

# Test 1 Review

Covers start of semester up through stellar spectra (Chs 1-5).  
All constants needed will be given on test.

Two equations from basic astronomy:

Small angle formula  $D = \frac{\alpha d}{206265}$

Parallax  $d = 1/p$

# Test #1

Covers material from first day of class, all the way through  
Spiral Galaxies

Supporting reading chapters 1-5

Some questions are “concept” questions, some involve  
working with equations, calculations

Study your lecture notes, homeworks, worksheets, review  
supporting reading

There will be one question from a paper we read

Know equations/constants on the sheet just handed out.

Bring calculator, something to write with. Closed book,  
closed notes

Attempt every question – show what you know

Don't get bogged down on a question.

## Mechanics

Newton II 
$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

For circular orbits, magnitude of  $F$  is 
$$F = m \frac{v^2}{r}$$

Torques and angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (=0 \text{ for central forces})$$

Gravity and  
gravitational  
acceleration

$$F = \frac{GMm}{r^2} \quad F = mg \Rightarrow g = \frac{GM}{r^2}$$

Energy of mass  $m$  in orbit around a fixed mass  $M$  (approximately true for, e.g. satellite orbiting Earth)

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

(Kinetic)      (Potential)

Escape velocity

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

But in general, both objects orbit center of mass, so for objects 1 and 2, total energy is

$$E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{r^2}$$

Total energy $E$	$< 0$	circular or elliptical (bound)
	$= 0$	parabolic (marginal)
	$> 0$	hyperbolic (unbound)

Virial theorem (for an ensemble of objects in equilibrium, but also true for the time average of an elliptical orbit)

$$2\langle KE \rangle + \langle U \rangle = 0$$

$$M = Rv^2/G$$

Tidal force

$$\frac{dF}{dr} = -\frac{2GMm}{r^3}$$

Roche limit

$$d \simeq 2.5 \left( \frac{\rho_p}{\rho_s} \right)^{1/3} R$$

Mean free path and time

$$\lambda = \frac{1}{n\sigma} \quad \tau = \frac{1}{n\sigma v}$$

## Luminosity

$$L = \int_A F_e \, dA$$

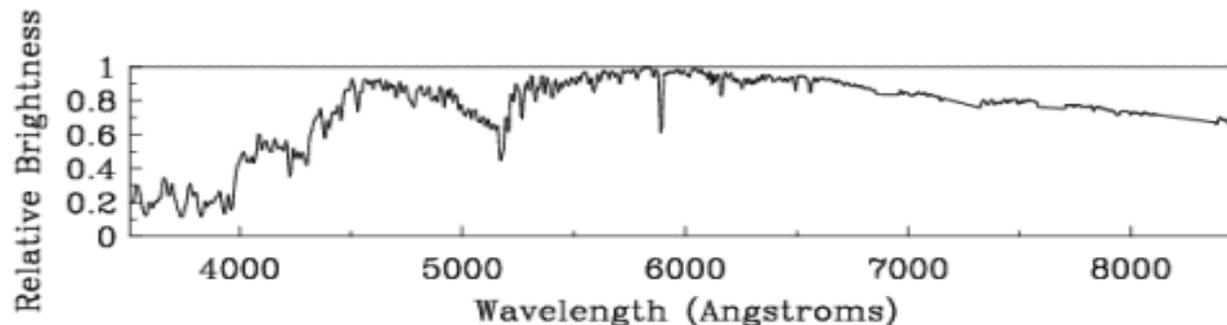
If  $F_e = \text{constant}$  over  $A$ , for a spherical blackbody:

$$L = 4\pi R^2 \sigma T^4$$

Since stars are not perfect blackbodies, define an 'effective' temperature,  $T_e$

$$L = 4\pi R^2 \sigma T_e^4$$

(the temperature of a blackbody of the same luminosity)



## Radiation

Planck radiation law

$$B_{\lambda}(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Wien and Stefan-Boltzmann laws

$$\lambda_{\max} = \frac{0.0028979}{T} \text{ m} \quad F_e = \sigma T^4$$

Luminosity of spherical BB

$$L = 4\pi R^2 \sigma T^4$$

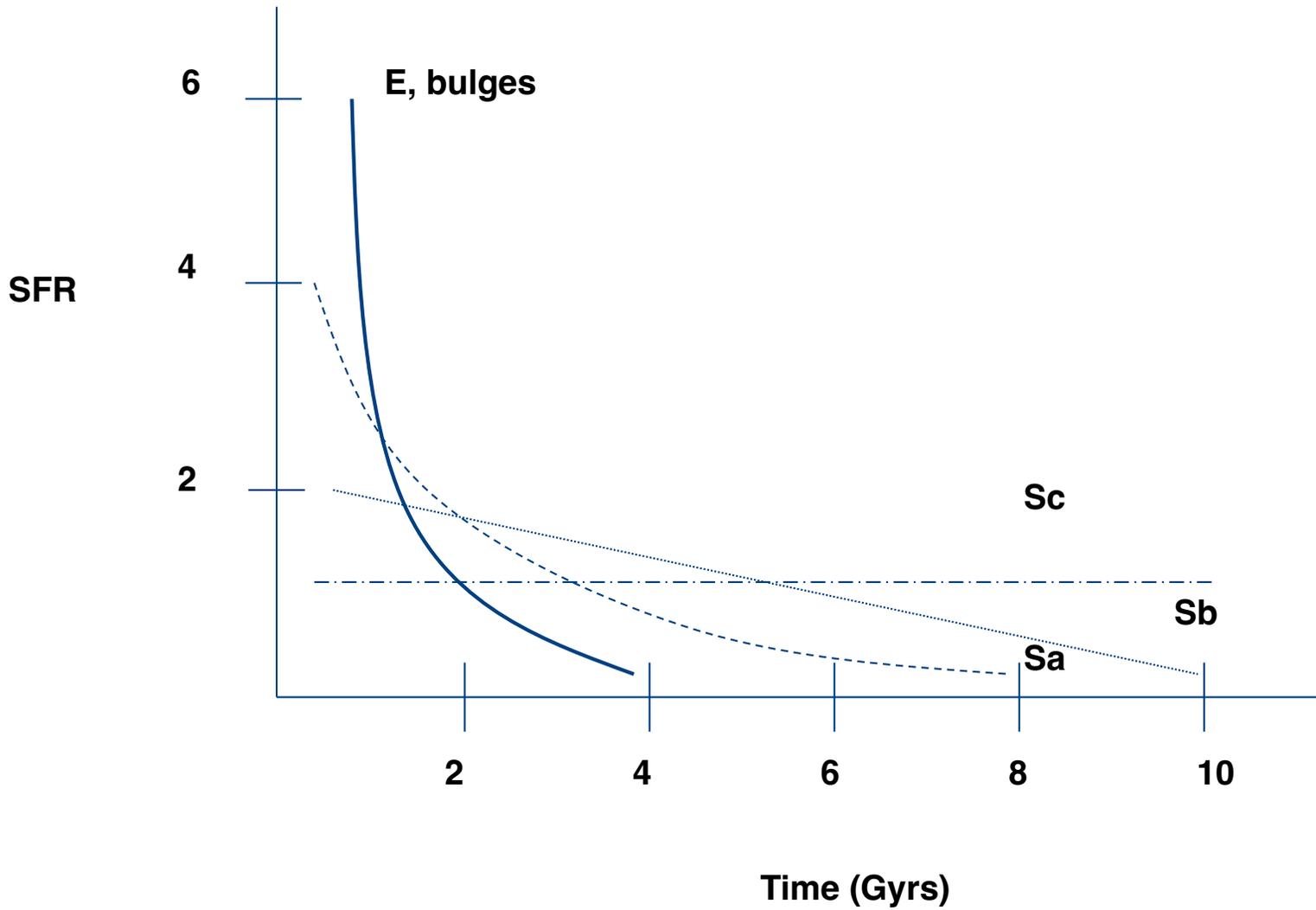
Incident flux at distance  $r$

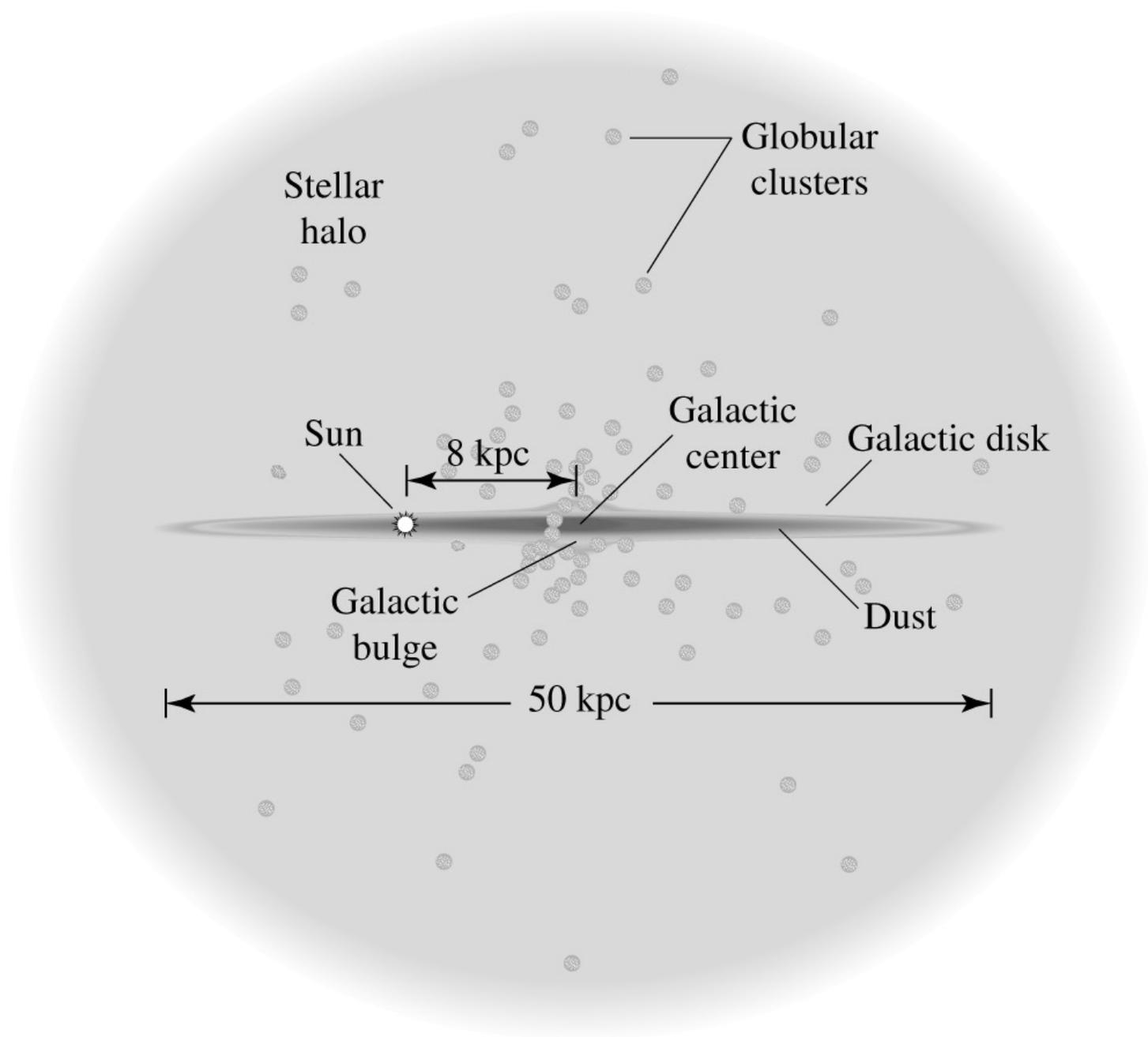
$$F_i = \frac{L}{4\pi r^2} = F_e \frac{R^2}{r^2}$$

# Overview of galaxy properties

	E	S0	Sa	Sb	Sc	Sd	Irr
Color	Red	—————→					Blue
Stellar pop	Old	Old/interm		Old + interm + young		Intermediate young	
SFR	None	Low	————→	Higher	————→		High
HI	None/Low	Low	————→	Modest	————→	High	Highest
Dust	None/Low	Higher		High	————	High	Lower (less metals)
Dyn	Bulge/halo dominated		Disk dom (rot)	————		Disk dom	

# Schematic star formation histories:





# Gaia's Milky Way map

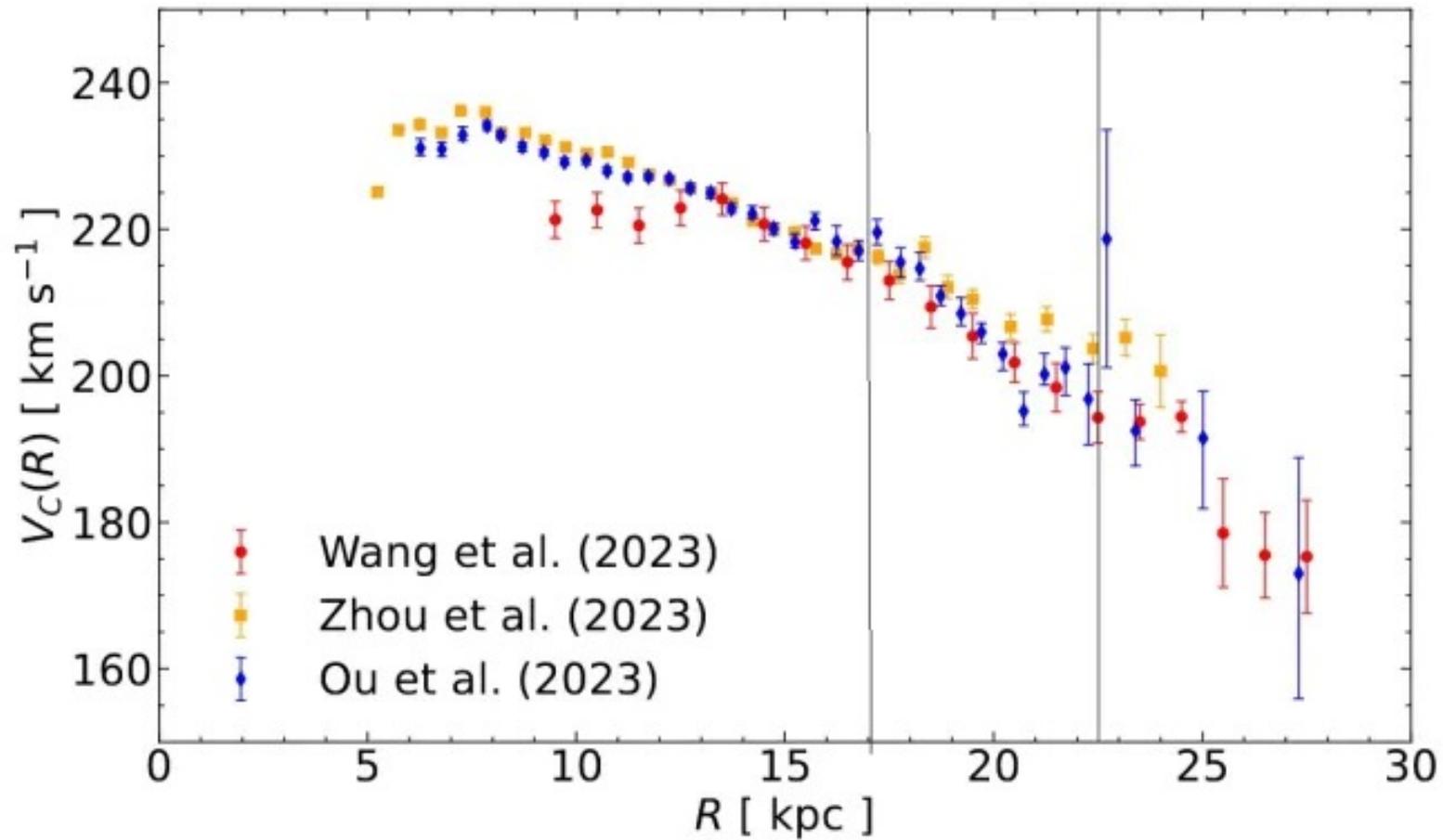


[www.esa.int/gaia](http://www.esa.int/gaia)

Credit: ESA/Gaia/DPAC, Stefan Payne-Wardenaar

Credit: ESA/Gaia/DPAC, Stefan Payne-Wardenaar ([source](#))

# MW Rotation Curve from Gaia

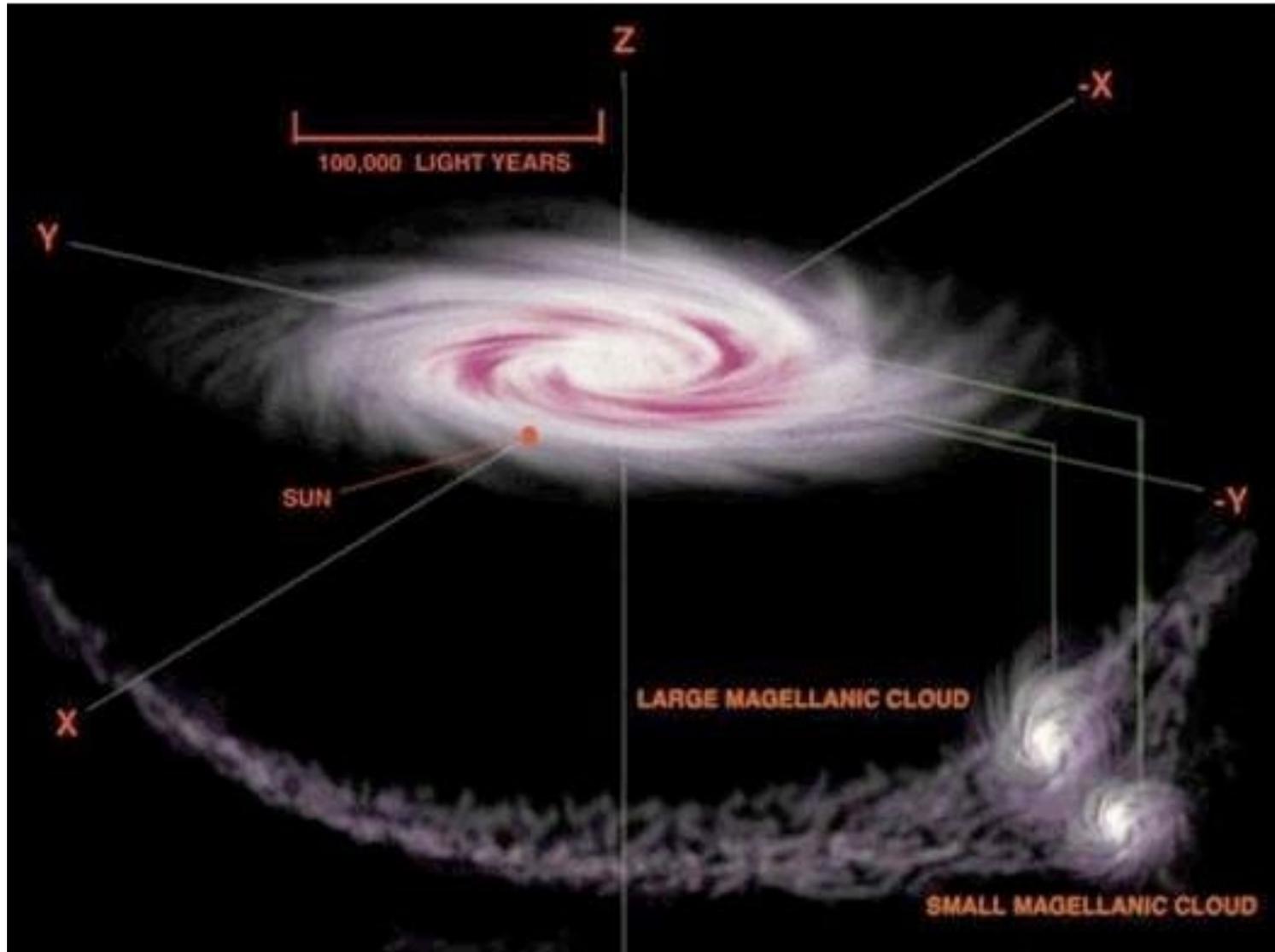


**Fig. 1.** Comparison of the 3 different measurements of the MW rotation curve based on *Gaia* DR3.

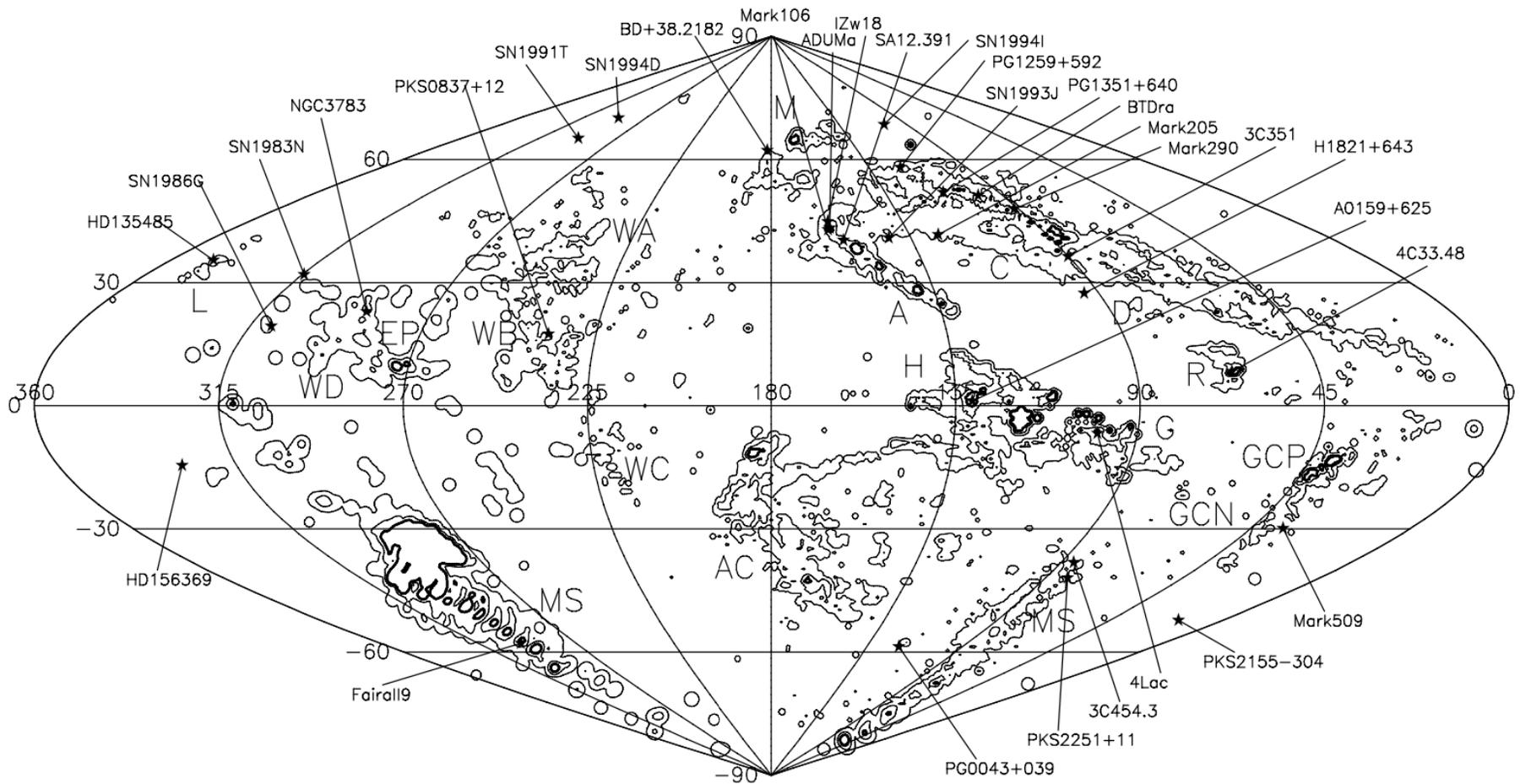
## Magellanic stream

21cm emission, about 180 deg across. Tidal debris tail. Gas falling into the Milky Way

Could be as much as 0.4 Msun/year(van Woerden et al. 2004)

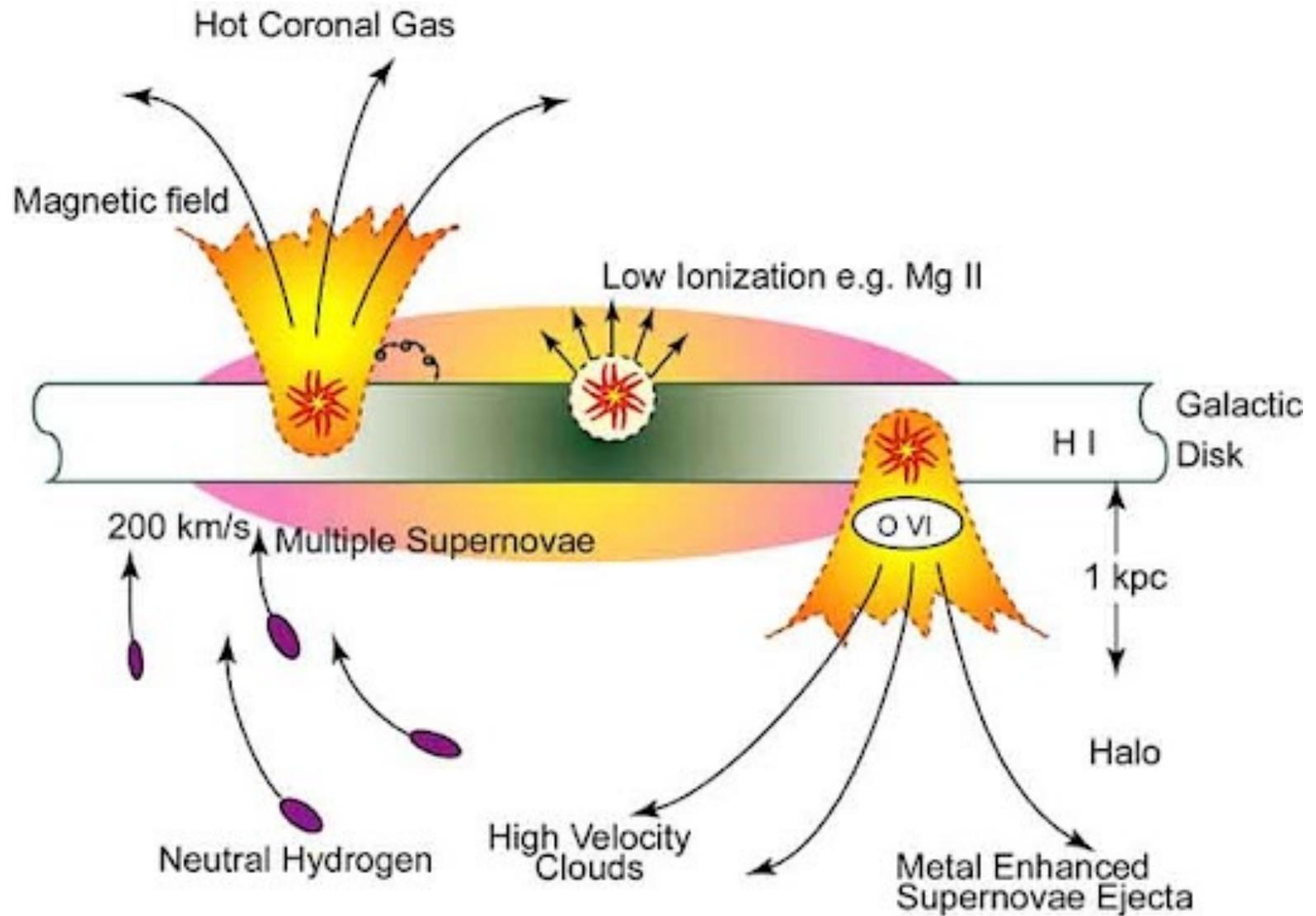


# High Velocity Clouds



Wakker et al. 1997

# High Velocity Clouds



- Presence of dust inferred from emission from dust grains:
  - Mid-IR to far-IR thermal continuum emission from heating by a radiation field
  - Thermal continuum from non-equilibrium heating (*Sellgren grains*) at near-IR to mid-IR (1-25 $\mu\text{m}$ )
  - IR emission bands from heated grains (some of which are the 'DIBs')
  - Radio continuum emission from rotating grains

### How important is dust?

- Gas to dust ratio in the MW  $\sim 100:1$ . ISM is about 10% of the baryonic mass, so dust grains  $\sim 0.1\%$  of total mass.
- However, they absorb 30-50% of the starlight emitted, and re-radiate in the IR  $\Rightarrow 0.1\%$  of the baryons are responsible for  $\sim 30\text{-}40\%$  of the bolometric luminosity of the Galaxy.

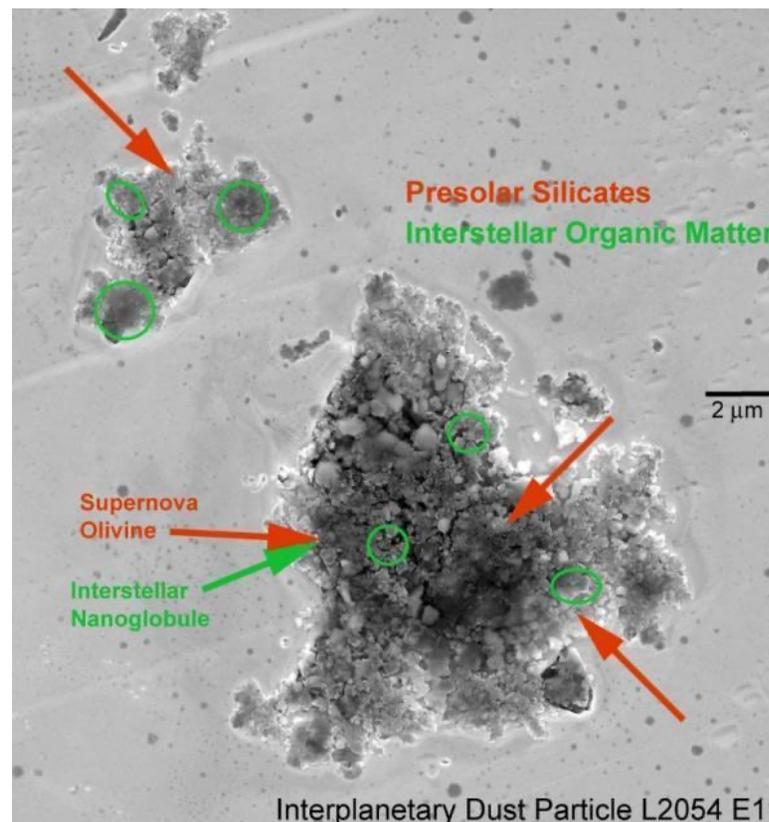
Dust grains are very important for chemistry, as they are the primary sites of molecular formation

Responsible for essentially all  $H_2$  in the ISM.

Grains *needed* for reactions.

Dust is also thought to be the starting points for coagulation into larger grains in protostellar disks, leading to planetesimals and eventually to planets

Carrying complex organic molecules with them

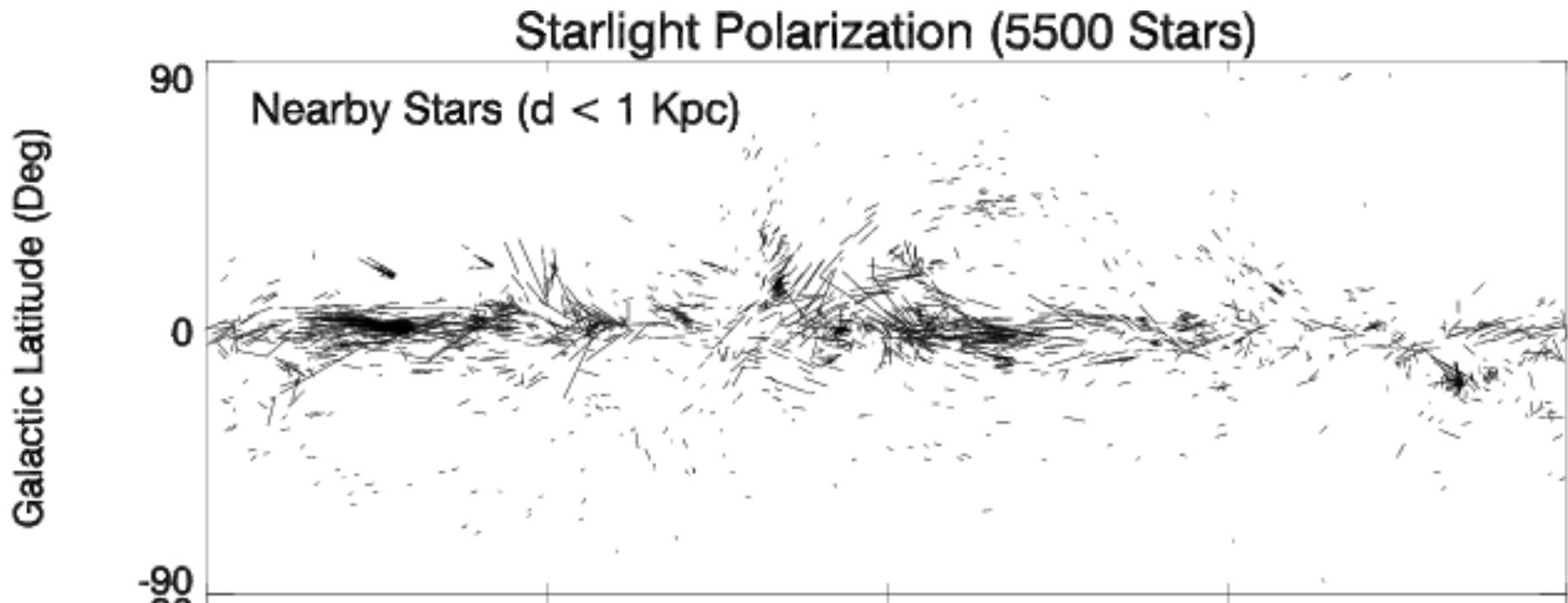


Busemann et al.  
2003

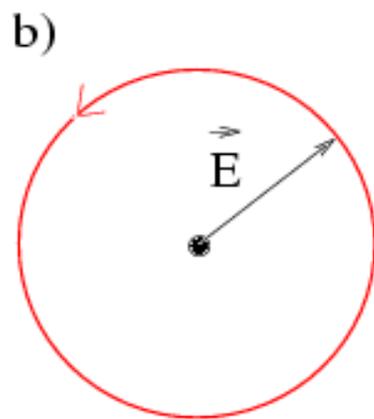
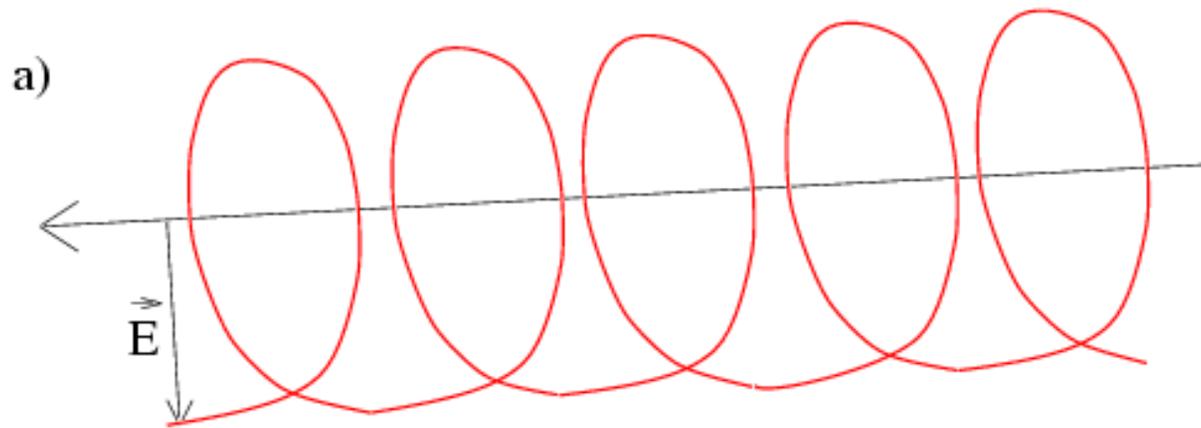
## Interstellar polarization

In addition to absorbing and scattering light passing through the ISM, elongated dust grains that are aligned can polarize light.

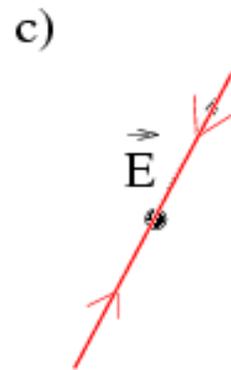
Observations of polarized light demonstrates that dust grains can not be spherical, and that they must be globally aligned, presumably by large-scale magnetic fields.



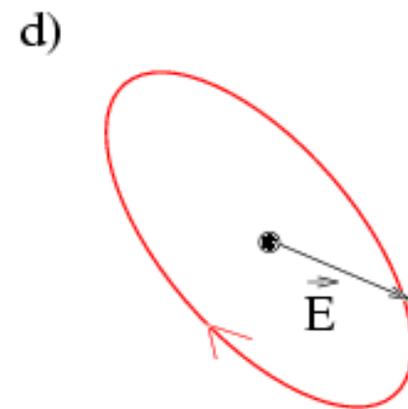
# Polarization of Light



Circular



Linear



Elliptical

# What is Polarized Light?

- Light is oscillating electric and magnetic fields
- Polarization is labeled by the shape of the trace of the tip of the **E** vector
- Each polarization has an orthogonal state
- Incoherent light can contain many polarization states

Stokes Parameters describe partially polarized light

$$I = RR + LL$$

$$Q = RL + LR$$

$$U = i(LR - RL)$$

$$V = LL - RR$$

For circular feeds

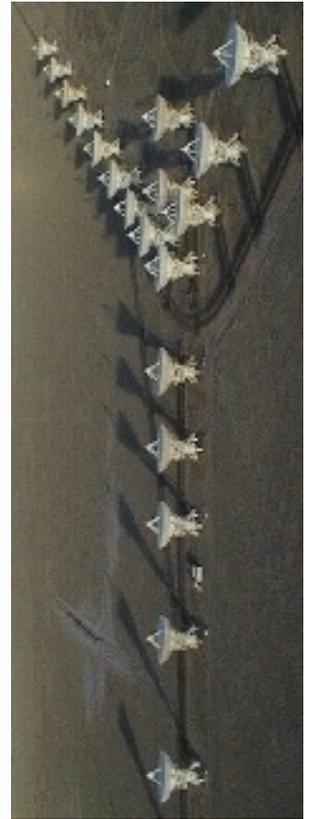
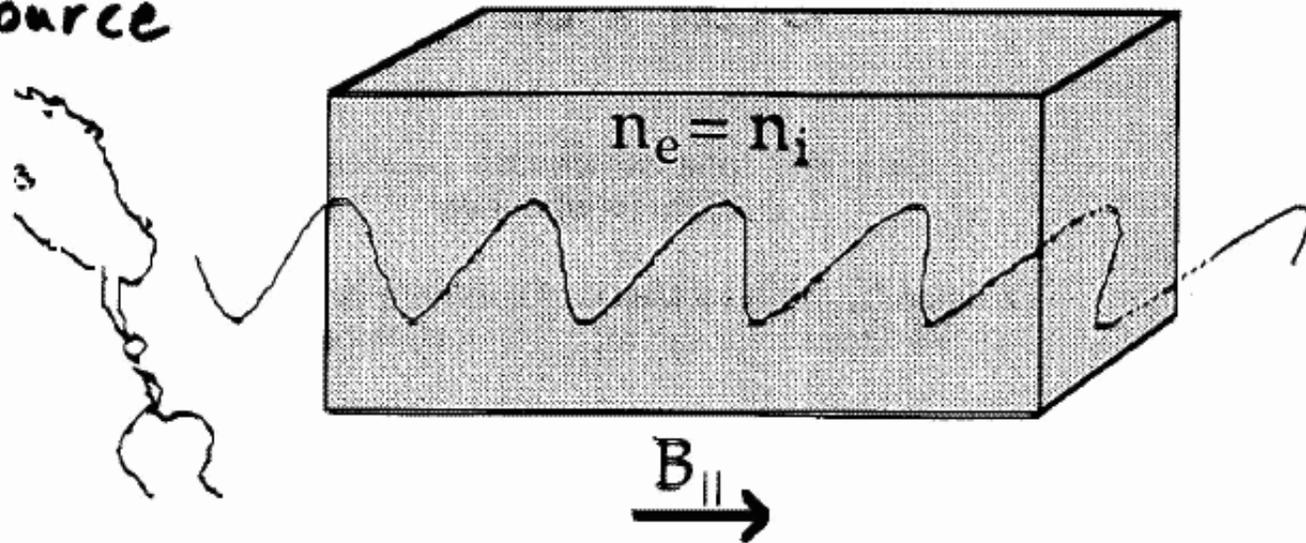
Alternate representation:

- |                       |                                       |
|-----------------------|---------------------------------------|
| • pol. angle (EVPA)   | $\phi = 0.5 \operatorname{atan}(U/Q)$ |
| • polarized intensity | $p = \sqrt{Q^2 + U^2}$                |
| • fractional linear   | $m = p / I$                           |
| • fractional circular | $v =  V  / I$                         |

# Faraday Rotation

Polarized  
Source

Plasma

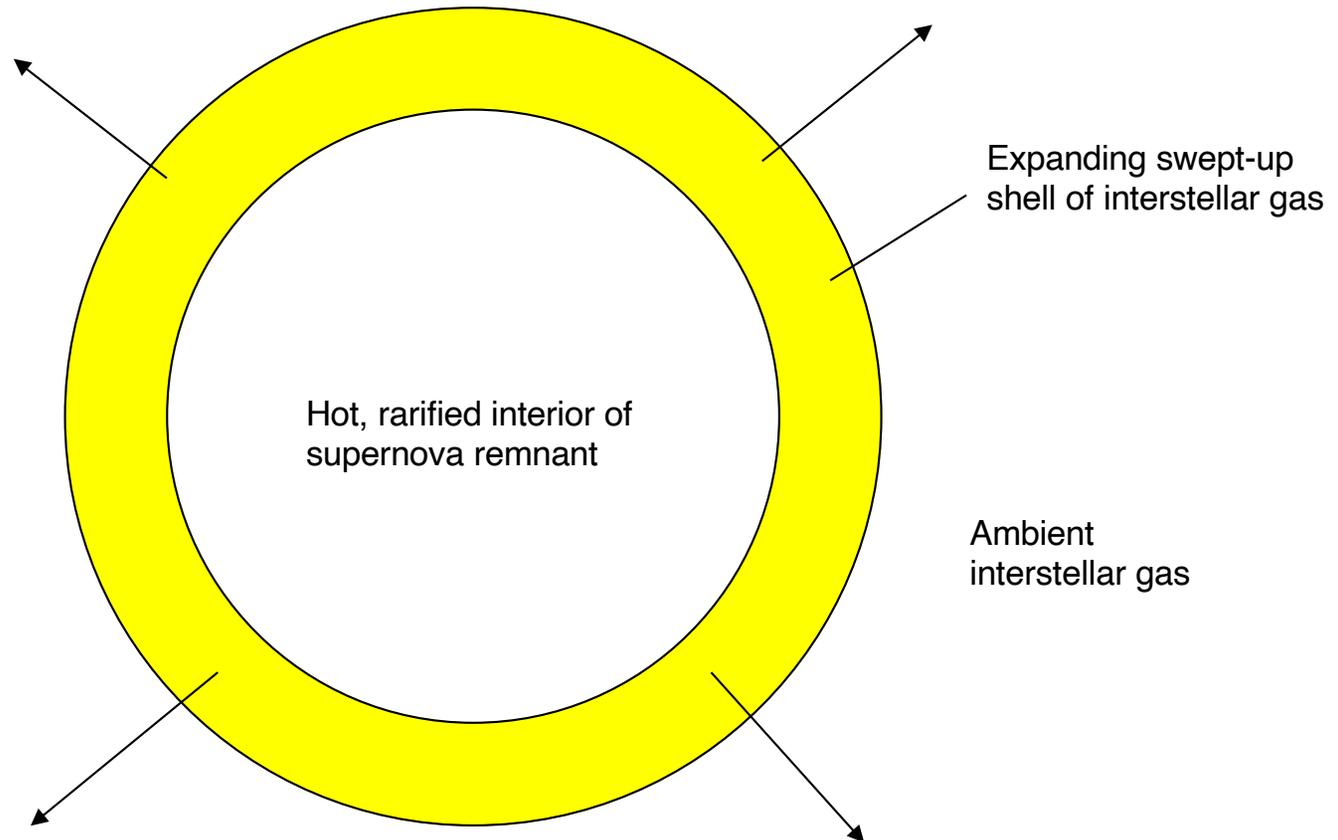


$$\Psi = \Psi_0 + RM\lambda^2$$

$$RM = 812 \int_0^L n_e B_{||} dl \text{ radians/m}^2$$

Handwritten annotations for the equation above:  
-  $L$  is labeled with  $\text{rpl}$   
-  $B_{||}$  is labeled with  $\text{mGauss}$   
-  $n_e$  is labeled with  $\text{cm}^{-3}$

How did the HIM get as hot as  $10^6$  K? By supernovae:



Expanding ejecta sweeps up ambient gas into a thin shell, imparts momentum into shell, so that the interior heats up.

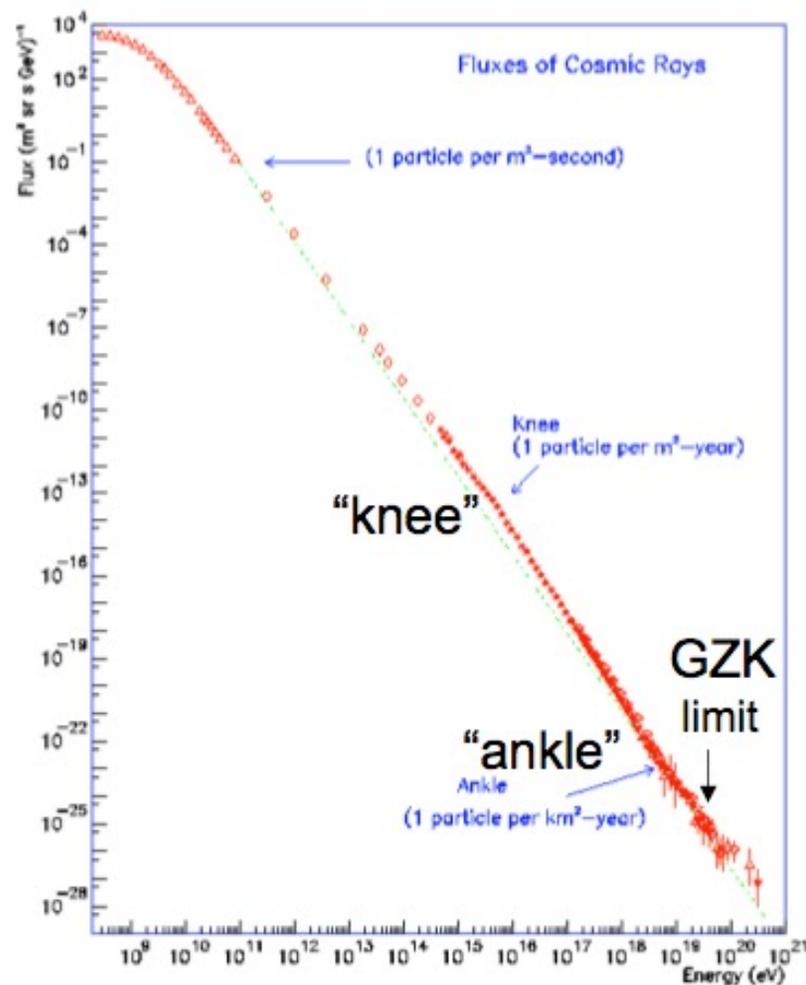
Initial speed  $\sim 5,000$  km/s, slows down to  $\sim 10$  km/s. Collisions between particles convert KE into thermal energy behind the shell.

## Cosmic Rays (CR)

Follows a power law, defined from  $10^{10}$ - $10^{15}$  eV. Steeper beyond the knee, back after the ankle.

The origin of ultra high energy CRs are not known:

- cannot be confined by galactic B-fields
- can't be accelerated by SNe.
- can't be coming from too far away, since they interact with CMB photons.
  
- Might be coming from nearby AGN, TDEs, FRBs



## Synchrotron emission

Relativistic charged particles (e, p) radiate when traveling in a magnetic field.

$$\vec{F} = q \frac{\vec{v}}{c} \times \vec{B}$$

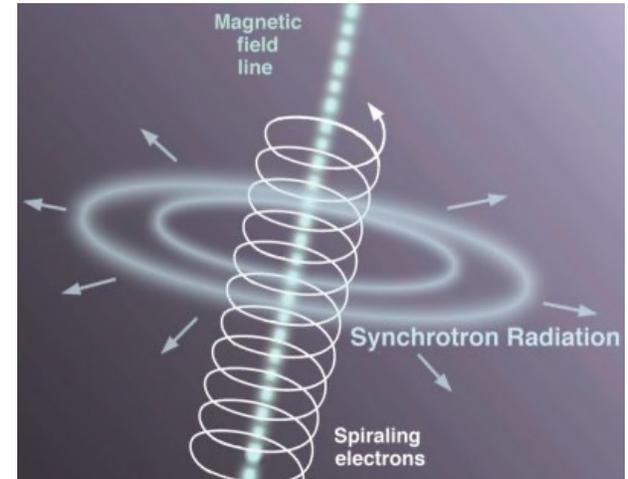
If  $\vec{v} \times \vec{B} \neq 0$ , acceleration  $\neq 0$

Accelerated charges radiate.

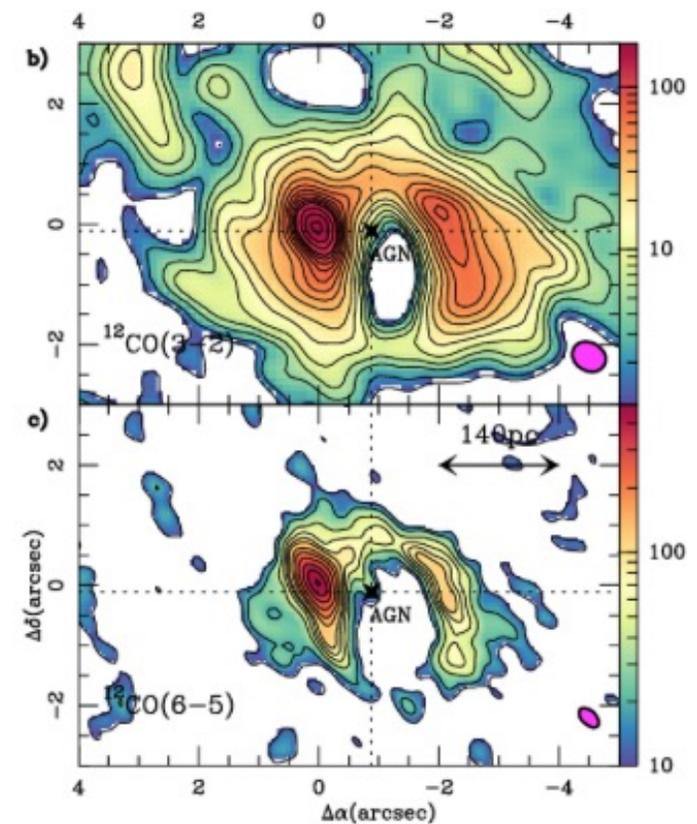
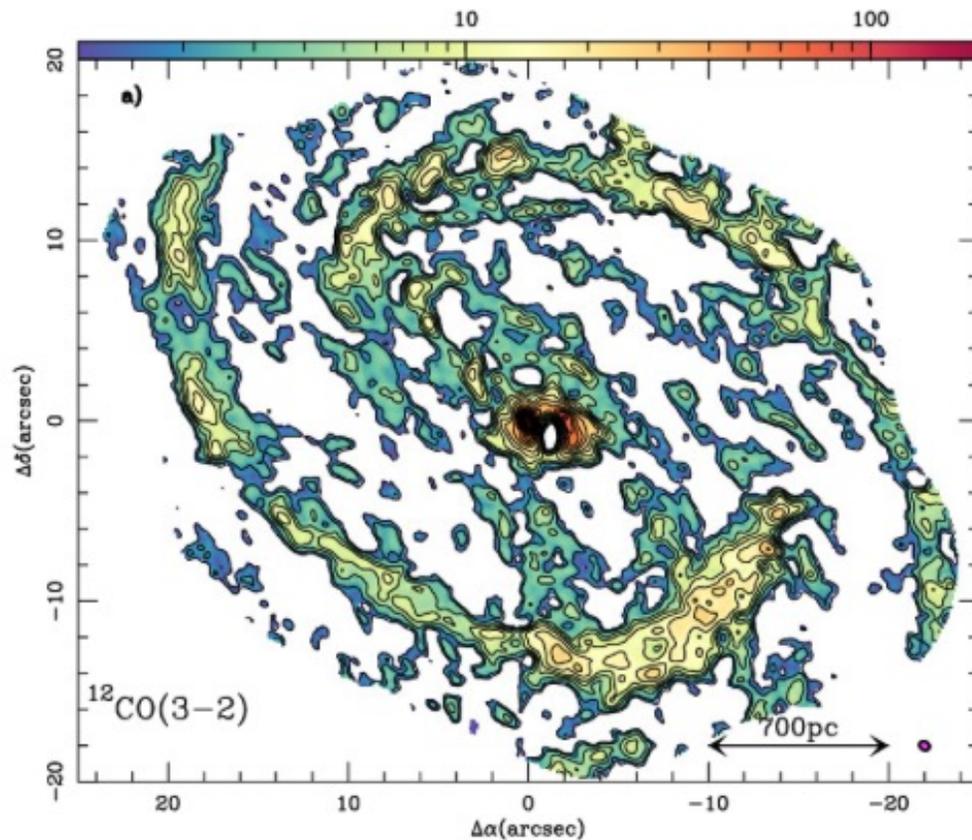
At non-relativistic velocities this results in *cyclotron radiation*, and at relativistic velocities it results in *synchrotron radiation*.

Consider acceleration in a magnetic field, described by the equation of motion:

$$F = ma = m \frac{dv}{dt} = \frac{q}{c} v \times B$$



Molecular clouds  
Due to the new mm-telescopes like ALMA, molecular studies are the most common way of studying the ISM.



NGC 1068

Garcia-Burillo et al. 2014

## HII regions

Ionized gas near hot, young massive stars.

Compared to the WIM, more well defined and denser.

$$n_e \sim 10^2 - 10^4 \text{ cm}^{-3}.$$

Example: Orion nebula. Red  $\Rightarrow$   $H\alpha$  dominates emission.

Why near massive stars only?

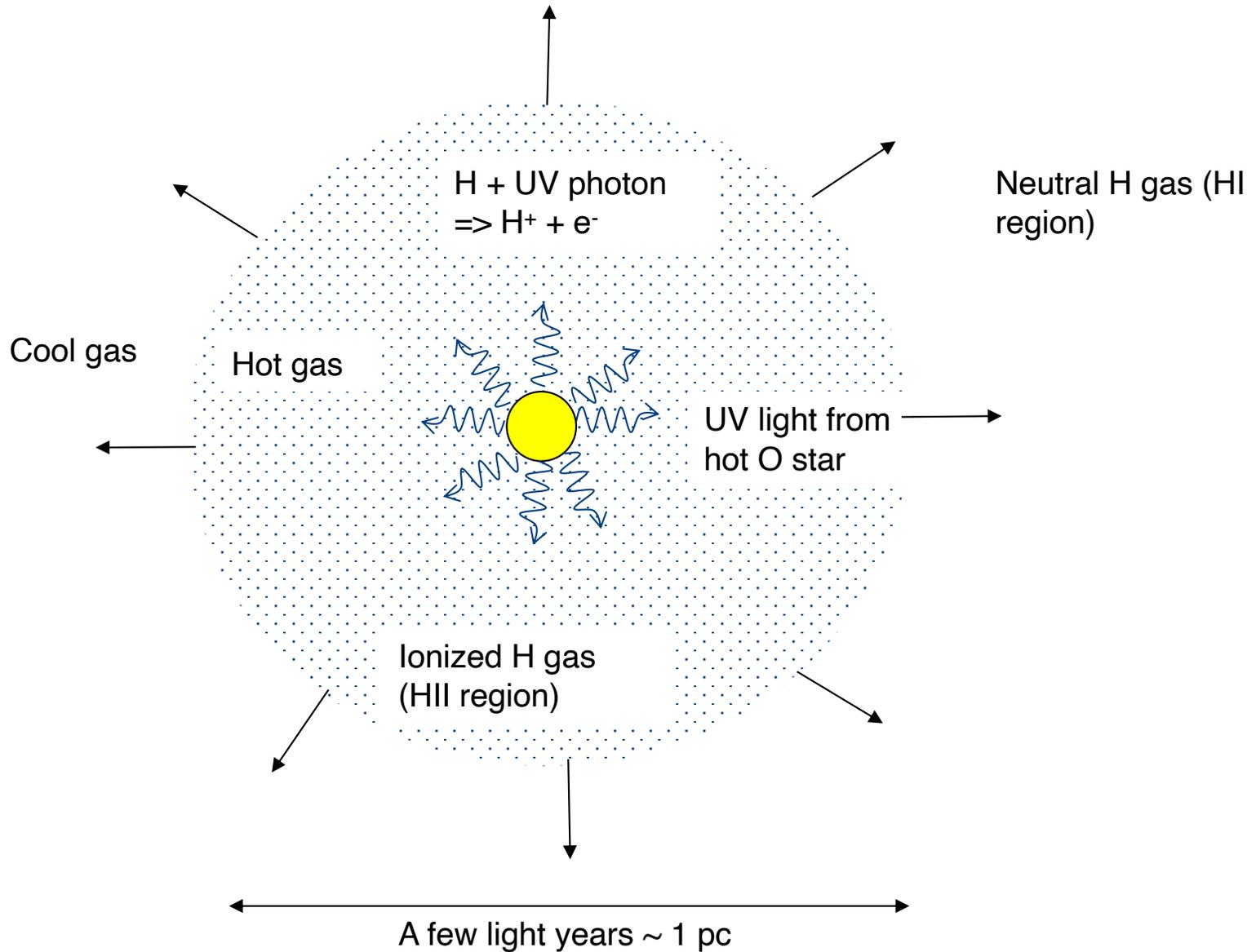
Need 13.6eV  $\Rightarrow$  912 Å (UV)

Use Wien's law  $\lambda_{\max} = \frac{0.29 \text{ cmK}}{T}$

$$T = \frac{0.29}{912 \times 10^{-4}} = 32,000 \text{ K}$$

This is a B0 or B1 star. Thus, HII regions surround only O and B stars, and share their short lifetimes.

Schematic diagram: Balance of ionization and recombination sets up HII region.



# The Mass Function

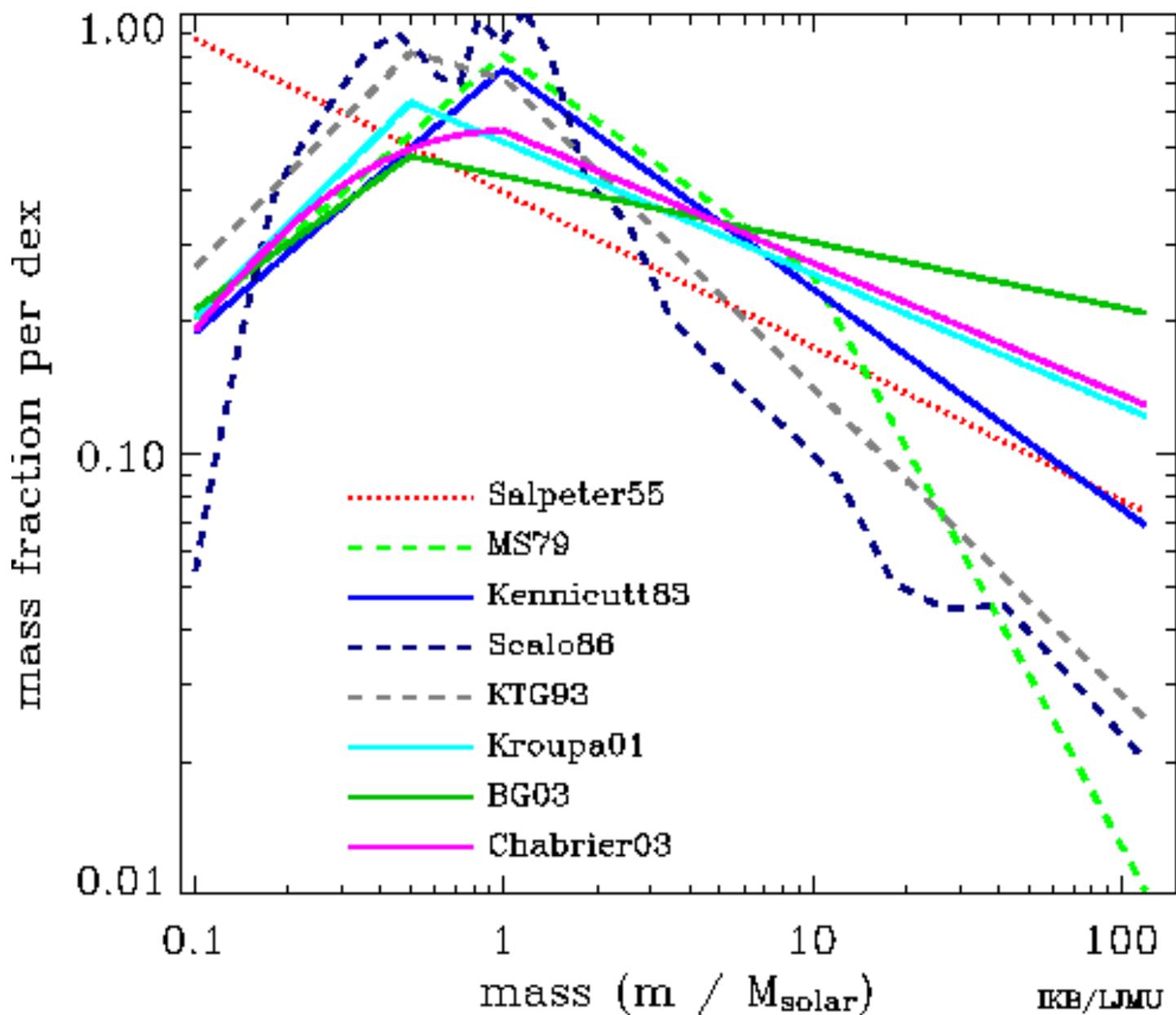
Whenever you have a large number of objects with various masses, useful to describe the number as a function of mass,  $N(M)$ , or size,  $N(R)$ . Constrains theories of their origin. Useful for Kuiper Belt, asteroids, impact craters, Saturn's ring particles, stars, gas clouds, galaxies.

Often have many small objects and a few large ones, which we can try to describe with a “power law” mass function:

$$N(M) \propto M^{-\beta}$$

Gives relative importance of large and small objects

# Stellar Initial Mass Functions



## Initial Mass Function for Stars

By observing the relative numbers of various masses of stars, we can deduce something about the cloud fragmentation process.

The *initial mass function* (IMF) describes the relative numbers of each stellar mass. Defined for stars in the Solar neighborhood by Salpeter (1955):

$$\xi(M) = \xi_0 M^{-2.35}$$

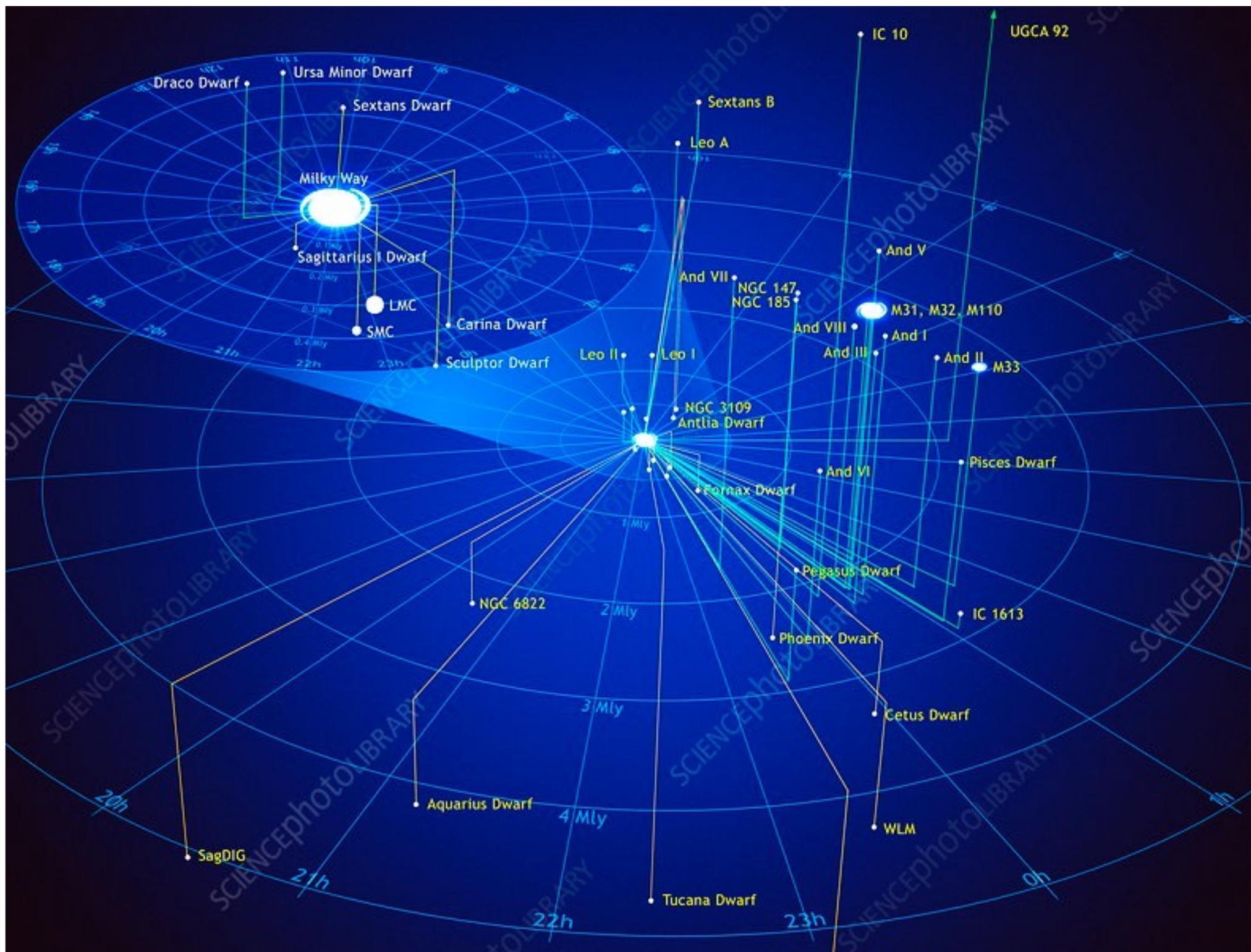
$M$  = mass in solar units.

Thus, the number of stars that form with masses between  $M$  and  $\Delta M$ :

$$\xi(M)\Delta M$$

Total number of stars formed with masses  $M_1$  and  $M_2$ :

$$N = \int_{M_1}^{M_2} \xi(M) dM = \xi_0 \int_{M_1}^{M_2} M^{-2.35} dM = \frac{\xi_0}{1.35} [M_1^{-1.35} - M_2^{-1.35}]$$



**Homologous collapse** Virial Theorem  $U + 2K = 0$  equilibrium cond.

The condition that  $U > 2K$  leads to

$$\frac{3M_c kT}{\mu m_H} < \frac{3}{5} \frac{GM_c^2}{R_c}.$$

Assuming constant density,  $R$  is given by:

$$R_c = \left( \frac{3M_c}{4\pi\rho_0} \right)^{1/3}$$

So, if  $M_J > M_c$  we have collapse and the minimum mass that does this is the Jean's mass:

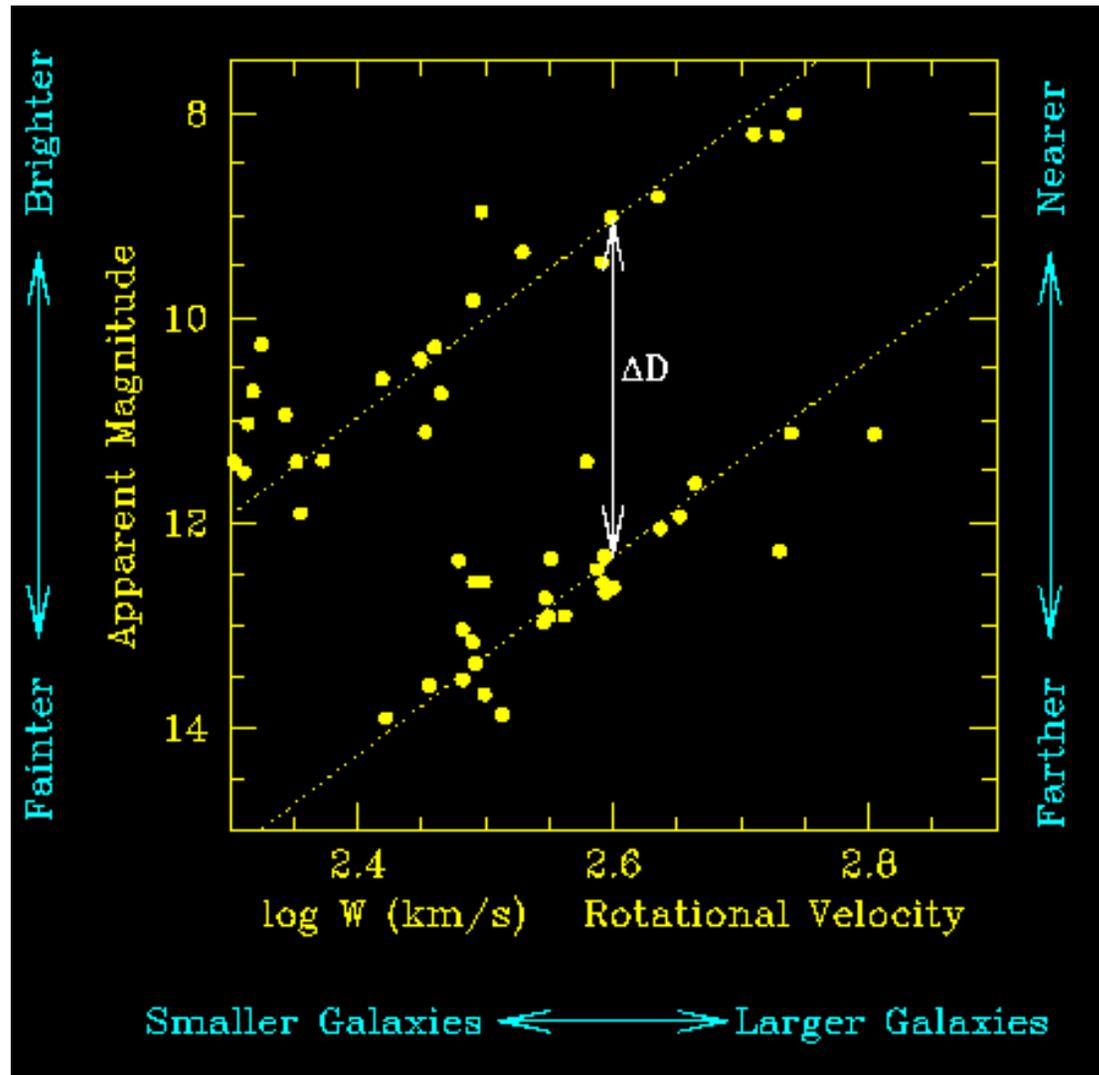
$$M_J \simeq \left( \frac{5kT}{G\mu m_H} \right)^{3/2} \left( \frac{3}{4\pi\rho_0} \right)^{1/2}$$

Which has a radius:

$$R_J \simeq \left( \frac{15kT}{4\pi G\mu m_H \rho_0} \right)^{1/2}$$

At  $z=1100$   
 $T \sim 3000$  K  
 $M_J \sim 10^7 M_{\text{Sun}}$   
 $R_J \sim 1$  kpc

Tully-Fisher relations of two clusters at different distances.



Also note, the more massive a galaxy is, the faster it rotates.

## Where does Tully-Fisher derive from?

Start with gravitational force = centripetal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow M = \frac{v^2 R}{G}$$

Assume roughly constant mass-to-light ratio for spirals

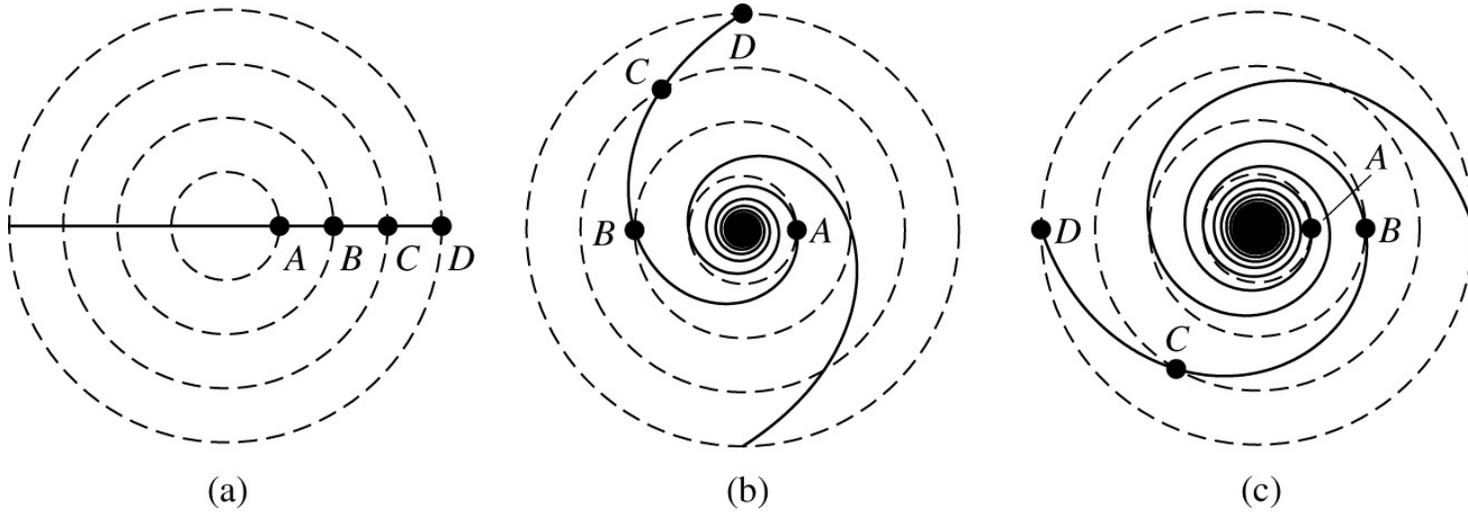
$$C_{ML} = \frac{L}{M}$$

Roughly constant surface brightness for spirals

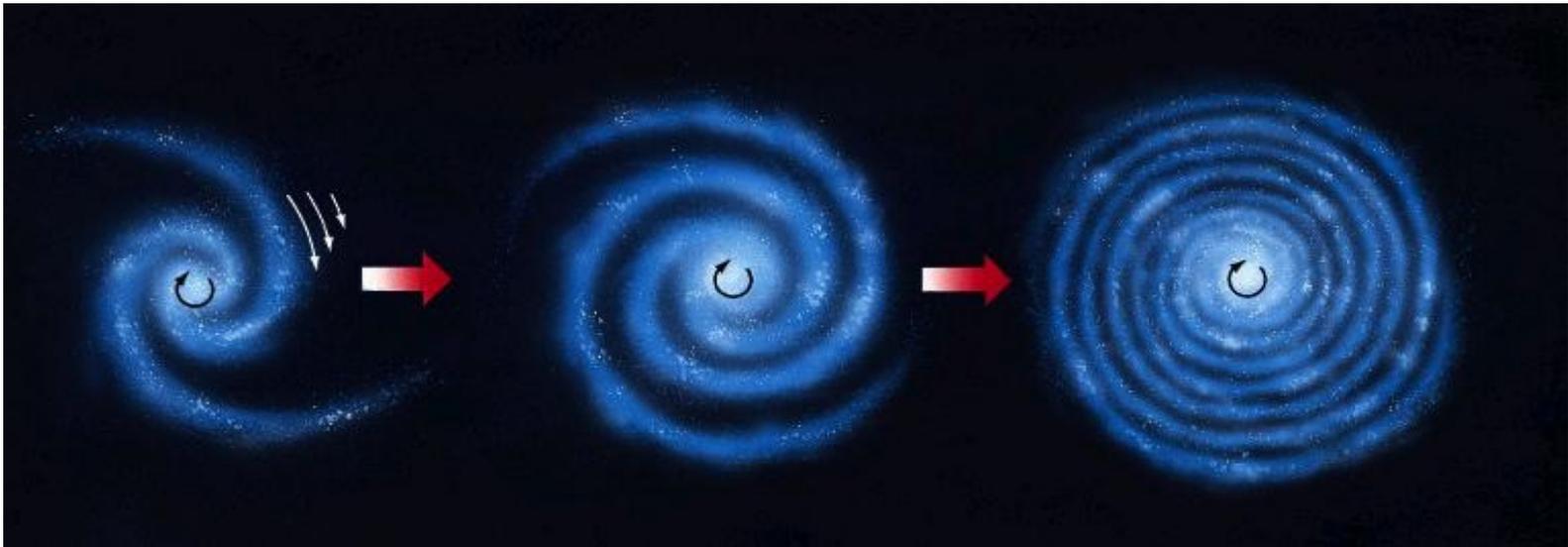
$$C_{SB} = \frac{L}{R^2}$$

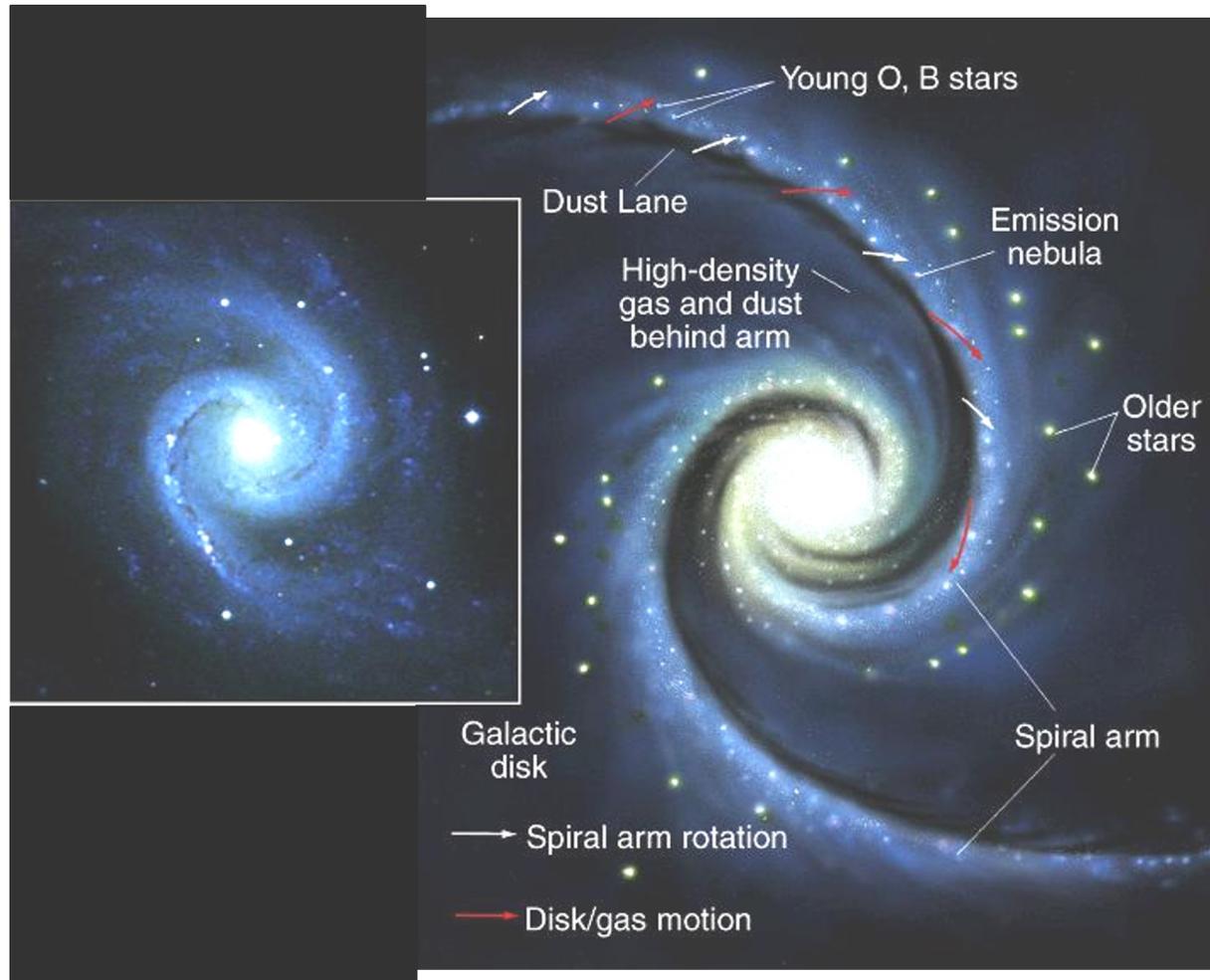
$$L = \frac{C_{ML}^2 v^4}{C_{sB} G^2}$$

## The winding dilemma



Assuming flat rotation curves, the arms rapidly get tightly wound.

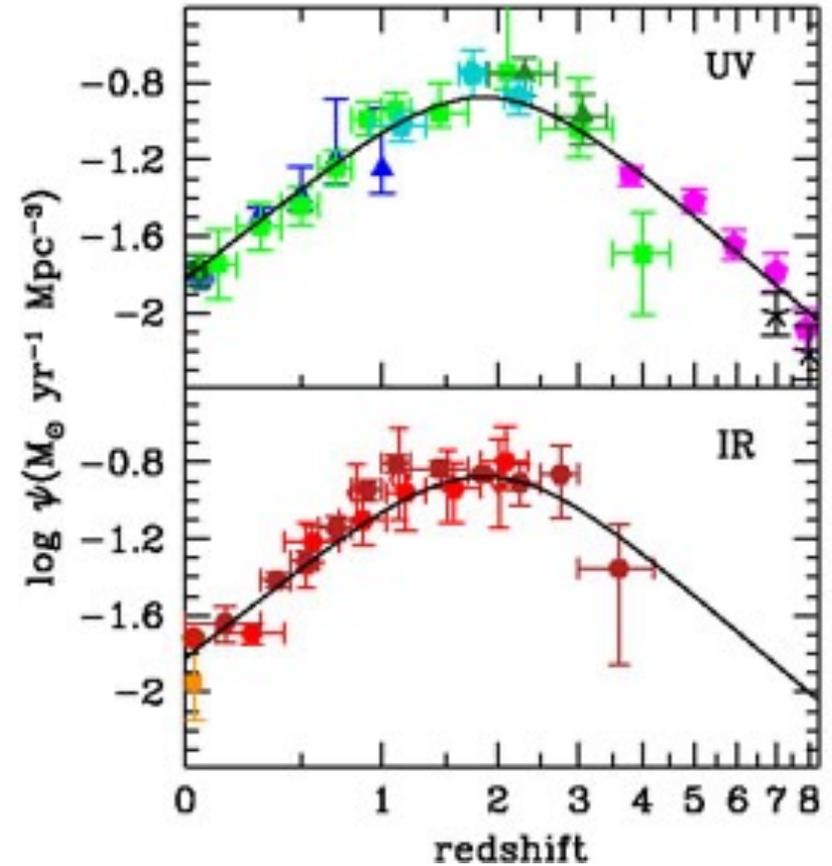
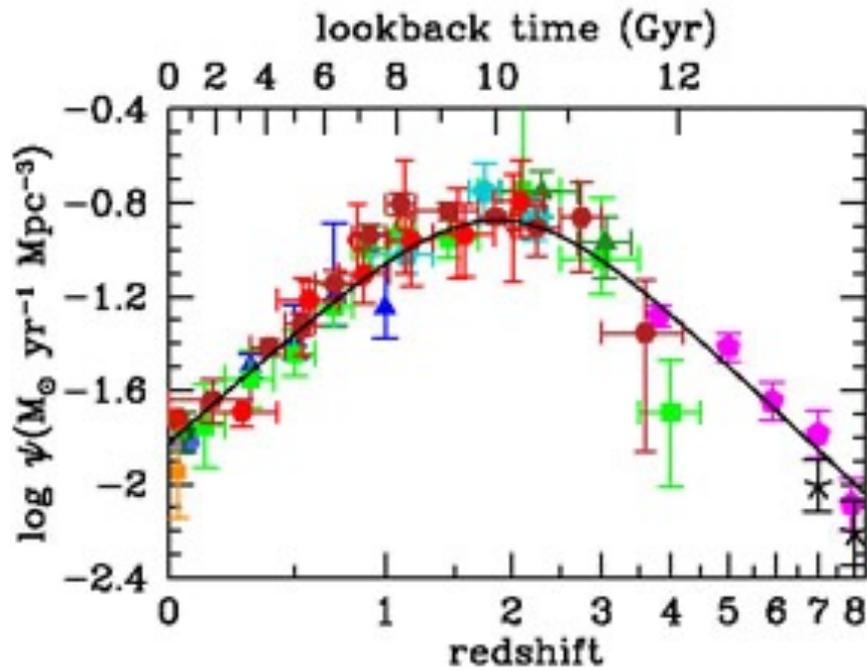




Also, squeezing of clouds initiates collapse within them => star formation. Bright young massive stars live and die in spiral arms. Emission nebulae mostly in spiral arms.

So arms always contain same types of objects, but individual objects come and go.

# Madau Plots



Star formation history of the Universe  
Peaks at  $z=1.9$  at an age of 3.5 billion years

Madau et al. (1996); Madau & Dickinson (2014)