Non-Imaging Data Analysis

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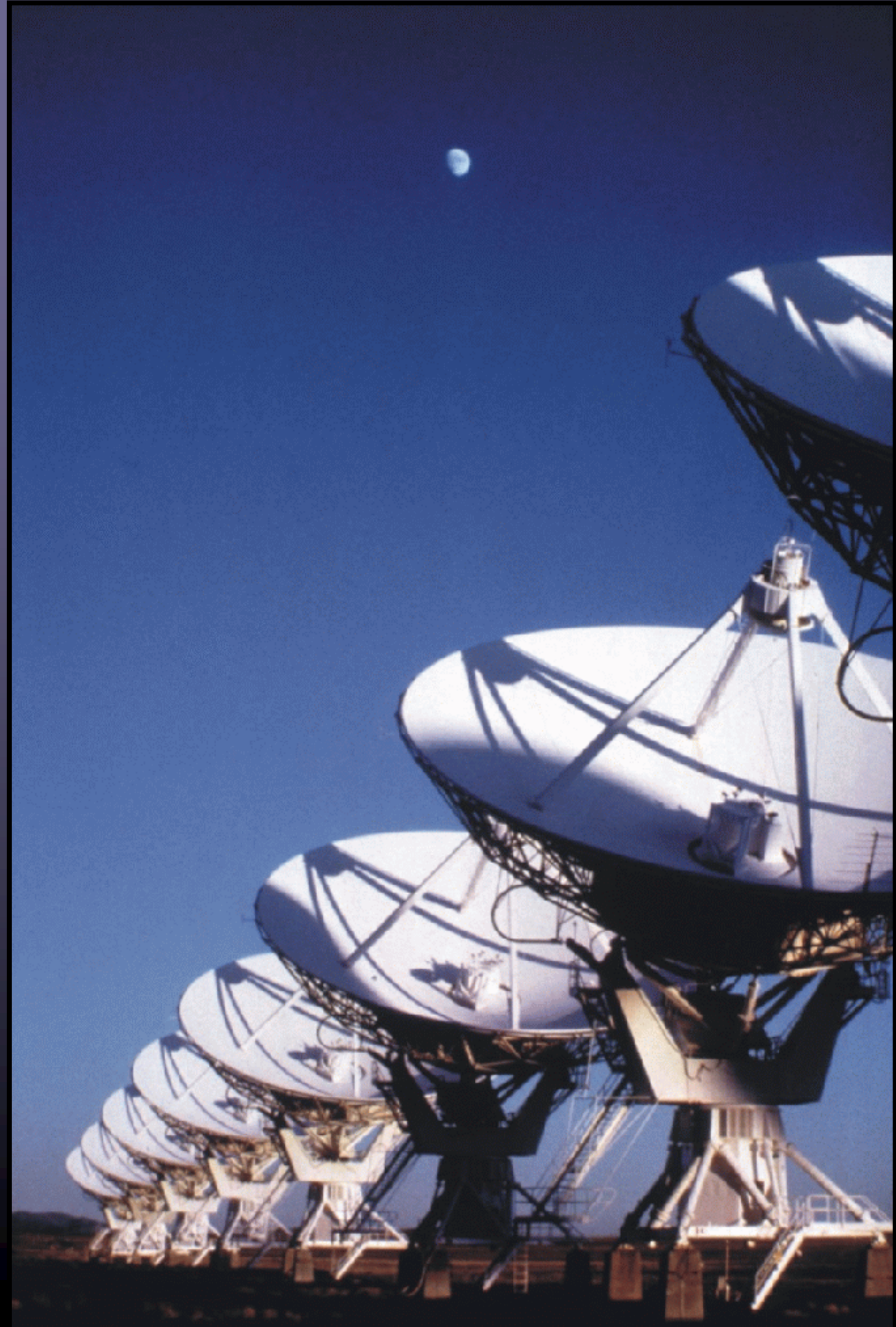


Outline

- Introduction
- Inspecting visibility data
- Model fitting
- Some applications
  - Component motion
  - Gamma-ray bursts
  - Blazars
  - Binary stars
  - Gravitational lenses

Introduction

Reasons for model fitting visibility data

- Insufficient (*u,v*)-plane coverage to make an image
- Inadequate calibration
- Missing data (e.g. no phases)
- Quantitative analysis
- Direct comparison of two data sets
- Error estimation
  - Usually, visibility measurements are independent gaussian variates
  - Systematic errors are usually localized in the (*u,v*) plane
- Statistical estimation of source parameters

Inspecting Visibility Data

- Fourier imaging

$$V(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m)I(l,m)\exp[-2\pi i(ul+vm)]\,dl\,dm$$

- Problems with direct inversion
  - Sampling
    - Poor (*u,v*) coverage
  - Missing data
    - e.g., no phases (speckle imaging)
  - Calibration
    - Closure quantities are independent of calibration
  - Non-Fourier imaging
    - e.g., wide-field imaging; time-variable sources (SS433)
  - Noise
    - Noise is uncorrelated in the (*u,v*) plane but correlated in the image

Inspecting Visibility Data

Useful displays

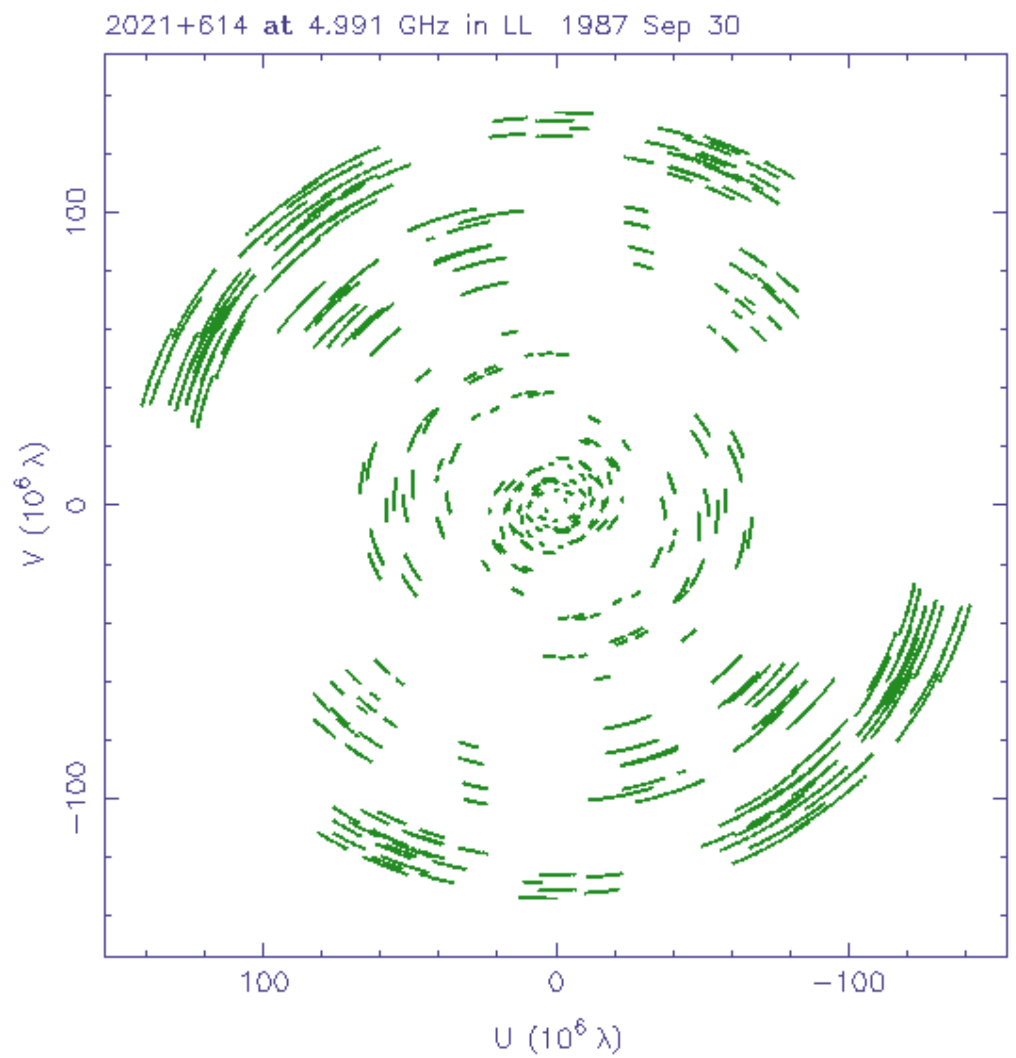
- Sampling of the (u,v) plane
- Amplitude and phase *vs*. radius in the (u,v) plane
- Amplitude and phase *vs*. time on each baseline
- Amplitude variation across the (u,v) plane
- Projection onto a particular orientation in the (u,v) plane

Example: 2021+614

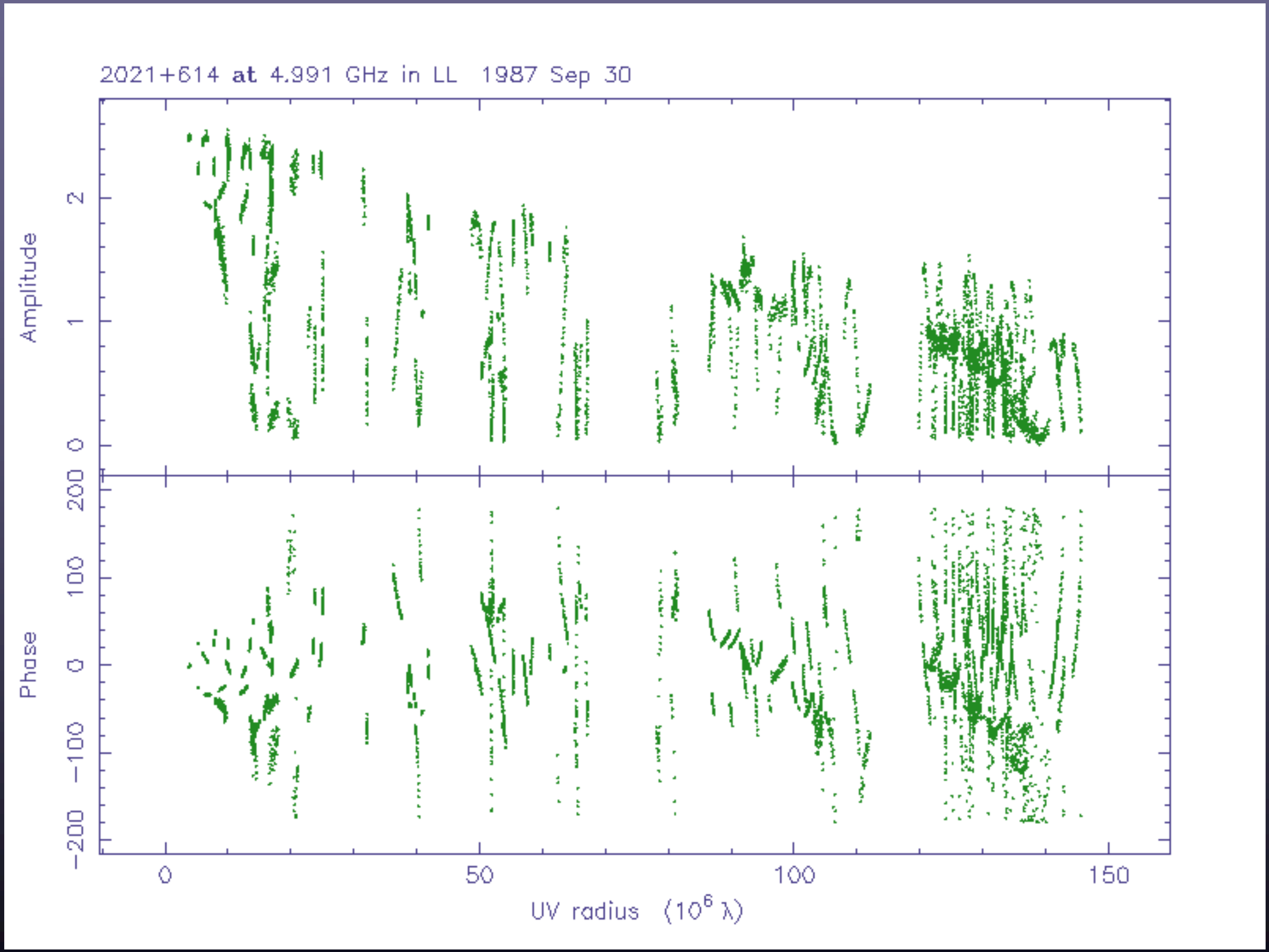
- GHz-peaked spectrum radio galaxy at z=0.23
- A VLBI dataset with 11 antennas from 1987
- VLBA only in 2000

The University of New Mexico

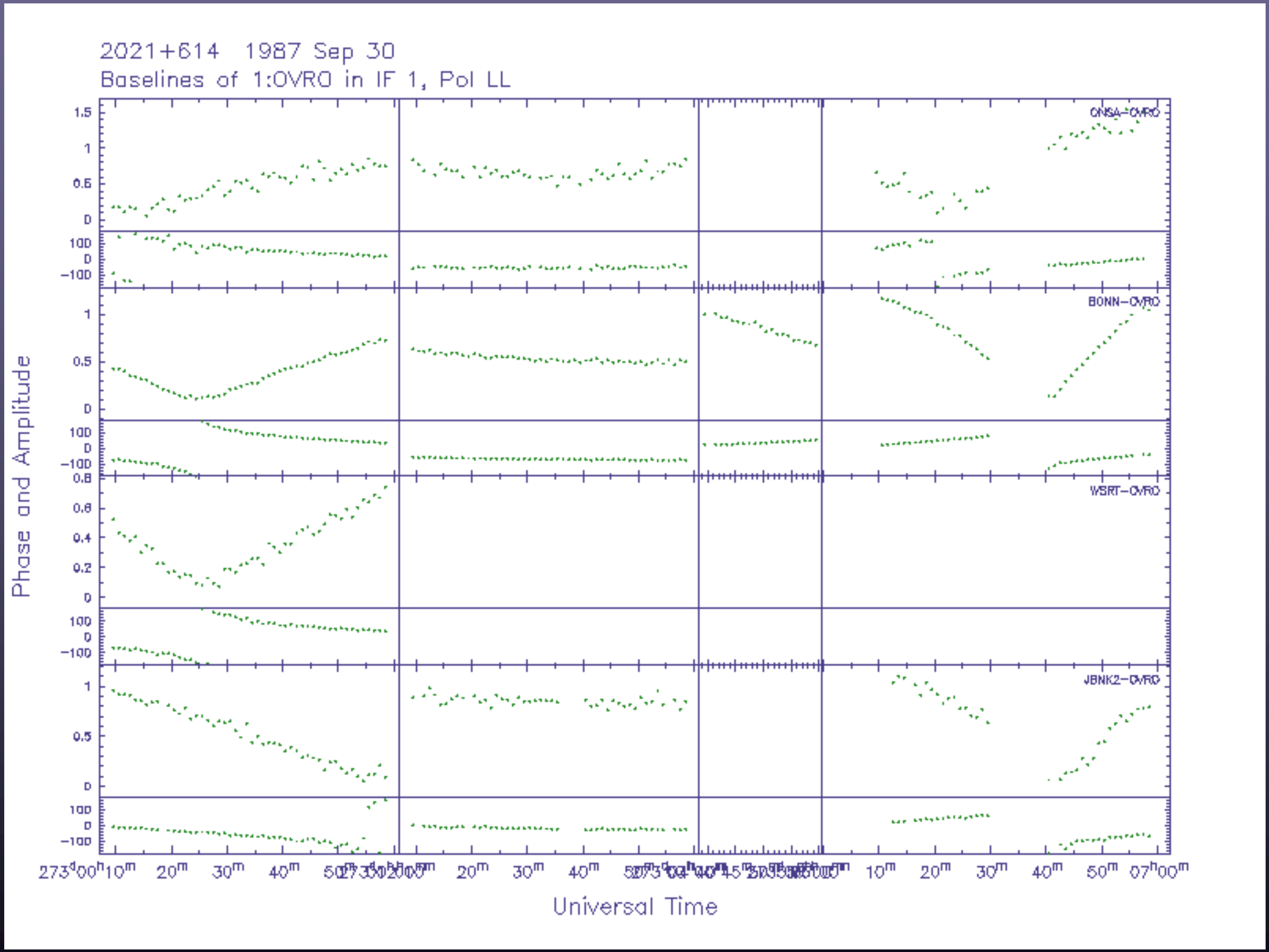
Sampling of the (u,v) plane



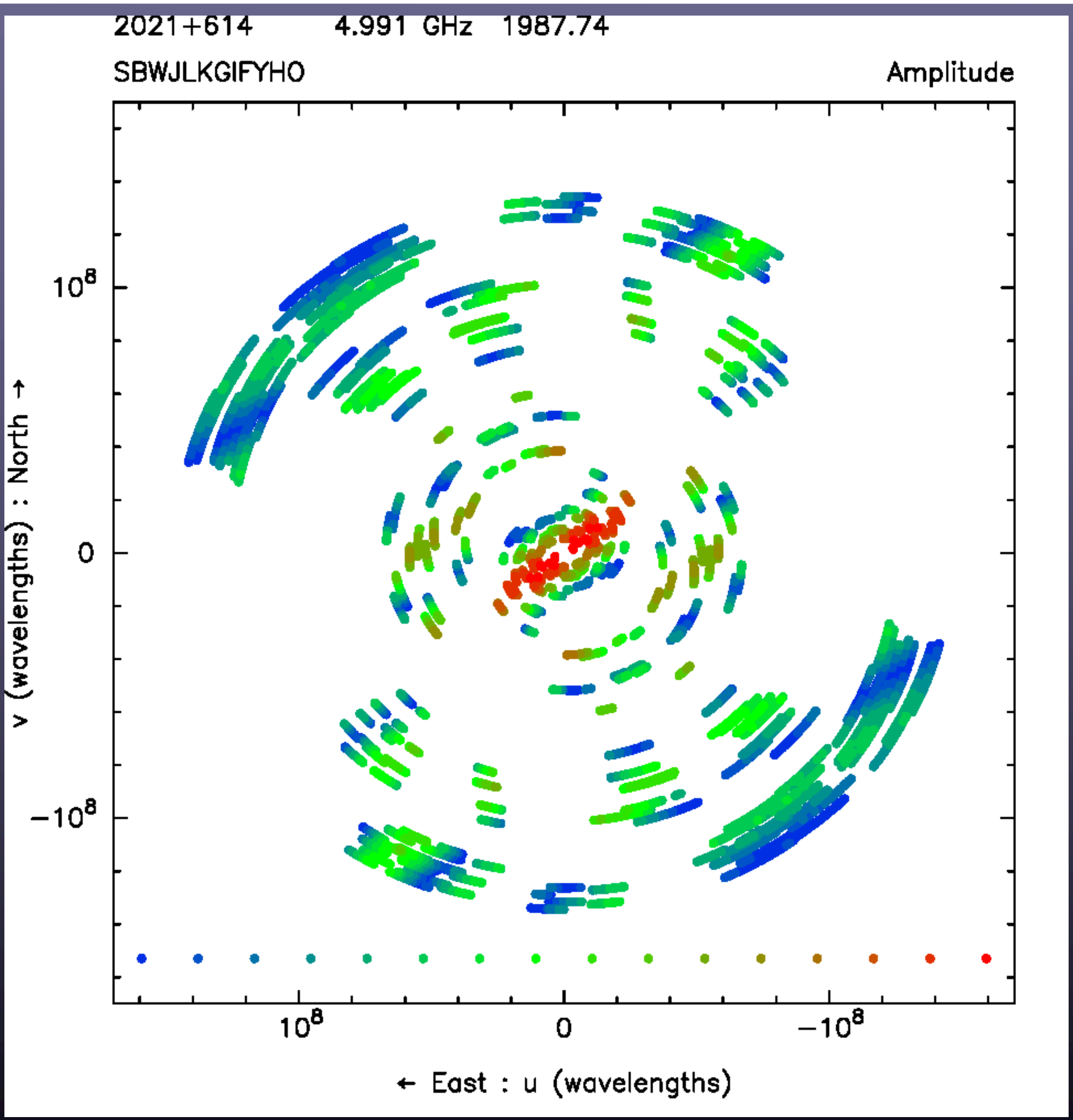
Visibility versus (u,v) radius



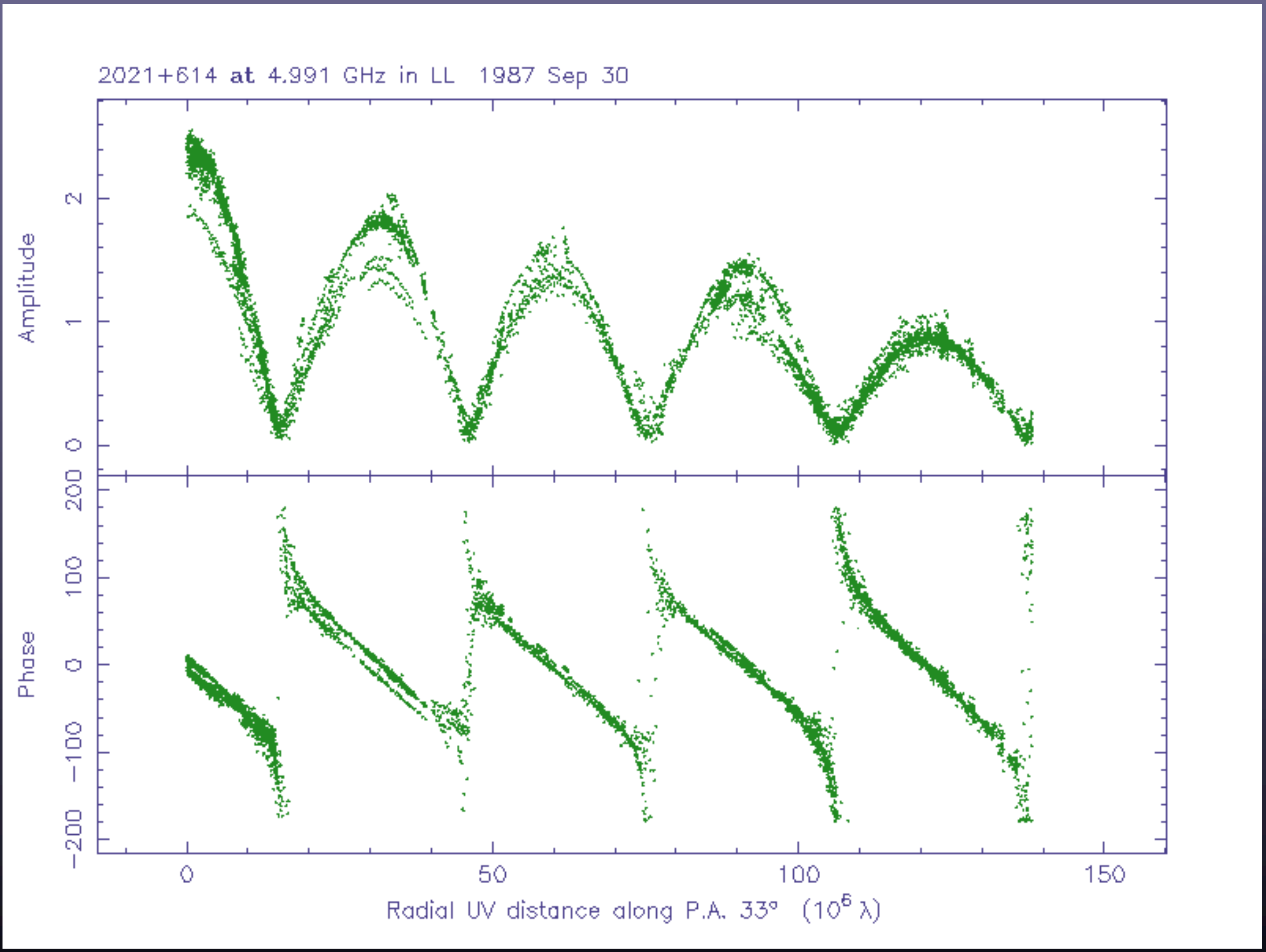
Visibility versus time

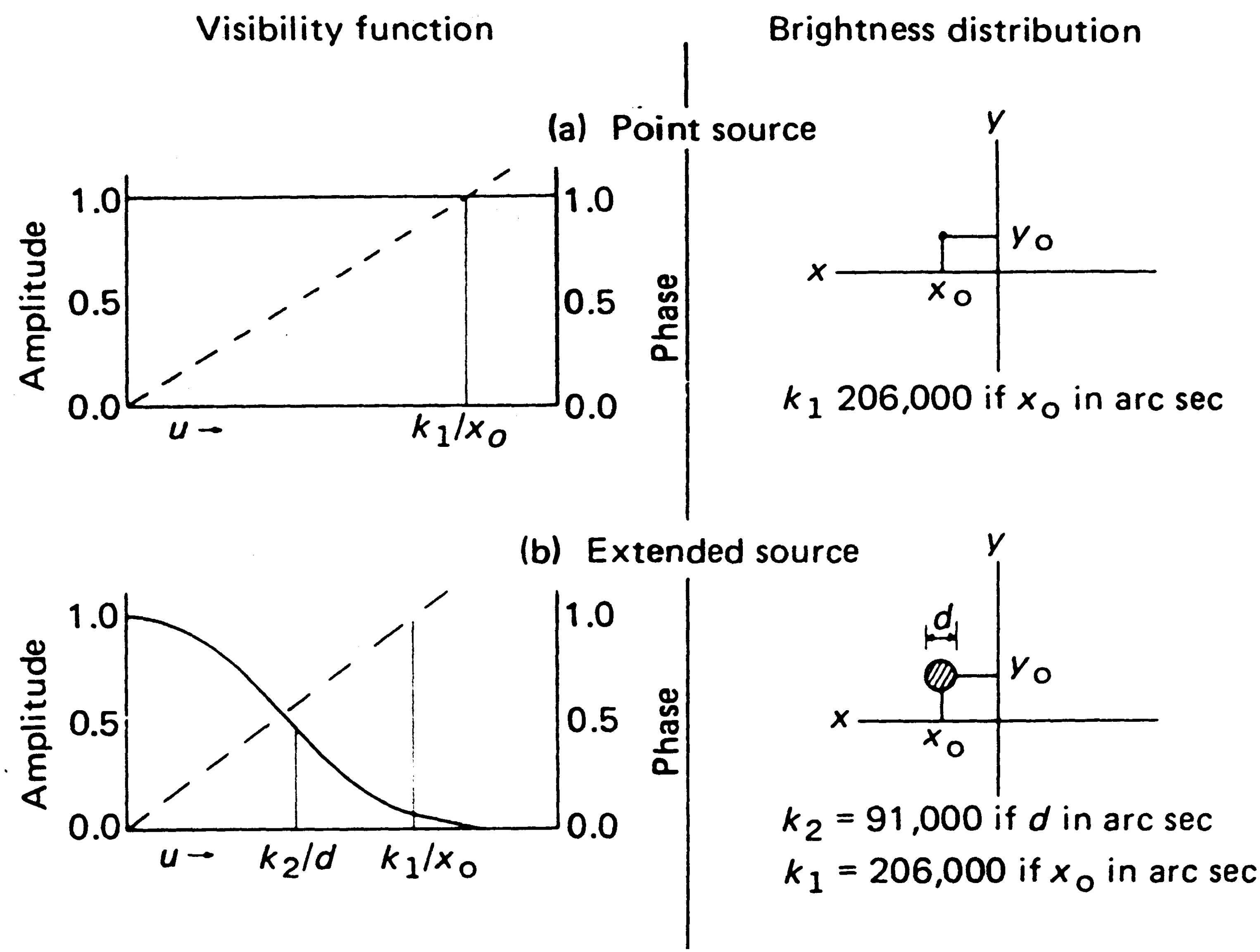


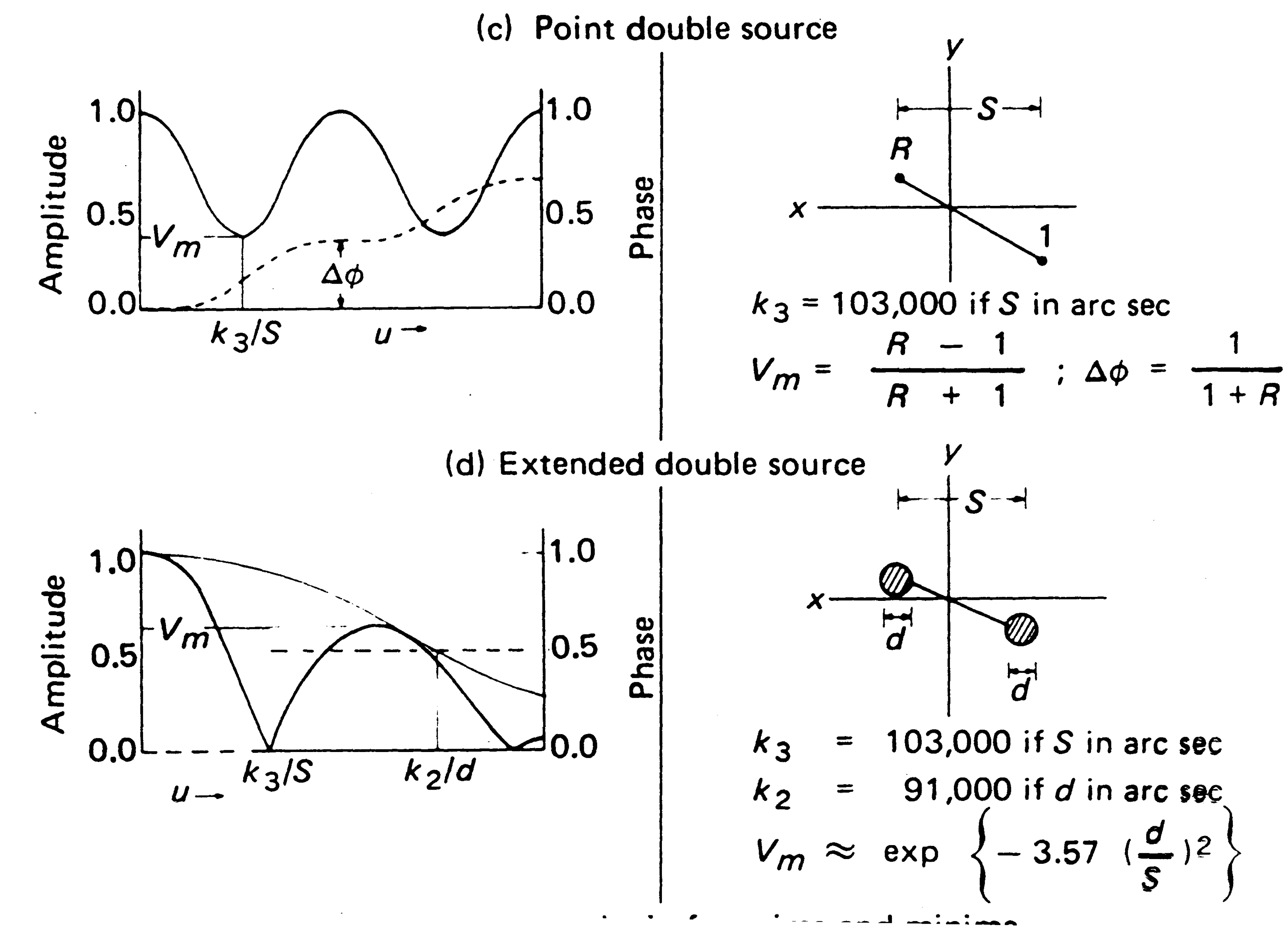
Amplitude across the (u,v) plane



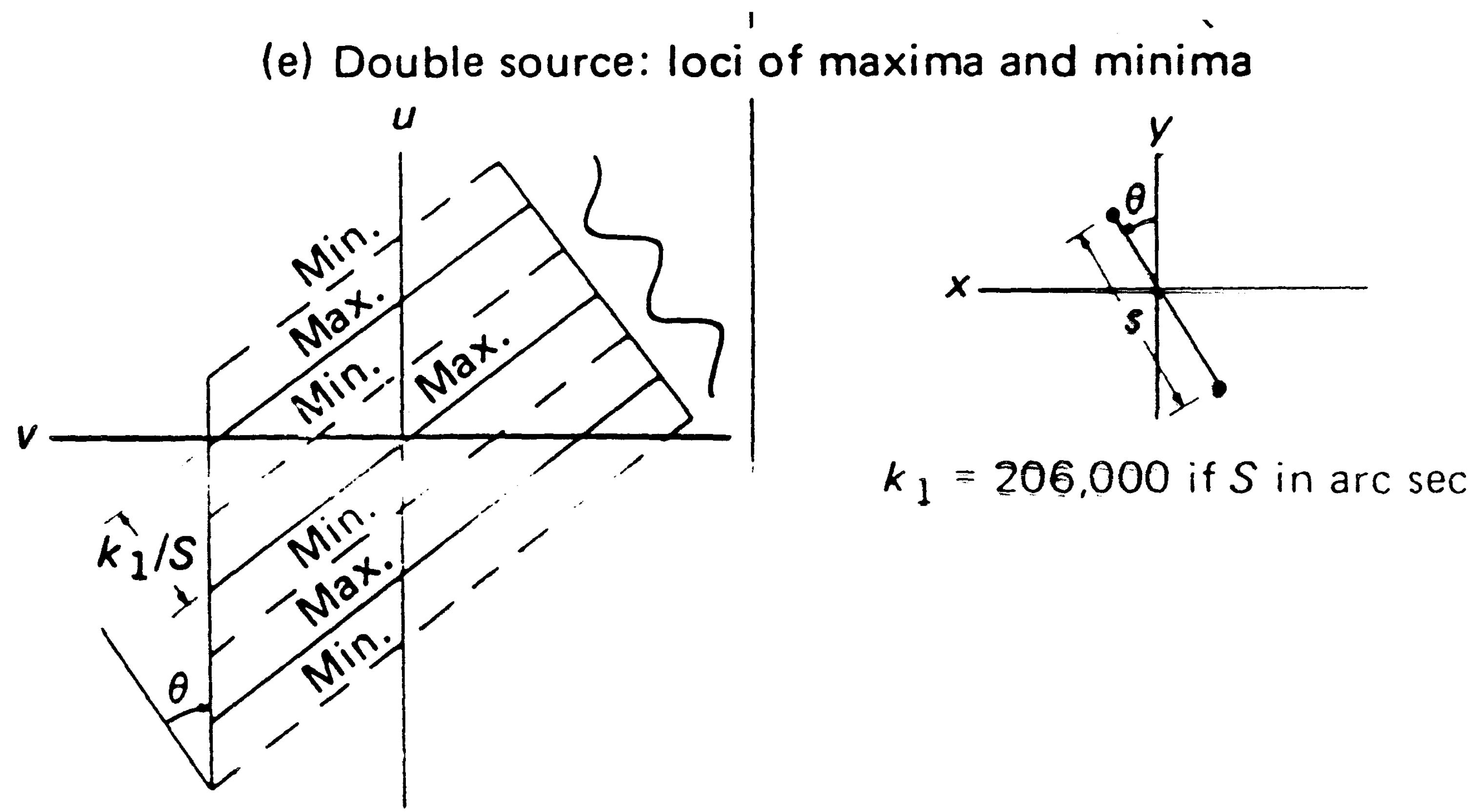
Projection in the (u,v) plane




Visibility function
Brightness distribution
(a) Point source
1.0
0.5
0.0
Amplitude
Phase
u →
k1/xo
y
x
yo
xo
k1 206,000 if xo in arc sec
(b) Extended source
d
k2/d
k2 = 91,000 if d in arc sec
k1 = 206,000 if xo in arc sec

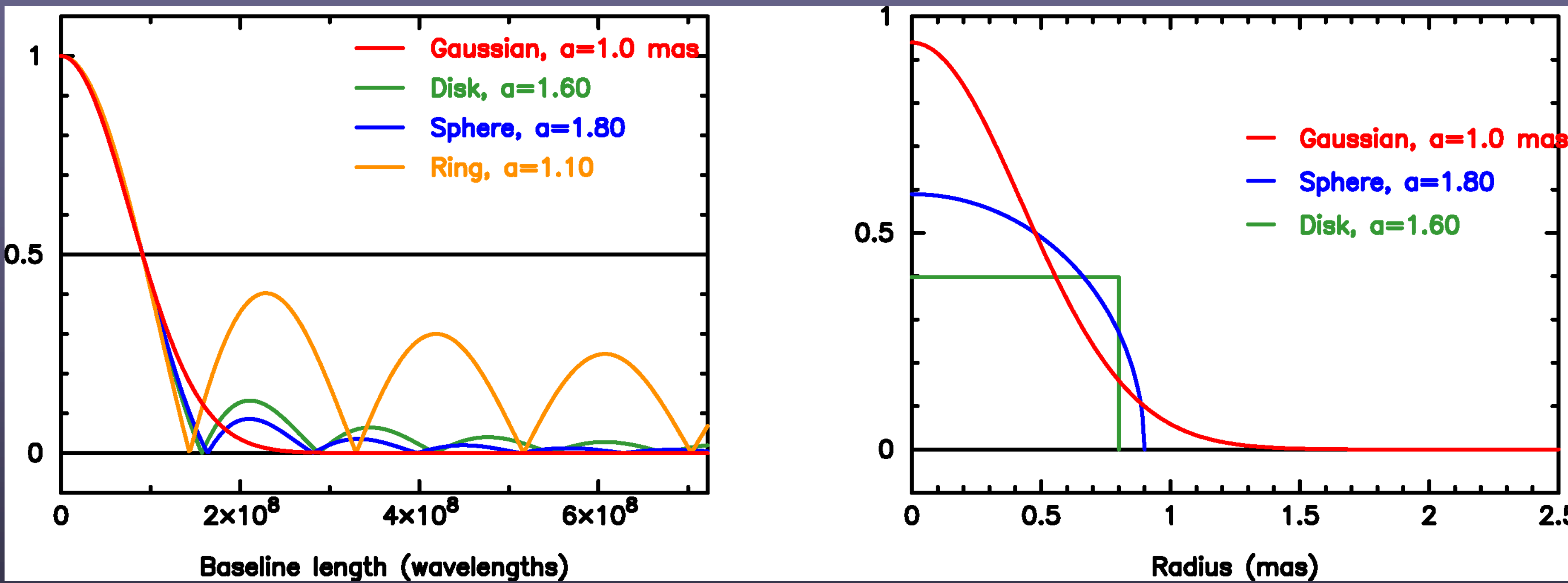


(c) Point double source
Amplitude
Phase
1.0
0.5
0.0
Vm
Δφ
k3/S
u →
y
x
S
R
1
k3 = 103,000 if S in arc sec
Vm = (R − 1)/(R + 1) ; Δφ = 1/(1 + R)
(d) Extended double source
d
k2/d
k2 = 91,000 if d in arc sec
Vm ≈ exp {− 3.57 (d/S)²}



(e) Double source: loci of maxima and minima
u
v
Min.
Max.
Min.
Max.
Min.
Max.
Min.
k_1/S
θ
y
θ
x
s
k_1 = 206,000 if S in arc sec

Simple models



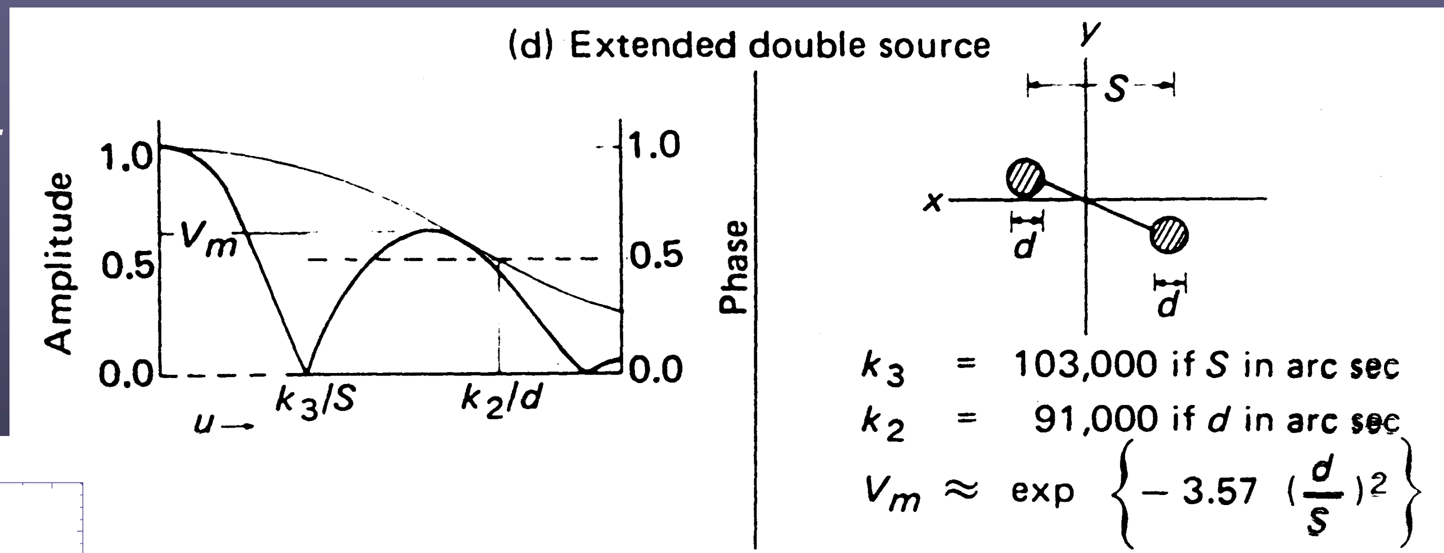
Visibility at short baselines contains little information about the profile of the source.

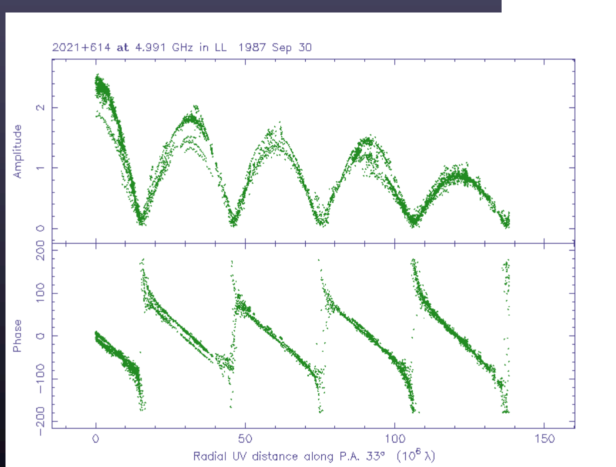
Trial model

By inspection, we can derive a simple model:

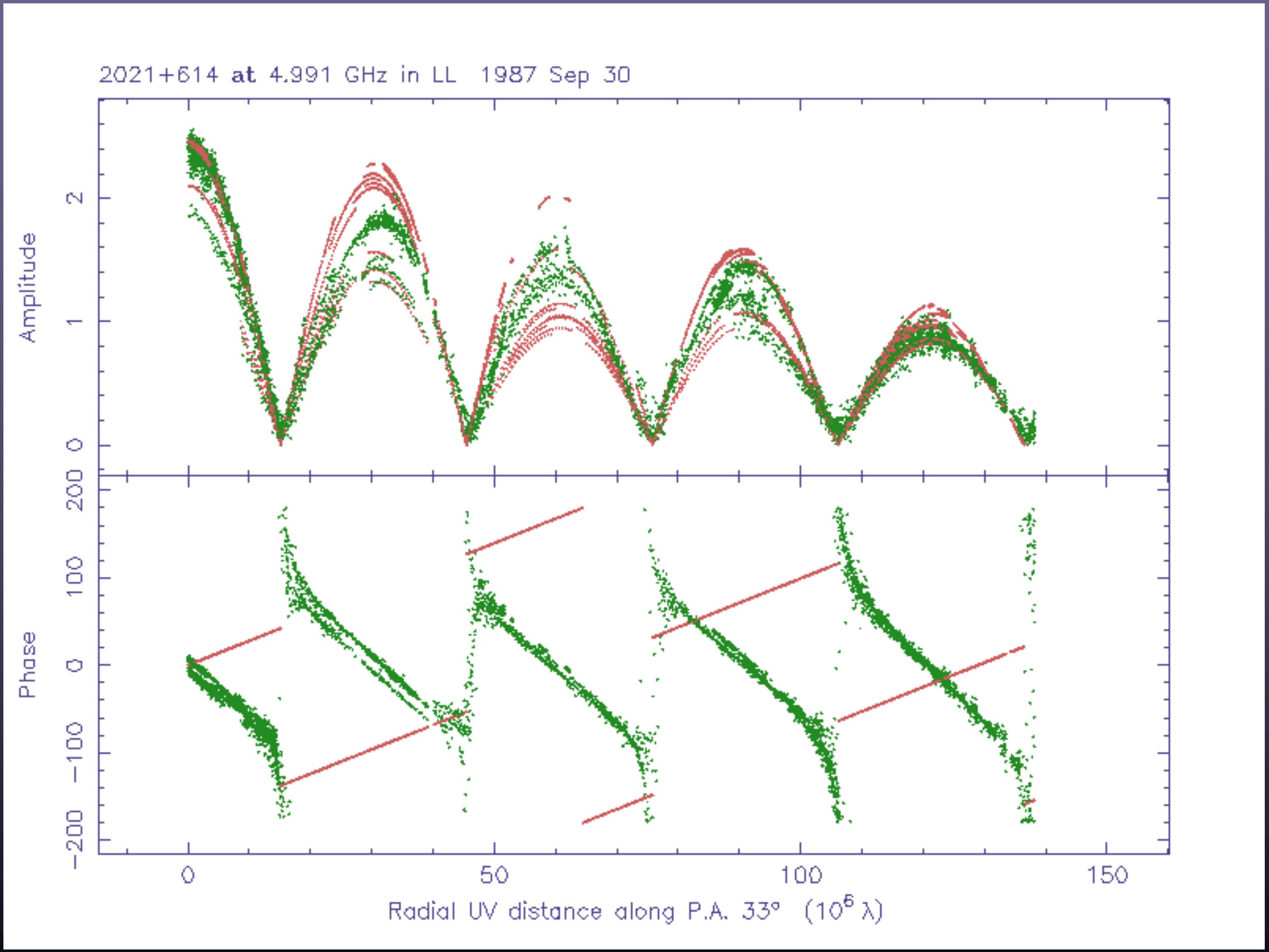
Two equal components, each 1.25 Jy, separated by about 6.8 milliarcsec in p.a. 33º, each about 0.8 milliarcsec in diameter (gaussian FWHM)

To be refined later…

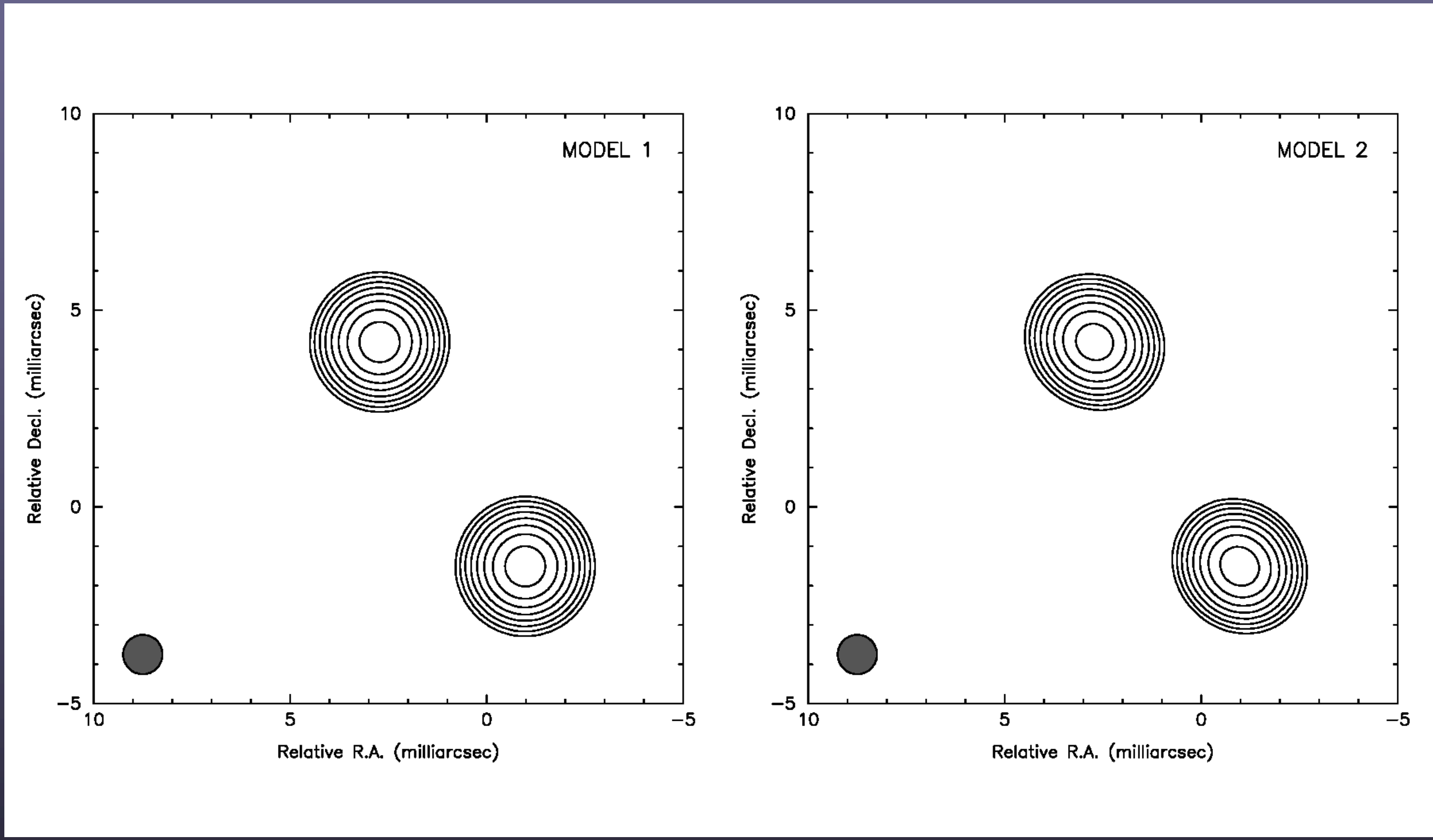




Projection in the (u,v) plane

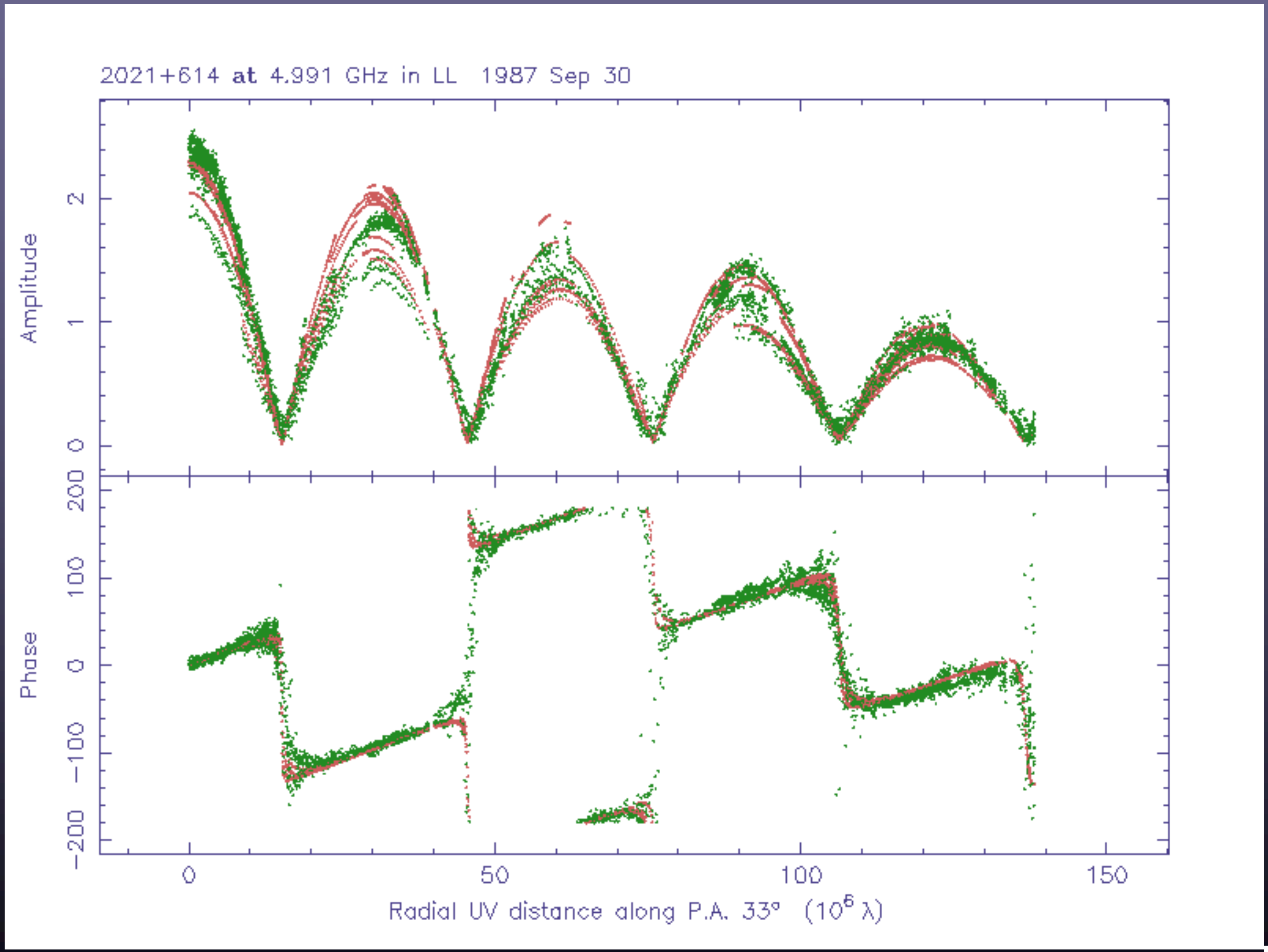


Practical model fitting: 2021



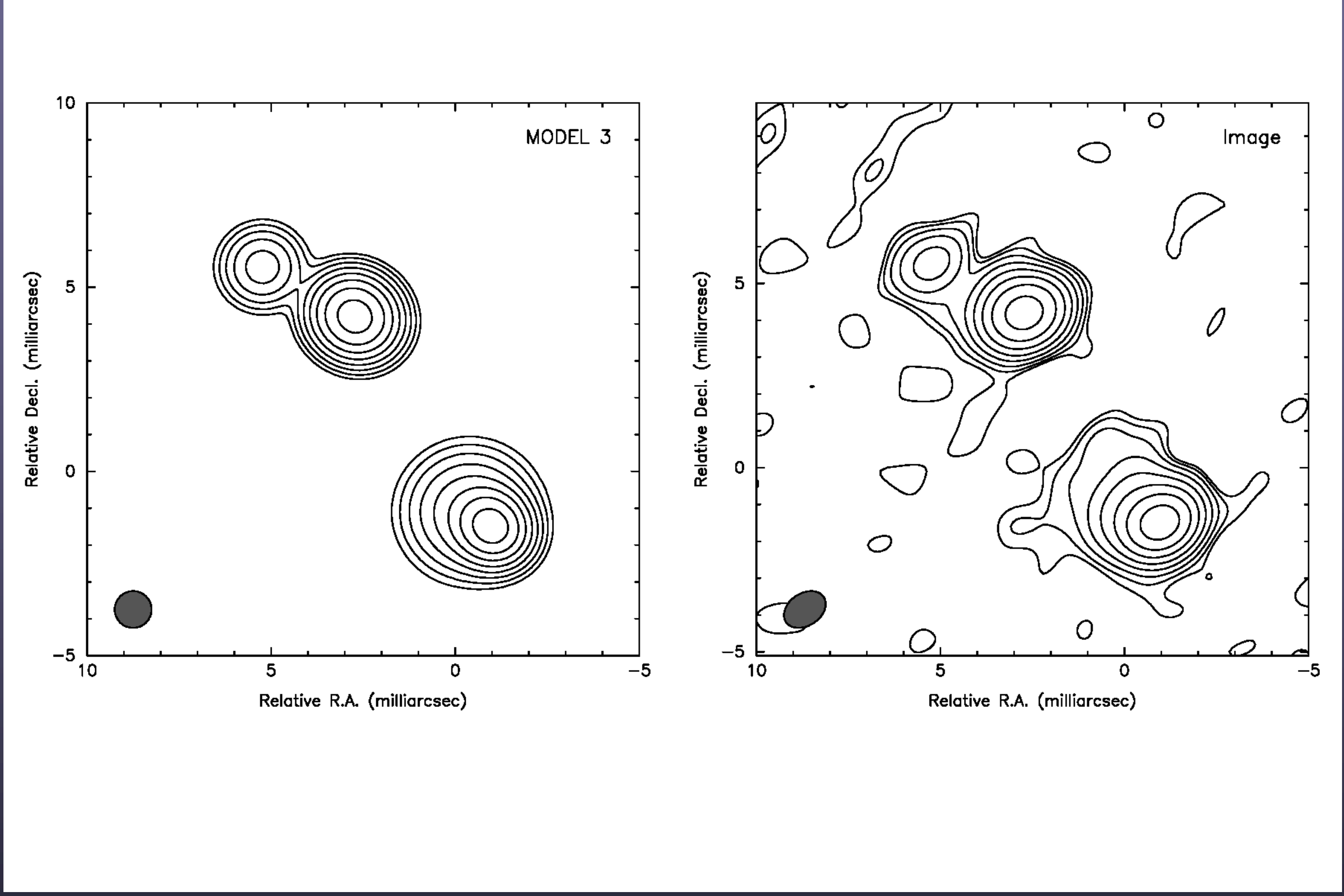
! Flux (Jy)	Radius (mas)	Theta (deg)	Major (mas)	Axial ratio	Phi (deg)	T
1.15566	4.99484	32.9118	0.867594	0.803463	54.4823	1
1.16520	1.79539	-147.037	0.825078	0.742822	45.2283	1

2021: model 2



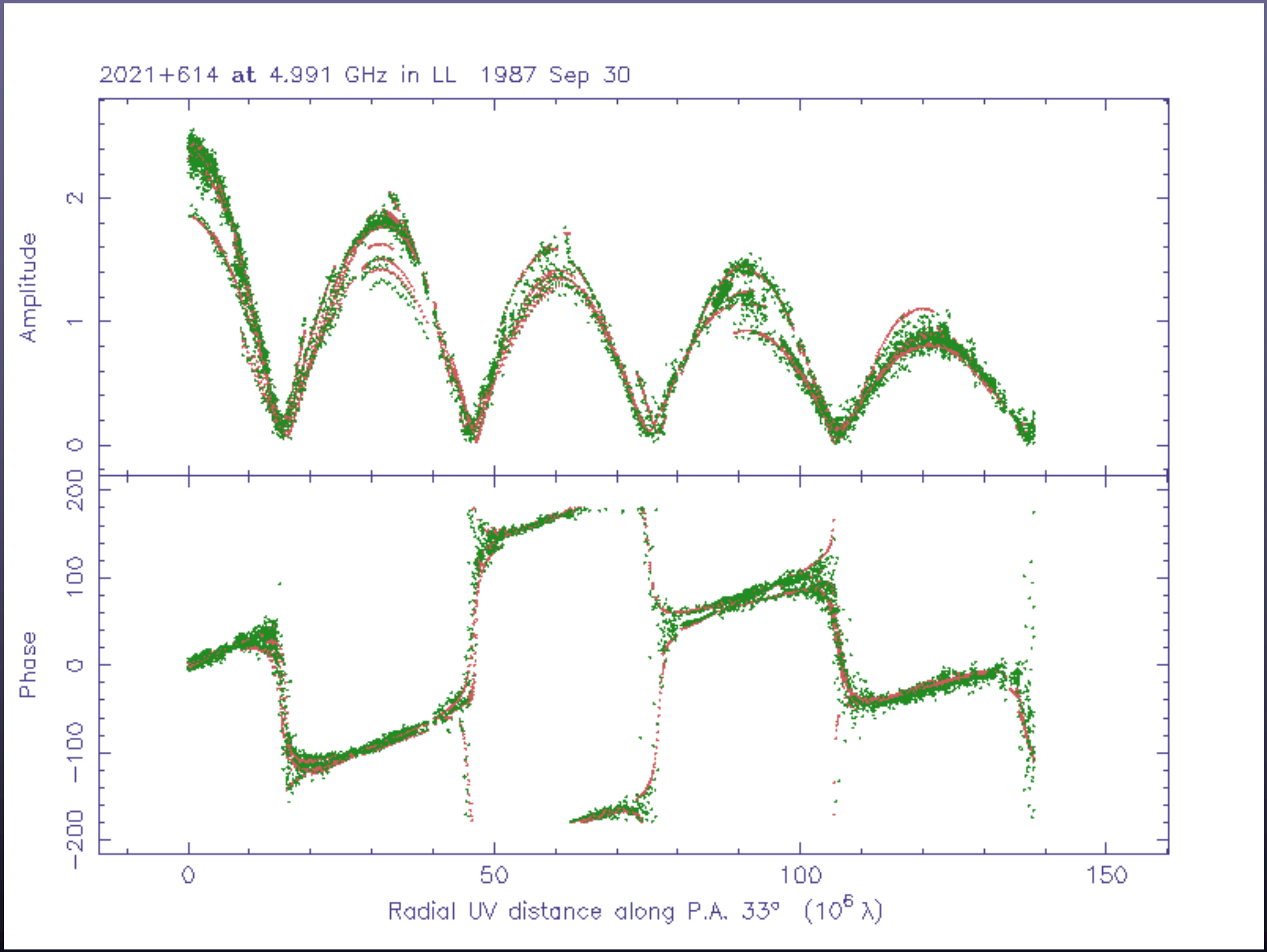
2021+614 at 4.991 GHz in LL 1987 Sep 30
Amplitude
Phase
0
1
2
200
100
0
−100
−200
0
50
100
150
Radial UV distance along P.A. 33° (10^6 λ)

Model fitting 2021



! Flux (Jy)	Radius (mas)	Theta (deg)	Major (mas)	Axial ratio	Phi (deg)	T
1.10808	5.01177	32.9772	0.871643	0.790796	60.4327	1
0.823118	1.80865	-146.615	0.589278	0.585766	53.1916	1
0.131209	7.62679	43.3576	0.741253	0.933106	-82.4635	1
0.419373	1.18399	-160.136	1.62101	0.951732	84.9951	1

2021: model 3



2021+614 at 4.991 GHz in LL 1987 Sep 30
Amplitude
Phase
Radial UV distance along P.A. 33° (10^6 λ)

Model fitting

Imaging as an Inverse Problem

- In synthesis imaging, we can solve the **forward problem**: given a sky brightness distribution, and knowing the characteristics of the instrument, we can predict the measurements (visibilities), within the limitations imposed by the noise.
- The **inverse problem** is much harder, given limited data and noise: the solution is rarely unique.
- A general approach to inverse problems is **model fitting**. See, e.g., Press et al., *Numerical Recipes*.
  1. Design a model defined by a number of adjustable parameters.
  2. Solve the forward problem to predict the measurements.
  3. Choose **a figure-of-merit** function, e.g., rms deviation between model predictions and measurements.
  4. Adjust the parameters to **minimize the merit function**.
- Goals:
  1. Best-fit values for the parameters.
  2. A measure of the goodness-of-fit of the optimized model.
  3. Estimates of the uncertainty of the best-fit parameters.

Uses of model fitting

Model fitting is most useful when the brightness distribution is simple.

- Checking amplitude calibration
- Starting point for self-calibration
- Estimating parameters of the model (with error estimates)
- In conjunction with CLEAN or MEM
- In astrometry and geodesy

Programs

- AIPS UVFIT
- Difmap (Martin Shepherd)

Parameters

Example

- Component position: (*x*,*y*) or polar coordinates
- Flux density
- Angular size (e.g., FWHM)
- Axial ratio and orientation (position angle)
  - For a non-circular component
- 6 parameters per component, plus a "shape"

- This is a conventional choice: other choices of parameters may be better!
- (Wavelets; shapelets* [Hermite functions])
  - * Chang & Refregier 2002, ApJ, 570, 447

Limitations of least squares

Assumptions that may be violated

- The model is a good representation of the data
  - Check the fit
- The errors are gaussian
  - True for real and imaginary parts of visibility
  - Not true for amplitudes and phases (except at high SNR)
- The variance of the errors is known
  - Estimate from T_{sys}, rms, etc.
- There are no systematic errors
  - Calibration errors, baseline offsets, etc. must be removed before or during fitting
- The errors are uncorrelated
  - Not true for closure quantities
  - Can be handled with full covariance matrix

Applications: Gravitational Lenses

Gravitational Lenses

- Single source, multiple images formed by intervening galaxy.
- Can be used to map mass distribution in lens.
- Can be used to measure distance of lens and H_0: need redshift of lens and background source, model of mass distribution, and a **time delay**.

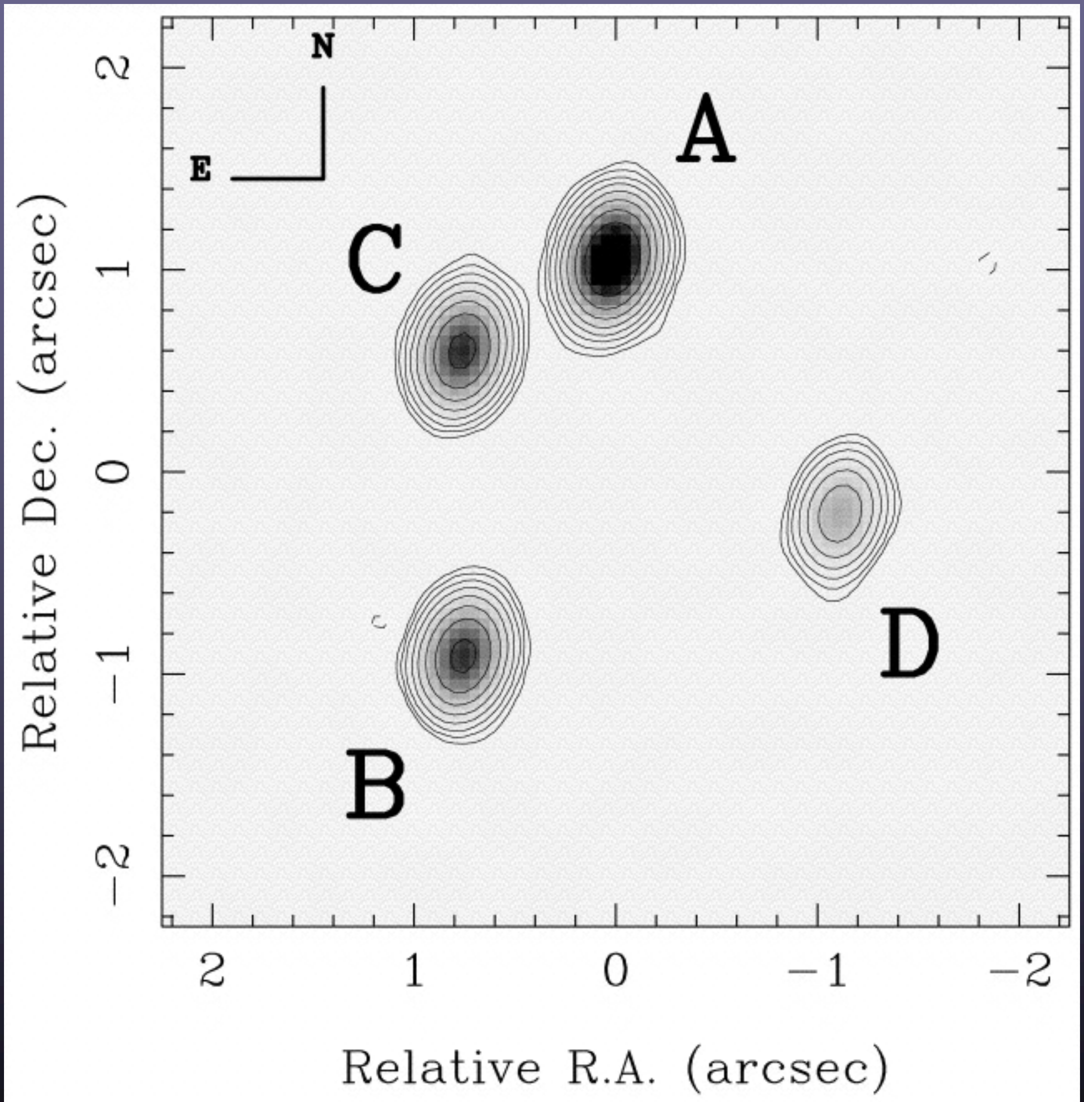
Application of model fitting

- Lens monitoring to measure flux densities of components as a function of time.
- Small number of components, usually point sources.
- Need error estimates.

Example: VLA monitoring of B1608+656 (Fassnacht et al. 1999, ApJ)

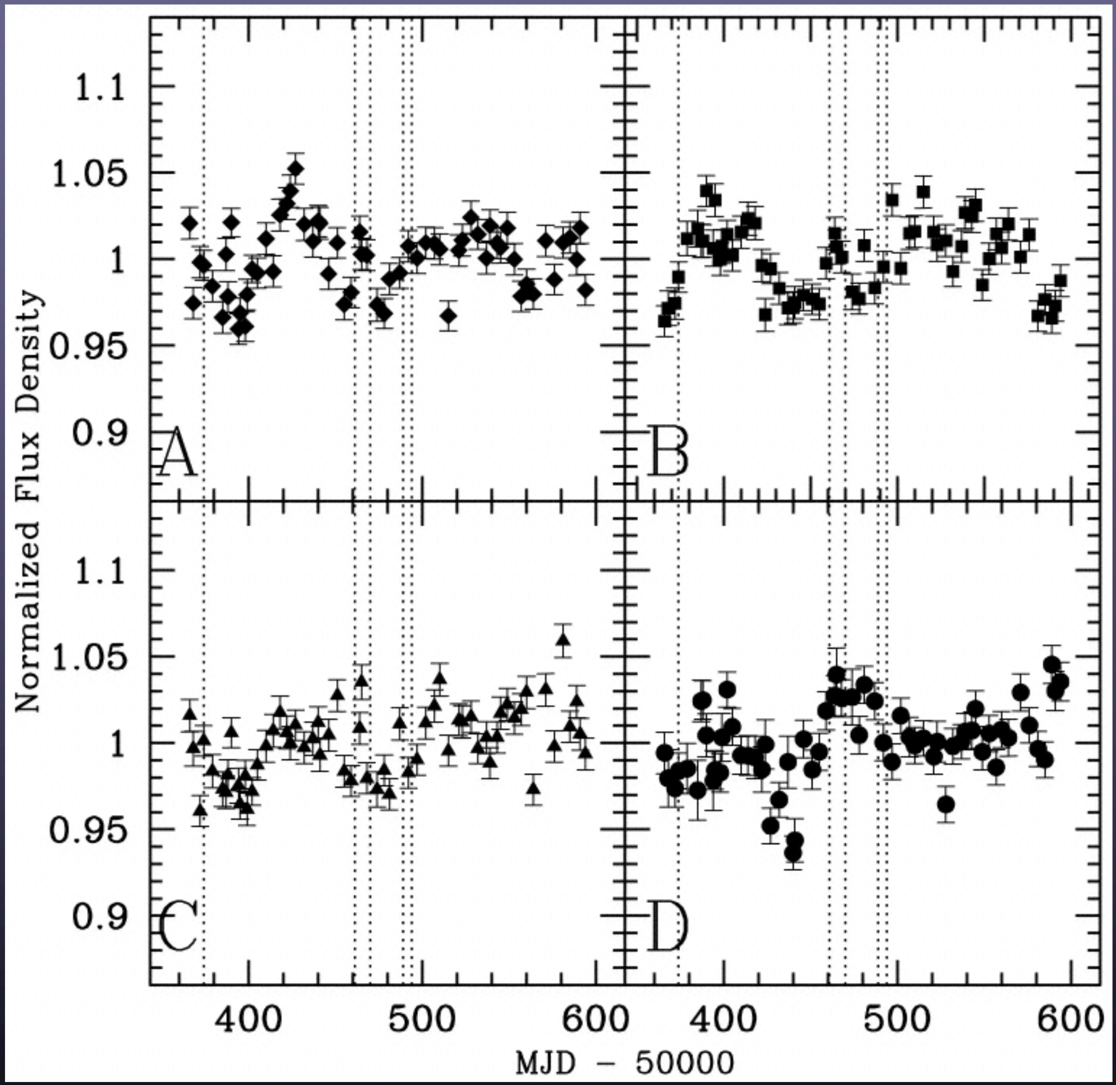
- VLA configuration changes: different HA on each day
- Other sources in the field

VLA image of 1608



1608 monitoring results

B – A = 31 days
B – C = 36 days
H_0 = 59 ± 8 km/s/Mpc



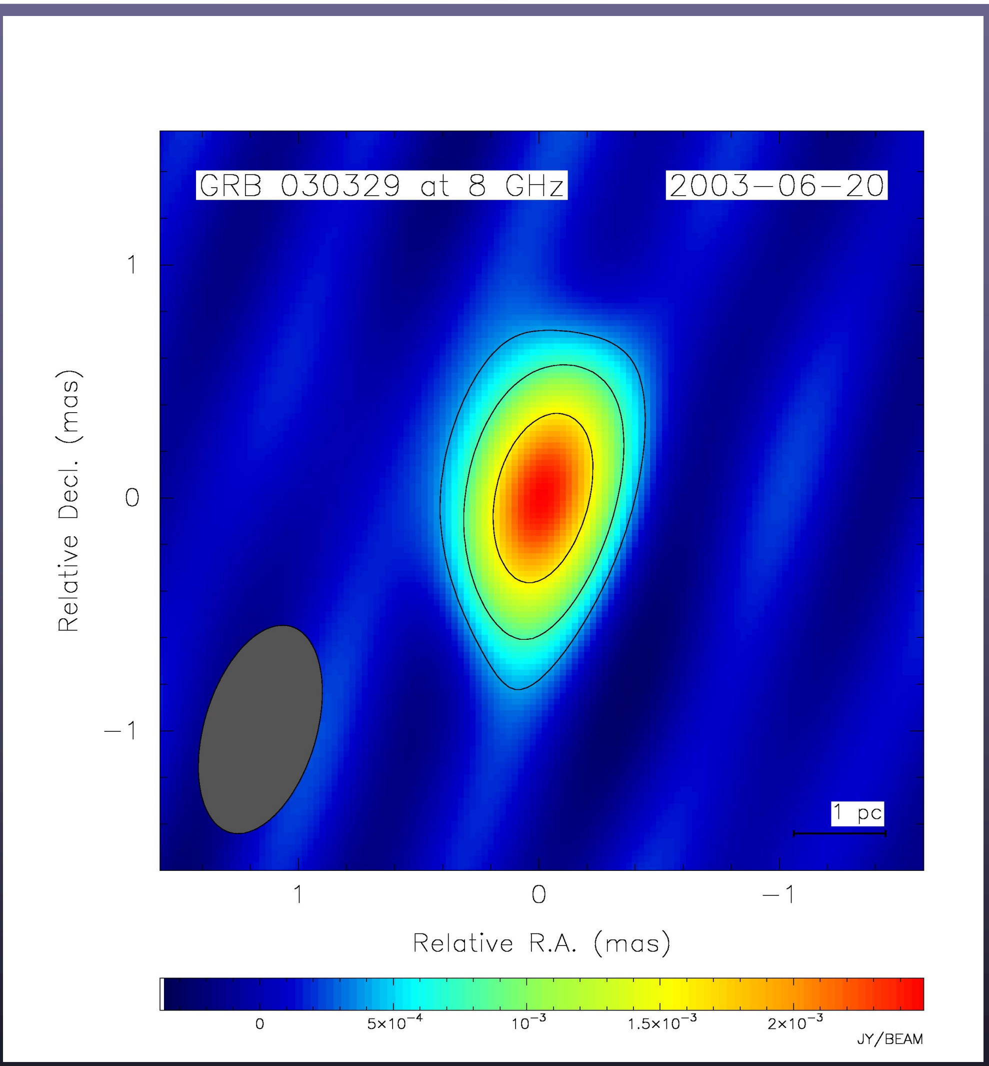
Applications - GRB030329

June 20, 2003

t+83 days

Peak ~ 3 mJy
Size 0.172 +/- 0.043 mas
0.5 +/- 0.1 pc
average velocity = 3c

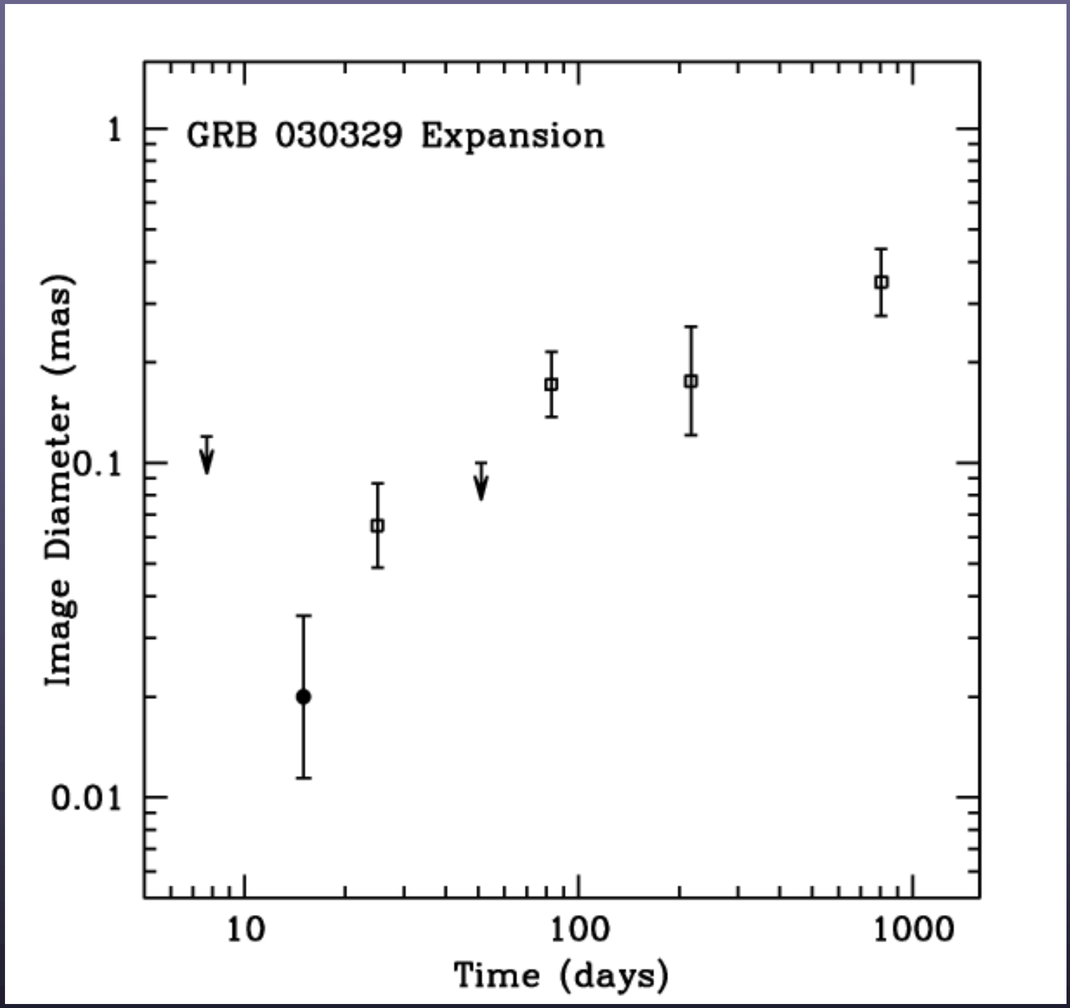
Taylor et al. 2004



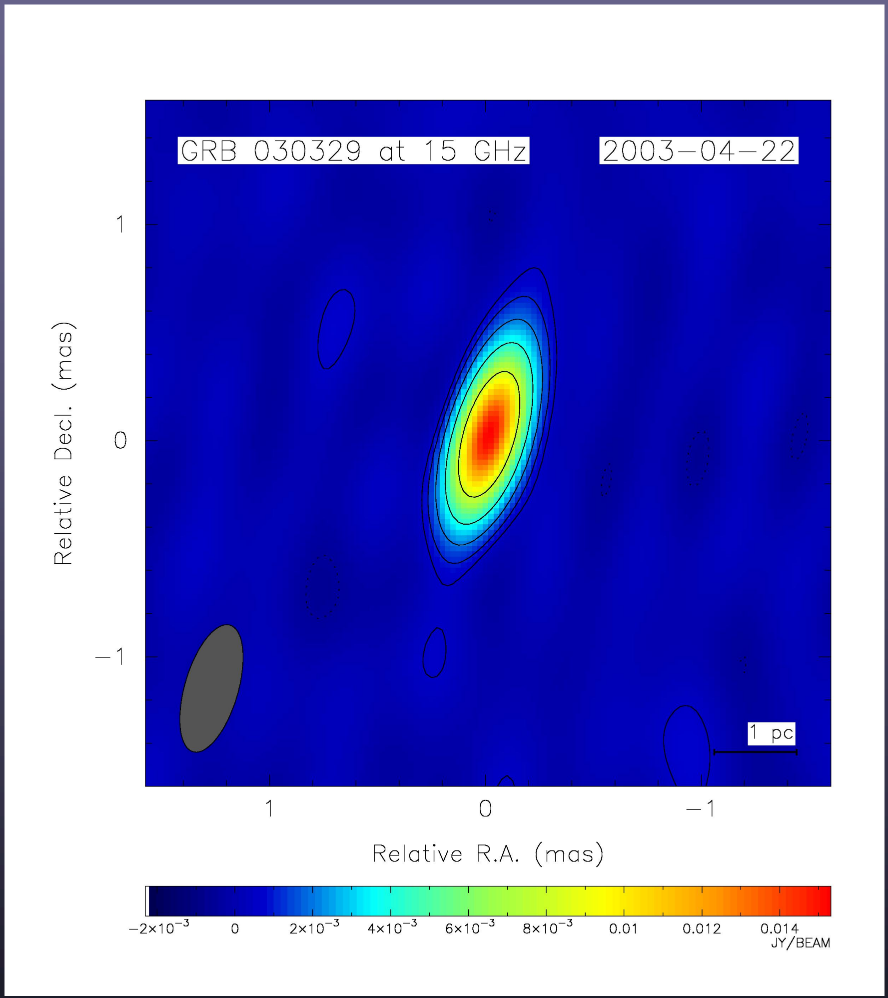
GRB 030329

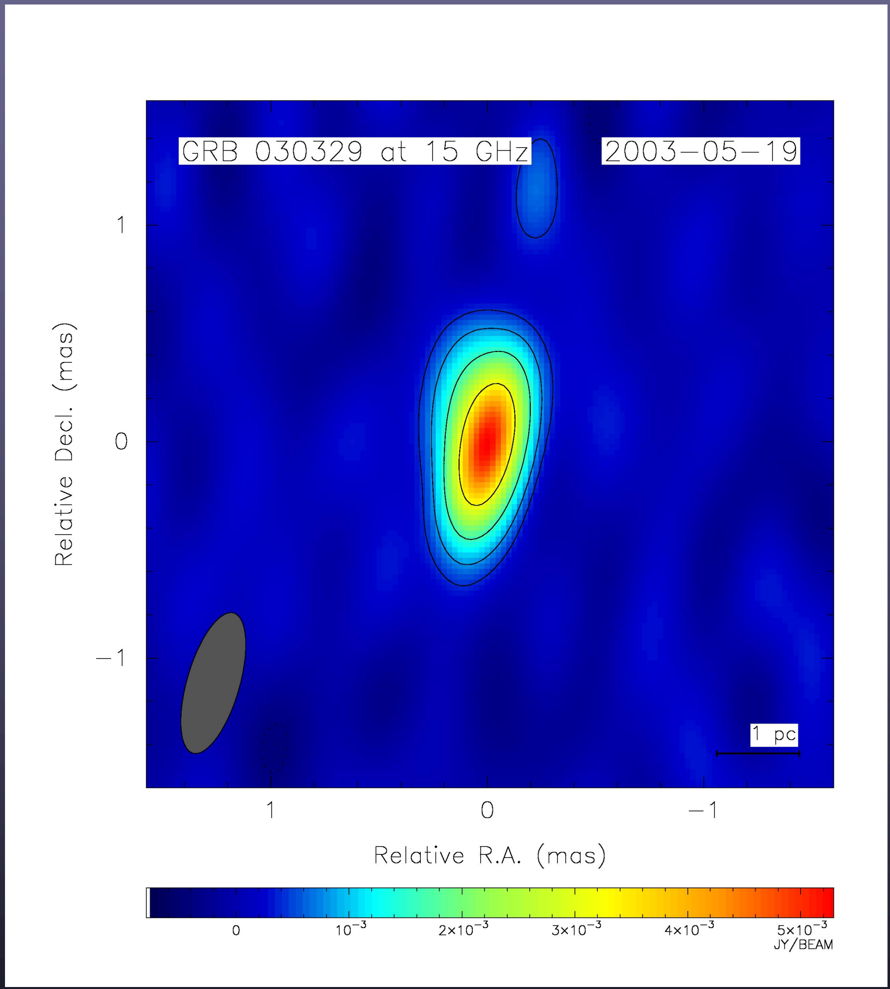
Expansion over 3 years

Apparent velocity ranging from
8c at 25 days to
1.2c after 800 days

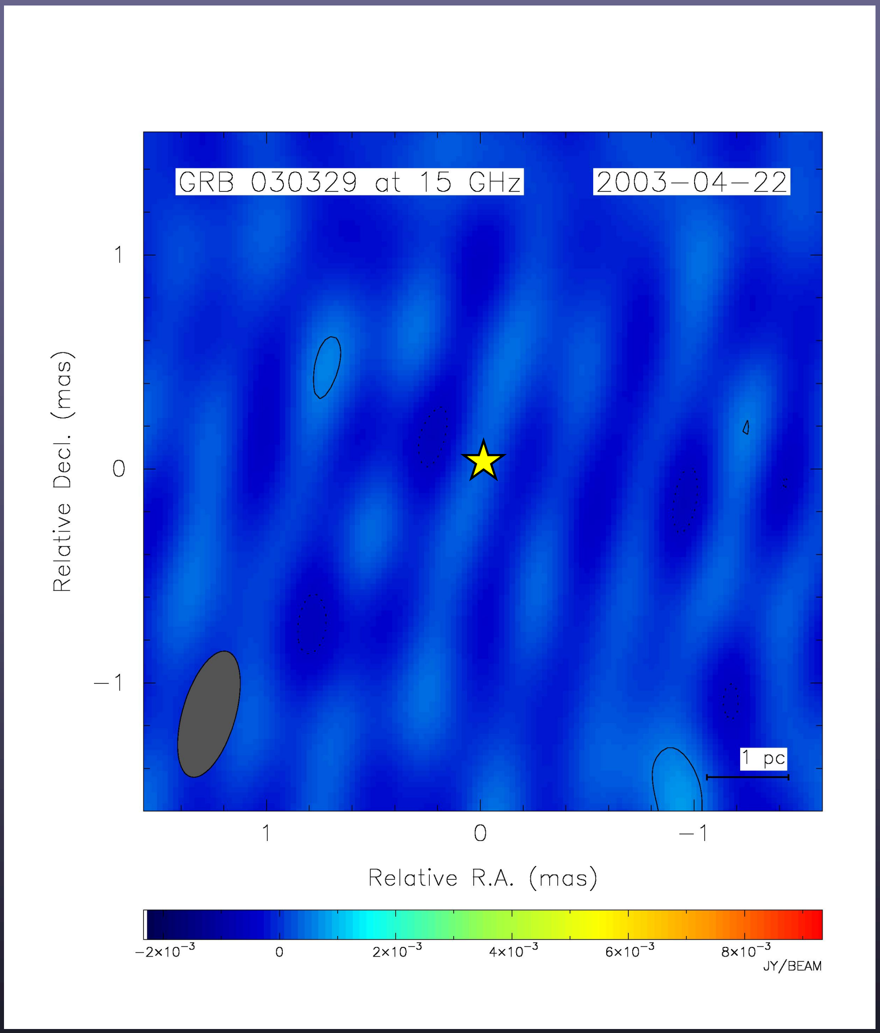


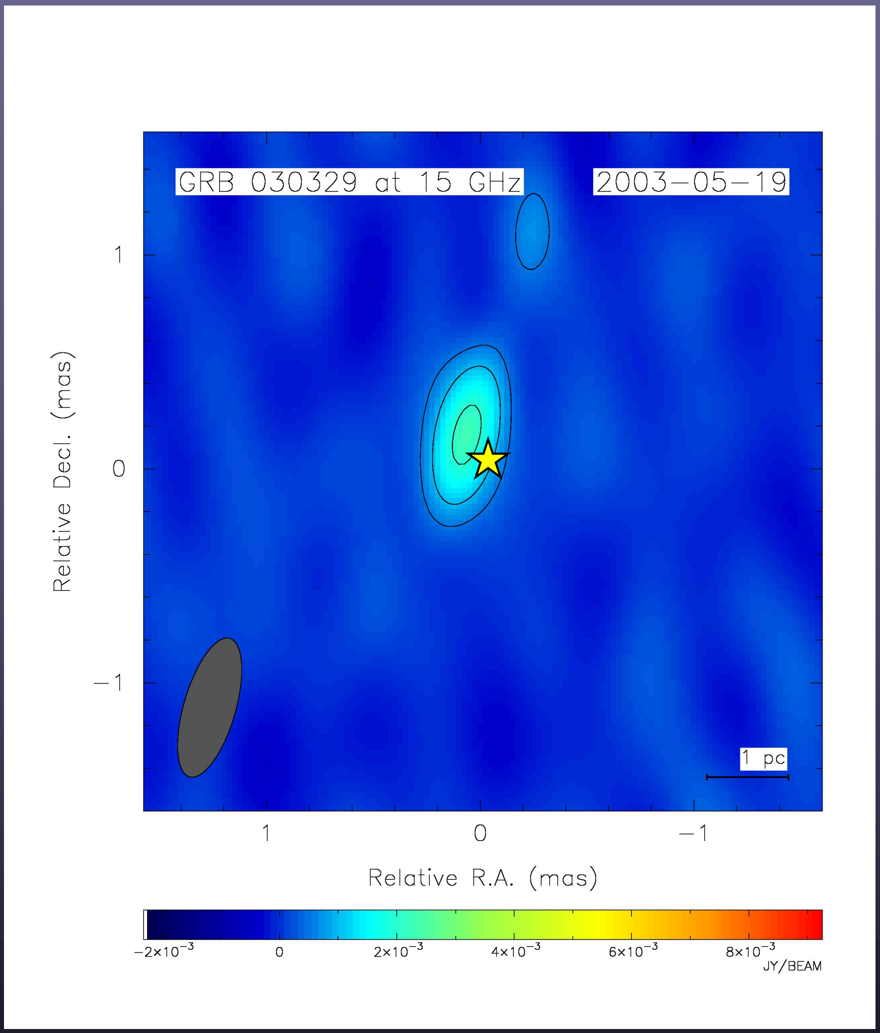
GRB030329


GRB 030329 at 15 GHz
2003−04−22
Relative Decl. (mas)
1
0
−1
1 pc
1
0
−1
Relative R.A. (mas)
−2×10⁻³
0
2×10⁻³
4×10⁻³
6×10⁻³
8×10⁻³
0.01
0.012
0.014
JY/BEAM


GRB 030329 at 15 GHz
2003−05−19
Relative Decl. (mas)
1
0
−1
1 pc
1
0
−1
Relative R.A. (mas)
0
10⁻³
2×10⁻³
3×10⁻³
4×10⁻³
5×10⁻³
JY/BEAM

GRB030329 subtracted

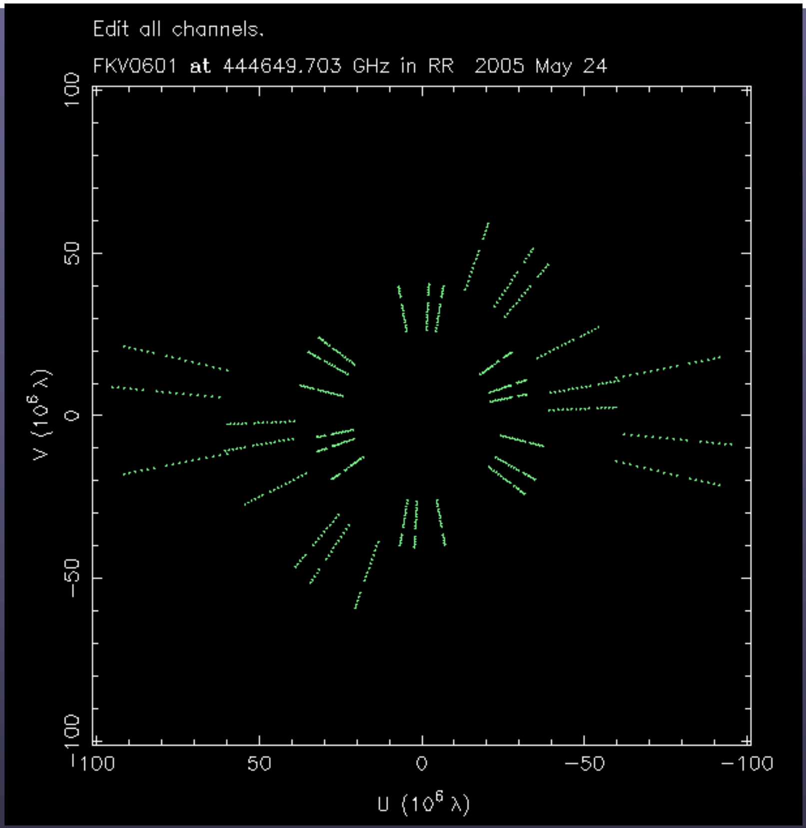


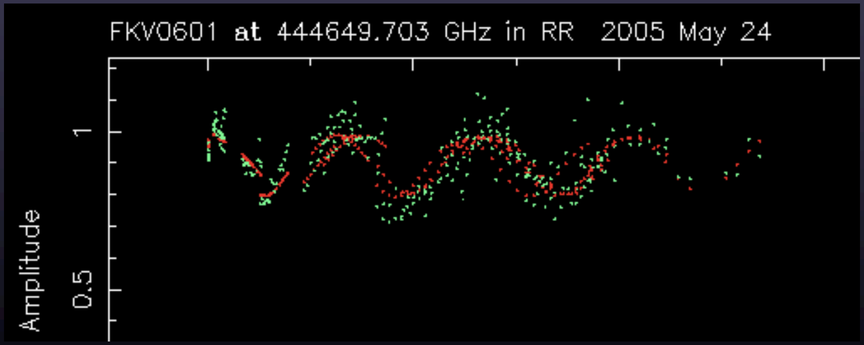


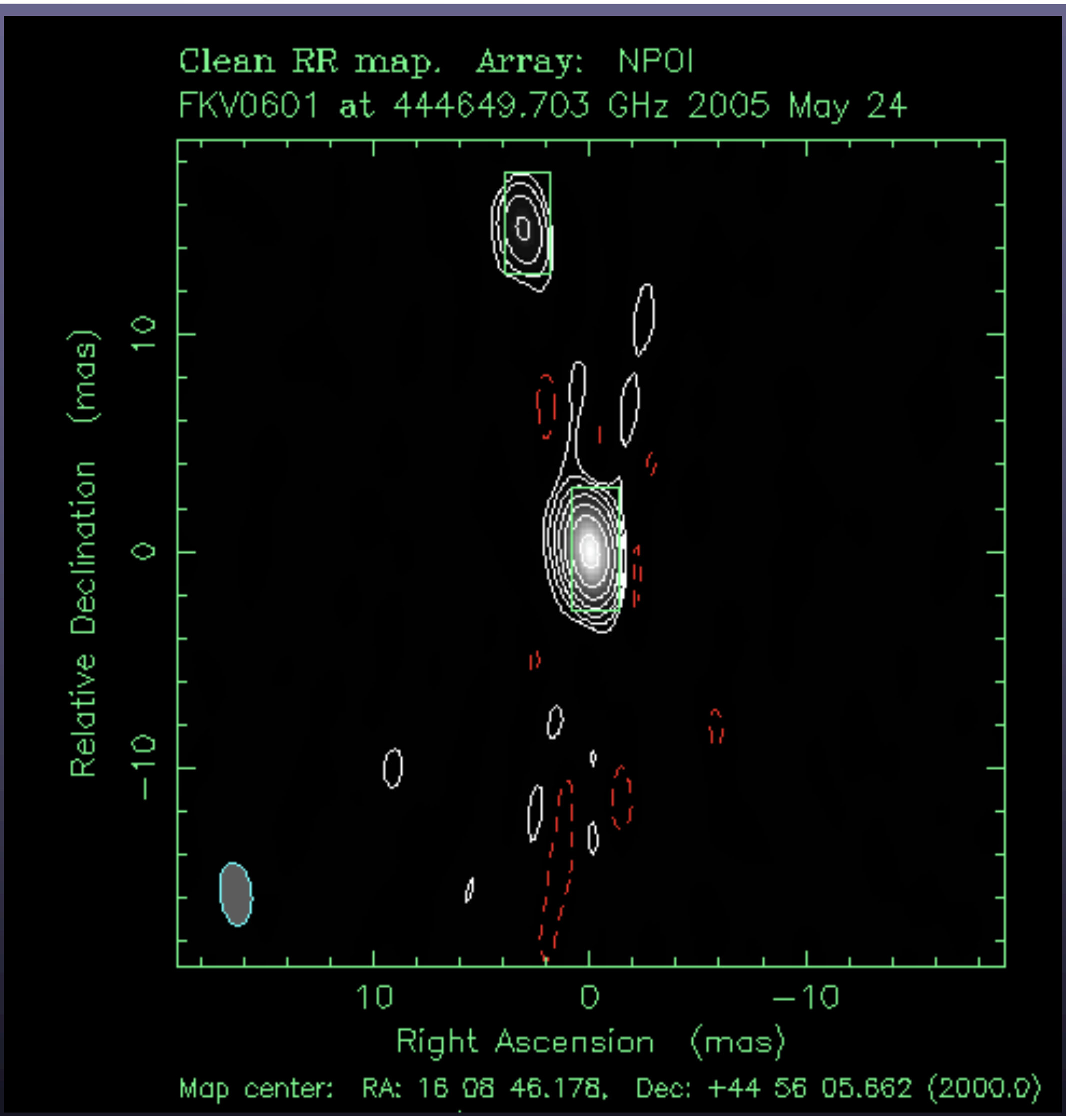
Applications: A Binary Star

- Binary Stars
  - Many stars are in binary systems
  - Orbital parameters can be used to measure stellar masses
  - Astrometry can provide direct distances via parallax and proper motions.
- Application of model fitting
  - Optical interferometry provides sparse visibility coverage
  - Small number of components
  - Need error estimates.
- Example: NPOI observations of Phi Herculis (Zavala et al. 2006)
  - Multiple observations map out the orbit

NPOI Observations of Phi Her







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  - VLA configuration changes: different HA on each day
  - Other sources in the field

Summary

- For simple sources observed with high SNR, much can be learned about the source (and observational errors) by inspection of the visibilities.
- Even if the data cannot be calibrated, the **closure quantities** are good observables, and modelfiting can help to interpret them.
- Quantitative data analysis is best regarded as an exercise in **statistical inference**, for which the maximum likelihood method is a general approach.
- For gaussian errors, the ML method is the **method of least squares**.
- Visibility data (usually) have uncorrelated gaussian errors, so analysis is most straightforward in the (*u,v*) plane.
- Consider visibility analysis when you want a quantitative answer (with error estimates) to a simple question about a source.
- Visibility analysis is inappropriate for large problems (many data points, many parameters, correlated errors); standard imaging methods can be much faster.

Further Reading

- http://www.nrao.edu/whatisra/
- www.nrao.edu

- Synthesis Imaging in Radio Astronomy
- ASP Vol 180, eds Taylor, Carilli & Perley
- Numerical Recipes, Press et al. 1992