Radio Astronomy Synchrotron Emission

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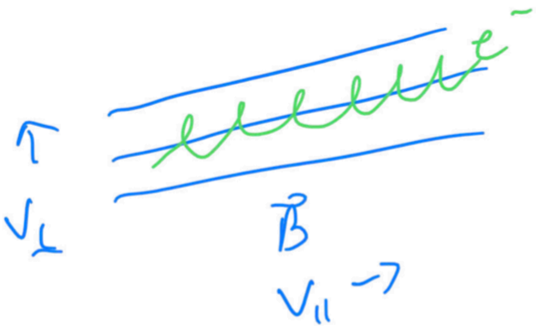
Astronomy 423 at UNM

Radio Astronomy

$\boxed{\text{NW } 10.7}$



relativistic e^-s in a magnetic field

1) Lorentz transformations
2) Laws of nature are invariant in a moving frame
3) The speed of light is the same in all frames

Consider frames k and k' with relative velocity v

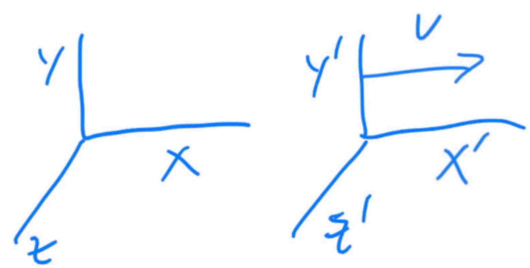
$$x = \gamma (x' + v t')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma \left(t' + \beta \frac{x'}{c} \right)$$

$$dt = \gamma \, dt'$$



$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Synchrotron radiation of a single e^- in a constant B field

$$a = F/m$$

$$\frac{d}{dt}(\gamma \vec{v}) = \frac{e(\vec{v} \times \vec{B})}{m}$$

If there is no $\vec{E}$ field $(\vec{E} = 0)$ then

Energy $\rightarrow$ $\frac{dE}{dt} = \frac{d}{dt}(\gamma m c^2) = 0$ Energy conservation

γ = constant v = constant

consider $V_{\parallel}$ and $V_{\perp}$

$$\frac{dV_{\parallel}}{dt} = 0 \qquad a_{\perp} = \frac{dv_{\perp}}{dt} = \frac{e}{\gamma m}(v_{\perp} \times \vec{B})$$

$$\underset{\text{const}}{v} = \sqrt{\underset{\text{const}}{V_{\parallel}^2} + v_{\perp}^2} \quad \therefore \; V_{\perp} \text{ must be constant as well}$$

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$V_\perp \sim$ constant

$$a_\perp = \frac{V_\perp^{\cancel{2}}}{r} = \frac{e\cancel{V_\perp}B}{\gamma m}$$

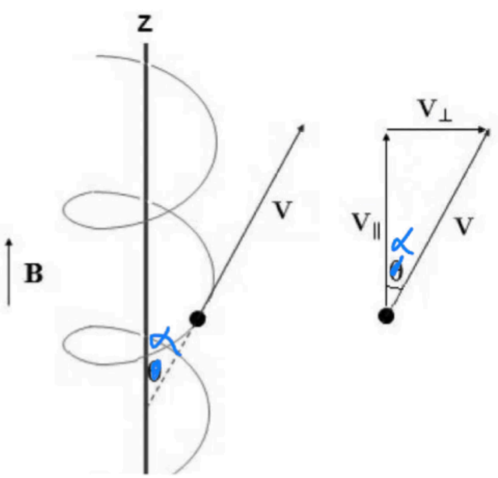
$$\frac{V}{R} = \omega_B = \frac{eB}{\gamma m}$$

$$\omega_G = \frac{eB}{m} = 17.6 \frac{B}{1\,\text{Gauss}}\ \text{MHz}$$

the gyro frequency

$$\nu_G = \frac{\omega_G}{2\pi}$$

$$\omega_B = \frac{\omega_G}{\gamma}$$



α = pitch angle

$$\tan\alpha = \frac{V_\perp}{V_\parallel}$$

$$a = V_\perp \omega_B$$

Total Power radiated:

assume $v_{\parallel} = 0$

$$P' = \frac{2}{3}\frac{e^2}{c^3} a_\perp'^2$$ for a dipole in frame k' at velocity v

$dt = \gamma\, dt'$ $\quad P = \frac{dE}{dt}$ $\quad E = \gamma E'$

$$P = \frac{dE}{dt} = \frac{dE}{dt'} \cdot \frac{dt'}{dt}$$

$$= \frac{\gamma\, dE'}{dt'} \cdot \frac{1}{\gamma} = P'$$ (invariant under Lorentz transform)

$E = \gamma m_e c^2$ $\quad \frac{E}{m_e c^2} = \gamma$

$$P = \frac{2e^4 v_\perp^2 B^2}{3m^2c^2}\left(\frac{E}{mc^2}\right)^2$$

$$u_B = \frac{B^2}{8\pi}$$

$$= \sigma_T \gamma^2 c\, u_B$$

$$\sigma_T = 6.65 \times 10^{-25}\ \mathrm{cm}^2$$

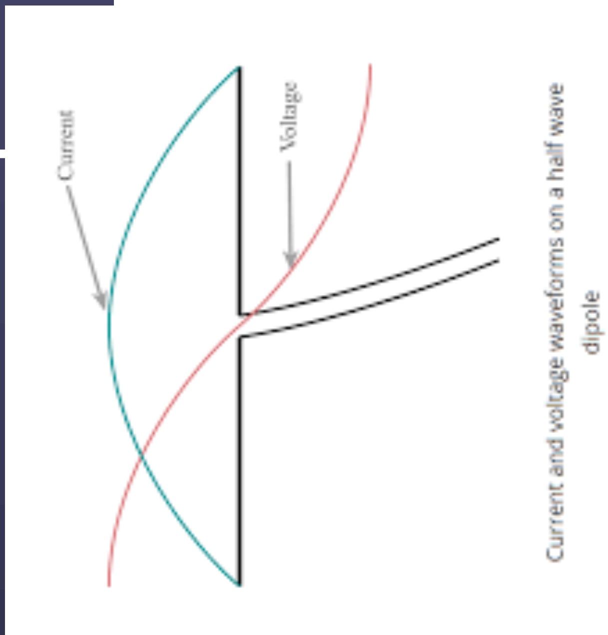
for single electron at velocity v

Lifetime estimate

$$\text{lifetime} = \frac{1}{2}\frac{E}{\frac{dE}{dt}} = \frac{1}{2}\frac{E}{P} = \frac{16.4\ \mathrm{yr}}{\left(\frac{B}{1G}\right)^2 \gamma} \quad (B \text{ in Gauss})$$

Example: $B = 10\ \mu G$ $\quad \nu_g \doteq 30\ \mathrm{Hz}$ $\quad \gamma = 10^4$

$$\text{age} = \frac{16.4\ \mathrm{yr}}{(10^{-5})^2 \cdot 10^4} = 16 \times 10^6\ \mathrm{yr}$$



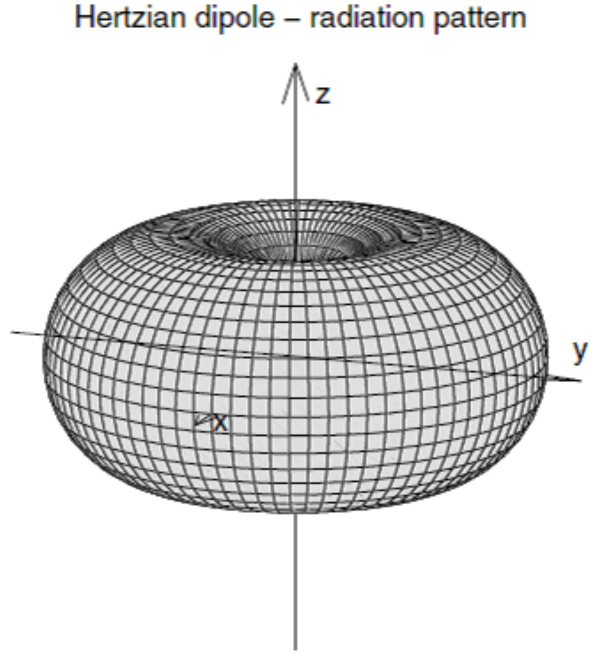


Figure 12.6 MATLAB plot of the 3-D normalized field polar radiation pattern of a Hertzian dipole (Fig.12.1); for MATLAB Exercise 12.17. *(color figure on CW)*

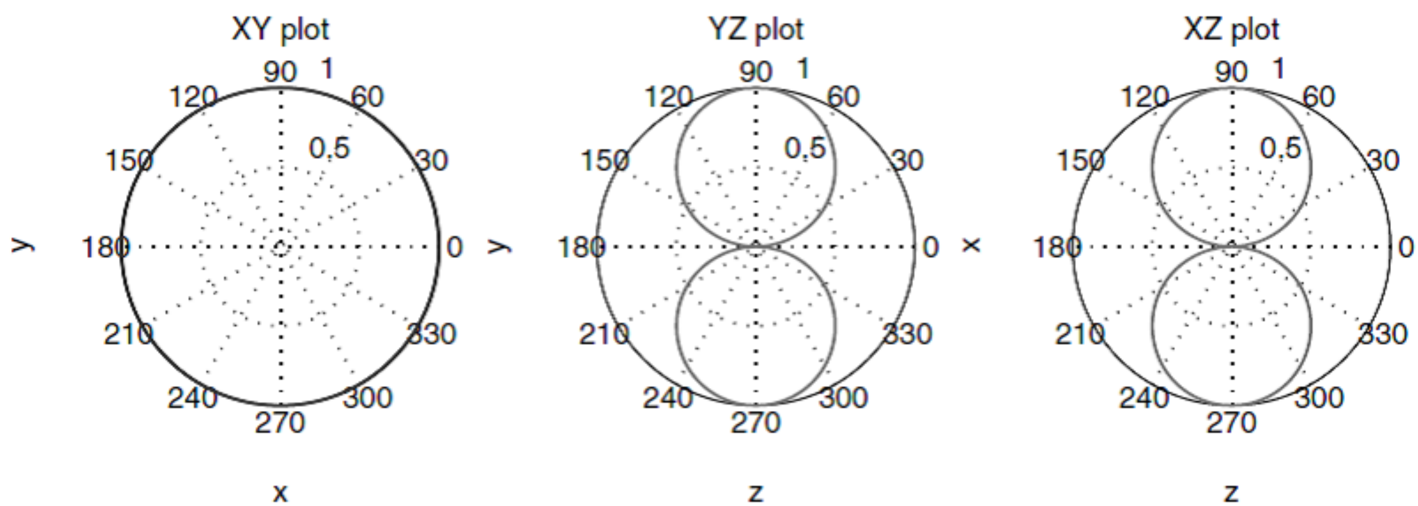


Figure 12.7 Cuts in three characteristic planes of the radiation pattern in Fig.12.6; for MATLAB Exercise 12.17. *(color figure on CW)*

Angular distribution:  in e^- frame

"Doppler boosting"

 in observer's frame

$$\sin\theta = \frac{1}{\gamma}\,\frac{\sin\theta'}{1+\beta\cos\theta'}$$

as $\beta \to 1$

$\tan\theta = \dfrac{1}{\gamma}$ $\quad \theta \sim \dfrac{1}{\gamma}$

Frequency Distribution

$$\Delta t' = \frac{2\pi}{\omega_G}$$

Observer sees a pulse during angle $\theta \sim \dfrac{1}{\gamma}$

using Doppler equation $\Delta t = \gamma(1-\beta)\Delta t' = \dfrac{1}{\cancel{2}\gamma^2}\cdot\dfrac{\cancel{2}\pi}{\omega_G}$

$$= \frac{\pi}{\gamma^2 \omega_G}$$ (see RW 10.7.3)

short pulse $\Rightarrow$ broad frequency

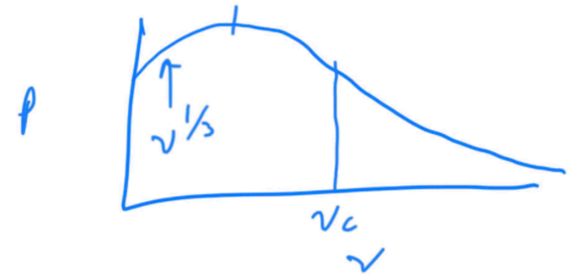
Spectrum

Bessel function

$$P(\nu) = \frac{\sqrt{3}\, e^3 B \sin\alpha}{mc^2} \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta$$

$$\nu_{max} = 0.29\, \nu_c$$

$$\nu_c = \frac{3}{2} \gamma^2 \nu_G \sin\alpha$$
$$= \frac{3}{2} \gamma^3 \nu_B \sin\alpha$$



Example: $B = 10\ \mu G$ what γ produces photons at 5 GHz?

see worksheet

what is the spectrum for an ensemble of rel. e^-s?

$$\epsilon_\nu = \int_E P(\nu)\, N(E)\, dE$$

Worksheet #4

- Download the worksheet from:

http://www.phys.unm.edu/~gbtaylor/astr423/WS4.pdf

Solve it in class.

Ask questions if you are stuck

Tell me when you have the answer.

what is the spectrum for an ensemble of rel. e^-s?

$$\epsilon_\nu = \int_E P(\nu)\, N(E)\, dE$$

$$N(E)\, dE = K E^{-\delta}\, dE$$ in astrophysics (power law)

$$\epsilon_\nu = \int_{E_1}^{E_2} P(\nu/\nu_c)\, E^{-\delta}\, dE$$

$$\epsilon_\nu = \nu^\alpha \quad \text{where} \quad \alpha = \frac{1}{2}(\delta - 1)$$

$\alpha = -0.75 \quad \delta = -0.5$

Polarization in a uniform field:

$$\% \text{ linear} = m = \frac{-\alpha+1}{-\alpha+5/3} = 72\% \quad \text{for } \alpha = -0.75$$

in a random field:

$$m = 0$$

Energy requirements:

$$W_{tot} = W_{part} + W_{mag} = V(u_p + u_B)$$

$$u_p = \eta k \int_{E_1}^{E_2} E^{1-\delta}\, dE \qquad \eta = \text{filling factor}$$

$$u_{mag} = \frac{B^2}{8\pi} \qquad E = \gamma m c^2$$

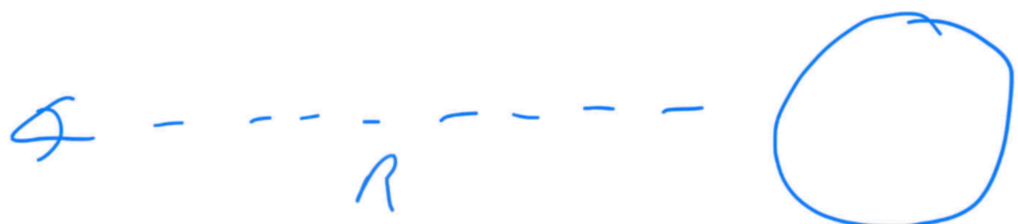
$$\nu_c = \frac{3}{2}\gamma^2 \nu_G = \frac{3}{2}\frac{eB}{m^3c^5}E^2$$

$$u_p = K \cdot G \cdot B^{\alpha - 1/2}$$

$$\text{where } G = \left(\frac{m}{1+2\alpha}\right)\left(\frac{e}{m^3c^5}\right)^{-\alpha-1/2}\left(\nu_{max}^{1/2+\alpha} - \nu_{min}^{1/2+\alpha}\right)$$

$$W_{tot} = K\,G\,B^{\alpha-1/2} + B^2/8\pi$$

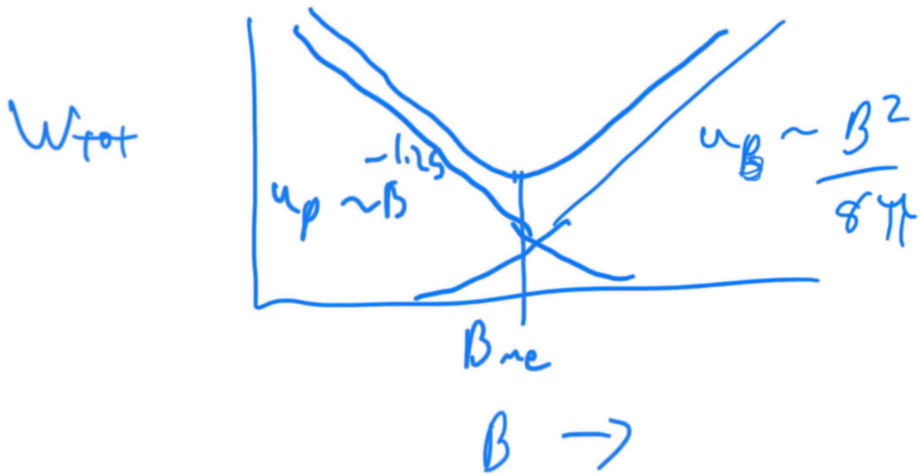
assume a spherical source with volume V



$$S_\nu = K H \frac{V}{R^2} B^{-\alpha+1/2}\nu^{\alpha}$$

$$H = b(\alpha)\frac{e^3}{mc^2}\left(\frac{3e}{4\pi m^3c^5}\right)^{\alpha}$$

next eliminate k and solve for B when W_{tot} is minimum



$$B_{me} = \left(6\pi \frac{G}{H} \frac{R^2}{V} S_\nu \nu^{-\alpha} \right)^{2/7}$$

However there is no guarantee that the system is at the minimum energy

an alternative assumption is equipartition $u_p = u_B$

Synchrotron Self-absorption (SSA):

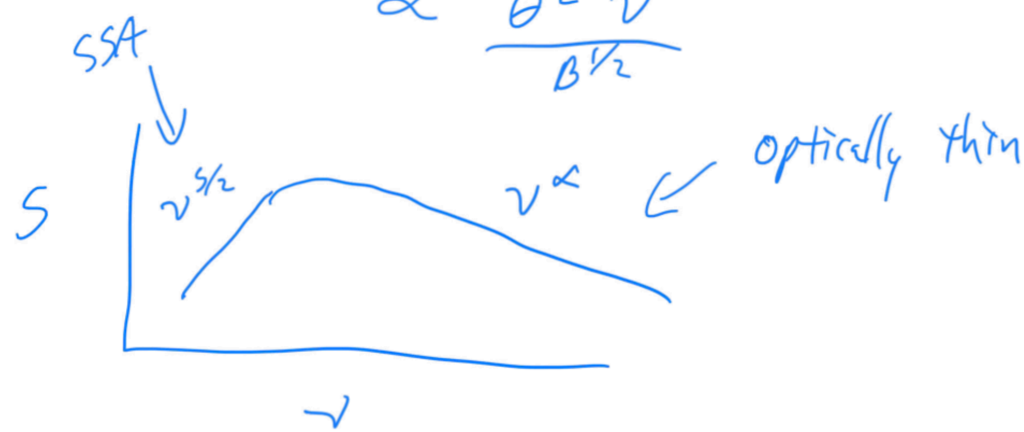
when energy density is high ($T_b \sim T_{kin}$)

$$\nu_c = \frac{3}{2}\gamma^2 \nu_G \; ; \; KE = \text{Thermal } E$$

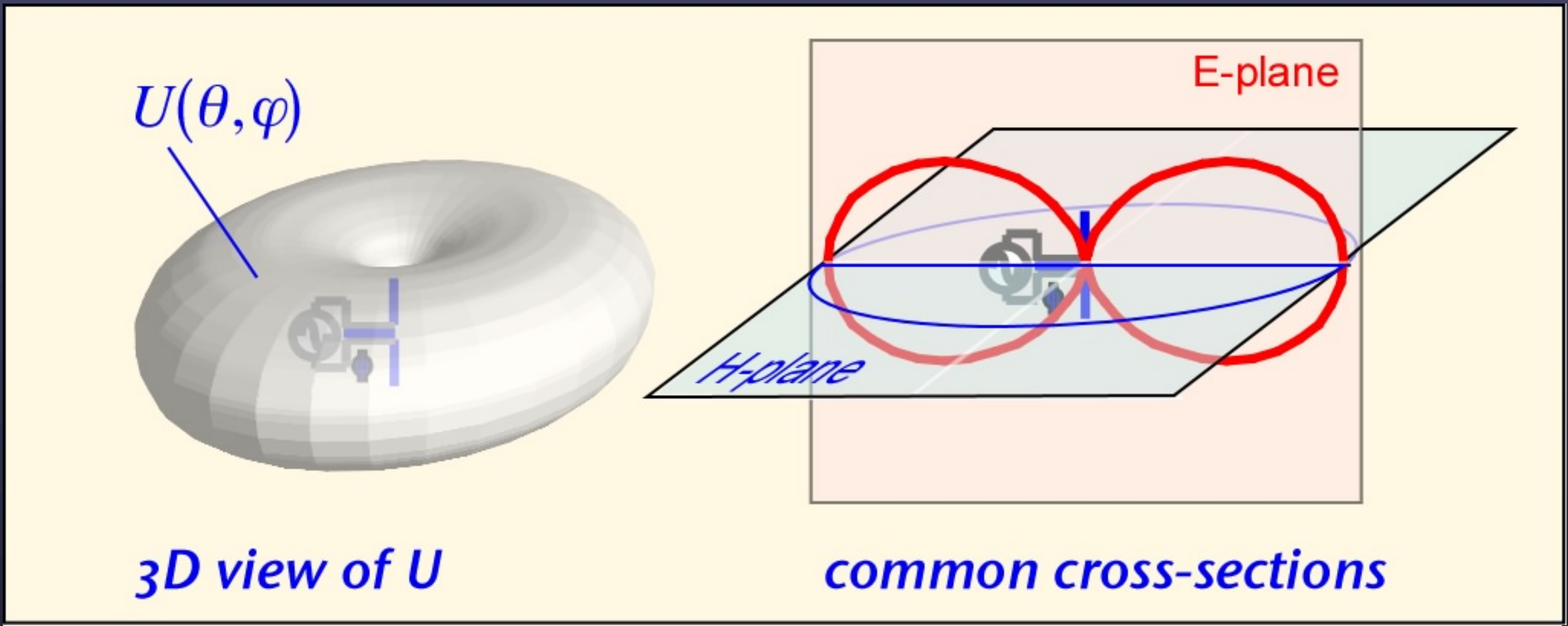
$$\gamma m_e c^2 = 3kT_e \qquad T_e \sim \frac{m_e c^2}{3k}\left(\frac{\nu}{\nu_G}\right)^{1/2} \sim T_b$$

$$S_\nu = \frac{2kT_b \Omega}{\lambda^2} = \frac{2 m_e \Omega \nu^{5/2}}{3 \nu_G^{1/2}} \qquad \Omega \sim \theta^2$$

$$\propto \frac{\theta^2 \nu^{5/2}}{B^{1/2}}$$



Hertz Dipole



$A_e = G\lambda^2/4\pi$ G=1.5 for Hertz Dipole

G = 2.5 at 20 MHz for LWA

G = 4.0 at 60 MHz for LWA