u$

The inverse of $\mathbf{\Phi}$ can be calculated with the help of the following **Matrix Inversion Lemma:** if **B** is a nonsingular $N \times N$ matrix, **Z** is an $N \times 1$ column vector, and β is a scalar, then the inverse of $\mathbf{Q} = \mathbf{B} - \beta\,\mathbf{Z}^*\mathbf{Z}^T$ is given by

$$\mathbf{Q}^{-1} = \mathbf{B}^{-1} - \alpha\,\mathbf{B}^{-1}\mathbf{Z}^*\mathbf{Z}^T\mathbf{B}^{-1}$$

where $\alpha^{-1} + \beta^{-1} = \mathbf{Z}^T\mathbf{B}^{-1}\mathbf{Z}^*$

LARGE ARRAY SIGNAL PROCESSING

Applying this lemma to $\mathbf{\Phi}$, we find its inverse as

$$\mathbf{\Phi}^{-1} = \mathbf{\Phi}_u^{-1} - \alpha \mathbf{\Phi}_u^{-1} \mathbf{U}_d^* \mathbf{U}_d^T \mathbf{\Phi}_u^{-1}$$

Evaluating α and substituting, after some algebra we get

$$\mathbf{\Phi}^{-1} \mathbf{U}_d^* = (\text{constant}) \ \mathbf{\Phi}_u^{-1} \mathbf{U}_d^*$$

Since multiplying the weight vector by a constant does not affect the ***SINR***, these two weight vectors produce identical ***SINR***.

LARGE ARRAY SIGNAL PROCESSING

Eigenvector Approach from linear algebra:

Recall that the ***SINR*** can be expressed as the ratio of two "quadratic forms":

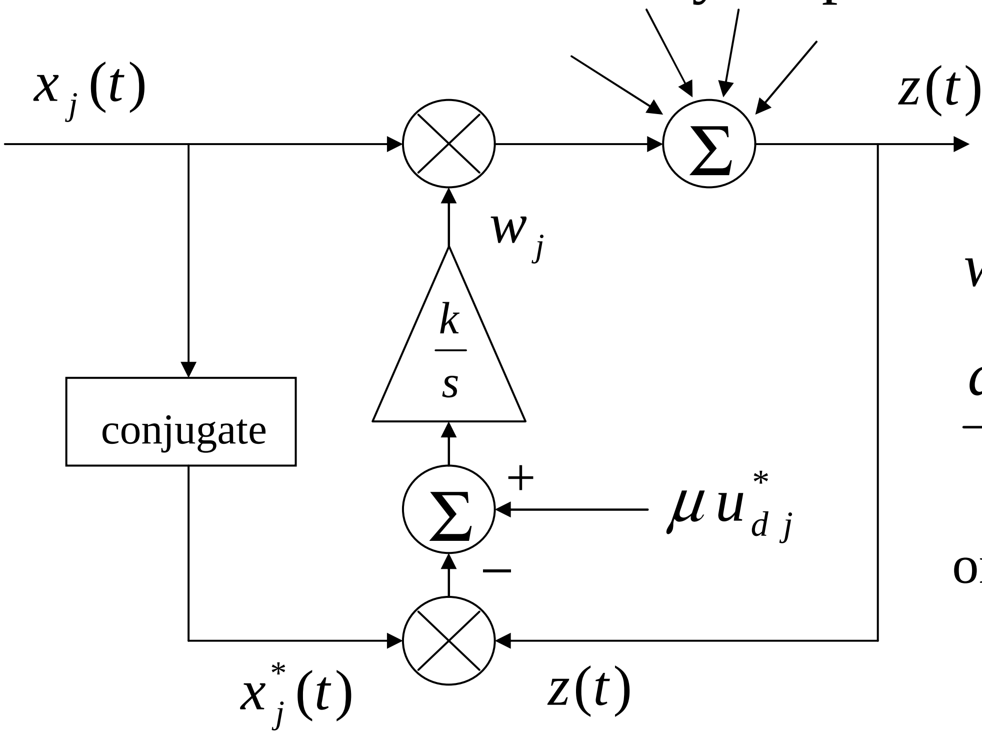
$$SINR = \frac{P_d}{P_u} = \frac{\mathbf{W}^{*T}\mathbf{\Phi}_d\,\mathbf{W}}{\mathbf{W}^{*T}\mathbf{\Phi}_u\,\mathbf{W}}$$

The ratio of two quadratic forms attains its maximum value when **W** is the eigenvector associated with the largest eigenvalue of $[\mathbf{\Phi}_u^{-1}\mathbf{\Phi}_d]$. This approach will be detailed and demonstrated in Part II.

LARGE ARRAY SIGNAL PROCESSING

CLOSED-LOOP ESTIMATION OF OPTIMUM WEIGHTS:

1. THE (MODIFIED) **APPLEBAUM** LOOP: consider a single branch of the array, with weights determined as a real-time correlation of the array output with each elemental signal



(AC notation)

$$w_j(t) = k\int_{-\infty}^{t}\left[\mu u^*_{dj} - x^*_j(\tau) z(\tau)\right] d\tau$$

$$\frac{dw_j(t)}{dt} = k\left[\mu u^*_{dj} - x^*_j(t) z(t)\right]$$

or, in vector form:

$$\frac{d\,\mathbf{W}}{dt} = k\left[\mu\,\mathbf{U}^*_d - \mathbf{X}^*\, z(t)\right]$$

Need the desired signal direction in advance.

LARGE ARRAY SIGNAL PROCESSING

From previous slide: $$\frac{d\,\mathbf{W}}{dt} = k\left[\mu\,\mathbf{U}_d^* - \mathbf{X}^*\,z(t)\right] \qquad (6)$$

Recalling that $z(t) = \mathbf{X}^T\mathbf{W}$ and substituting into (6), yields

$$\frac{d\,\mathbf{W}}{dt} = k\left[\mu\,\mathbf{U}_d^* - \mathbf{X}^*\mathbf{X}^T\mathbf{W}\right], \qquad \frac{d\,\mathbf{W}}{dt} + k\,\mathbf{X}^*\mathbf{X}^T\mathbf{W} = k\mu\,\mathbf{U}_d^*$$

Using the approximation $\hat{\mathbf{\Phi}} \equiv \mathbf{X}^*\mathbf{X}^T \rightarrow E\left(\mathbf{X}^*\mathbf{X}^T\right)$, we get

$$\frac{d\,\mathbf{W}}{dt} + k\,\hat{\mathbf{\Phi}}\mathbf{W} \cong k\mu\,\mathbf{U}_d^* \qquad (7)$$

In the steady-state $d\,\mathbf{W}/dt = 0$, yielding $\hat{\mathbf{\Phi}}\mathbf{W}_{ss} = \mu\,\mathbf{U}_d^*$, or

$$\boxed{\mathbf{W}_{ss} = \mu\,\hat{\mathbf{\Phi}}^{-1}\,\mathbf{U}_d^* \cong \mathbf{W}_{opt}}$$

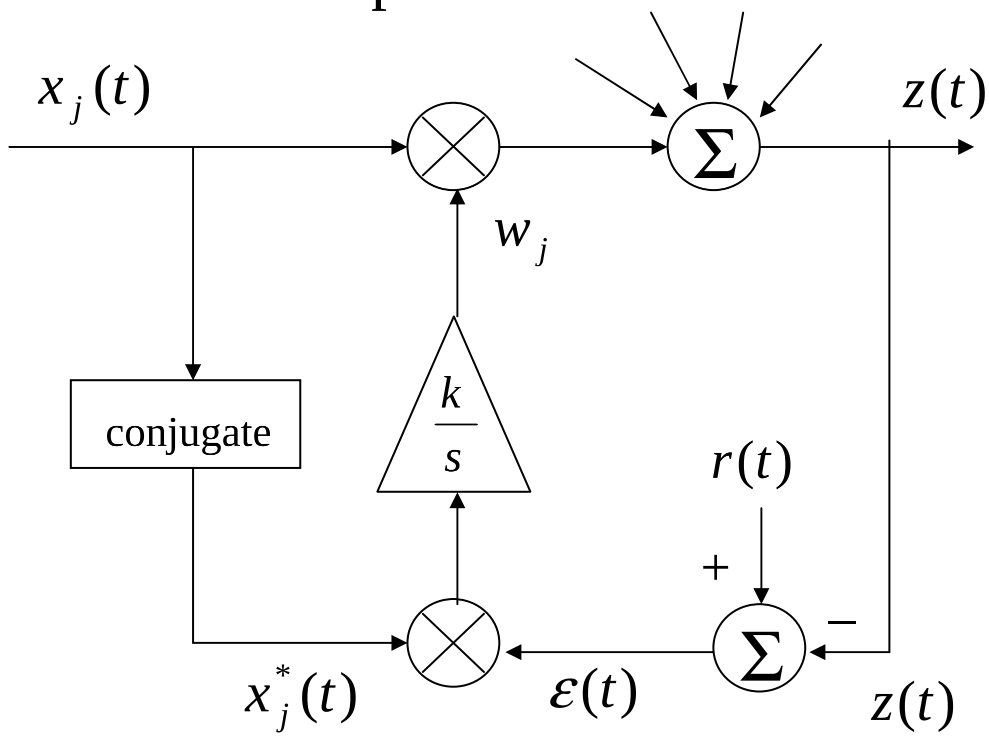
LARGE ARRAY SIGNAL PROCESSING

2. THE **LMS** LOOP: if the differential equation describing the **APPLEBAUM** loop is modified slightly, we obtain the differential equation for the LMS loop:

$$\frac{d\,\mathbf{W}}{dt} + k\,\hat{\mathbf{\Phi}}\mathbf{W} \cong k\,\hat{\mathbf{P}} \qquad (8)$$

$$\mathbf{P} = E[\mathbf{X}^* r(t)] \cong \mathbf{X}^* r(t) \equiv \hat{\mathbf{P}}$$

$$\varepsilon(t) = r(t) - z(t)$$



Needs reference signal, but not the desired signal direction.

LARGE ARRAY SIGNAL PROCESSING

RELATIONSHIP BETWEEN **APPLEBAUM** AND **LMS** ARRAYS

Weights of the **Applebaum** array satisfy the differential equation

$$\frac{d\,\mathbf{W}}{dt} + k\,\hat{\mathbf{\Phi}}\mathbf{W} \cong k\mu\;\mathbf{U}_d^*$$

Weights of the **LMS** array satisfy the differential equation

$$\frac{d\,\mathbf{W}}{dt} + k\,\hat{\mathbf{\Phi}}\mathbf{W} \cong k\,\hat{\mathbf{P}}. \qquad \text{But } \mathbf{P} = E[\mathbf{X}^* r(t)] = E[s^*(t) r(t)]\mathbf{U}_d^*$$

It follows that if $\mu\;\mathbf{U}_d^* = \hat{\mathbf{P}}$, the **LMS** and **Applebaum** arrays will perform identically, both maximizing ***SINR***. However, the LMS array does not need to know the source direction to track the signal.

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3. THE DISCRETE VERSION OF THE **LMS** LOOP:

Starting with $\frac{d\,\mathbf{W}}{dt} + k\,\hat{\mathbf{\Phi}}\mathbf{W} \cong k\,\hat{\mathbf{P}}$ and substituting for the estimates, we get

$$\frac{d\,\mathbf{W}}{dt} \cong k[\hat{\mathbf{P}} - \hat{\mathbf{\Phi}}\mathbf{W}] = k[\,\mathbf{X}^{*} r(t) - \mathbf{X}^{*}\mathbf{X}^{T}\mathbf{W}]$$

$$= k\,\mathbf{X}^{*}[r(t) - z(t)] = k\,\mathbf{X}^{*}\varepsilon(t)$$

which means that each component satisfies

$$\frac{d\,w_i(t)}{dt} \cong k\,x_i^{*}(t)\,\varepsilon(t) \qquad (9)$$

LARGE ARRAY SIGNAL PROCESSING

Next, approximate the derivative with the difference

$$\frac{dw_i}{dt} \cong \frac{w_i(n+1) - w_i(n)}{\Delta t}$$

where $w_i(n)$ is a sample of the *i*-th weight at time $t_n = n\Delta t$.

Rewriting (9) in terms of the difference yields

$$w_i(n+1) - w_i(n) = \gamma x_i^*(n)\,\varepsilon(n); \quad \gamma \equiv k\Delta t$$

which can be put into a form known as the "**LMS algorithm**"

$$\boxed{w_i(n+1) = w_i(n) + \gamma x_i^*(n)\,\varepsilon(n)}$$

where $\varepsilon(n) = r(n) - z(n)$.

LARGE ARRAY SIGNAL PROCESSING

SOME PROPERTIES OF THE **LMS** ALGORITHM:

$$\boxed{w_i(n+1) = w_i(n) + \gamma x_i^*(n)\,\varepsilon(n)}$$

- Needs a reference signal (correlated with the received signal)
- Does not need to know the source direction
- Complexity per update: order *N* (for *N* antennas)
- Magnitude of updates diminish as output signal “approaches” the reference signal (error approaches zero)

LARGE ARRAY SIGNAL PROCESSING

THE "CONSTANT MODULUS ALGORITHM" (**CMA**)

Recall the form of the discrete **LMS** algorithm derived before:

$$w_i(n+1) = w_i(n) + \gamma x_i^*(n)\,\varepsilon(n)$$

If we let $\varepsilon(n) = \left(|\,s(n)\,|^2 - s_0^2\right) s(n)$, the resulting algorithm is known as the **CMA**:

$$\boxed{w_i(n+1) = w_i(n) + \gamma\left(|\,s(n)\,|^2 - s_0^2\right) x_i^*(n)\,s(n)}$$

LARGE ARRAY SIGNAL PROCESSING

SOME PROPERTIES OF THE **CMA**:

$$w_i(n+1) = w_i(n) + \gamma\left(|\,s(n)\,|^2 - s_0^2\right)x_i^*(n)\,s(n)$$

- The **CMA** does **not need** either a **source direction** or a **reference signal**, only the “target” power of the desired source
- Any change in source power is attributed to interference, which the CMA attempts to cancel
- Order *N* complexity (multiplies per update)

LARGE ARRAY SIGNAL PROCESSING

SUMMARY OF REPRESENTATIVE "ORDER N" ALGORITHMS FOR DSN APPLICATIONS

- **LMS ALGORITHM:**
  - Needs a **reference signal** (filtered residual carrier, or other correlated reference derived via signal processing)
  - Adaptively maximizes ***SINR*** (nulls interference)
- **CMA:**
  - Needs estimate of desired **signal power** only
  - Adaptively maximizes ***SINR*** (nulls interference)

OPEN ISSUES: convergence rate under "realistic" DSN spacecraft tracking conditions

LARGE ARRAY SIGNAL PROCESSING

PART II: REAL-TIME DEMONSTRATIONS

LMS ALGORITHM (F. Pollara):

- Real-time convergence from initial weight vector to optimum, with and without noise
- Demonstration of gradient descent (min. of error surface)

NORMALIZED CMA (M. Srinivasan):

- New algorithm, needs estimate of **average signal power**
- Can phase up array with “noise-like” signals from quasars

EFFICIENT EIGENVECTOR ALGORTHM (C. Lee):

- Based on matrix theory result on maximization of ratio of two quadratic forms
- Efficient, iterative implementation