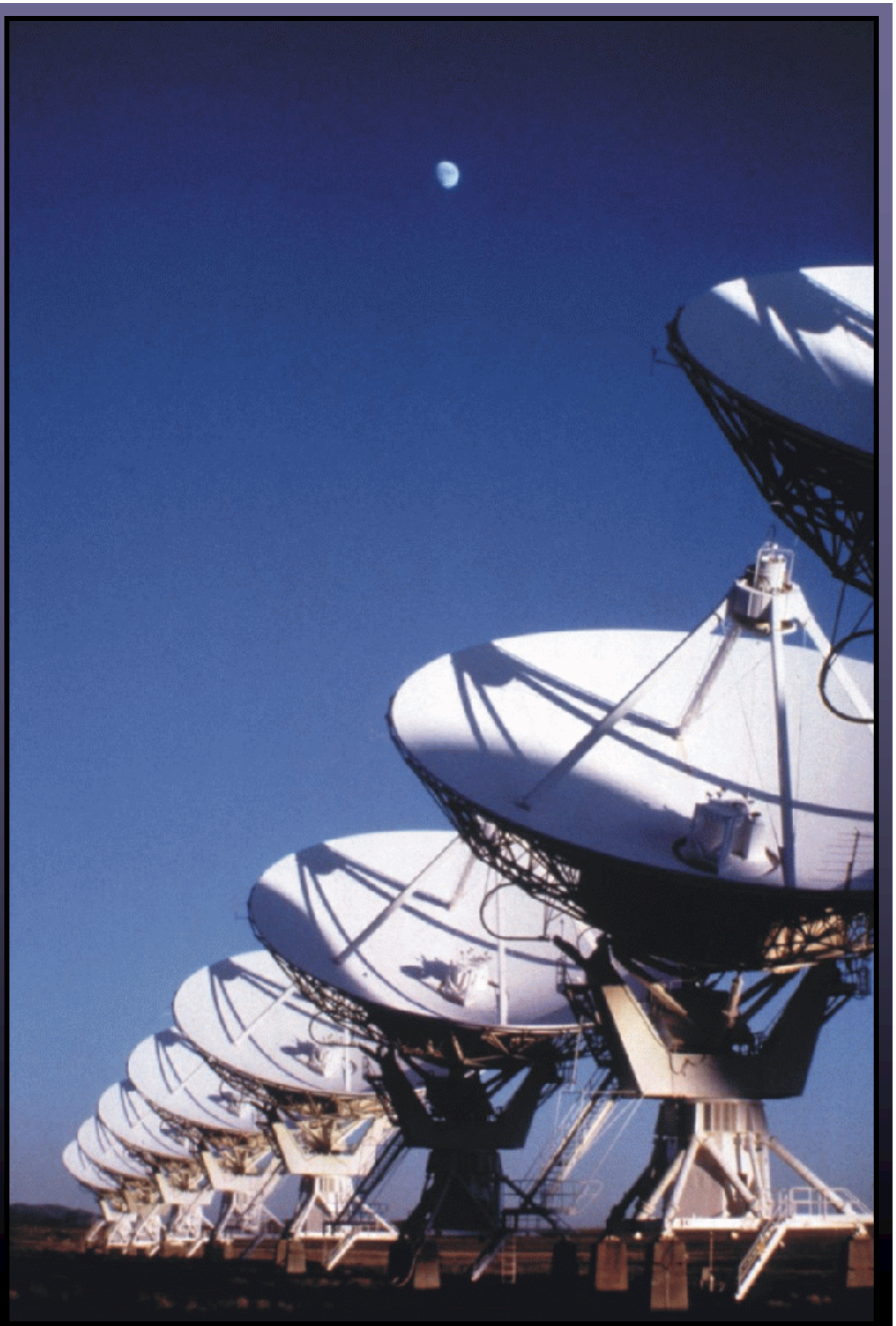
# Polarization in Interferometry

Greg Taylor

*University of New Mexico*

***Astronomy 423 at UNM***

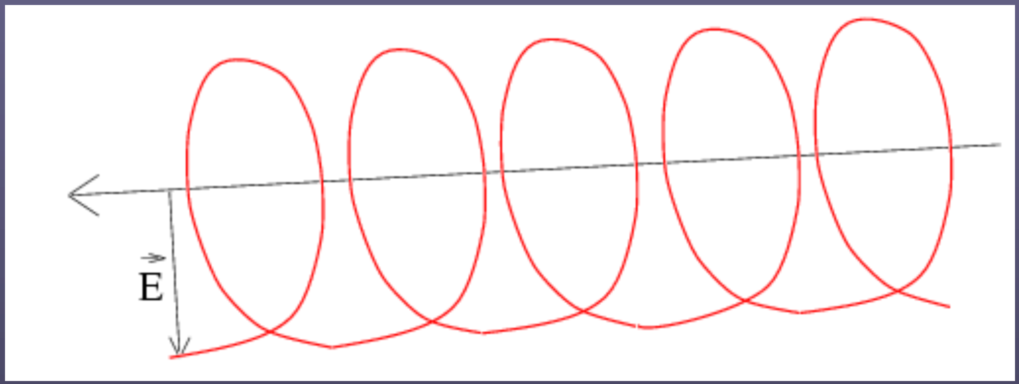
***Radio Astronomy***



# Projects

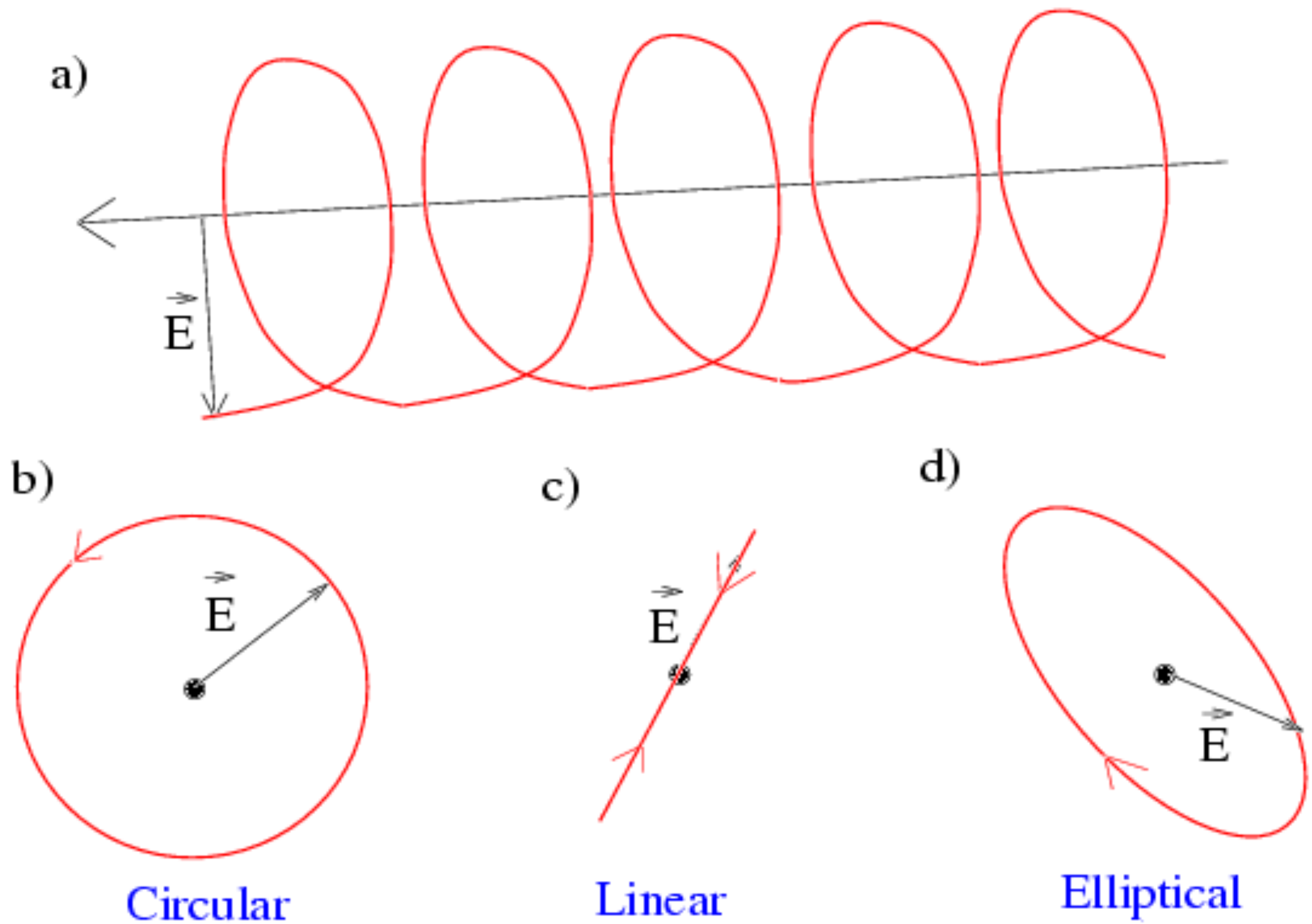
- You can start now. See handouts on class web page
- Imaging tutorial on March 22nd, short tutorial today
- HW6 (VLA calibration) due on March 24
- Writeup – look at the example. Be sure to include:
  - Title, Authors, Abstract, Introduction, Observations, Analysis, Results, Discussion, Conclusions and Future Work, Acknowledgements, References
  - One write-up for each project. Everybody in the group should be involved
- Presentation – Will be in powerpoint, keynote, or similar. Includes:
  - Title, Authors, Acknowledgements, Introduction, Observations, Analysis, Results, Discussion, Conclusions and Future Work
  - Plan for ~30 minutes total with everybody getting involved in the presentation
- Outline due Monday, April 12

# Outline




- What is polarized light?
- What fun science can be done with polarimetry?
- How do interferometers measure polarization?
- How do you calibrate and image in full polarization?

# Polarization of Light



# What is Polarized Light?



- Light is oscillating electric and magnetic fields
- Polarization is labeled by the shape of the trace of the tip of the **E** vector
- Each polarization has an orthogonal state
- Incoherent light can contain many polarization states

Stokes Parameters describe partially polarized light

$I = RR + LL$
$Q = RL + LR$
$U = i(LR - RL)$
$V = LL - RR$

For circular feeds

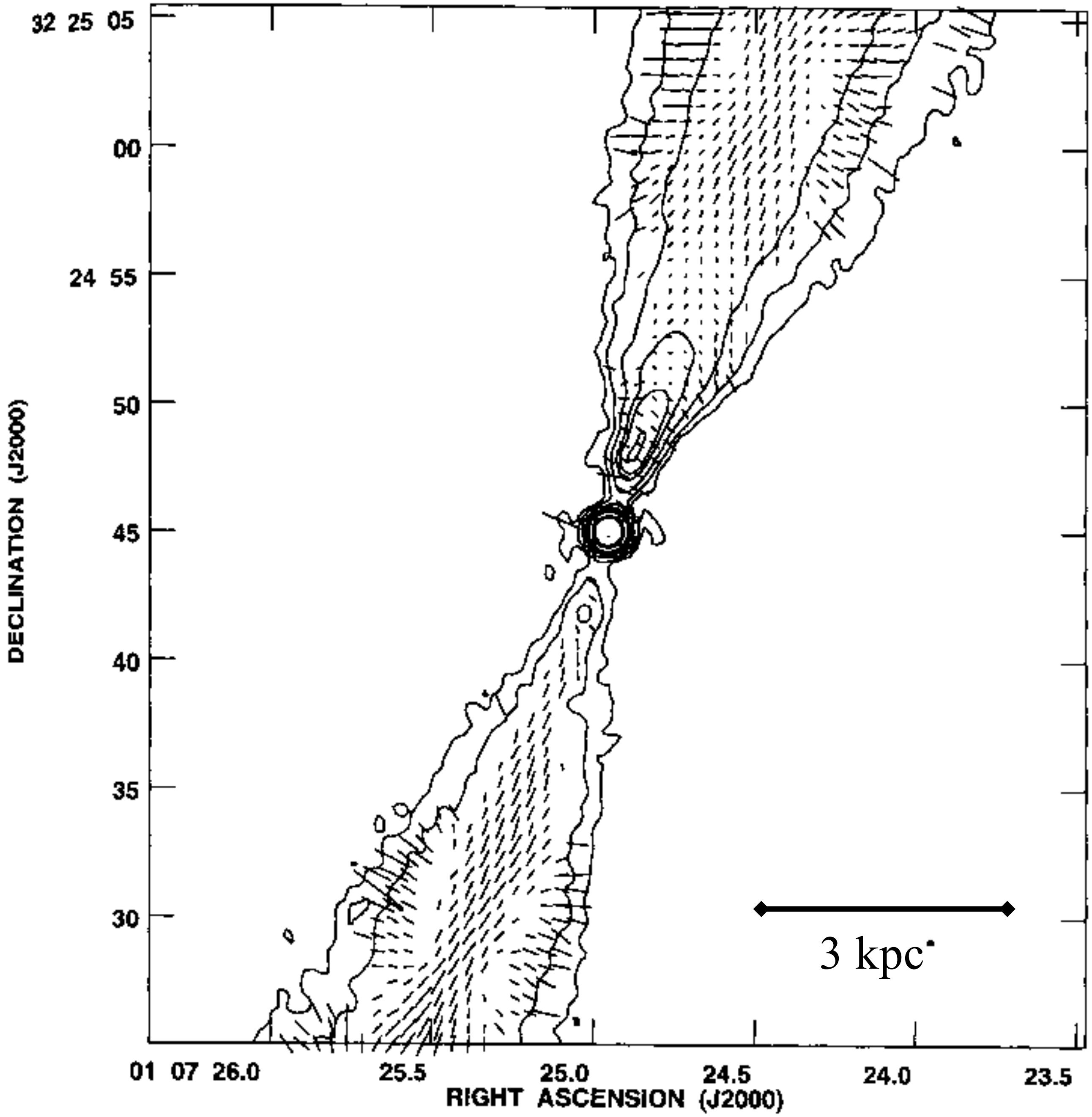
Alternate representation:

- pol. angle (EVPA) $\phi = 0.5 \text{ atan } (U/Q)$
- polarized intensity $p = \text{sqrt}(Q^2 + U^2)$
- fractional linear $m = p / I$
- fractional circular $v = |V| / I$

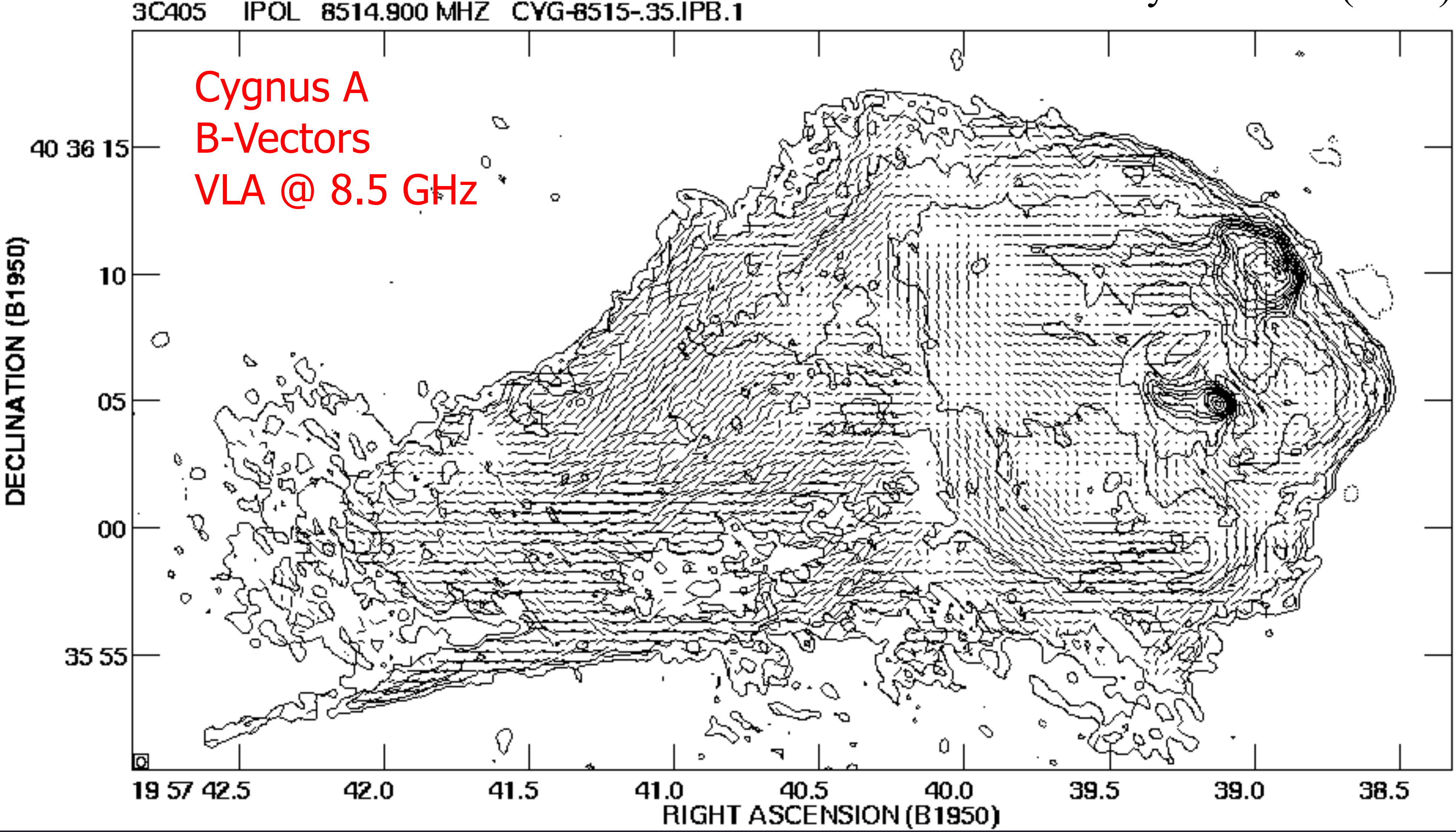
# 3C 31
# VLA @ 8.4 GHz

E-Vectors
Laing (1996)

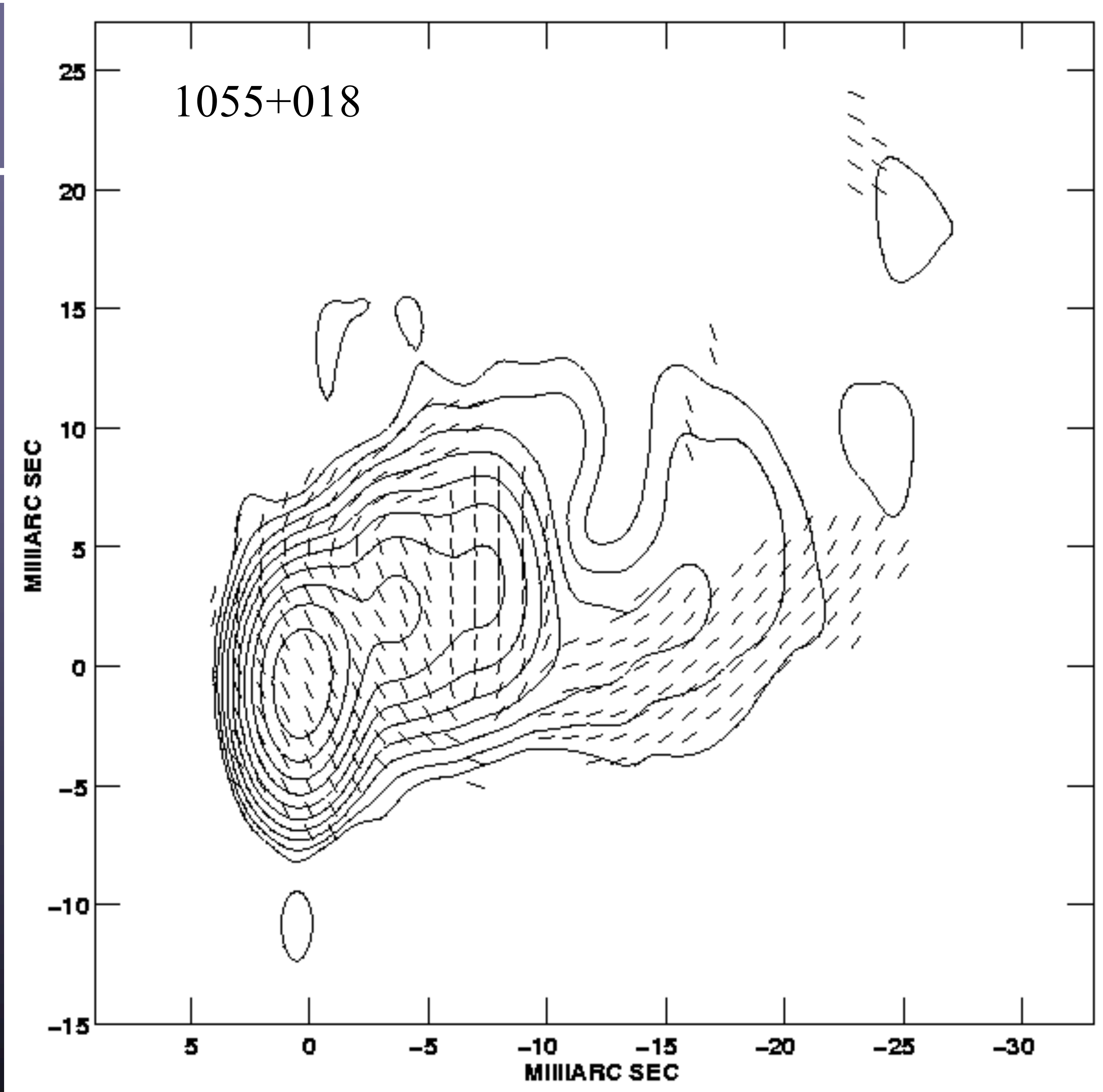




Perley & Carilli (1996)




10 kpc



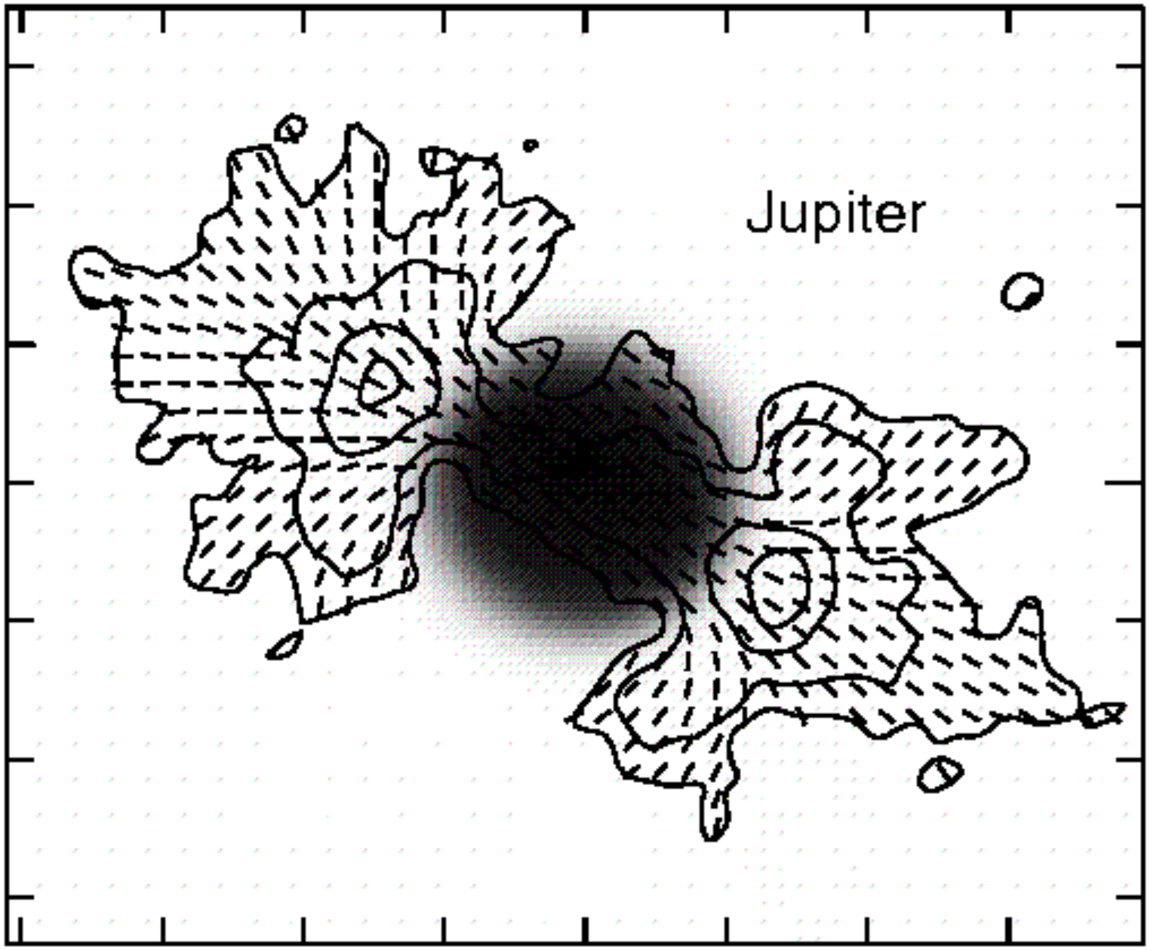

Attridge et al 1999



VLBA @ 5 GHz

# >SNTHS IMAGN SUMMR SCHUL



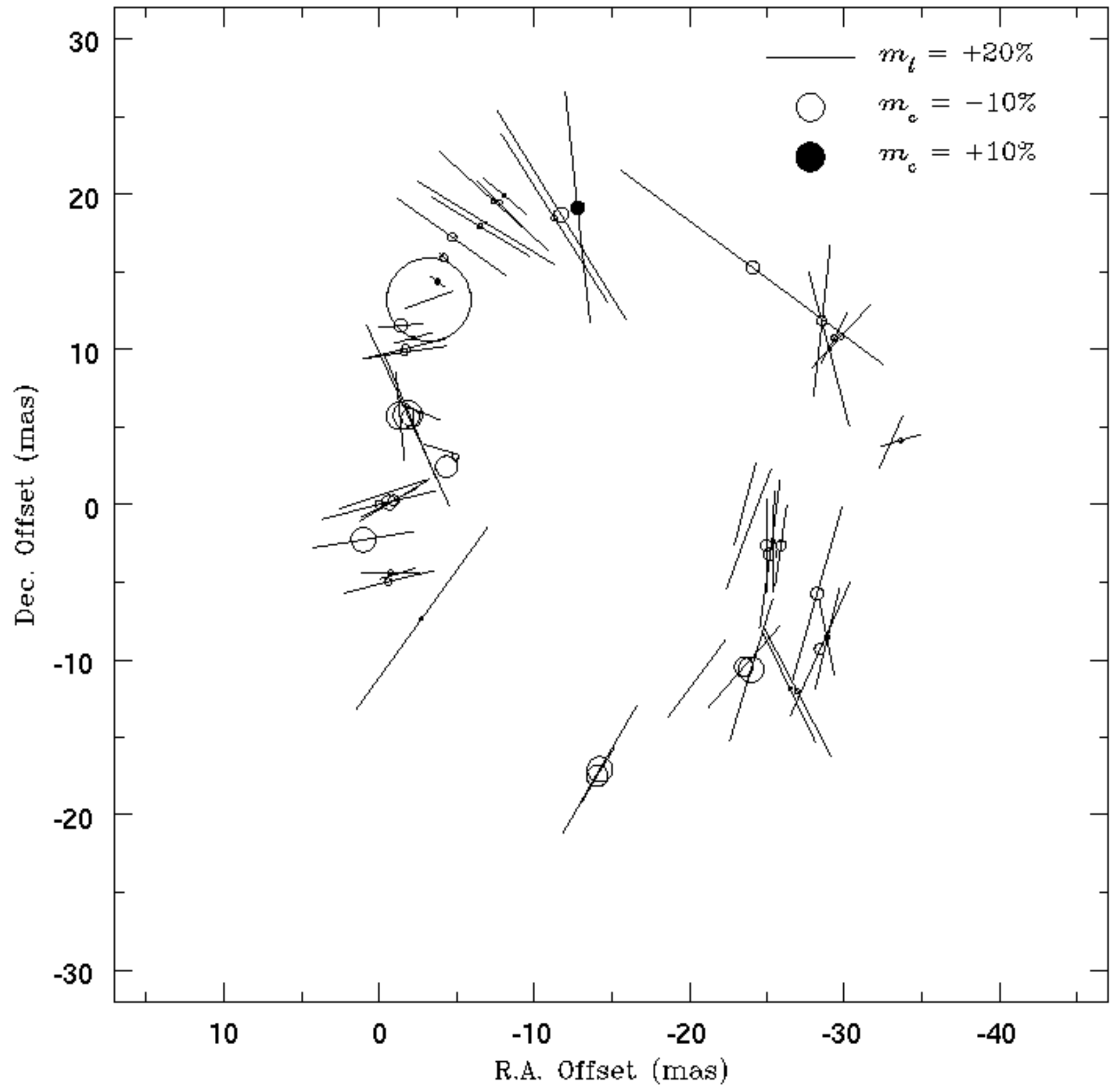



June 20-27, 2000
Socorro, NM, USA

# R Aquirrii

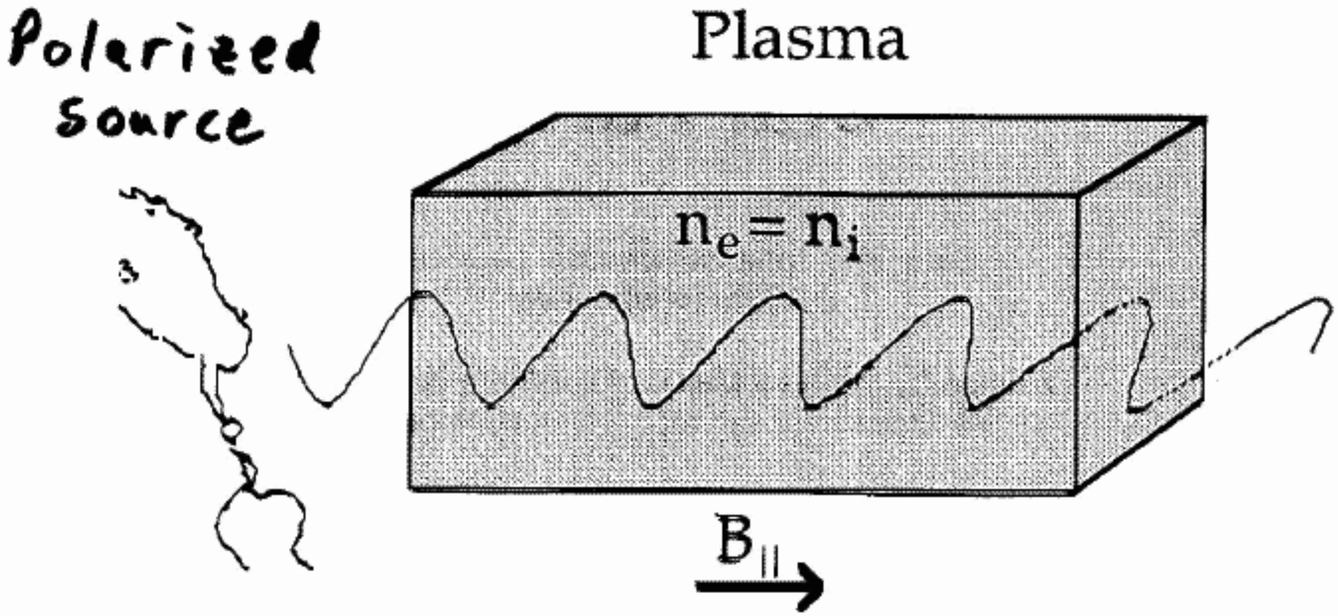
Stellar SiO Masers
Boboltz et al 1998

VLBA @ 43 GHz




(b) Epoch 2 (29 Dec. 1995, $\phi = 0.78$)

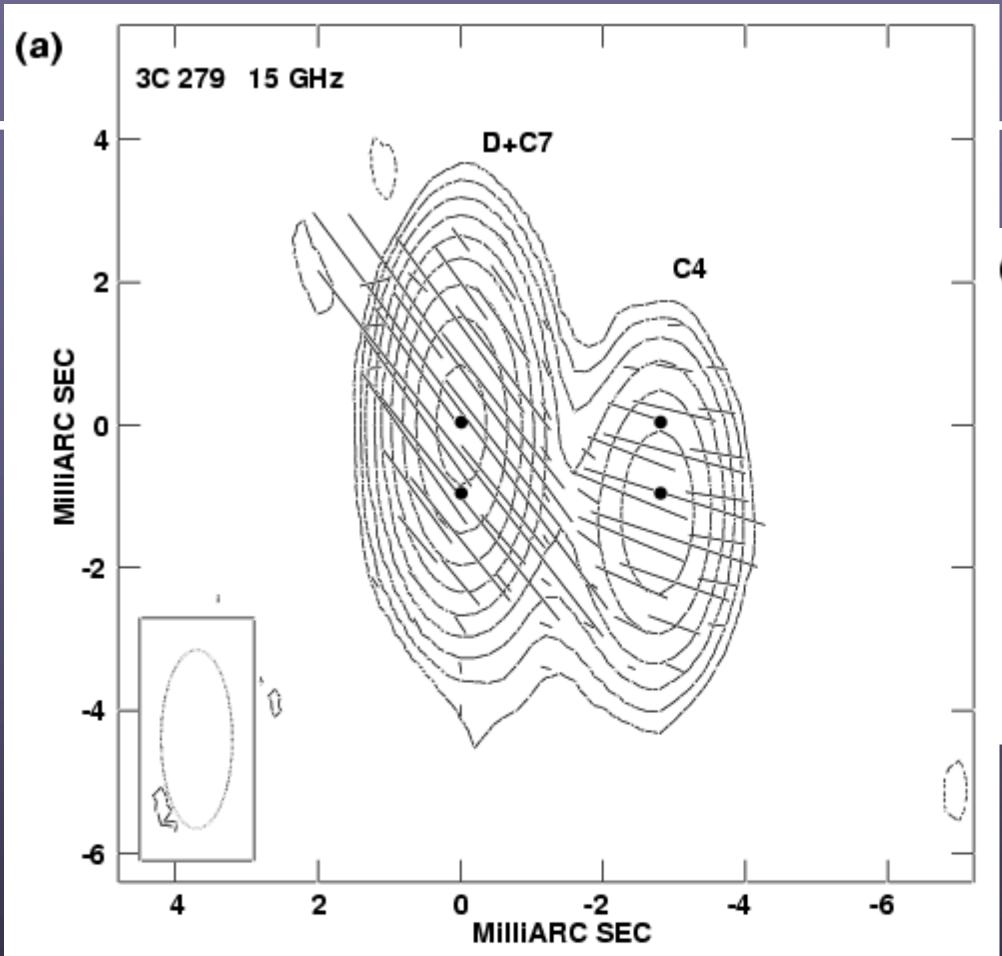
# Faraday Rotation

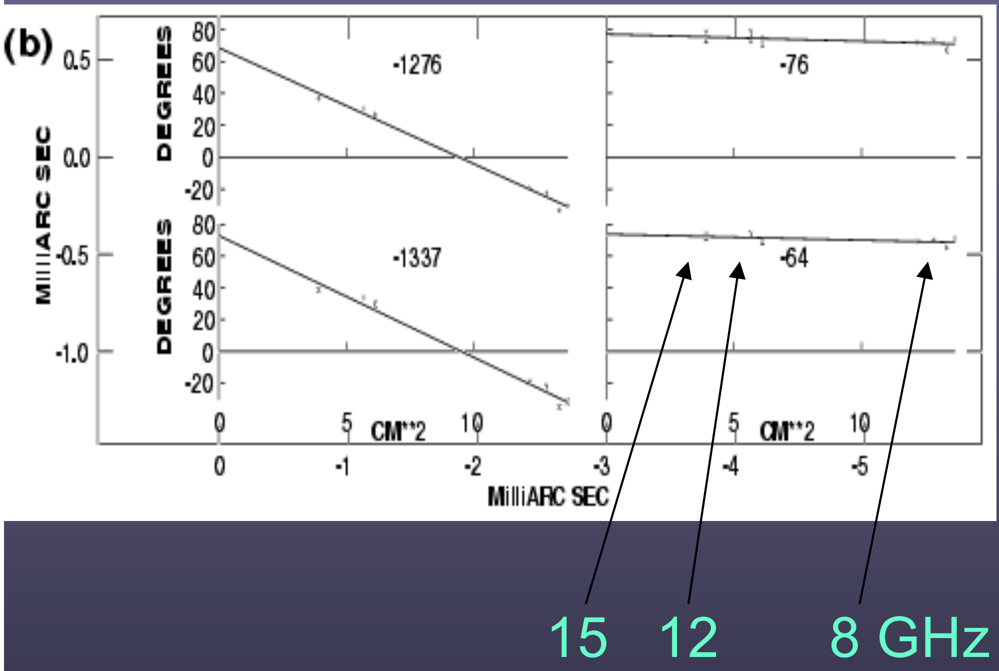


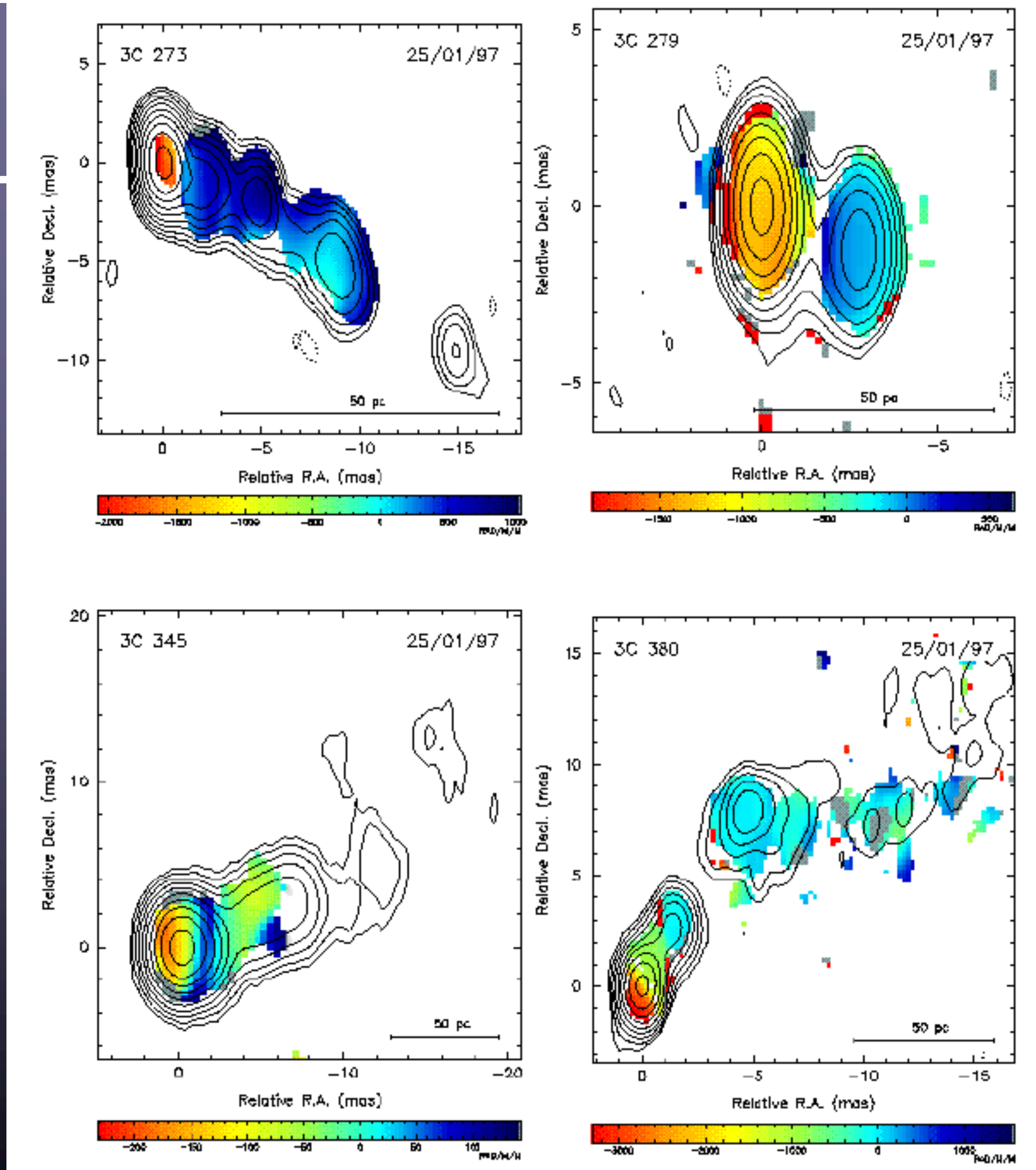

$$\Psi = \Psi_0 + RM\,\lambda^2$$

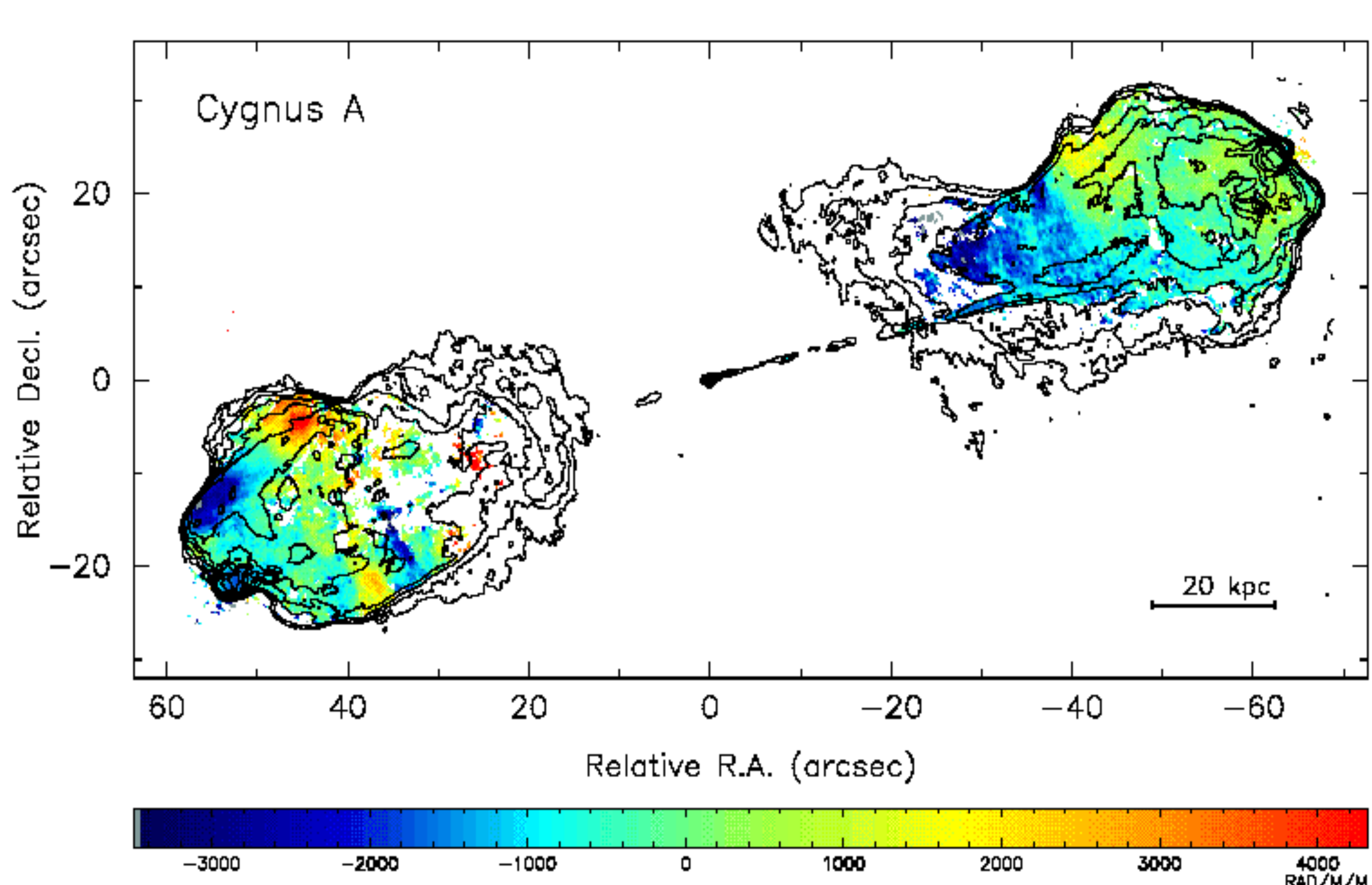
$$RM = 812 \int_0^L n_e B_{||}\, dl \text{ radians/m}^2$$

(L: kpc; $B_{||}$: μGauss; $n_e$: $cm^{-3}$)


(a)
3C 279 15 GHz
D+C7
C4
MilliARC SEC
MilliARC SEC


(b)
DEGREES
-1276
-76
-1337
-64
CM**2
MilliARC SEC
15 12 8 GHz


3C 273
25/01/97
Relative Decl. (mas)
Relative R.A. (mas)
50 pc
3C 279
25/01/97
Relative Decl. (mas)
Relative R.A. (mas)
50 pc
3C 345
25/01/97
Relative Decl. (mas)
Relative R.A. (mas)
50 pc
3C 380
25/01/97
Relative Decl. (mas)
Relative R.A. (mas)
50 pc




See Review of "Cluster Magnetic Fields" by Carilli & Taylor 2002 (ARA&A)

# Antenna Response

- **Jones' matrix:**

$$J_i = G_i D_i P_i.$$

- $G_i$, **is the "gain" given by**

$$G_i = \begin{pmatrix} g_{ip} & 0 \\ 0 & g_{iq} \end{pmatrix},$$

**where** $g_{ip}$ **and** $g_{iq}$ **are complex gain factors for the two orthogonally polarized signals.**

- $D_i$, **models imperfections in the feed polarization response.**

$$D_i = \begin{pmatrix} 1 & d_{ip} \\ -d_{iq} & 1 \end{pmatrix},$$

**where** $d_{ip}$ **and** $d_{iq}$ **are complex "leakage" terms.**

# Antenna Response continued



- $P_i$, includes effects of parallactic angle, $\chi$

  Antennas with equatorial mounts have $\chi = 0$.

  For alt-az mounted antennas, Parallactic angle has an effect on the measured signals which depends on the feed polarization type:

  $$P_i^{+} = \begin{pmatrix} \cos(\chi) & -\sin(\chi) \\ \sin(\chi) & \cos(\chi) \end{pmatrix}$$ for linear or

  $$P_i^{\odot} = \begin{pmatrix} e^{-j\chi} & 0 \\ 0 & e^{j\chi} \end{pmatrix}$$ for circular feeds

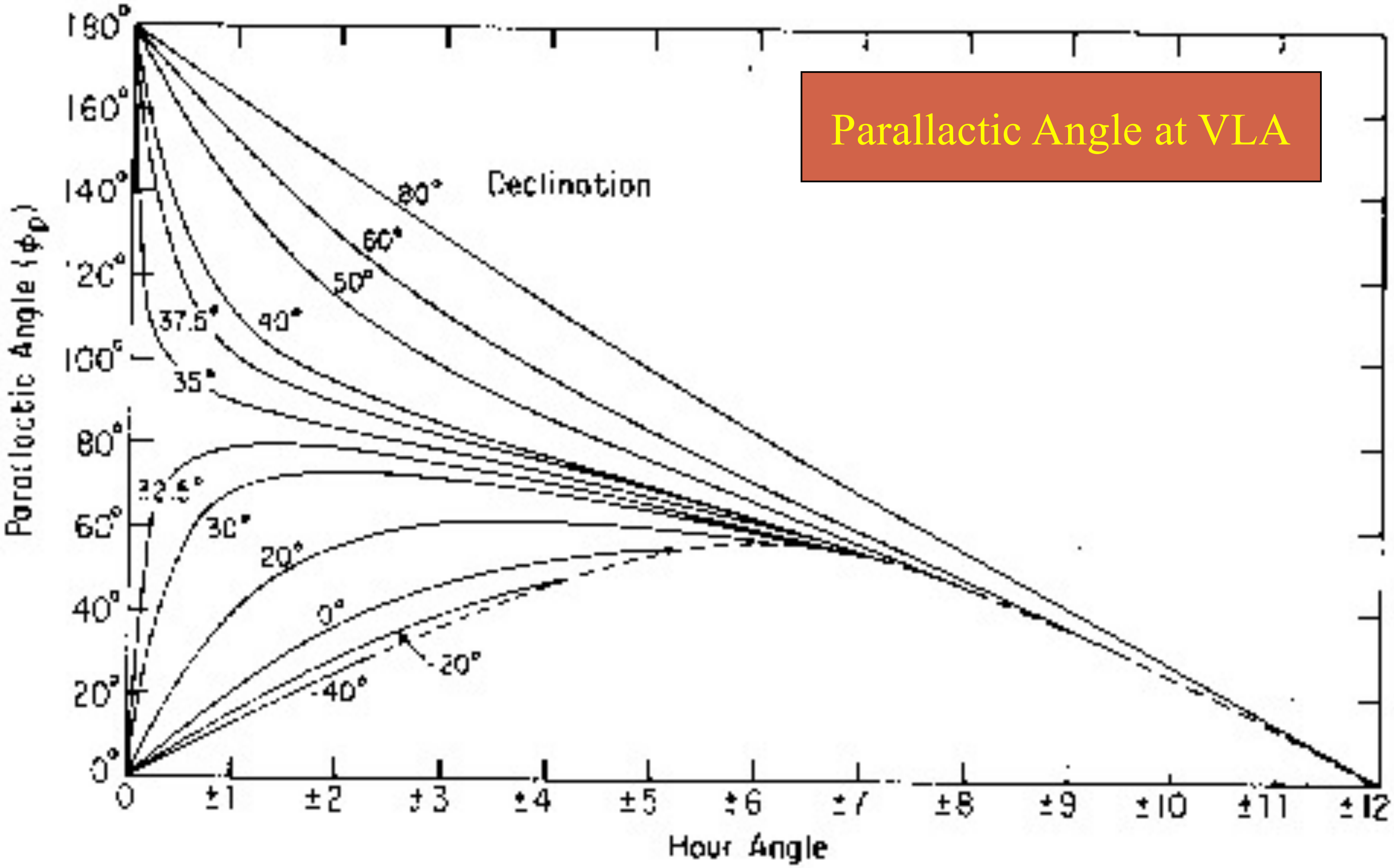
  where $j = \sqrt{-1}$.

  The parallactic angle is given by

  $$\chi = \tan^{-1}\left(\frac{\cos(\lambda)\sin(h)}{\sin(\lambda)\cos(\delta) - \cos(\lambda)\sin(\delta)\cos(h)}\right)$$

  where $\delta$ is the source declination, $\lambda$ is the latitude of the antenna, and $h$ is the source hour angle.

# Parallactic Angle at VLA



# Linearized Interferometer Response

- **For crossed linearly polarized feeds**

$$
\begin{aligned}
v_{pp} &= \tfrac{1}{2} g_{ip} g_{kp}^{*} (I + Q\ \cos 2\chi + U\ \sin 2\chi),\\
v_{pq} &= \tfrac{1}{2} g_{ip} g_{kq}^{*} ((d_{ip} - d_{kq}^{*}) I - Q\ \sin 2\chi + U\ \cos 2\chi + jV),\\
v_{qp} &= \tfrac{1}{2} g_{iq} g_{kp}^{*} ((d_{kp}^{*} - d_{iq}) I - Q\ \sin 2\chi + U\ \cos 2\chi - jV),\\
v_{qq} &= \tfrac{1}{2} g_{iq} g_{kq}^{*} (I - Q\ \cos 2\chi - U\ \sin 2\chi),
\end{aligned}
$$

- **for circularly polarized feeds:**

$$
\begin{aligned}
v_{pp} &= \tfrac{1}{2} g_{ip} g_{kp}^{*} (I + V),\\
v_{pq} &= \tfrac{1}{2} g_{ip} g_{kq}^{*} ((d_{ip} - d_{kq}^{*}) I +\ e^{-2j\chi} (Q + jU)),\\
v_{qp} &= \tfrac{1}{2} g_{iq} g_{kp}^{*} ((d_{kp}^{*} - d_{iq}) I +\ e^{2j\chi} (Q - jU)),\\
v_{qq} &= \tfrac{1}{2} g_{iq} g_{kq}^{*} (I - V).
\end{aligned}
$$

- **Linearized equations may limit dynamic range**

# Calibration

- Corrupting effects:
  - Atmosphere
  - Instrumental gain variations
  - Instrumental imperfections
- Details depend on feed polarization type
- Astronomical and other measurements needed to calibrate
- Astronomical calibration sources
  - Preferably unresolved
  - Synchrotron emission usually has:
    1) weak circular polarization (<0.1%)
    2) significant linear polarization (1–10%)
  - Physically small means time variable

# Calibration of Circular Feeds

- Parallel correlations sensitive to Stokes $I$ & $V$

$$v_{pp} = \tfrac{1}{2} g_{ip} g^*_{kp} (I + V),$$
$$v_{qq} = \tfrac{1}{2} g_{iq} g^*_{kq} (I - V).$$

- Assume $V = 0$ for calibrator
- Can seperate and solve for gains for $p$ and $q$
- Instrumental ($d$) and source polarization ($Q$,$U$) sum of two vectors:

$$v_{pq} = \tfrac{1}{2} g_{ip} g^*_{kq} ((d_{ip} - d^*_{kq}) I + e^{-2j\chi}(Q + jU)),$$
$$v_{qp} = \tfrac{1}{2} g_{iq} g^*_{kp} ((d^*_{kp} - d_{iq}) I + e^{2j\chi}(Q - jU))$$

- Calibrator observations of a range of PA gives clean separation
- Independent gain calibration for $p$ and $q$ allows arbitrary phase offset – refer all phases to same "reference" antenna
- $p - q$ phase difference is that of the reference antenna
- Need observations of calibrator of known polarization angle aka Electric Vector Position Angle (EVPA)

# Calibration procedure (Circular Feeds)



- From frequent observations of a calibrator:

  1) determine $p, q$ gains ($g$) assuming $V = 0$

  2) determine source ($Q, U$) and instrumental ($d$) polarization

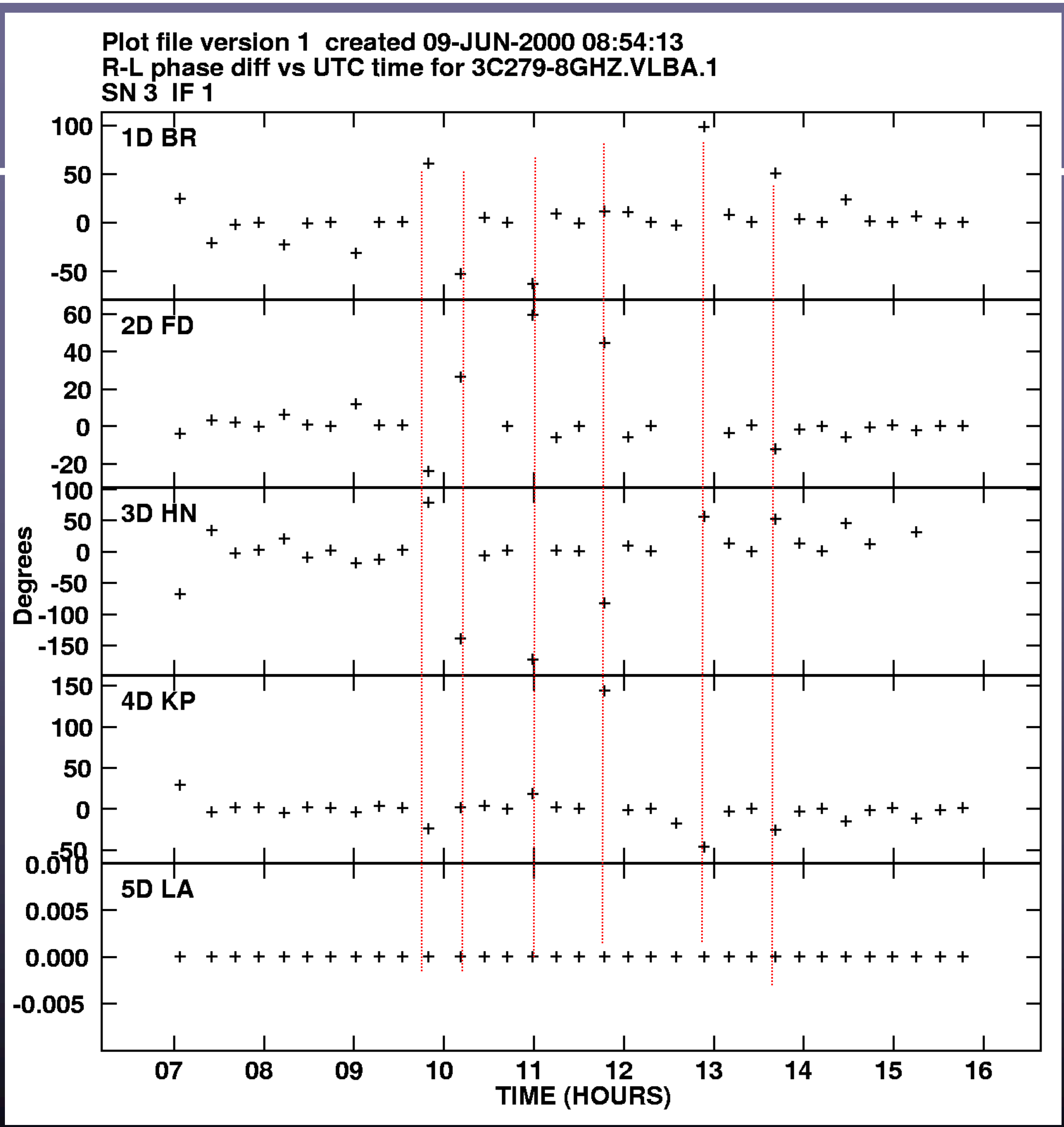
- From observations of a calibrator with known polarization angle:

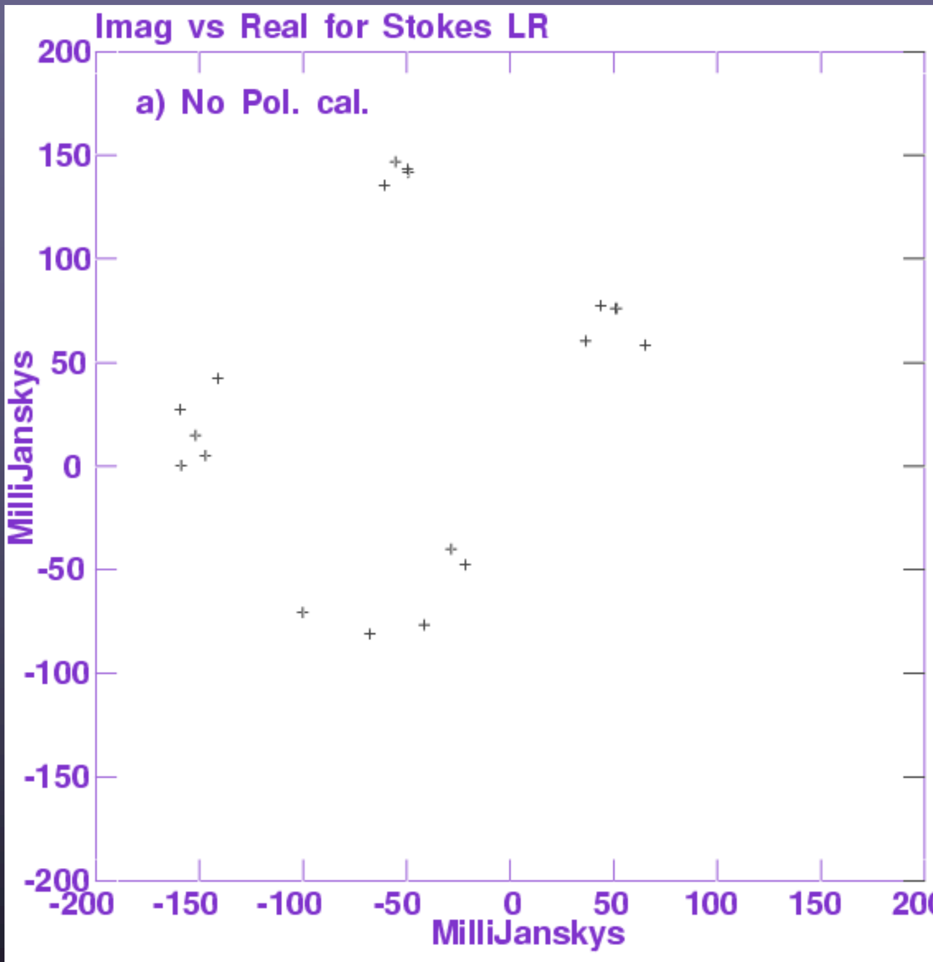
  3) determine $p - q$ phase difference of reference antenna

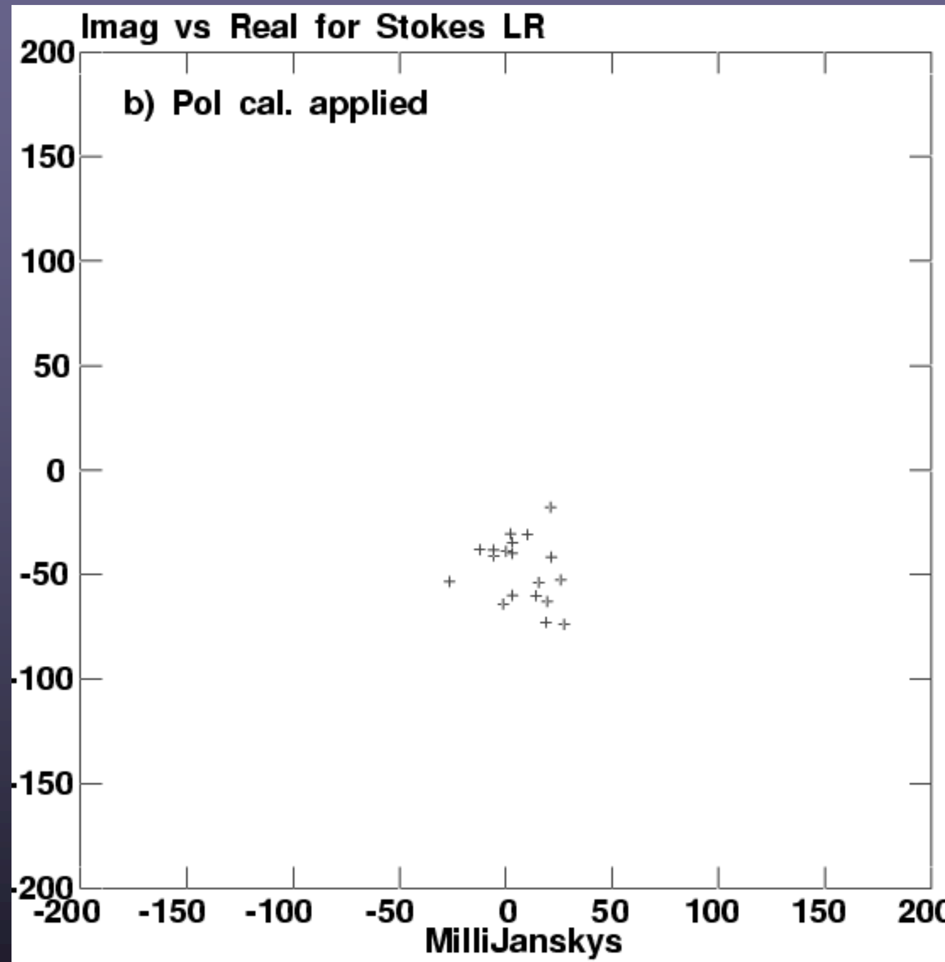
- Things that can go wrong:

  1) time variable $p - q$ phase difference

  2) time variable $d$ terms


Plot file version 1 created 09-JUN-2000 08:54:13
R-L phase diff vs UTC time for 3C279-8GHZ.VLBA.1
SN 3 IF 1
1D BR
2D FD
3D HN
4D KP
5D LA
Degrees
TIME (HOURS)
07
08
09
10
11
12
13
14
15
16


Imag vs Real for Stokes LR
a) No Pol. cal.
MilliJanskys
MilliJanskys
200
150
100
50
0
-50
-100
-150
-200
-200
-150
-100
-50
0
50
100
150
200


Imag vs Real for Stokes LR
b) Pol cal. applied
MilliJanskys
200
150
100
50
0
-50
-100
-150
-200
-200
-150
-100
-50
0
50
100
150
200

## Calibration of Linear Feeds



- Parallel correlations sensitive to $I, Q$, & $U$

$$
\begin{aligned}
v_{pp} &= \tfrac{1}{2} g_{ip} g^*_{kp} (I + Q\ \cos 2\chi + U\ \sin 2\chi), \\
v_{qq} &= \tfrac{1}{2} g_{iq} g^*_{kq} (I - Q\ \cos 2\chi - U\ \sin 2\chi),
\end{aligned}
$$

- Calibrator $Q$ and $U$ usually cannot be ignored (few %)
- Phase unaffected by polarization of a point source at the phase center
- Cannot seperate $p, q$ gains and calibrator polarization
- $p - q$ phase offset not known
- May be unknown orientation error of $p$ and $q$
- Need obs of source with known polarization

$$
\begin{aligned}
v_{pq} &= \tfrac{1}{2} g_{ip} g^*_{kq} ((d_{ip} - d^*_{kq}) I - Q\ \sin 2\chi + U\ \cos 2\chi + jV), \\
v_{qp} &= \tfrac{1}{2} g_{iq} g^*_{kp} ((d^*_{kp} - d_{iq}) I - Q\ \sin 2\chi + U\ \cos 2\chi - jV),
\end{aligned}
$$

- Calibrator $Q$ and $U$ affect real part of cross pol. correlations
- Calibrator $V$ affects imaginary part of cross pol. correlations but unaffected by PA

# Calibration procedure (Linear Feeds)



- From frequent observations of a calibrator solve jointly for:

  1) Gains ($g$)
  2) Calibrator polarization ($Q, U$)
  3) Instrumental polarization ($d$)

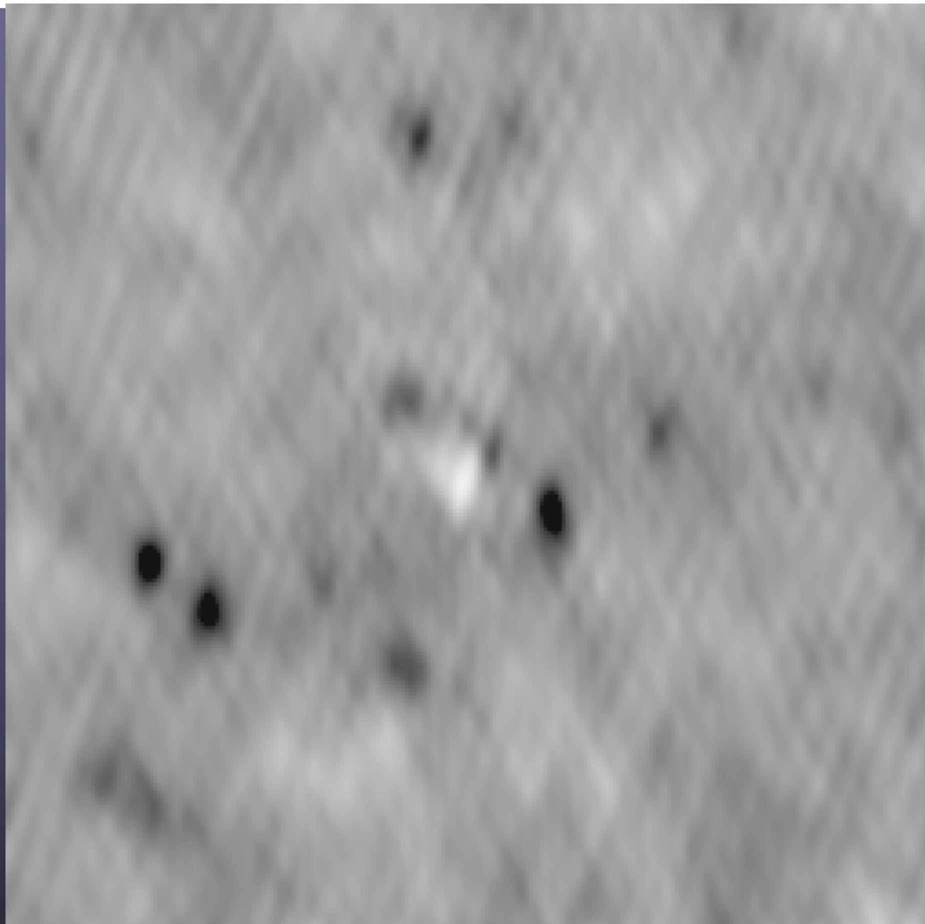
- Iterative scheme may work (e.g., ATCA)
- From obs. of a calibrator of known EVPA

  4) Determine feed orientations
  5) Determine $p - q$ phase difference

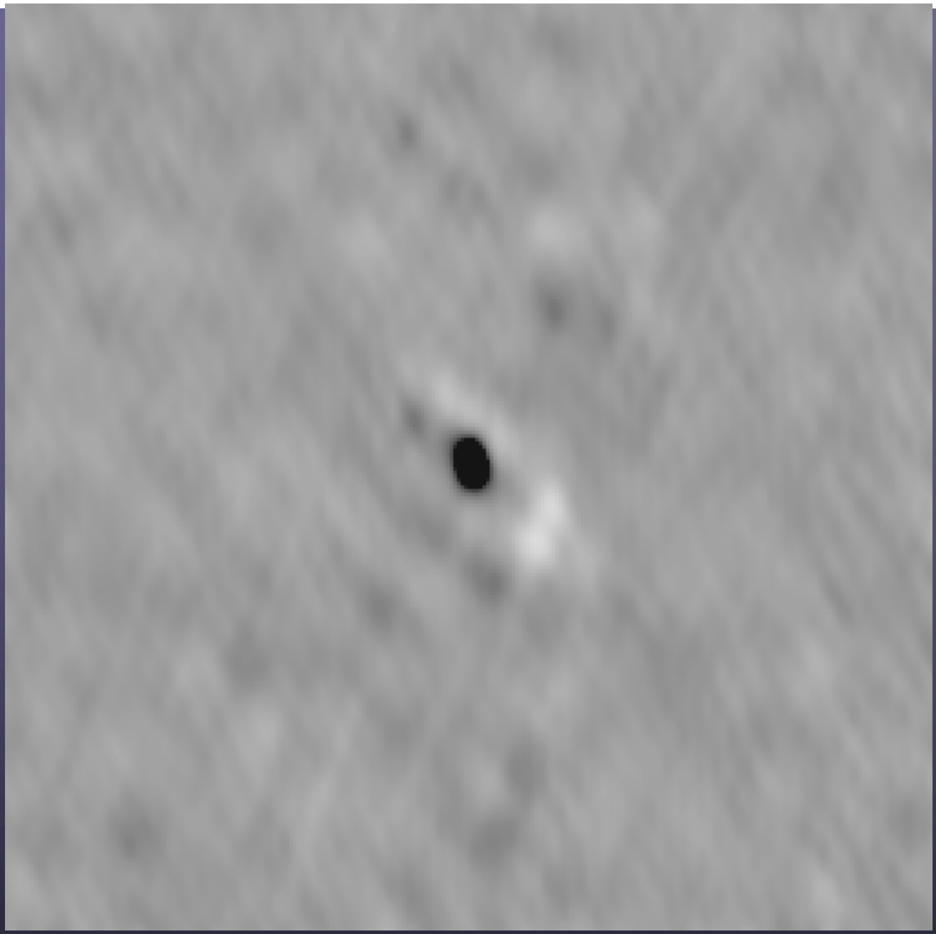
- Things that can go wrong:

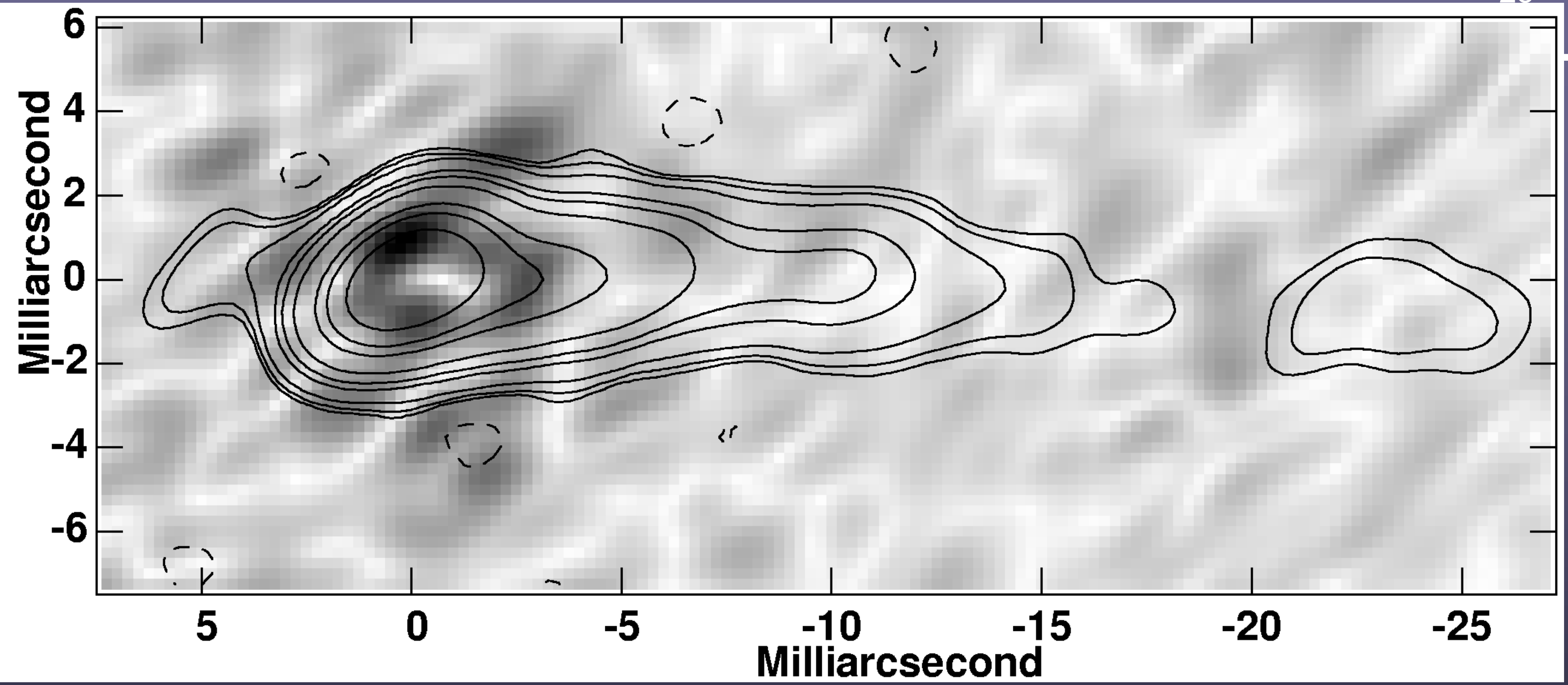
  1) time variable $p - q$ phase difference
  2) time variable $d$ terms

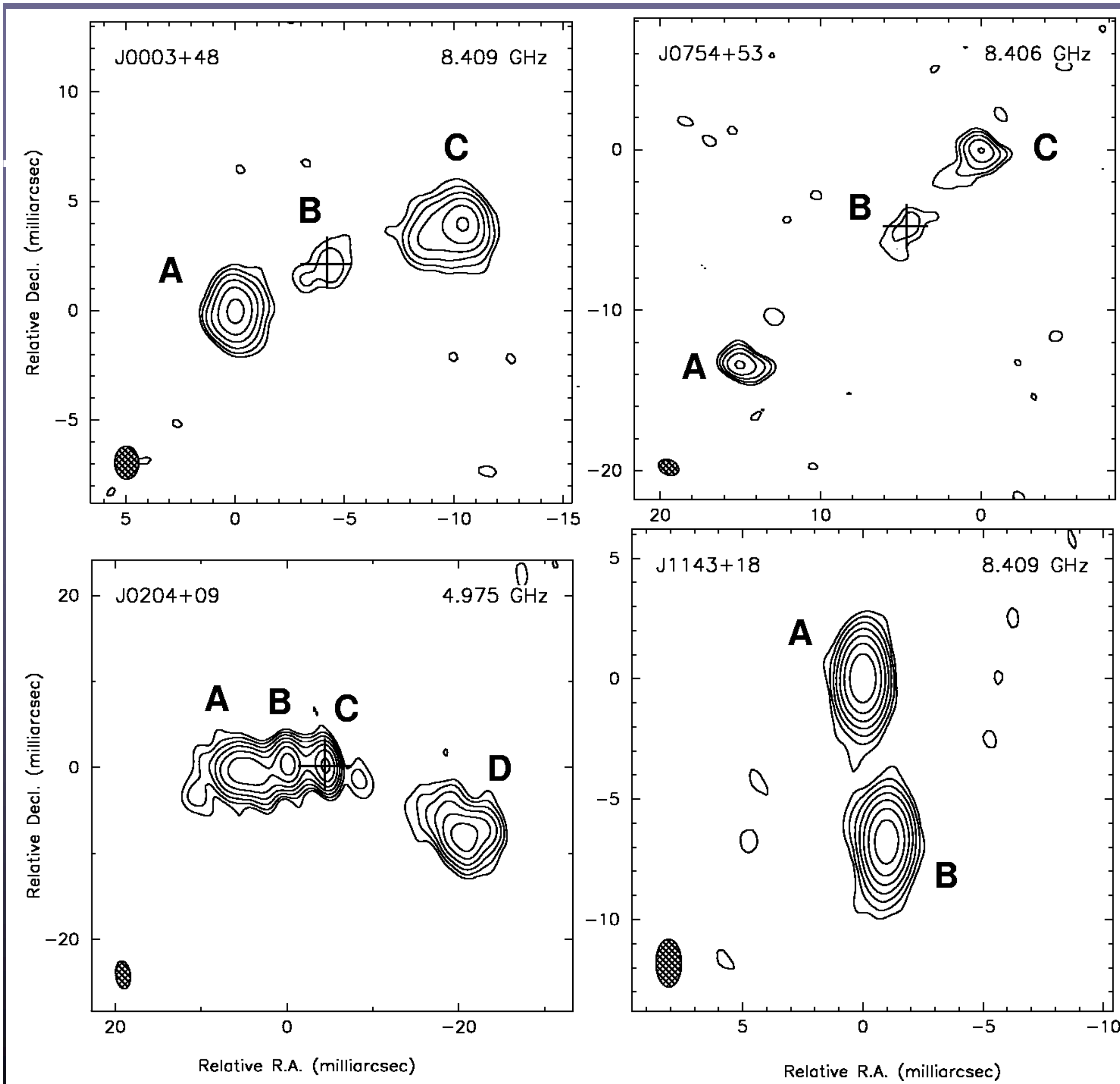
Bad D-term solution



Good D-term solution




6
4
2
0
-2
-4
-6
Milliarcsecond
5
0
-5
-10
-15
-20
-25
Milliarcsecond

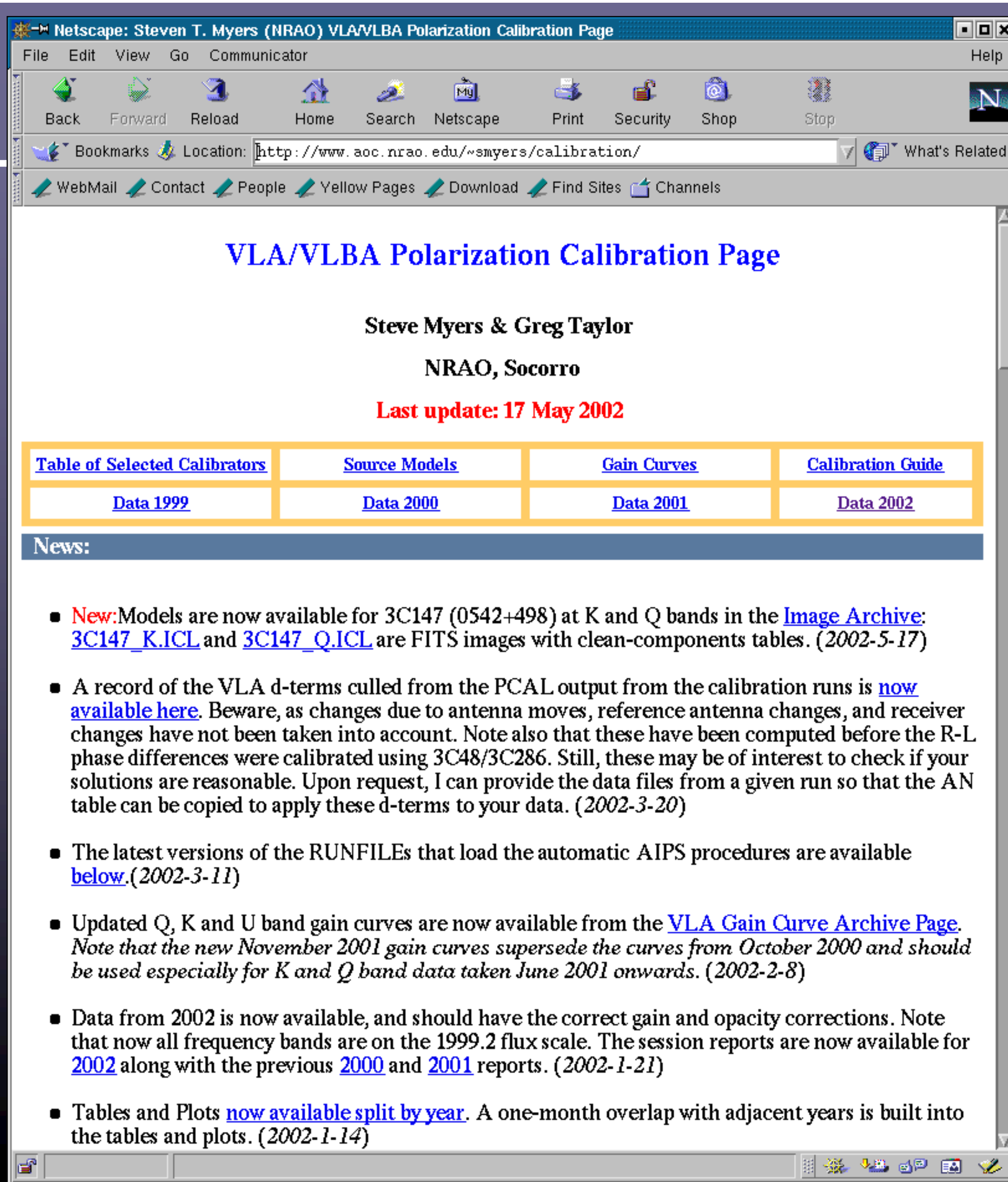



## Compact Symmetric Objects (CSOs)

VLBA @ 8.4 GHz

Peck & Taylor (2001)

# Practical VLBI polarization angle calibration



Netscape: Steven T. Myers (NRAO) VLA/VLBA Polarization Calibration Page

http://www.aoc.nrao.edu/~smyers/calibration/

# VLA/VLBA Polarization Calibration Page

**Steve Myers & Greg Taylor**

**NRAO, Socorro**

**Last update: 17 May 2002**

| | | | |
|---|---|---|---|
| Table of Selected Calibrators | Source Models | Gain Curves | Calibration Guide |
| Data 1999 | Data 2000 | Data 2001 | Data 2002 |

## News:

- New:Models are now available for 3C147 (0542+498) at K and Q bands in the Image Archive: 3C147_K.ICL and 3C147_Q.ICL are FITS images with clean-components tables. (*2002-5-17*)
- A record of the VLA d-terms culled from the PCAL output from the calibration runs is now available here. Beware, as changes due to antenna moves, reference antenna changes, and receiver changes have not been taken into account. Note also that these have been computed before the R-L phase differences were calibrated using 3C48/3C286. Still, these may be of interest to check if your solutions are reasonable. Upon request, I can provide the data files from a given run so that the AN table can be copied to apply these d-terms to your data. (*2002-3-20*)
- The latest versions of the RUNFILEs that load the automatic AIPS procedures are available below.(*2002-3-11*)
- Updated Q, K and U band gain curves are now available from the VLA Gain Curve Archive Page. *Note that the new November 2001 gain curves supersede the curves from October 2000 and should be used especially for K and Q band data taken June 2001 onwards.* (*2002-2-8*)
- Data from 2002 is now available, and should have the correct gain and opacity corrections. Note that now all frequency bands are on the 1999.2 flux scale. The session reports are now available for 2002 along with the previous 2000 and 2001 reports. (*2002-1-21*)
- Tables and Plots now available split by year. A one-month overlap with adjacent years is built into the tables and plots. (*2002-1-14*)

# 1) Find a Calibrator

Netscape: VLA/VLBA Polarization Calibration Database 2002

File Edit View Go Communicator Help

Back Forward Reload Home Search Netscape Print Security Shop Stop

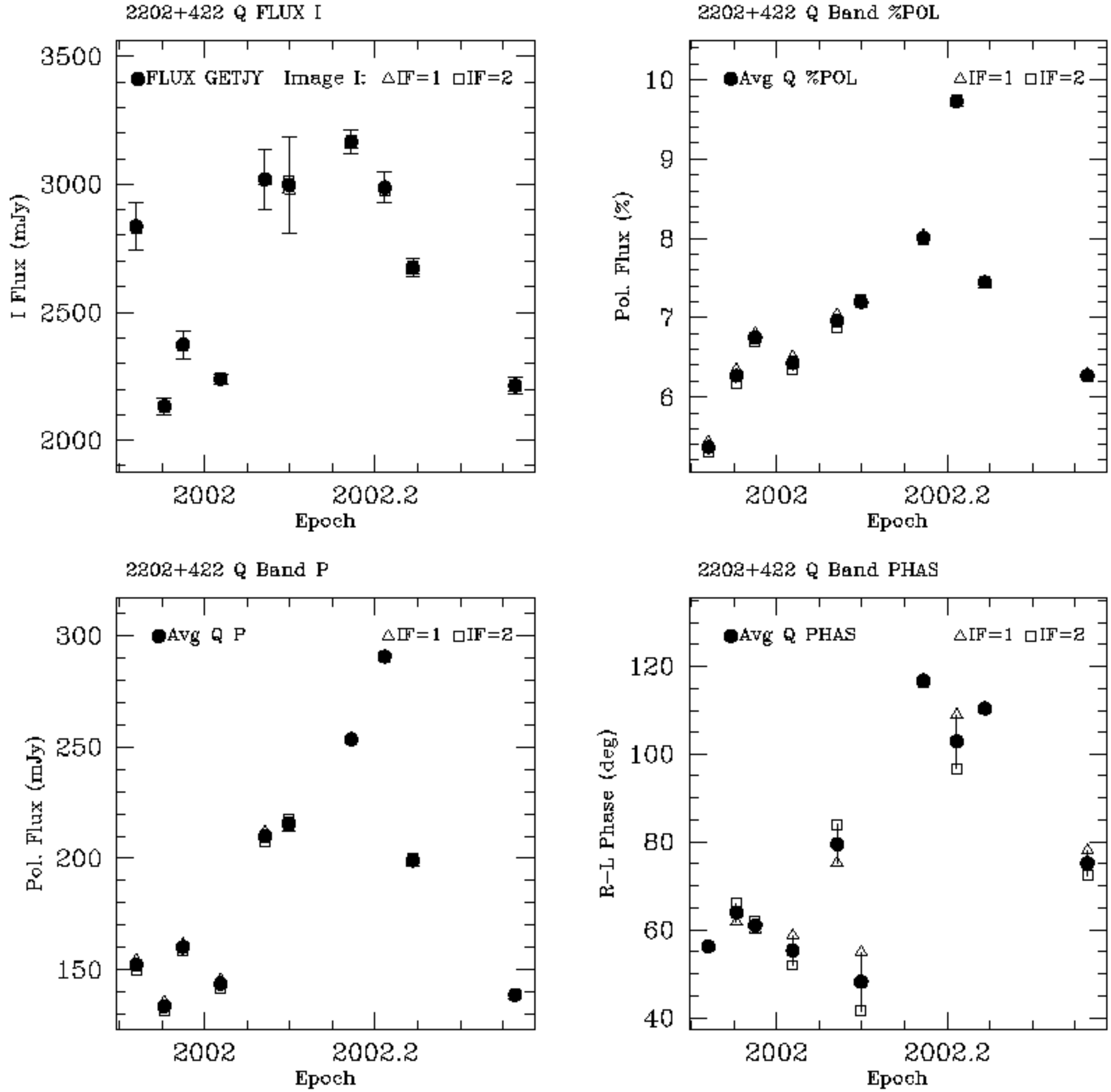
Bookmarks Location: http://www.aoc.nrao.edu/~smyers/calibration/selected_2002.html What's Related

WebMail Contact People Yellow Pages Download Find Sites Channels

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3 ± 1.32 | 68.09 ± 72.74 | all | 4.577 ± 0.911 | 4572.89 ± 910.68 | 165.26 ± 90.90 | 3.63 ± 1.95 | 31.25 ± 71.82 |
| | | | 2136+006 Q BAND | | | | |
| 9 ± 0.01 | 8.18 ± 0.28 | 20011202 | 2.898 ± 0.037 | 2897.50 ± 1.26 | 33.65 ± 0.22 | 1.16 ± 0.01 | 36.34 ± 3.65 |
| 8 ± 0.01 | 11.49 ± 0.55 | 20011214 | 2.877 ± 0.048 | 2877.13 ± 4.62 | 43.00 ± 1.31 | 1.49 ± 0.04 | 59.69 ± 2.06 |
| 4 ± 0.02 | 6.42 ± 2.09 | 20011222 | 2.787 ± 0.022 | 2785.25 ± 8.78 | 33.40 ± 1.26 | 1.20 ± 0.04 | 48.86 ± 3.54 |
| 8 ± 0.01 | -6.50 ± 3.74 | 20020107 | 2.720 ± 0.023 | 2717.28 ± 1.89 | 22.79 ± 0.62 | 0.84 ± 0.02 | -6.46 ± 5.47 |
| 7 ± 0.01 | 60.75 ± 0.42 | 20020126 | 2.956 ± 0.090 | 2955.37 ± 1.08 | 64.55 ± 2.36 | 2.18 ± 0.08 | 26.85 ± 6.71 |
| 1 ± 0.02 | 59.25 ± 1.79 | 20020205 | 3.170 ± 0.168 | 3169.27 ± 17.58 | 84.98 ± 3.21 | 2.68 ± 0.09 | 16.76 ± 7.59 |
| 6 ± 0.10 | 73.22 ± 1.48 | 20020304 | 2.746 ± 0.034 | 2745.33 ± 2.20 | 54.37 ± 2.43 | 1.98 ± 0.09 | 84.84 ± 0.92 |
| 8 ± 0.03 | 80.75 ± 0.05 | 20020318 | 3.227 ± 0.058 | 3225.06 ± 10.75 | 72.37 ± 0.45 | 2.24 ± 0.01 | 69.41 ± 7.39 |
| 0 ± 0.05 | 76.62 ± 0.52 | 20020330 | 2.874 ± 0.037 | 2872.52 ± 7.26 | 89.40 ± 4.83 | 3.11 ± 0.16 | 69.80 ± 0.57 |
| 5 ± 0.04 | 69.77 ± 0.56 | 20020513 | 2.905 ± 0.040 | 2903.34 ± 8.83 | 47.73 ± 1.25 | 1.64 ± 0.04 | 68.74 ± 2.46 |
| 4 ± 0.24 | 43.99 ± 32.78 | all | 2.916 ± 0.158 | 2914.80 ± 158.37 | 54.62 ± 21.52 | 1.85 ± 0.68 | 47.48 ± 27.17 |
| | | | 2202+422 Q BAND | | | | |
| 6 ± 0.00 | 81.05 ± 0.87 | 20011202 | 2.836 ± 0.092 | 2835.85 ± 8.14 | 152.18 ± 2.34 | 5.37 ± 0.07 | 56.19 ± 0.08 |
| 5 ± 0.00 | 71.50 ± 0.60 | 20011214 | 2.134 ± 0.032 | 2132.71 ± 0.13 | 133.68 ± 1.81 | 6.27 ± 0.09 | 63.93 ± 2.15 |
| 7 ± 0.02 | 71.72 ± 2.38 | 20011222 | 2.374 ± 0.054 | 2373.31 ± 1.71 | 160.21 ± 1.28 | 6.75 ± 0.05 | 61.03 ± 0.91 |
| 7 ± 0.10 | 86.95 ± 0.11 | 20020107 | 2.240 ± 0.019 | 2237.72 ± 4.35 | 143.77 ± 1.72 | 6.43 ± 0.09 | 55.27 ± 3.31 |
| 7 ± 0.05 | 123.03 ± 1.67 | 20020126 | 3.019 ± 0.118 | 3016.47 ± 4.07 | 209.87 ± 2.17 | 6.96 ± 0.08 | 79.51 ± 4.37 |
| 9 ± 0.08 | 104.00 ± 0.23 | 20020205 | 2.998 ± 0.188 | 2994.10 ± 16.85 | 215.65 ± 1.97 | 7.20 ± 0.03 | 48.16 ± 6.75 |
| 7 ± 0.06 | 130.09 ± 1.93 | 20020304 | 3.165 ± 0.046 | 3164.23 ± 7.11 | 253.39 ± 0.35 | 8.01 ± 0.03 | 116.67 ± 0.31 |
| 6 ± 0.07 | 124.16 ± 1.05 | 20020318 | 2.988 ± 0.060 | 2986.84 ± 9.30 | 290.61 ± 0.60 | 9.73 ± 0.01 | 102.86 ± 6.13 |
| 4 ± 0.11 | 118.05 ± 0.97 | 20020330 | 2.675 ± 0.036 | 2672.85 ± 7.19 | 199.01 ± 0.83 | 7.45 ± 0.01 | 110.42 ± 0.03 |
| 3 ± 0.01 | 105.55 ± 1.53 | 20020513 | 2.214 ± 0.033 | 2212.25 ± 6.33 | 138.75 ± 0.04 | 6.27 ± 0.02 | 75.17 ± 2.87 |
| 8 ± 1.29 | 101.61 ± 21.23 | all | 2.664 ± 0.370 | 2662.63 ± 370.11 | 189.71 ± 50.52 | 7.04 ± 1.13 | 76.92 ± 23.52 |
| | | | 2253+161 Q BAND | | | | |

# 2) Estimate ϕ_VLA

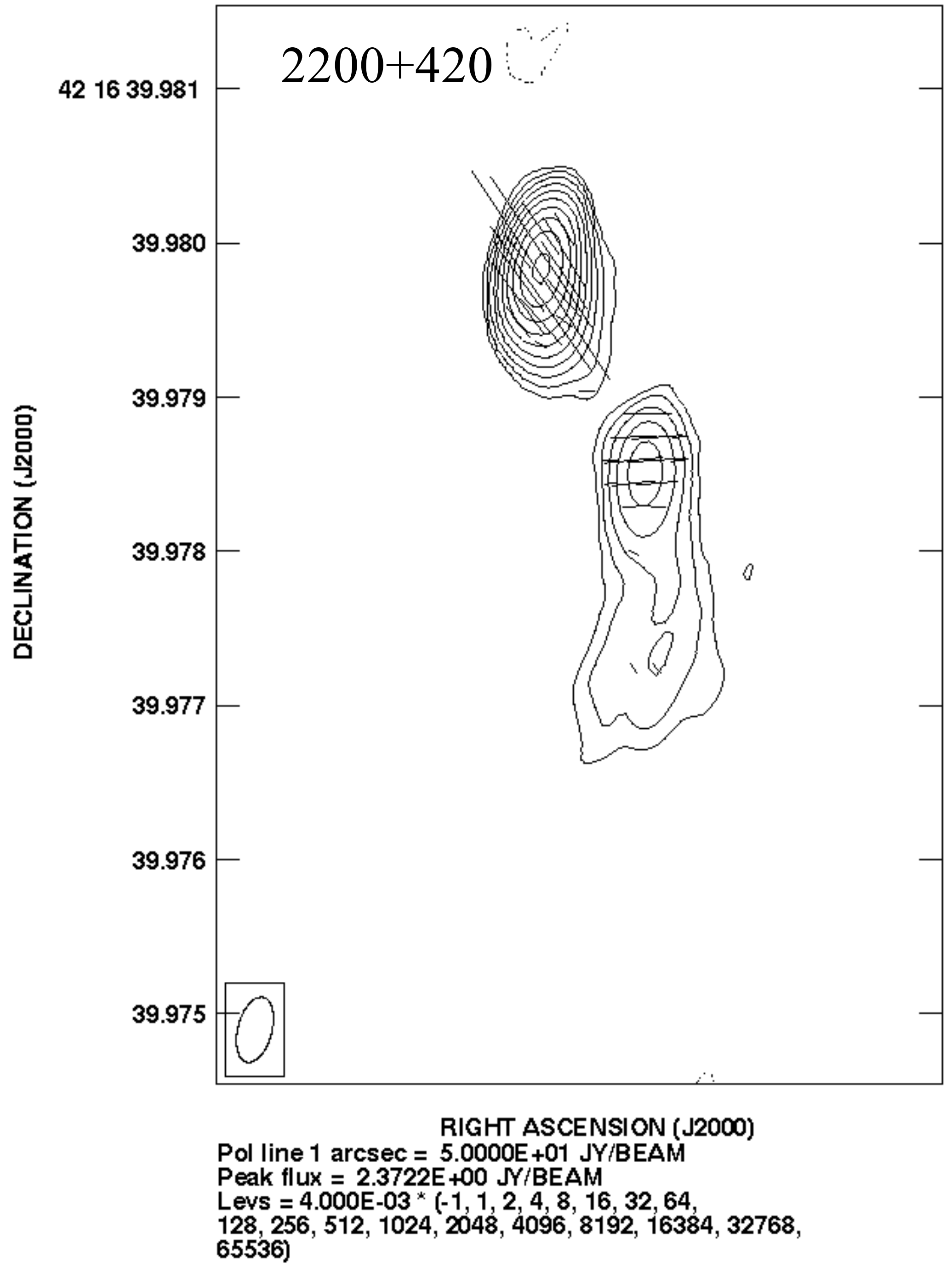




3) Sum Q and U over VLBA image.

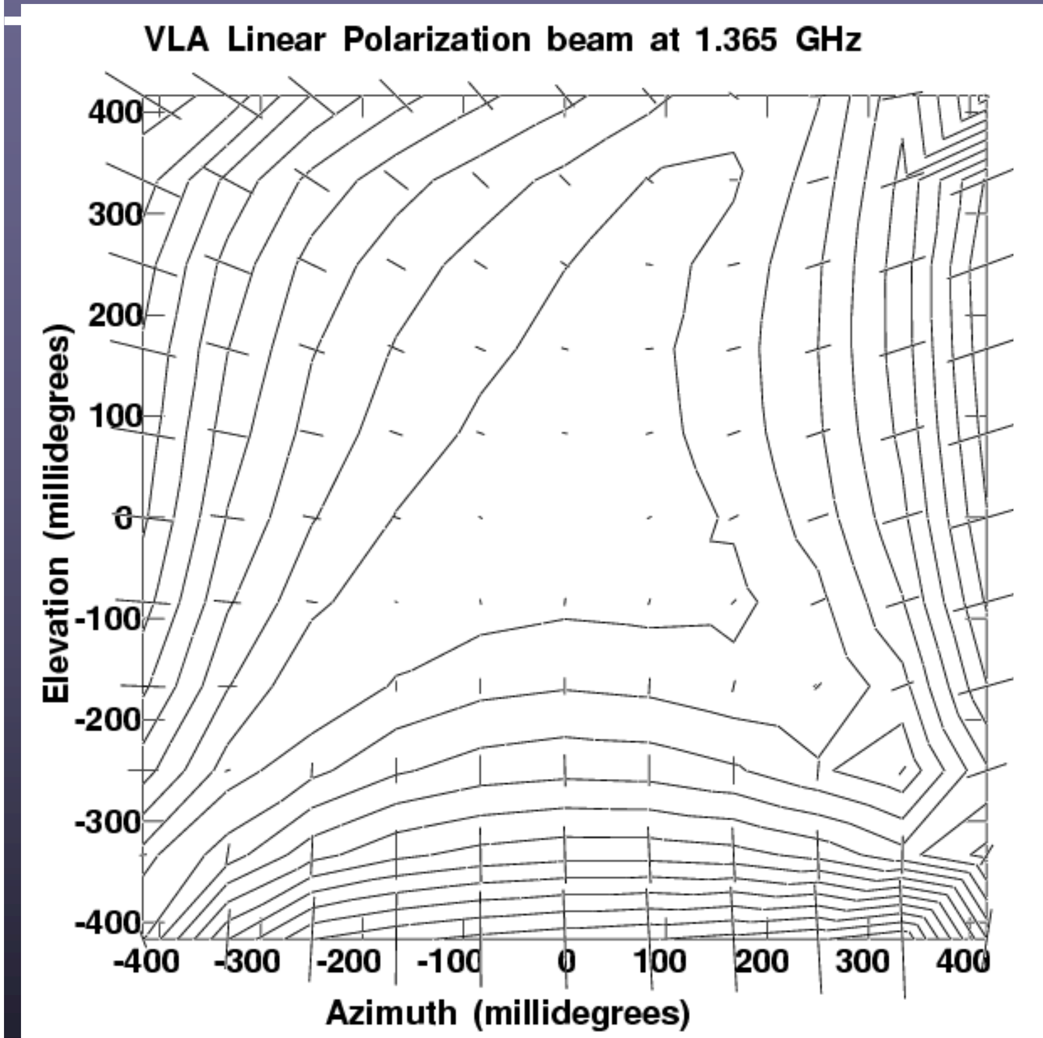
4) Compute
ϕ□VLBA = 0.5 atan (U/Q)

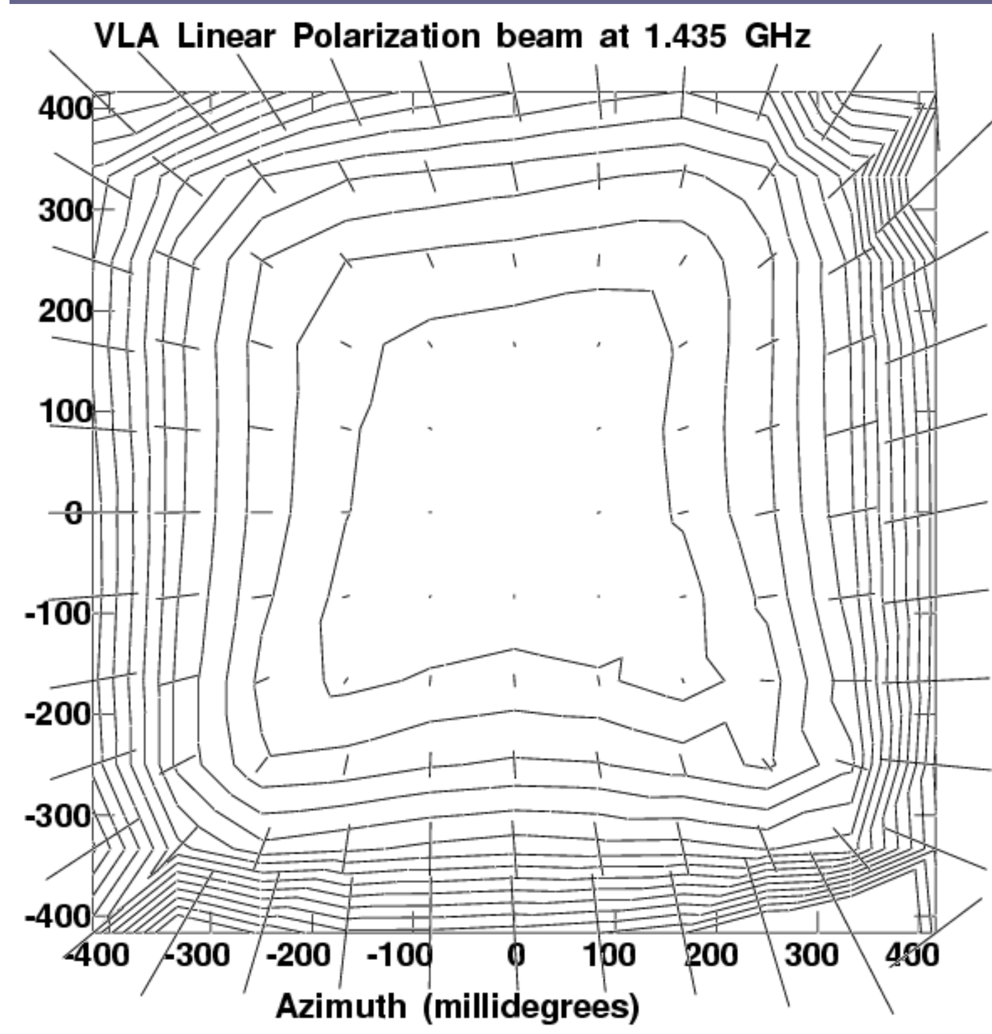
5) Rotate VLBA data by
(ϕ_VLA – ϕ_VLBA)

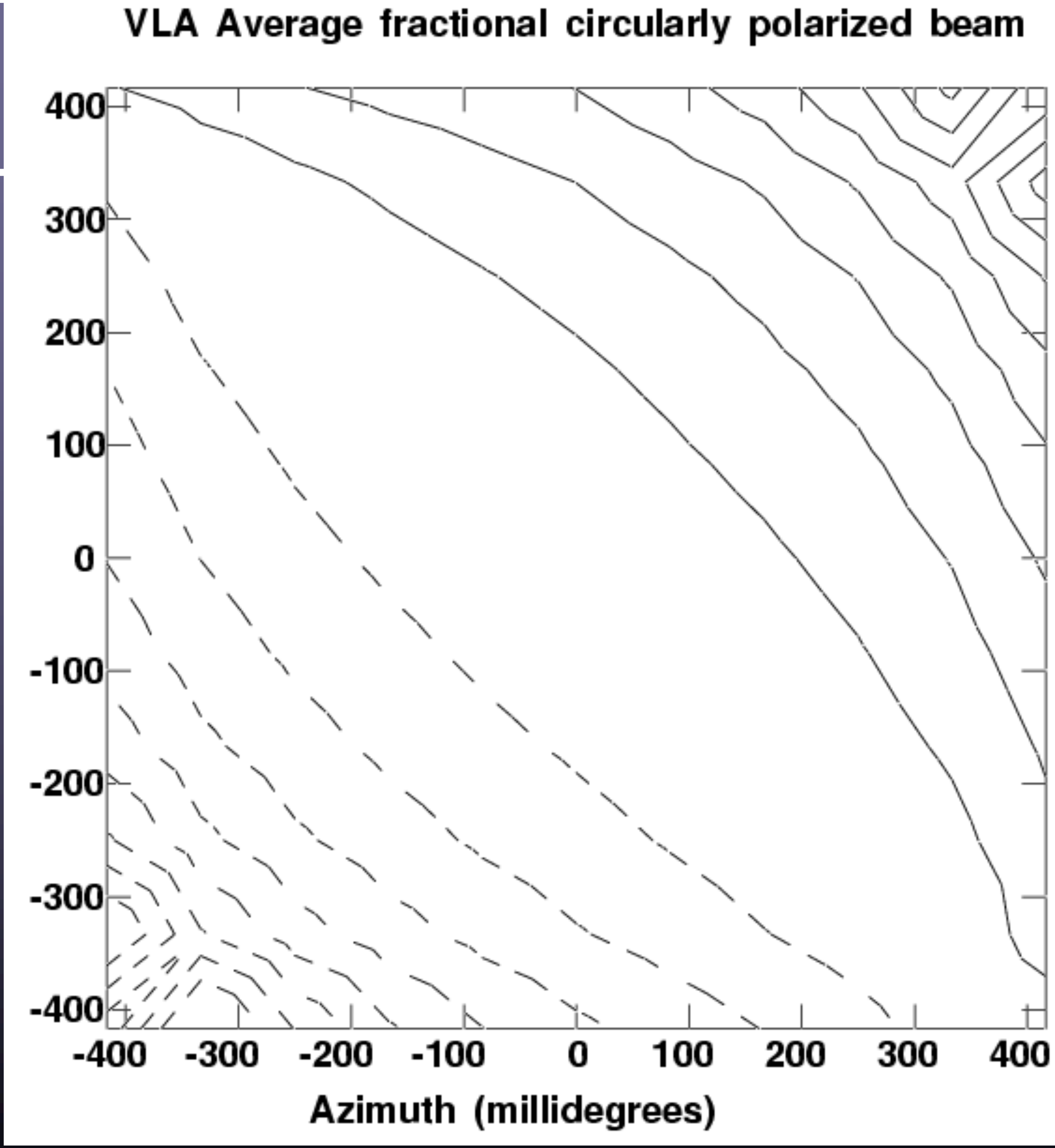


# Wide Field Polarimetry

- Instrumental poln. varies across antenna beam
- Instrumental poln. pattern rotates with PA
- "Snapshot" images can be corrected from $I$ image and antenna poln. pattern
- General correction involves imaging/deconvolutio
- Long synthesis provides some reduction
- Pattern may be a strong function of frequency
- Off axis feeds (e.g., VLA) cause "Beam squint"
- Circularly polarized beams aren't concentric, so beware of strong off-axis instrumental circular poln.


VLA Linear Polarization beam at 1.365 GHz
Elevation (millidegrees)
400
300
200
100
0
-100
-200
-300
-400
-400
-300
-200
-100
0
100
200
300
400
Azimuth (millidegrees)


VLA Linear Polarization beam at 1.435 GHz
400
300
200
100
0
-100
-200
-300
-400
-400
-300
-200
-100
0
100
200
300
400
Azimuth (millidegrees)


VLA Average fractional circularly polarized beam
400
300
200
100
0
-100
-200
-300
-400
-400 -300 -200 -100 0 100 200 300 400
Azimuth (millidegrees)

# Ionospheric Faraday Rotation

- Causes rotation of the polarization angle, $\Delta\Phi$

$$\Delta\Phi = \frac{0.93 \times 10^6 \int N\ B_{\parallel}\ ds}{(2\pi\nu)^2} \text{ radians} \qquad (1)$$

$N$ = electron density ($cm^{-3}$),
$B_{\parallel}$ = magnetic field $\parallel$ to propagation (Gauss),
$\nu$ = radio frequency (Hz),
$s$ = distance along the line of sight (cm).

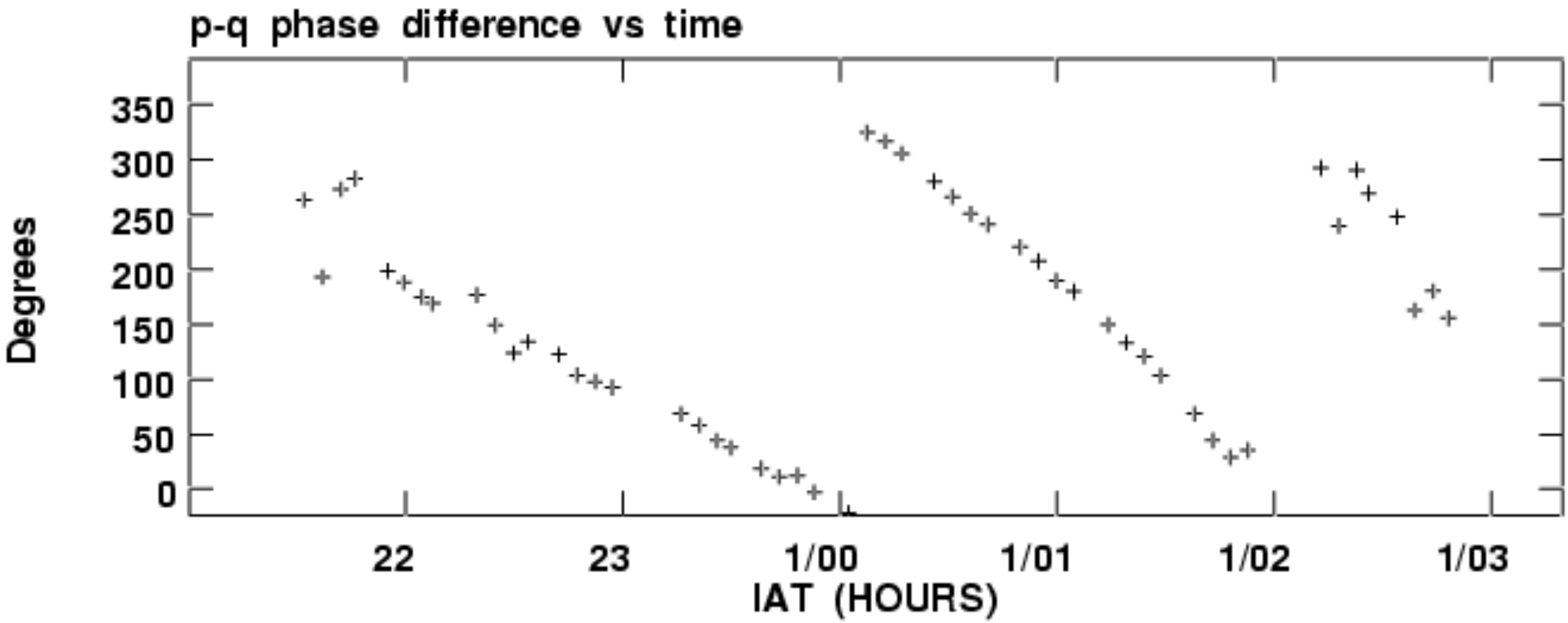
- Effect decreases rapidly with frequency
- Earth's ionosphere may be a problem at $\nu < 2$ GHz
- Effect depends on time of day and solar activity
- Effect depends on line of sight through magnetic field

# External Calibration of IFR

1) Measure zenith total electron content
2) Assume ionospheric structure
3) Assume magnetic field structure (offset dipole)
4) Calculate time dependent RM correction for line of sight to calibrators and sources

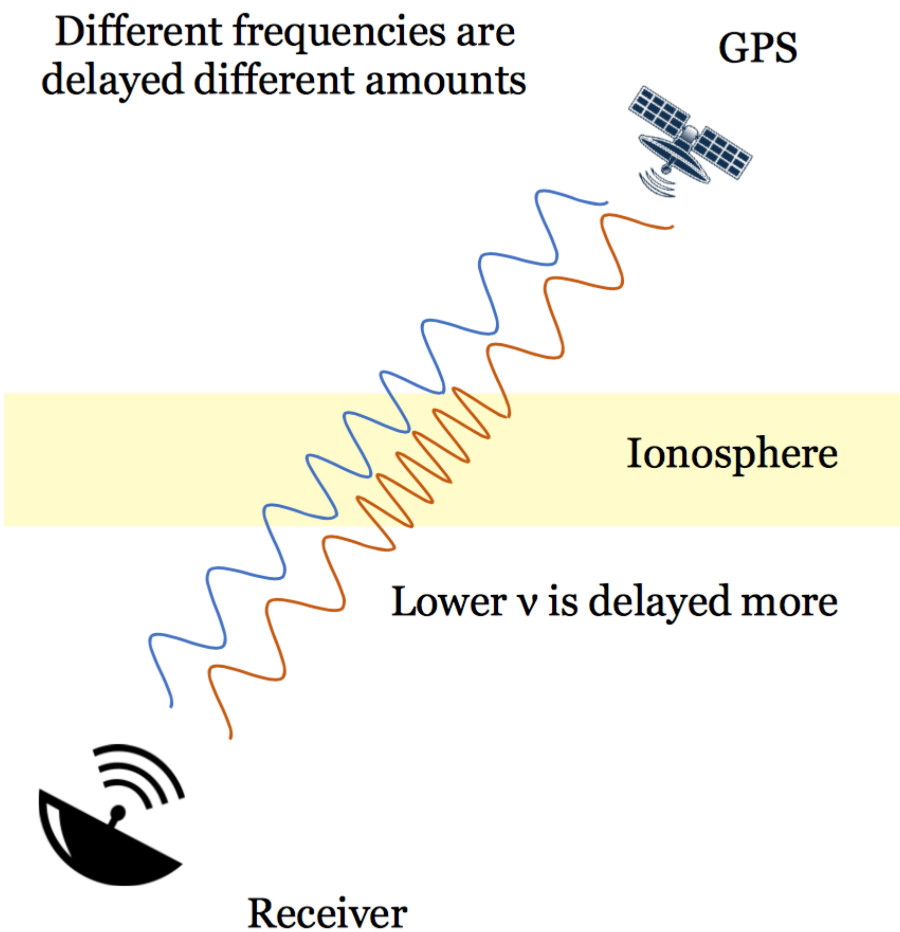
# Self Calibration of IFR

1) Make snapshot reference polarization image
2) Divide data into time segments and determine RM or $p - q$ phase difference
3) Apply corrections to the data



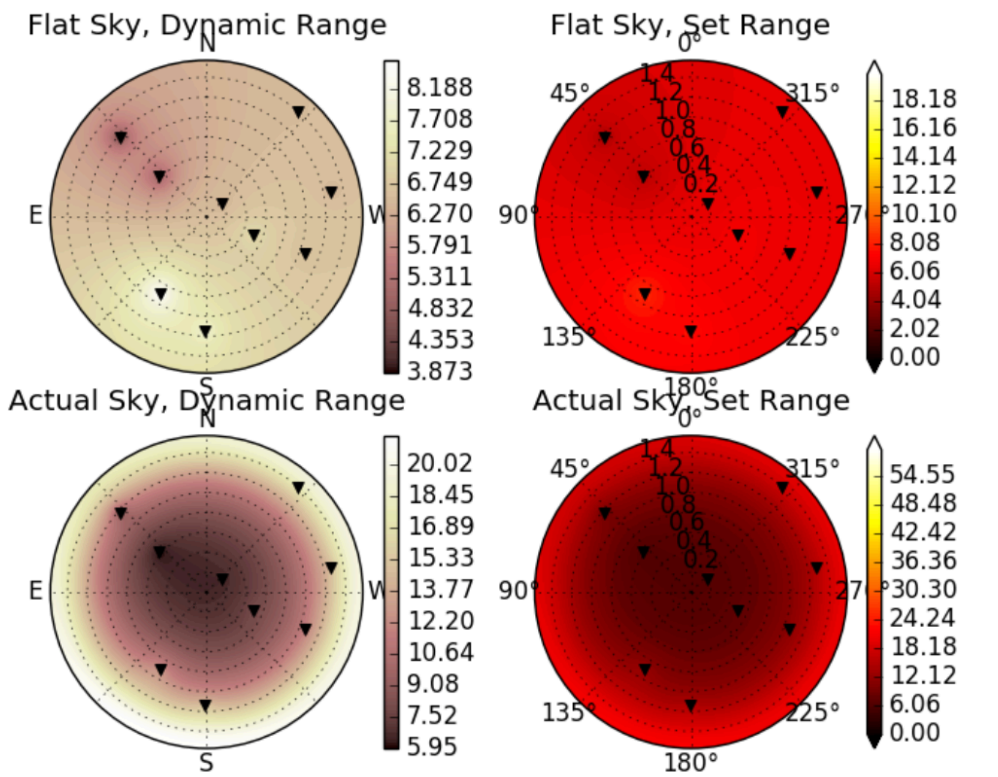

VLA observations at 330 MHz

# Calibration Techniques for IFR




- Dual Frequency: 1575.42 MHz and 1227.60 MHz
- Ionosphere is plasma
- Plasmas causes group velocity delay, phase velocity increase in EM waves
- Delays are strongly frequency dependent
- Integrated electron density: Total Electron Content (TEC)
- Measured in $n_e/m^2$ or TEC units (TECU) 1 TECU = $10^{16}$ $n_e/m^2$
- No profile information

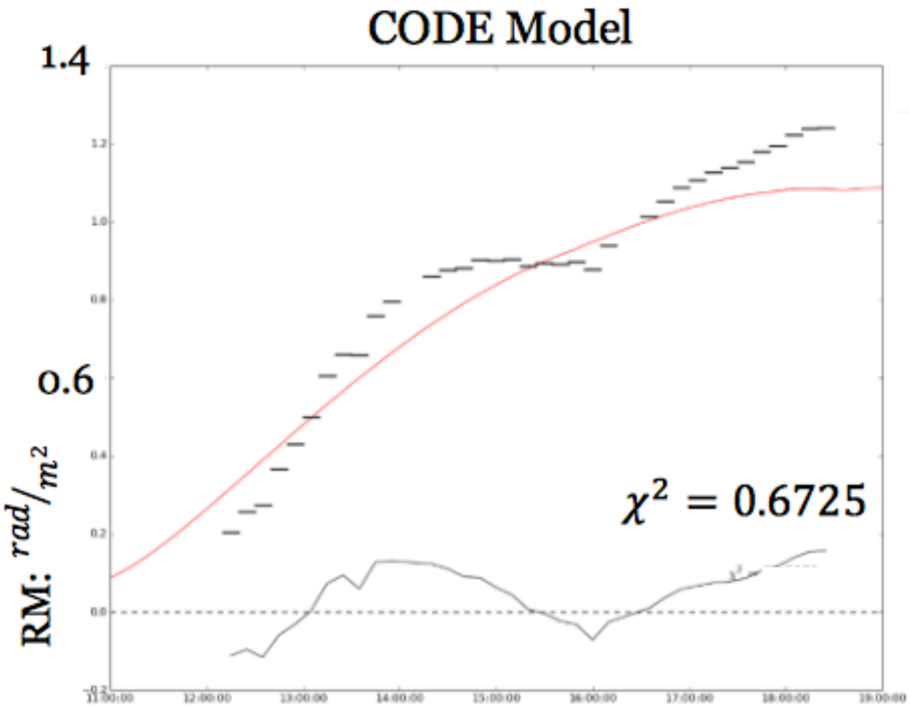
Interpolated Observation at LWA1
at 2016/10/14 12:00:00 UTC

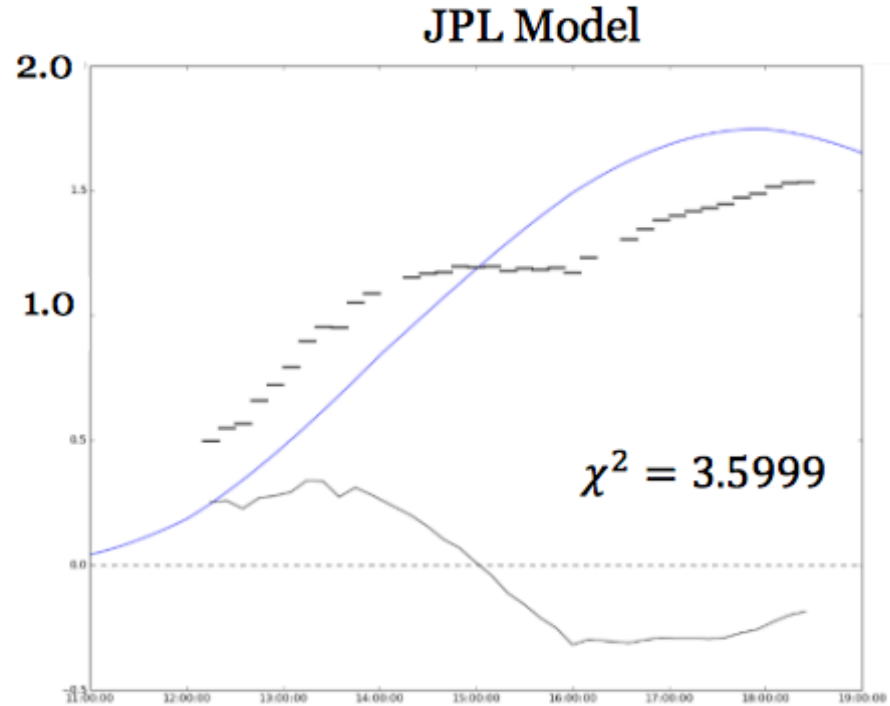



- Each satellite provides accurate data to a single point on the sky
- Between 7-14 satellites at any given time
- For points between satellites, use linear weighted average of satellites

$$TEC = \sum \frac{\rho}{d_l} TEC_1$$

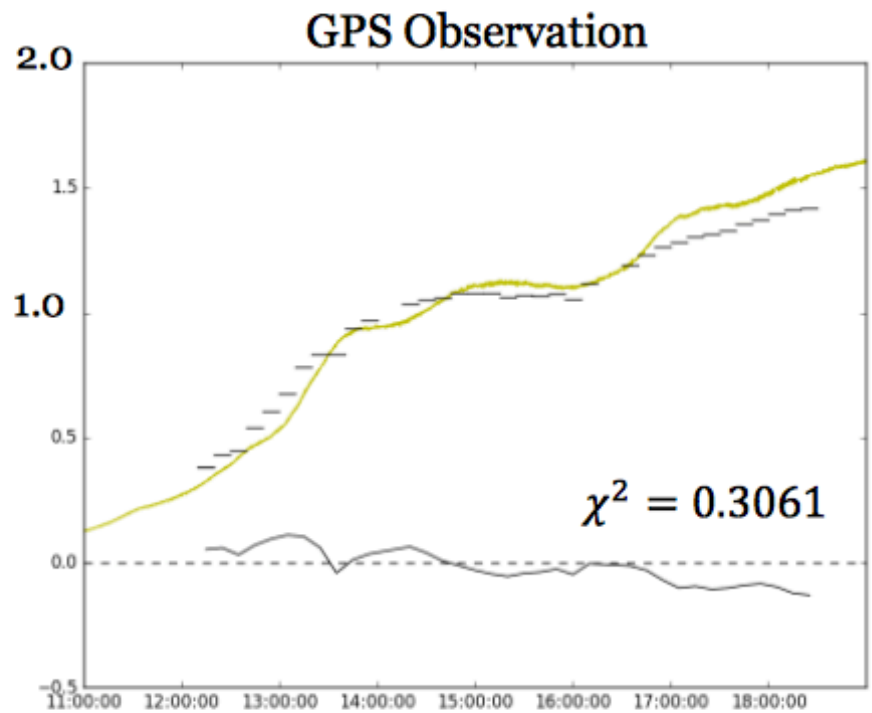
$$\rho = \frac{1}{\sum \frac{1}{d_l}}$$




Pulsar: B0950+08
3x 2Hour Observations
(Black Dashed Lines)

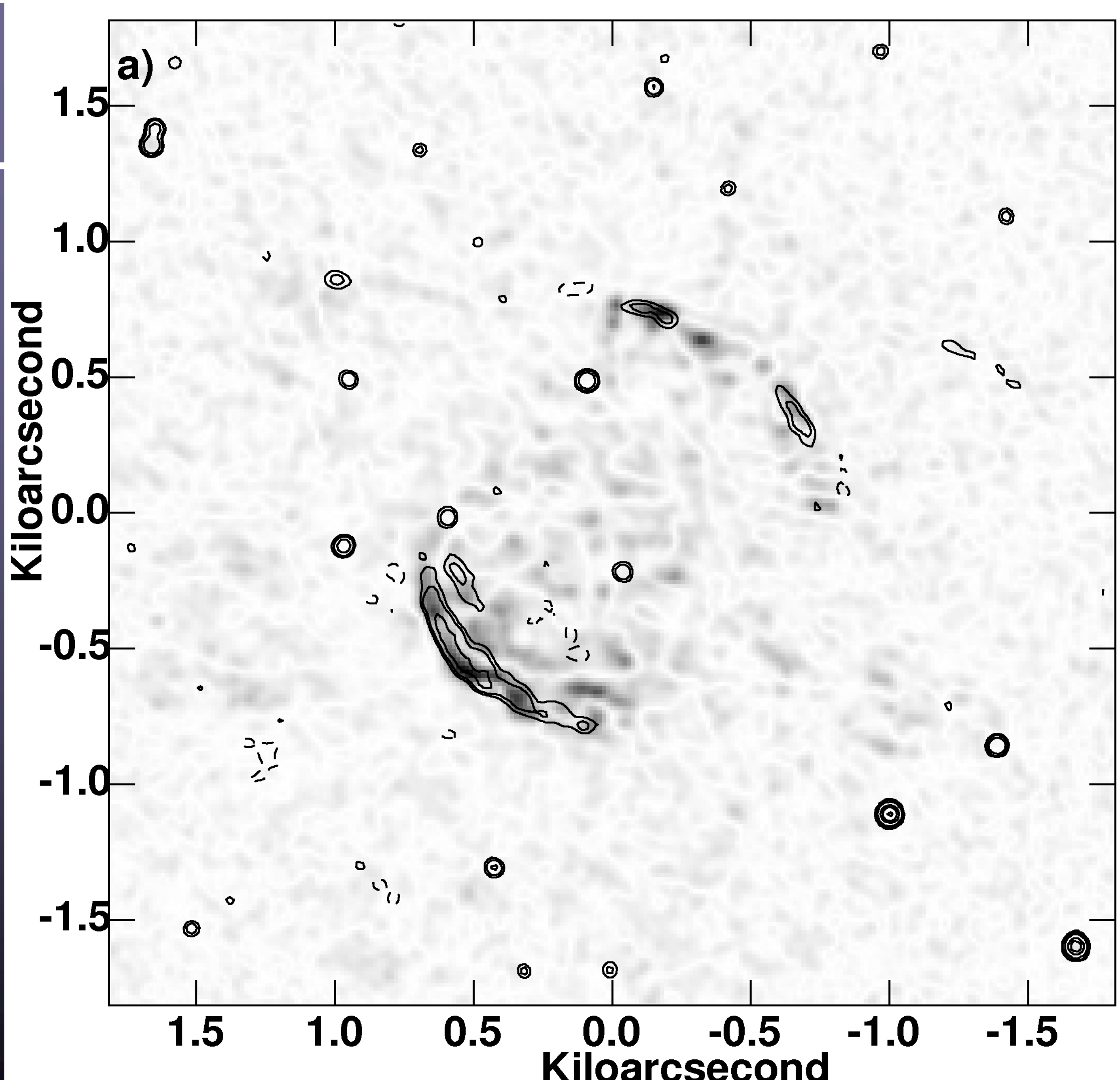
Date: 2016/09/23



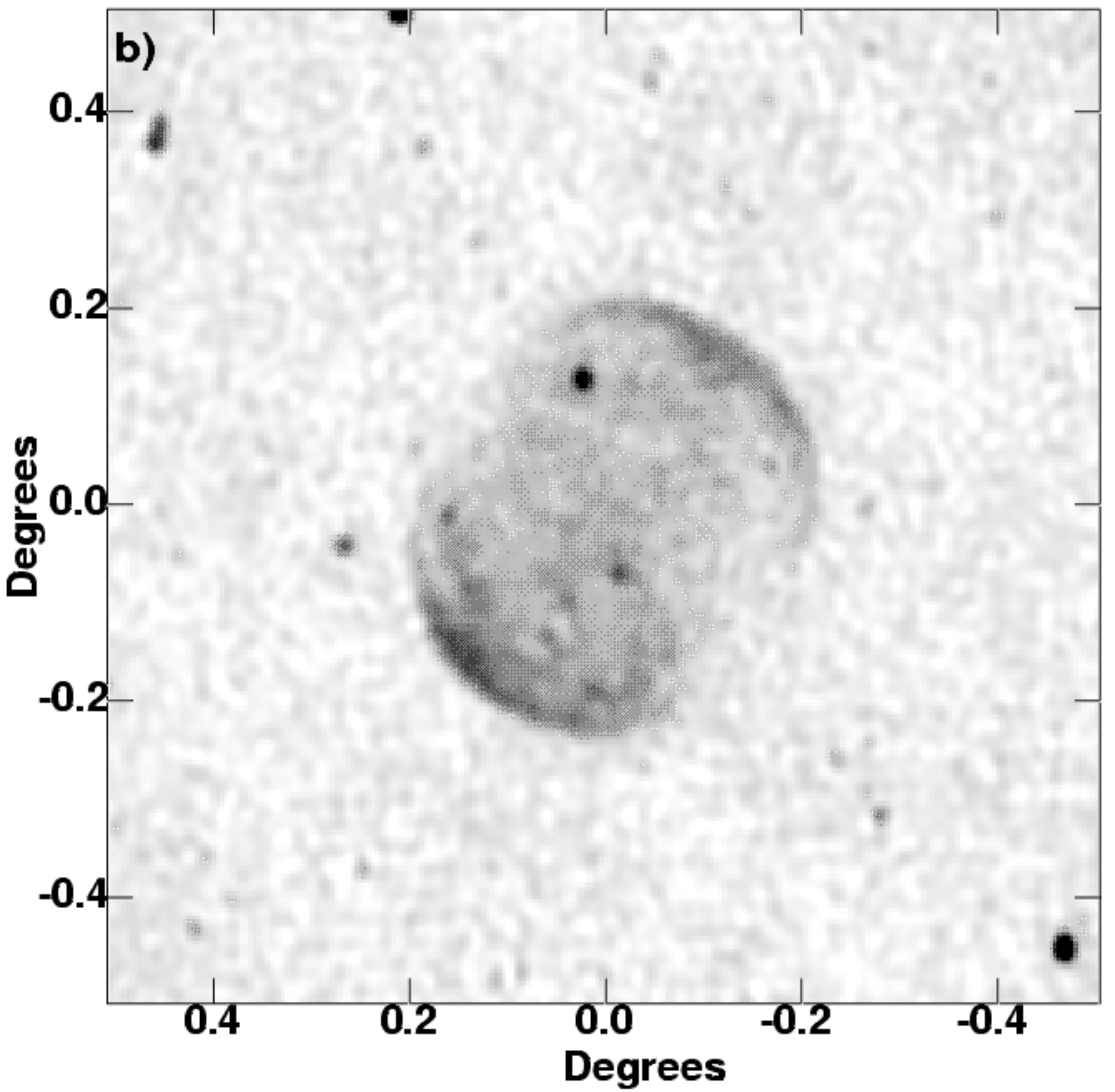
# Imaging

- Similar to total intensity ($I$) imaging
- Apply calibration
- Convert to Stokes' representation
- Image/deconvolve $I, Q, U$ and $V$ independently
- $I$ positive but $Q, U$, and $V$ may not be
- Special case:
  - If some but not all RL, LR correlations present, can't make independent $Q, U$ image
  - Make $Q + jU$ image & use complex deconvolution
- Spatial frequency filtering effects
  - May miss structure on scales resolved on all baselines
  - Filtering effects can be different for $I, Q, U$ and $V$ (e.g., poln. angle variations either intrinsic or from RMs)
  - May lead to $>$ 100% polarization

Total Intensity contours + polarized intensity greyscale



# Total Intensity With shorter baselines



# Further Reading

- http://www.nrao.edu/whatisra/mechanisms.shtml
- http://www.nrao.edu/whatisra/
- www.nrao.edu

- Cluster Magnetic Fields, **2002, ARA&A, 40, 319**
- Synthesis Imaging in Radio Astronomy
- ASP Vol 180, eds Taylor, Carilli & Perley