





Imaging and non-Imaging analysis

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#### Announcements

- VLA/LWA tour on Wednesday, March 26 departing 7am!
- Sign up sheet being passed around
- For credit. If you can't go provide a 4 page paper about a radio telescope.
- HW5 is due on Wednesday
- Start playing around with aips and HW6, this is going to take a while
- LST for LWA-SV:
- https://lwalab.phys.unm.edu/OpScreen/lwasv/index.html

#### Announcements

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How do we go from the measurement of the coherence function (the Visibilities) to the images of the sky?

First half of the lecture: Imaging

Measured Visibilities  $\rightarrow$  Dirty Image



#### Plan for the lecture-II

# Second part of the lecture: Deconvolution Dirty image → Model of the sky (+ residuals)



#### Plan for the lecture-III

## • Third part of the lecture: Modelfitting Measured Visibilities → Model of the sky



## Imaging

Interferometers are indirect imaging devices

$$V^{\circ}(u,v,w) = \int \int I(l,m) e^{2\pi i \left[ul + vm + w\left(\sqrt{1 - l^2 - m^2} - 1\right)\right]} \frac{dldm}{\sqrt{1 - l^2 - m^2}}$$

•For small w (small max. baseline) or small field of view ( $l^2 + m^2 << 1$ ) l(l,m) is 2D Fourier transform of V(u,v) $V^{\circ}(u,v) = \int \int \int I(l,m)e^{2\pi i [ul+vm]} dl dm$ 

$$I^{\circ}(u,v) = \int \int I(l,m)e^{2\pi \iota [ul+vm]} dl dm$$

$$V^{\circ}(u,v) \rightleftharpoons I(l,m)$$

#### Imaging: Ideal 2D Fourier relationship



• This is true ONLY if V is measured for all (u,v)!

## Imaging: (u,v) plane sampling

 With limited number of antennas, the (u,v) plane is sampled at discrete points:



Effect of sampling the (u,v) plane:

$$I^{d}(l,m) = FT^{-1}\left[V^{\circ}(u,v)S(u,v)\right]$$

Using the Convolution Theorem:

$$I^{d}(l,m) = B(l,m) \star I^{\circ}(l,m)$$

The Dirty Image (I<sup>d</sup>) is the convolution of the True Image (I<sup>o</sup>) and the Dirty Beam/Point Spread Function (B)

$$B = FT^{-1}(S)$$

In practice

 $I^{d} = B^{*}I^{o} + B^{*}I^{N}$  where  $I^{N} = FT^{-1}$ (Vis. Noise)

• To recover *I*°, we must deconvolve *B* from *I*<sup>d</sup>. The algorithm must also separate *B\*I*° from *B\*I*<sup>N</sup>.

## The Dirty Image



#### Making of the Dirty Image

- Fast Fourier Transform (FFT) is used for efficient Fourier transformation. It however requires regularly spaced grid of data.
- Measured visibilities are irregularly sampled (along u,v tracks).
- Convolutional gridding is used to effectively interpolate the visibilities everywhere and then resample them on a regular grid (the Gridding operation)

 PSF is a weighted sum of cosines corresponding to the measured fourier components:

$$B(l,m) = \frac{\sum_{k} W_k \cos(u_k l + v_k m)}{\sum_{k} W_k}$$

Visibility weights  $(w_i)$  are also gridded on a regular grid and FFT used to compute the Dirty Beam (aka the PSF).

- The peak of the PSF is normalized to 1.0
- The 'main lobe' has a size  $\Delta x \sim 1/u_{max}$  and  $\Delta y \sim 1/v_{max}$ This is the 'diffraction limited' resolution (the Clean Beam) of the telescope.

#### **Dirty Beam: Interesting properties**

## Side lobes extend indefinitely

• RMS ~ 1/N where N = No. of antennas



#### Close-in side lobes of the PSF

 Close-in sidelobes of the PSF are controlled by the (u,v) coverage envelope.



# Close-in side lobes: VLA (u,v) coverage



#### PSF forming: Weighting...

. Weighting function  $(W_k)$  can be chosen to modify resolution and side lobes

Natural Weighting

## $W_k = 1/\sigma_k^2$ where $\sigma_k^2$ is the RMS noise

→ Best RMS across the image.

- Jarge scales (smaller baselines) have higher weights.
- Jeffective resolution less than the inverse of the longest baseline.

## ...Weighting...

Uniform weighting

 $W_k = 1/\rho(u_k, v_k)$  where  $\rho(u_k, v_k)$  is the density of uppoints in the *k*<sup>th</sup> cell.

- Short baselines (large scale features in the image) are weighted down.
- Relatively better resolution
- Increases the RMS noise.
- Super uniform weighting:

Consider density over larger region. Minimize side lobes locally.



## ...Weighting

## • Robust/Briggs weighting: $W_k = 1/[S.\rho(u_k, v_k) + \sigma_k^2]$

 Parameterized filter – allows continuous variation between optimal resolution (uniform weighting) and optimal noise (natural weighting).



## **Examples of weighting**



#### **PSF Forming: Tapering**

 The PSF can be further controlled by applying a tapering function on the weights (e.g. such that the weights smoothly go to zero beyond the maximum baseline).

 $W_k = T(u_k, v_k) W_k(u_k, v_k)$ 

 Bottom line on weighting/tapering: These help a bit, but imaging quality is limited by the deconvolution process!

## Test 1 results



#### The missing information

- As seen earlier, not all parts of the uv-plane are sampled – the 'invisible distribution'
- 1. "Central hole" below  $u_{min}$  and  $v_{min}$ :
  - Image plane effect: Total integrated power is not measured.



- Upper limit on the largest scale in the image plane.
- 2. No measurements beyond  $u_{max}$  and  $v_{max}$ :
  - Size of the main lobe of the PSF is finite (finite resolution).
- 3. Holes in the (u,v) plane:
  - Contribute to the side lobes of the PSF.



#### Recovering the missing information

- For information beyond the max. baseline, one requires extrapolation. That's un-physical (unconstrained).
- Information corresponding to the "central hole": possible, but difficult (need extra information).
- Information corresponding to the (u,v) holes: requires interpolation. The measurements provide constraints – hence possible. But non-linear methods necessary.

#### **Deconvolution = interpolation in the visibility plane.**

#### Prior knowledge about the sky

- What can we assume about the sky emission:
  - 1. Sky does not look like cosine waves
  - 2. Sky brightness is positive (but there are exceptions)
  - 3. Sky is a collection of point sources (weak assertion)
  - 4. Sky could be smooth
  - 5. Sky is mostly blank (sometimes justifies "boxed" deconvolution)
- Non-linear deconvolution algorithms search for a model image I<sup>M</sup> such that the residual visibilities V<sup>R</sup>=V<sup>o</sup>-V<sup>M</sup> are minimized, subject to the constraints given by the (assumed) prior knowledge.

## The classic Clean algorithm (Hogborn, 1974)

- Prior knowledge:
  - sky is composed of point sources
  - mostly blank

#### • Algorithm:

- **1. Search for the peak in the dirty image.**
- 2. Add a fraction g (loop gain) of the peak value to  $I^{M}$ .
- 3. Subtract a scaled version of the PSF from the position of the peak.

 $I_{i+1}^{R} = I_{i}^{R} - g.B.max(I_{i}^{R})$ 

- 4. If residuals are not "noise like", goto 1.
- 5. Smooth *I*<sup>M</sup> by an estimate of the main lobe (the "clean beam") of the PSF and add the residuals to make the "restored image"

- It is a steepest descent minimization.
- Model image is a collection of delta functions a scale insensitive algorithm.
- A least square fit of sinusoids to the visibilities which is proved to converge (Schwarz 1978).
- Stabilized by keeping a small loop gain (usually g=0.1-0.2).
- Stopping criteria: either the max. iterations or max. residuals some multiple of the expected peak noise.
- Search space constrained by user defined windows.
- x Ignores coupling between pixels (extended emission) – assumes an orthogonal search space.

## Clean: Model



## **Clean: Restored**



## **Clean: Residual**



## **Clean: Model visibilities**

Model Vis.





## Role of boxes

• Limit the search for components to only parts of the image.

A way to regularize the deconvolution process.

- . Useful when small no. of visibilities (e.g. VLBI/snapshots).
- Do not over-Clean within the boxes (over-fitting).



- . Deeper Clean with no/loose boxes and lower loop gain can achieve similar (more objective) results.
- Stop when Cleaning within the boxes has no global effect (insignificant coupling of pixels due to the PSF).

## **Inspecting Visibility Data**

#### Useful displays

- Sampling of the (u,v) plane
- Amplitude and phase vs. radius in the (u,v) plane
- Amplitude and phase vs. time on each baseline
- Amplitude variation across the (u,v) plane
- Projection onto a particular orientation in the (u,v) plane

#### Example: 2021+614

- GHz-peaked spectrum radio galaxy at z=0.23
- A VLBI dataset with 11 antennas from 1987
- VLBA only in 2000

## Sampling of the (u,v) plane



## Visibility versus (u,v) radius



#### **Visibility versus time**



## Amplitude across the (u,v) plane



## **Projection in the (u,v) plane**









#### **Simple models**



Visibility at short baselines contains little information about the profile of the source.

## **Trial model**

By inspection, we can derive a simple model:

Two equal components, each 1.25 Jy, separated by about 6.8 milliarcsec in p.a. 33°, each about 0.8 milliarcsec in diameter

(gaussian FWHM) To be refined later...





# **Projection in the (u,v) plane**



## **Practical model fitting: 2021**



! Flux (Jy)	Radius (mas)	Theta (deg)	Major (mas)	Axial ratio	Phi (deg)	Т
1.15566	4.99484	32.9118	0.867594	0.803463	54.4823	1
1.16520	1.79539	-147.037	0.825078	0.742822	45.2283	1

## 2021: model 2



Model fitting 2021



## 2021: model 3

![](_page_47_Figure_1.jpeg)

#### Imaging as an Inverse Problem

- In synthesis imaging, we can solve the **forward problem**: given a sky brightness distribution, and knowing the characteristics of the instrument, we can predict the measurements (visibilities), within the limitations imposed by the noise.
- The **inverse problem** is much harder, given limited data and noise: the solution is rarely unique.
- A general approach to inverse problems is **model fitting**. See, e.g., Press et al., *Numerical Recipes*.
  - 1. Design a model defined by a number of adjustable parameters.
  - 2. Solve the forward problem to predict the measurements.
  - 3. Choose **a figure-of-merit** function, e.g., rms deviation between model predictions and measurements.
  - 4. Adjust the parameters to **minimize the merit function**.
- Goals:
  - 1. Best-fit values for the parameters.
  - 2. A measure of the goodness-of-fit of the optimized model.
  - 3. Estimates of the uncertainty of the best-fit parameters.

#### **Uses of model fitting**

# Model fitting is most useful when the brightness distribution is simple.

- Checking amplitude calibration
- Starting point for self-calibration
- Estimating parameters of the model (with error estimates)
- In conjunction with CLEAN or MEM
- In astrometry and geodesy

#### Programs

- AIPS UVFIT
- Difmap (Martin Shepherd)

#### **Parameters**

#### Example

- Component position: (x, y) or polar coordinates
- Flux density
- Angular size (e.g., FWHM)
- Axial ratio and orientation (position angle)
  - For a non-circular component
- 6 parameters per component, plus a "shape"
- This is a conventional choice: other choices of parameters may be better!
- (Wavelets; shapelets\* [Hermite functions])
  - \* Chang & Refregier 2002, ApJ, 570, 447

#### **Limitations of least squares**

Assumptions that may be violated

- The model is a good representation of the data
  - Check the fit
- The errors are gaussian
  - True for real and imaginary parts of visibility
  - Not true for amplitudes and phases (except at high SNR)
- The variance of the errors is known
  - Estimate from T<sub>sys</sub>, rms, etc.
- There are no systematic errors
  - Calibration errors, baseline offsets, etc. must be removed before or during fitting
- The errors are uncorrelated
  - Not true for closure quantities
  - Can be handled with full covariance matrix

## **Applications: Gravitational Lenses**

#### **Gravitational Lenses**

- Single source, multiple images formed by intervening galaxy.
- Can be used to map mass distribution in lens.
- Can be used to measure distance of lens and  $H_0$ : need redshift of lens and background source, model of mass distribution, and a **time delay**.

#### Application of model fitting

- Lens monitoring to measure flux densities of components as a function of time.
- Small number of components, usually point sources.
- Need error estimates.

#### Example: VLA monitoring of B1608+656 (Fassnacht et al. 1999, ApJ)

- VLA configuration changes: different HA on each day
- Other sources in the field

## VLA image of 1608

![](_page_53_Figure_1.jpeg)

## **1608 monitoring results**

B - A = 31 daysB - C = 36 days  $H_0 = 59 \pm 8 \text{ km/s/Mpc}$ 

![](_page_54_Figure_2.jpeg)