Cross Correlators

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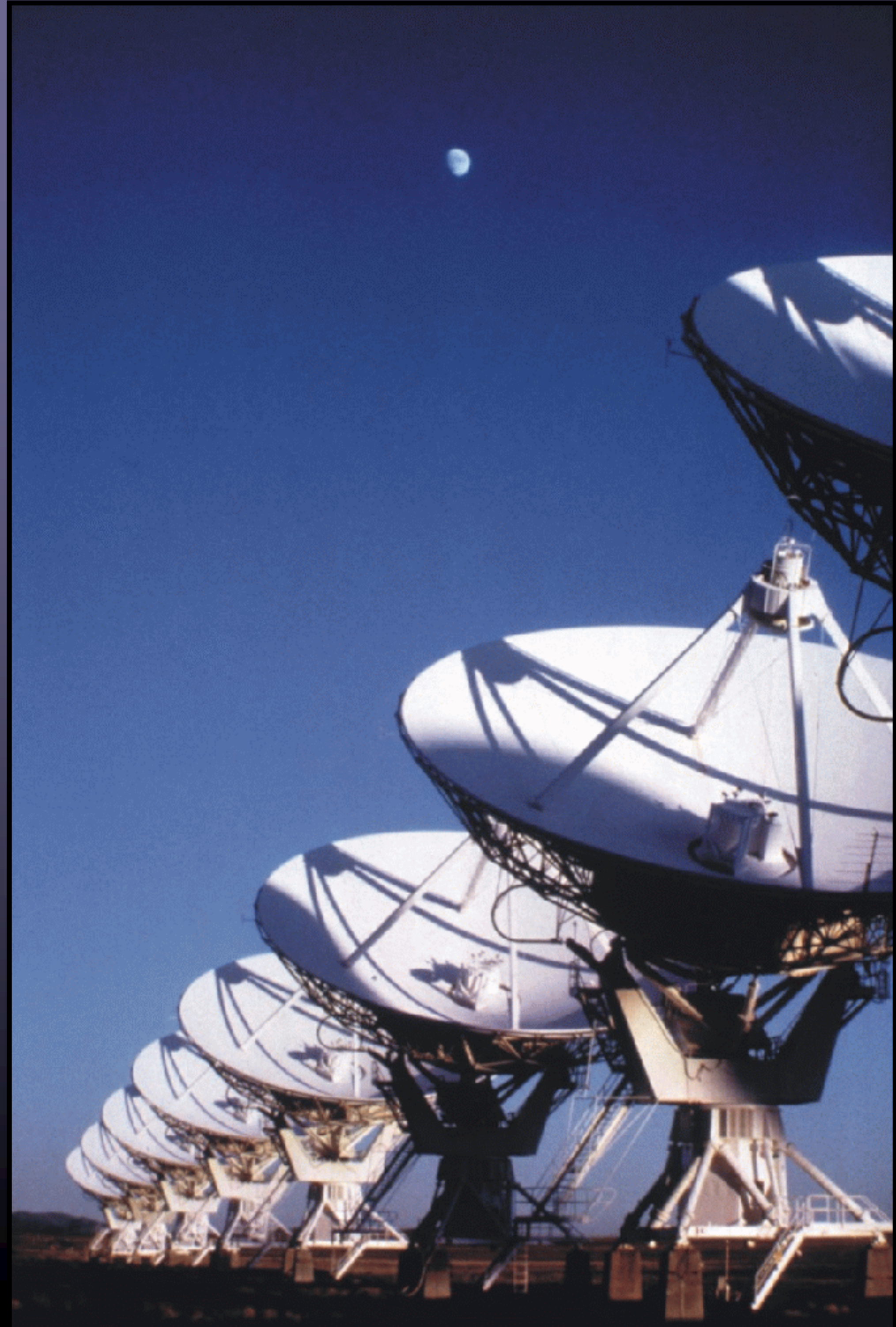


Astronomy 423 at UNM

Radio Astronomy

Announcements

- VLA observing tonight or tomorrow?
- LWA observing ?
- Interferometry by Thompson, Moran & Swenson:
- https://www.dropbox.com/s/tpt4l6tlner32zb/2017_book_interferometryandsynthesisinra.pdf?dl=0
- Exam 1 on Wednesday, March 3
- ALMA interferometry school Mar 15-26

Announcements

- ALMA interferometry school Mar 15-26

1) ALMA Basics, Cycle 8 Capabilities
2) Interferometry Basics
3) New Proposal Review
4) The OT
5) ALMA Data Products

6) CASA Simulations
7) What's new in CASA
8) Science-ready Data Products
9) Imaging with CASA

Outline

- Re-cap of interferometry
  - What is a correlator?
- The correlation function
- Simple correlators
- Spectral line correlators
- Details
  - Sampling and quantization
  - Delay model
- The VLA and LWA correlators

This lecture is complementary to Chapter 4 of ASP 180 and is based on a lecture by Walter Brisken

Re-cap of Interferometry

- What are we fundamentally trying to measure?
- How do we accomplish this in a traditional telescope?
  - Optical or radio
- What changes when we go to a interferometer?
  - A “sparse” telescope
- What do visibilities tell us about the sky?

What is a Correlator?

A correlator is a hardware or software device that combines sampled voltage time series from one or more antennas to produce sets of complex visibilities, V_{ij}.

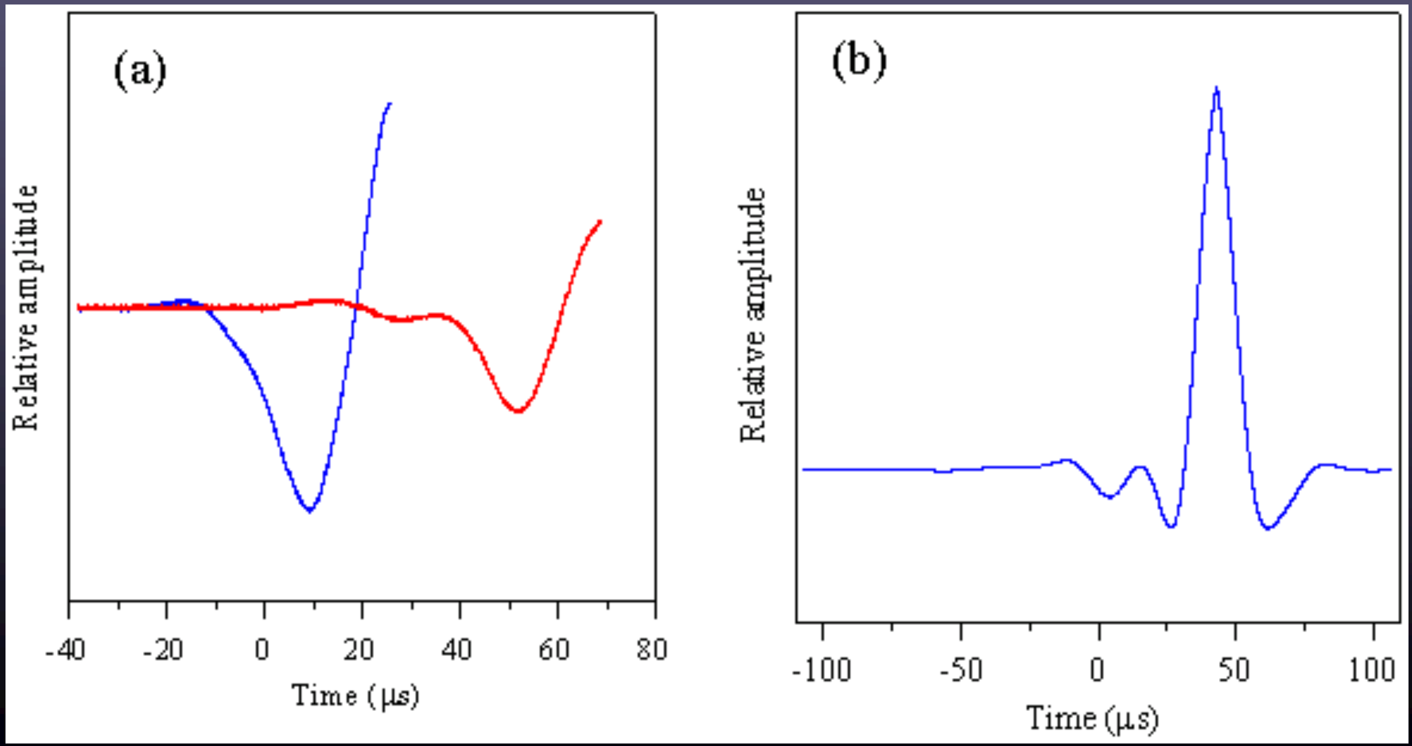
- Visibilities are in general a function of
  - Frequency
  - Antenna pair
  - Time
- They are used for
  - Imaging
  - Spectroscopy / polarimetry
  - Astrometry

The Correlation Function

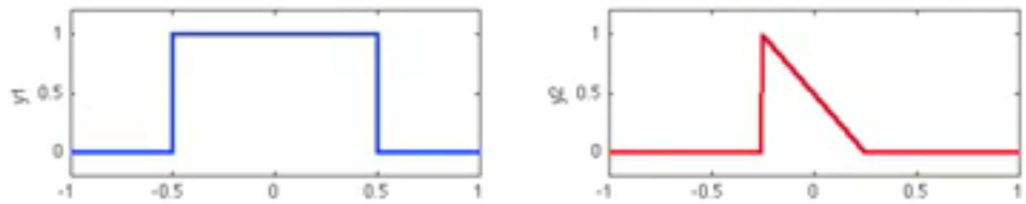
For continuous functions, *f* and *g*, the cross-correlation is defined as:

$$(f \star g)(t) \stackrel{\mathrm{def}}{=} \int_{-\infty}^{\infty} f^*(\tau)\ g(t+\tau)\,d\tau,$$

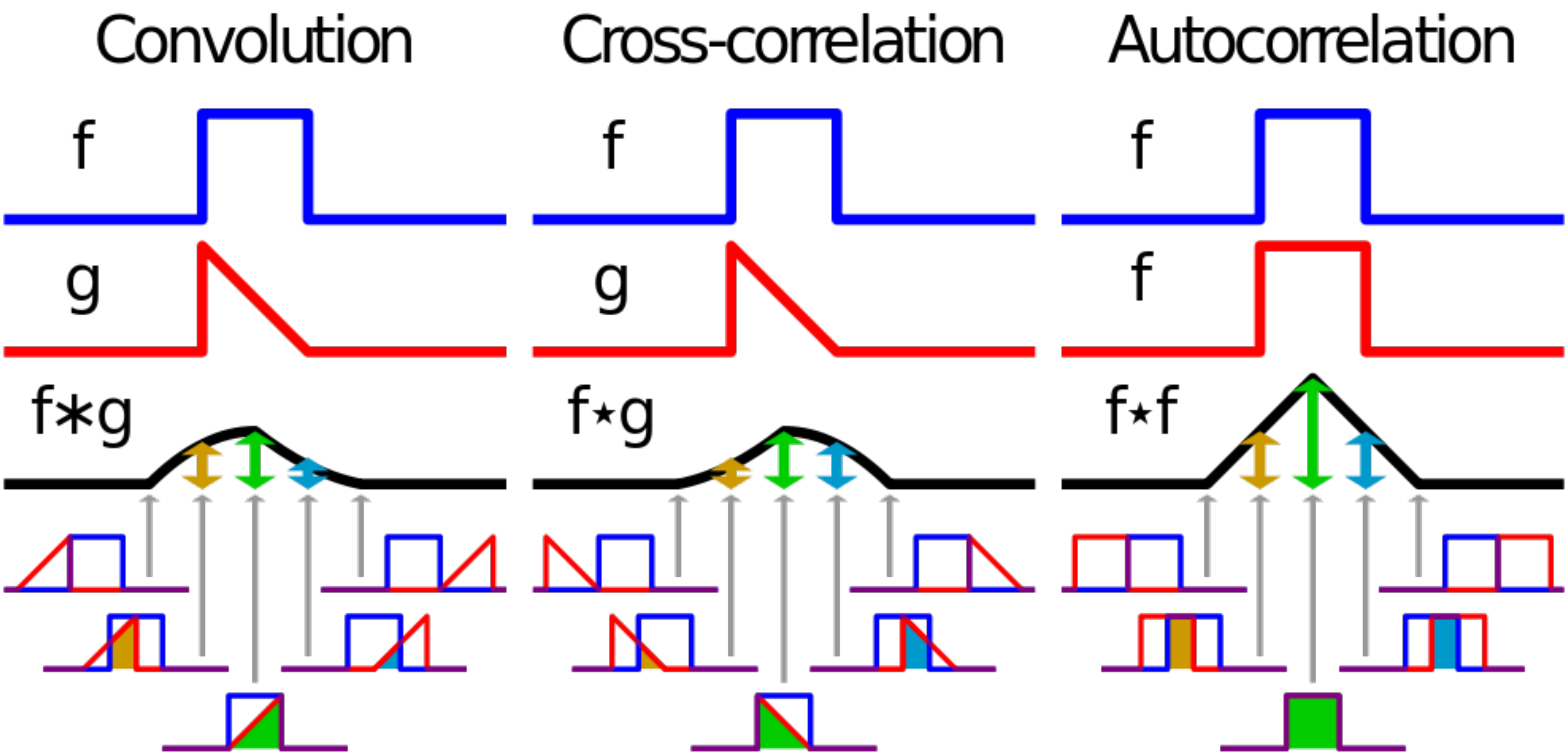
where f^* denotes the complex conjugate of *f*.



Auto-Correlation and Convolution Functions



Auto-Correlation and Convolution Functions



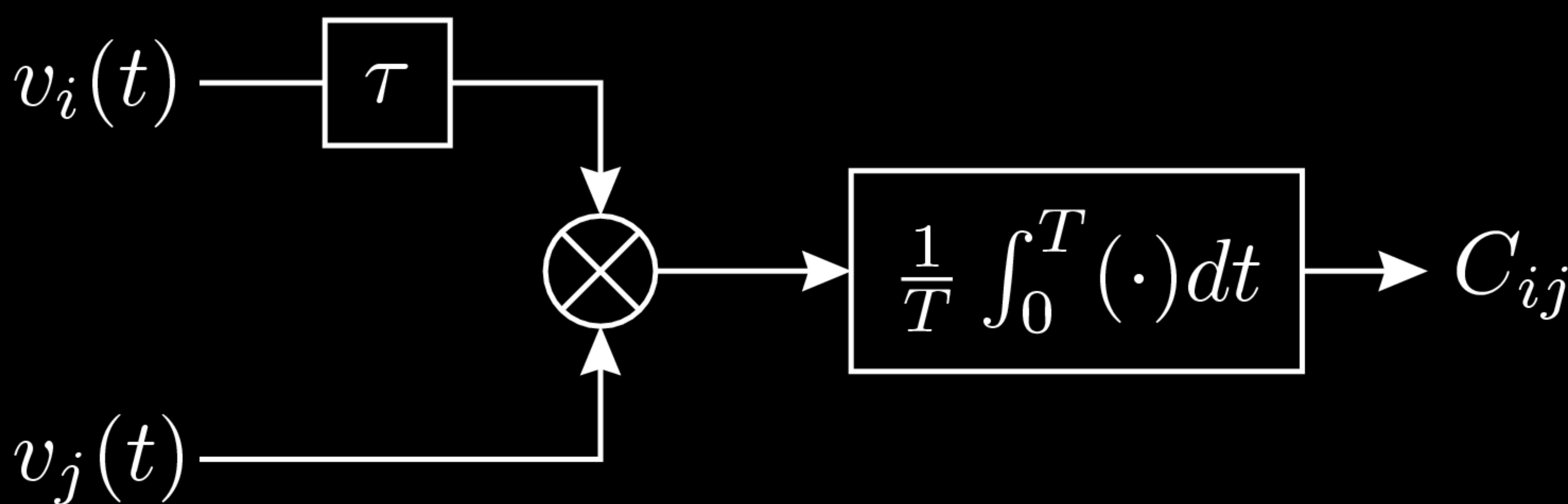
The Correlation Function

$$C_{ij}(\tau) = \langle v_i(t) v_j(t+\tau) \rangle_T$$

- If $i = j$ it is an auto-correlation (AC). Otherwise it is a cross-correlation (CC).
- Useful for
  - Determining timescales (CC and AC)
  - Motion detection (2-D CC)
  - Optical character recognition (2-D CC)
  - Pulsar timing
  - Template matching (CC)
    - Also called "matched filtering"

A Real (valued) Cross Correlator

$$C_{ij}(\tau) = \langle v_i(t) v_j(t+\tau) \rangle_T$$



Visibilities

What astronomers really want is the complex visibility

$$V_{ij} = \langle E_i(t) E_j^*(t+\tau) \rangle$$

where the real part of $E_i(t)$ is the voltage measured by antenna i.

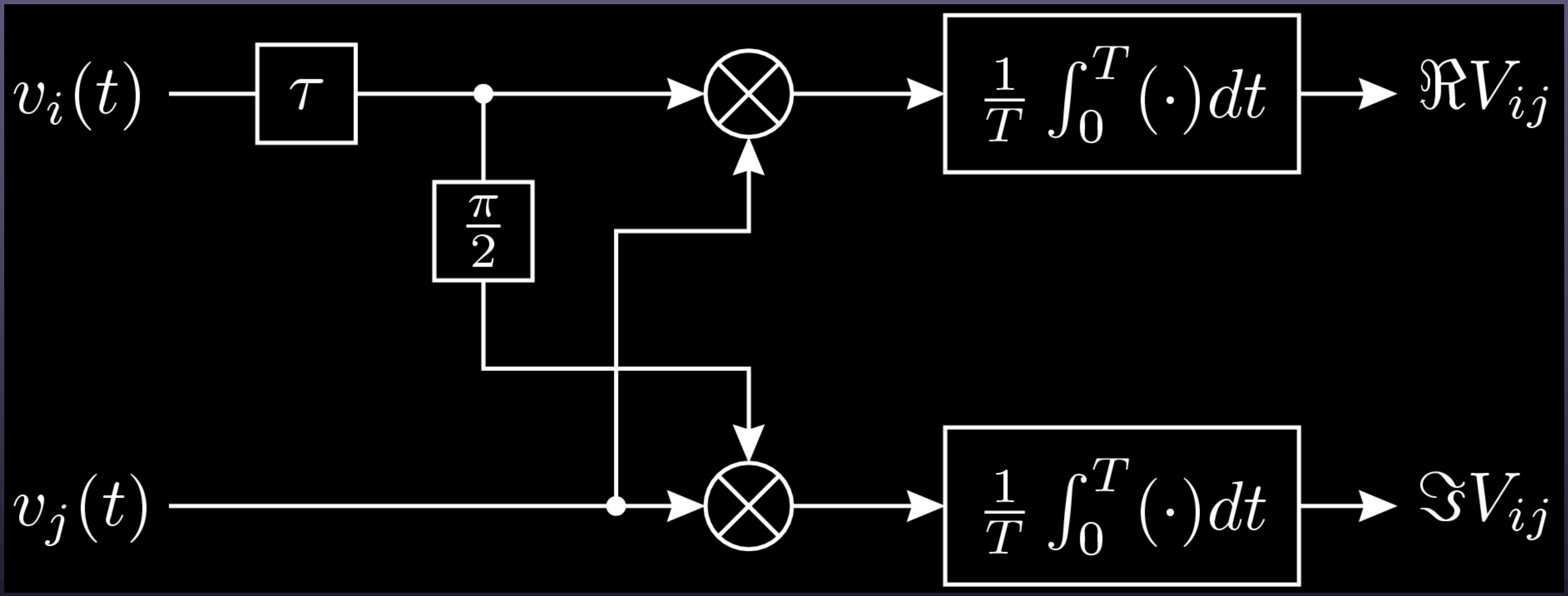
So what is the imaginary part of $E_i(t)$*?*

It is the same as the real part but with each frequency component *phase* lagged by 90 degrees.

$$E_i(t) = v_i(t) + \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{v_i(t')}{t-t'} dt'$$

The Complex Correlator

$$V_{ij} = \langle v_i(t) v_j(t+\tau) \rangle + i \langle \mathcal{H}\left[v_i(t)\right] v_j(t+\tau) \rangle$$



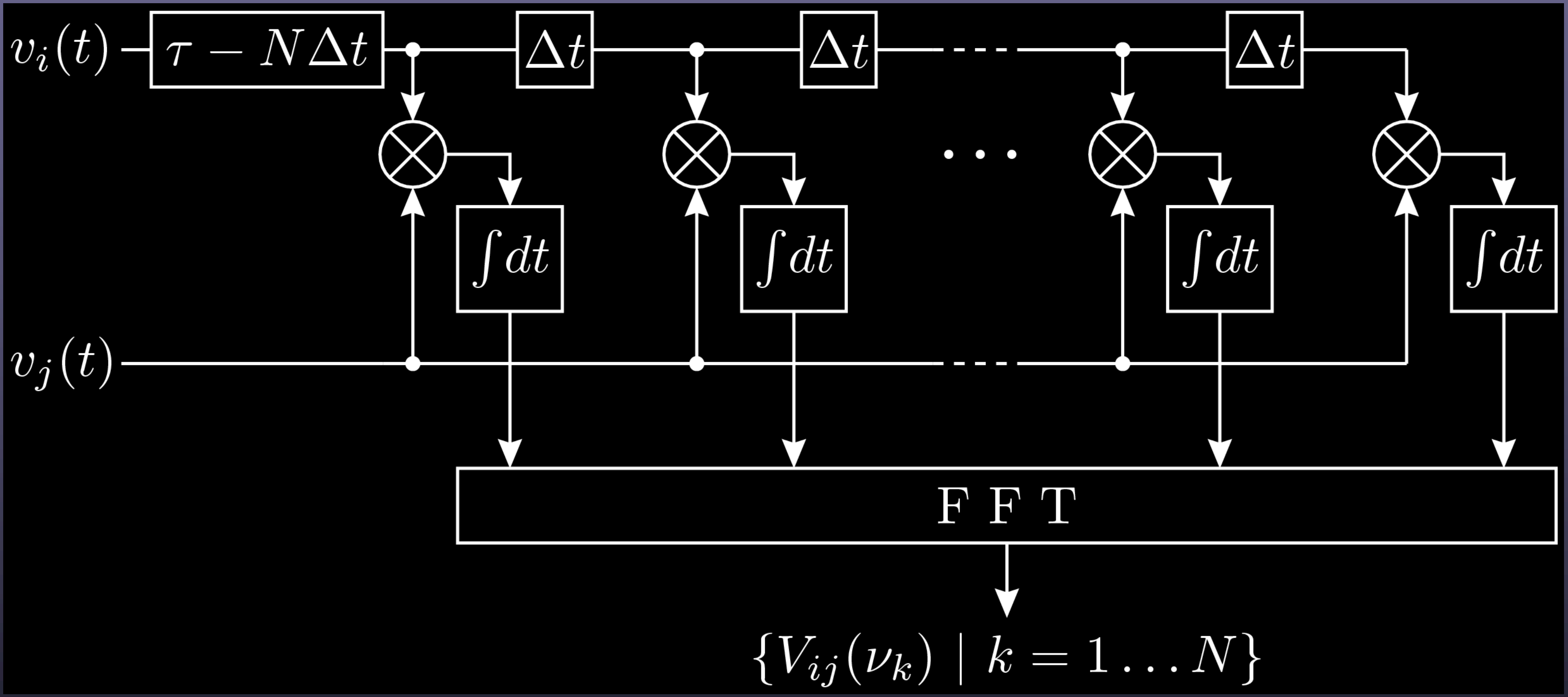
Spectral Line Correlators

- Chop up bandwidth for
  - Calibration
    - Bandpass calibration
    - Fringe fitting
  - Spectroscopy
  - Wide-field imaging
  - (Its all Spectral Line these days)
- Conceptual version
  - Build analog filter bank
  - Attach a complex correlator to each filter

Practical Spectral Line Correlators

- Use a single filter / sampler
  - Easier to calibrate
  - Practical, up to a point
- The FX architecture
  - F : Replace filterbank with digital Fourier transform
  - X : Use a complex-correlator for each frequency channel
  - Then integrate
- The XF architecture
  - X : Measure correlation function at many lags
  - Integrate
  - F : Fourier transform
- Other architectures possible

The XF Correlator



XF Spectral Response

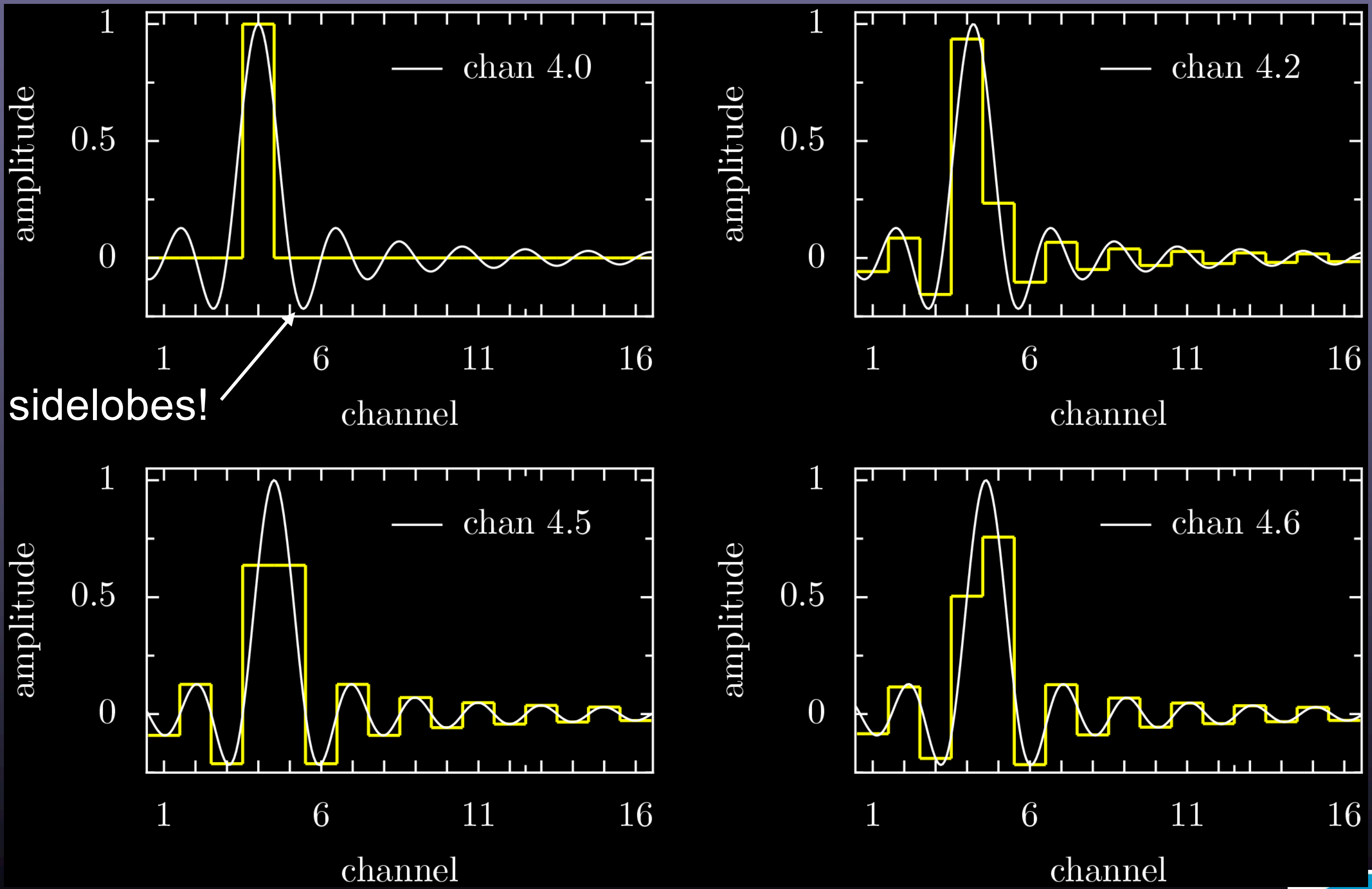
- XF correlators measure lags over a finite delay range

$$V_{ij}(\tau) \quad = \quad \langle v_i(t) v_j(t+\tau) \rangle \cdot \sqcap\left(\frac{t}{N\Delta t}\right)$$

- Results in convolved visibility spectrum

$$\begin{aligned} V_{ij}(\nu) \quad &= \quad \mathcal{F}\left[\langle v_i(t) v_j(t+\tau) \rangle \cdot \sqcap\left(\frac{t}{N\Delta t}\right)\right] \\ &= \quad \mathcal{F}\left[\langle v_i(t) v_j(t+\tau) \rangle\right] \star \mathrm{sinc}\left(N\Delta t\, \nu\right) \end{aligned}$$

XF Spectral Response (2)



22% sidelobes!

Hanning Smoothing

- Multiply lag spectrum by Hanning taper function

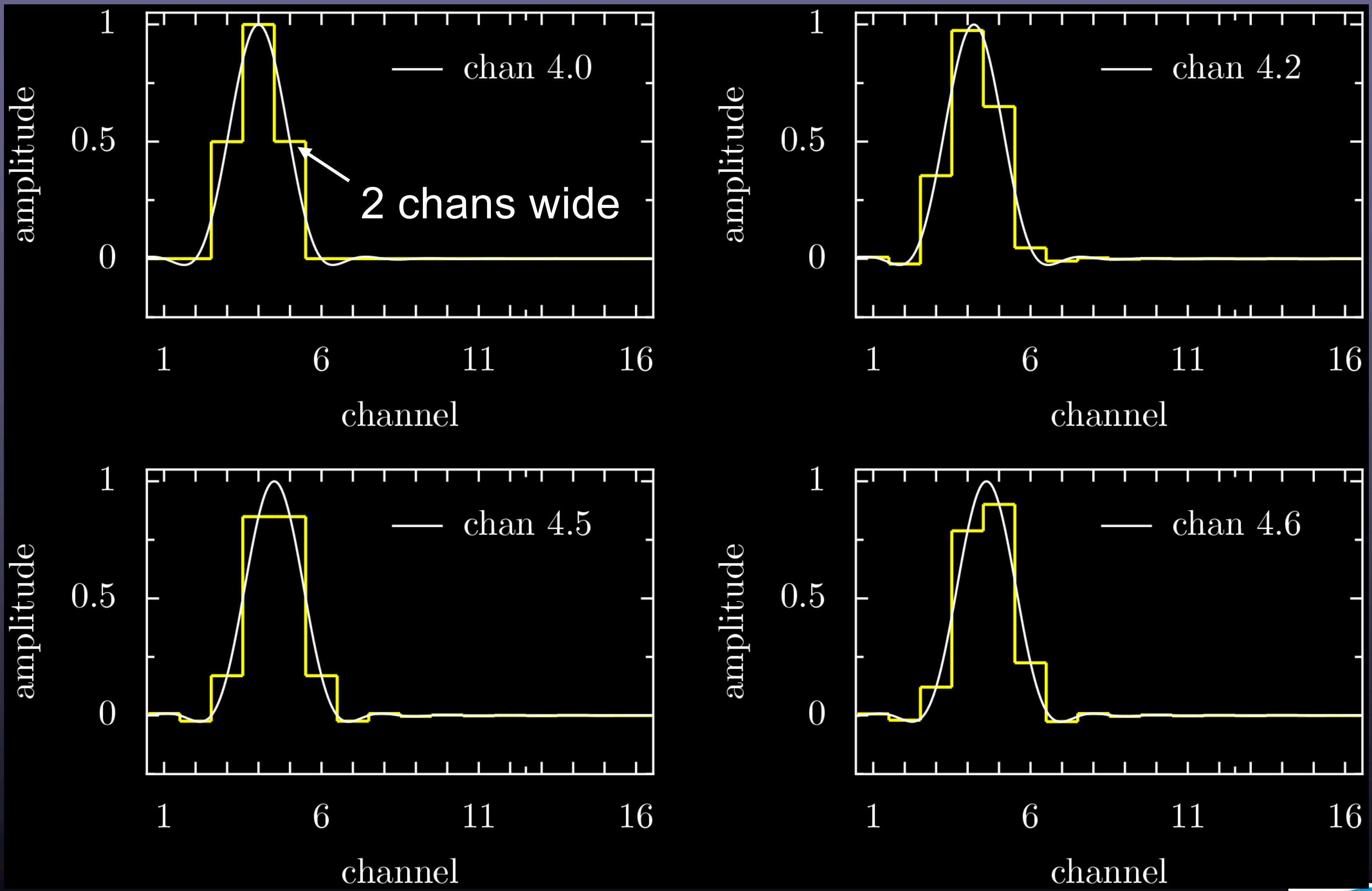
$$H(\tau) = \frac{1}{2}\left(1 + \cos\frac{\pi\tau}{N\Delta t}\right)$$

- This is equivalent to convolution of the spectrum by

$$H(\nu) = \delta(\nu) - \frac{1}{2}\delta\left(\nu - \frac{1}{2N\Delta t}\right) - \frac{1}{2}\delta\left(\nu + \frac{1}{2N\Delta t}\right)$$

- Note that sensitivity and spectral resolution are reduced.

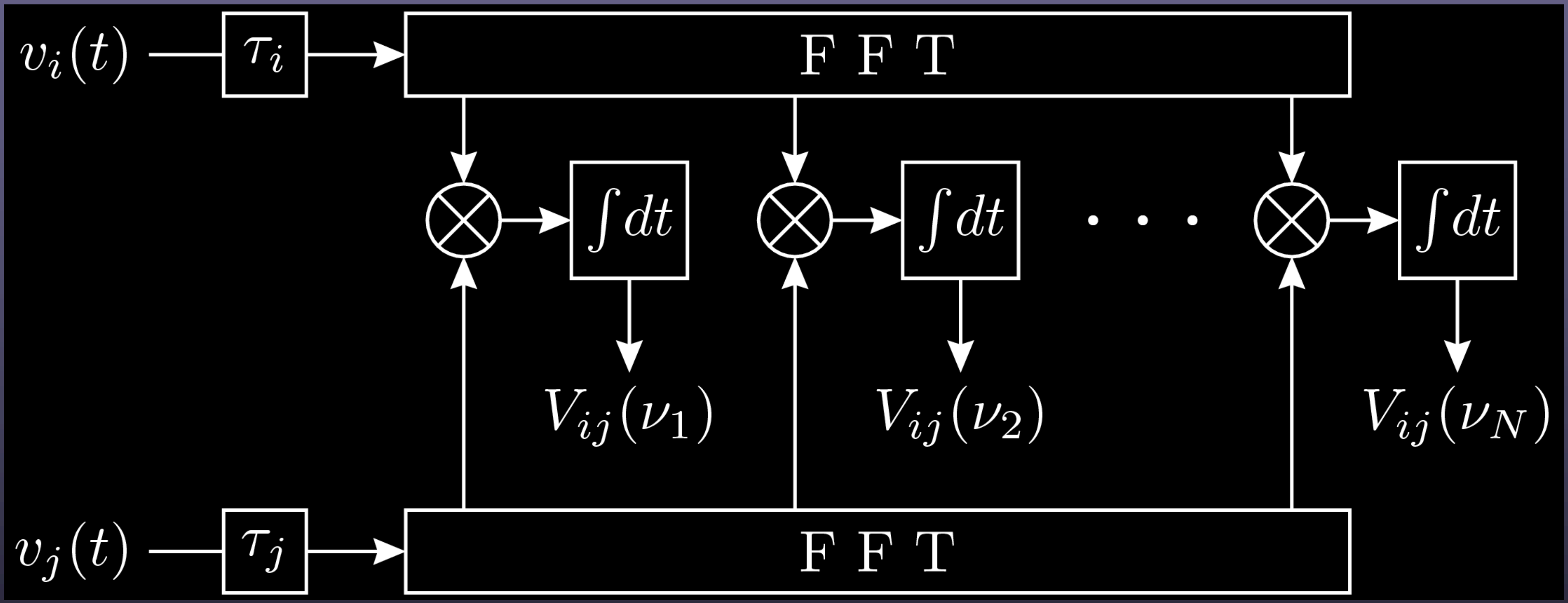
Hanning Smoothing (2)



FX Correlators

- Spectrum is available **before integration**
  - Can apply fractional sample delay per channel
  - Can apply pulsar gate per channel
- Most of the digital parts run *N* times slower than the sample rate
- Fewer computations (compared to XF)

The FX correlator



FX Spectral Response

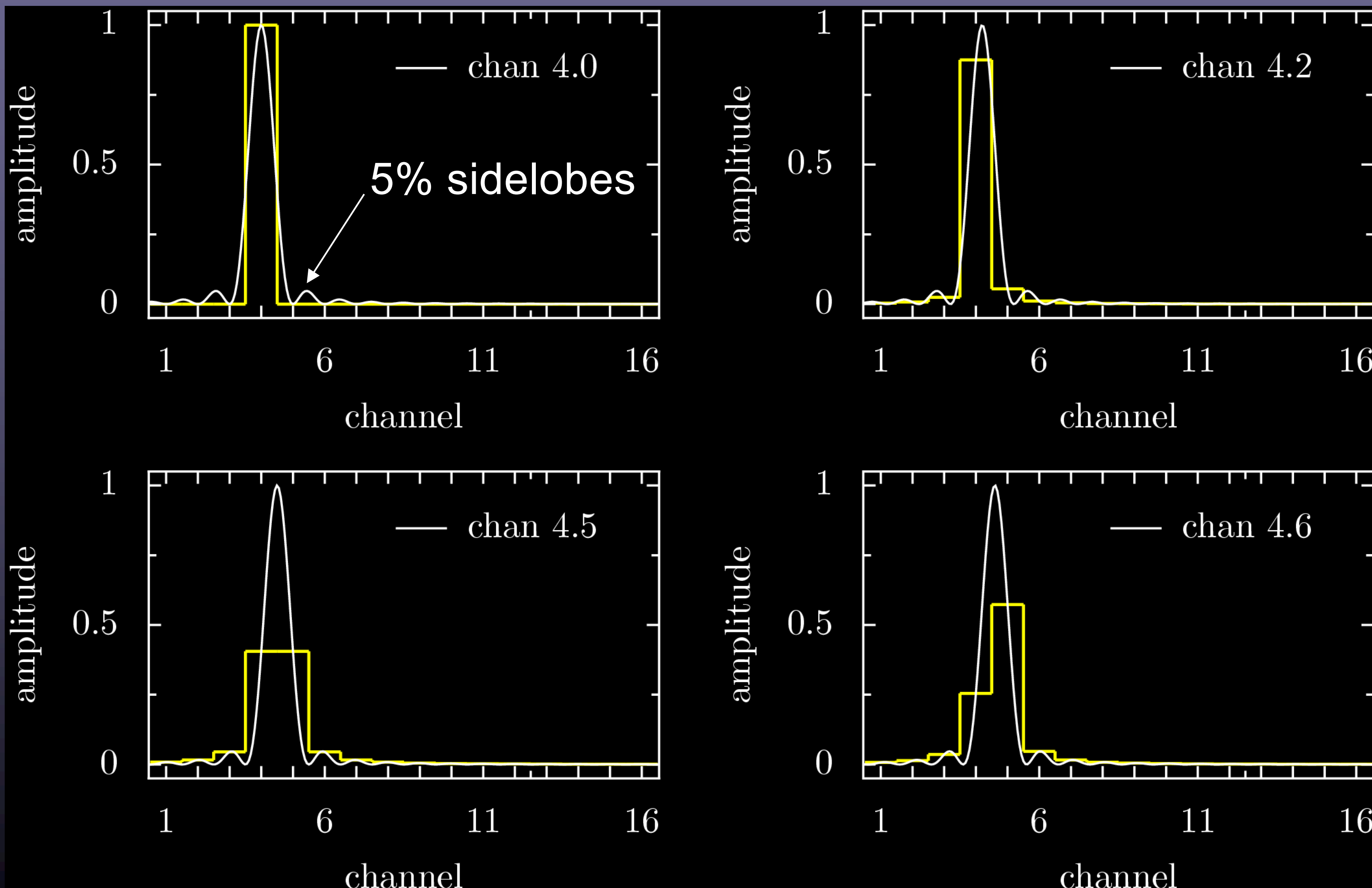
- FX Correlators derive spectra from truncated time series

$$
\begin{aligned}
v(\nu) &= \mathcal{F}\left[v(t)\cdot\sqcap\left(\frac{t}{N\Delta t}\right)\right] \\
&= \mathcal{F}[v(t)]\star\mathcal{F}\left[\sqcap\left(\frac{t}{N\Delta t}\right)\right] \\
&\propto \mathcal{F}[v(t)]\star\mathrm{sinc}\,(N\Delta t\nu)
\end{aligned}
$$

- Results in convolved visibility spectrum

$$
\begin{aligned}
V_{ij}(\nu) &= \left\langle\left(\mathcal{F}[v_i(t)]\star\mathrm{sinc}\,(N\Delta t\nu)\right)\left(\mathcal{F}[v_j(t)]\star\mathrm{sinc}\,(N\Delta t\,\nu)\right)^*\right\rangle \\
&= \left\langle\mathcal{F}[v_i(t)]\,\mathcal{F}[v_j(t)]^*\right\rangle\star\mathrm{sinc}^2\,(N\Delta t\,\nu)
\end{aligned}
$$

FX Spectral Response (2)



Time Series, Sampling, and Quantization

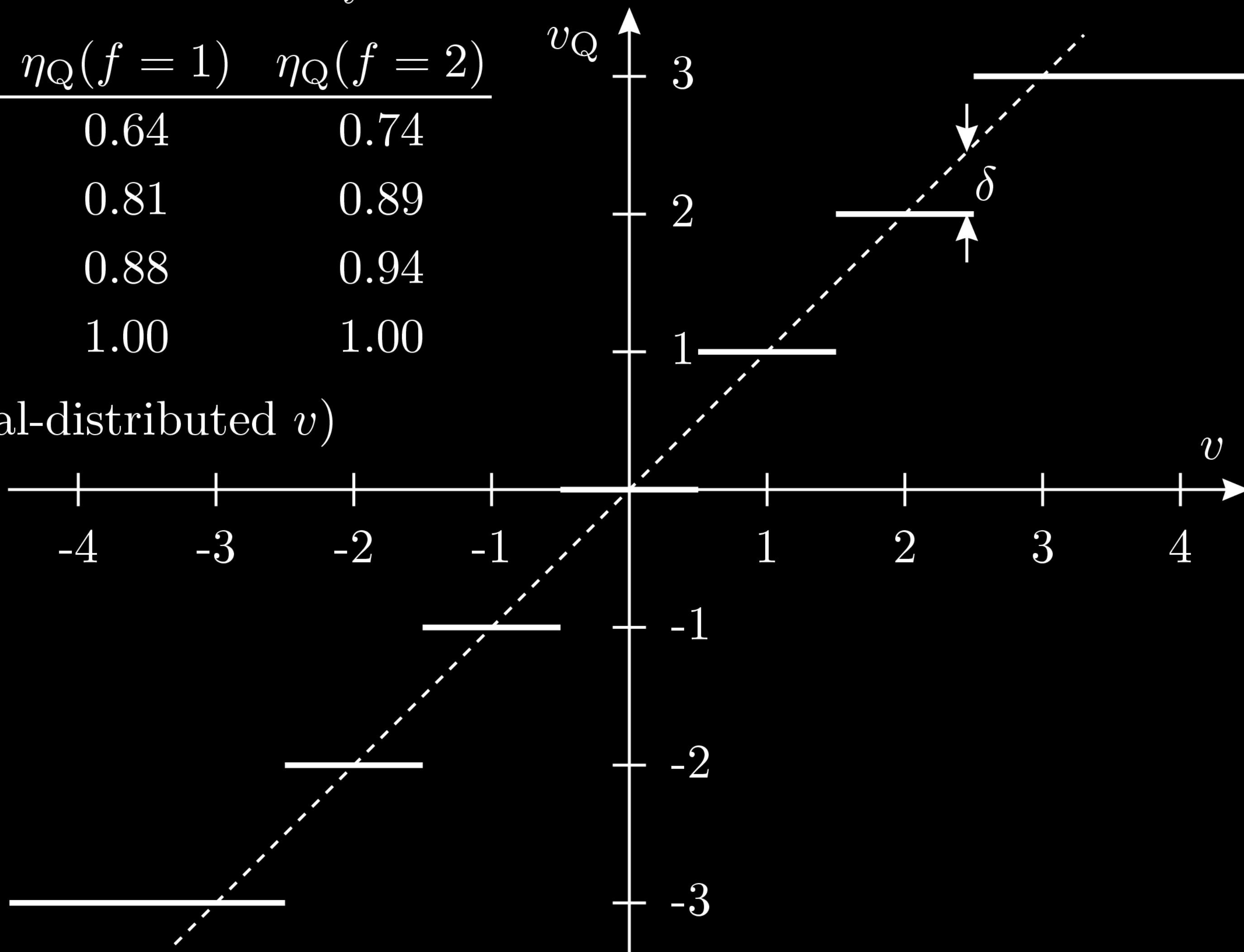
- $\{v_i(t)\}$ are real-valued time series sampled at "uniform" intervals, Δt.
- The sampling theorem allows this to accurately reconstruct a bandwidth of $\Delta\nu = \frac{1}{2\Delta t}$.
- Sampling involves quantization of the signal
  - Quantization noise
  - Strong signals become non-linear
  - Sampling theorem violated!

Quantization Noise

Quantization efficiency

N levels	$\eta_Q(f=1)$	$\eta_Q(f=2)$
2	0.64	0.74
3	0.81	0.89
4	0.88	0.94
∞	1.00	1.00

(For normal-distributed v)



Automatic Gain Control (AGC)

- Normally prior to sampling the amplitude level of each time series is adjusted so that quantization noise is minimized.
- This occurs on timescales very long compared to a sample interval.
- The magnitude of the amplitude is stored so that the true amplitudes can be reconstructed after correlation.

The Correlation Coefficient

- The correlation coefficient, ρ_{ij} measures the likeness of two time series in an amplitude independent manner:

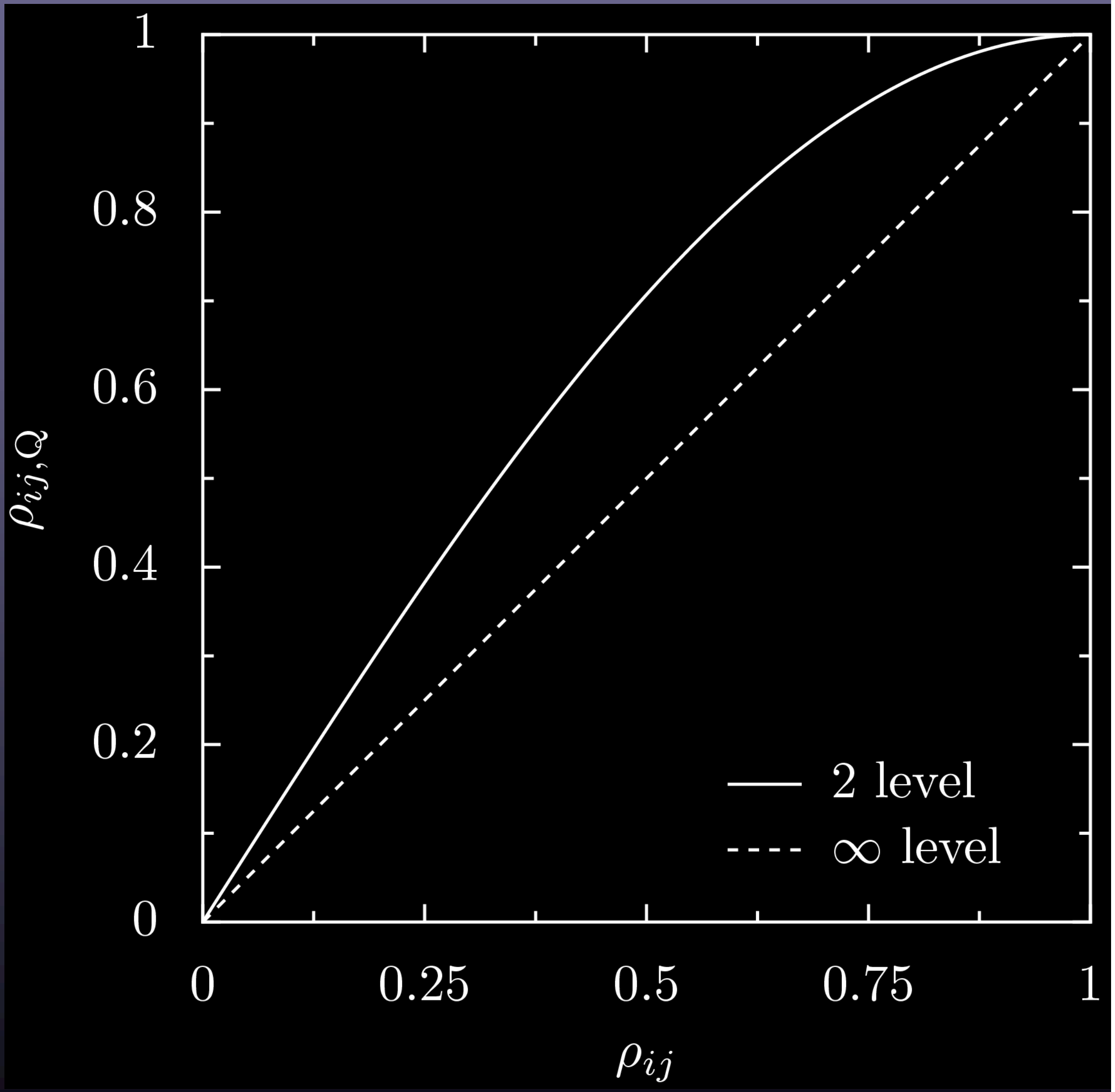
$$\rho_{ij} = \frac{|V_{ij}|}{\sqrt{V_{ii} V_{jj}}}$$

- Normally the correlation coefficient is much less than 1

- Because of AGC, the correlator actually measures the correlation coefficient. The visibility amplitude is restored by dividing by the AGC gain.

Van Vleck Correction

- At low correlation, quantization ***increases correlation***
- Quantization causes predictable non-linearity at high correlation V_{ij}
- Correction must be applied to the real and imaginary parts of separately
  - Thus the visibility phase is affected as well as the amplitude

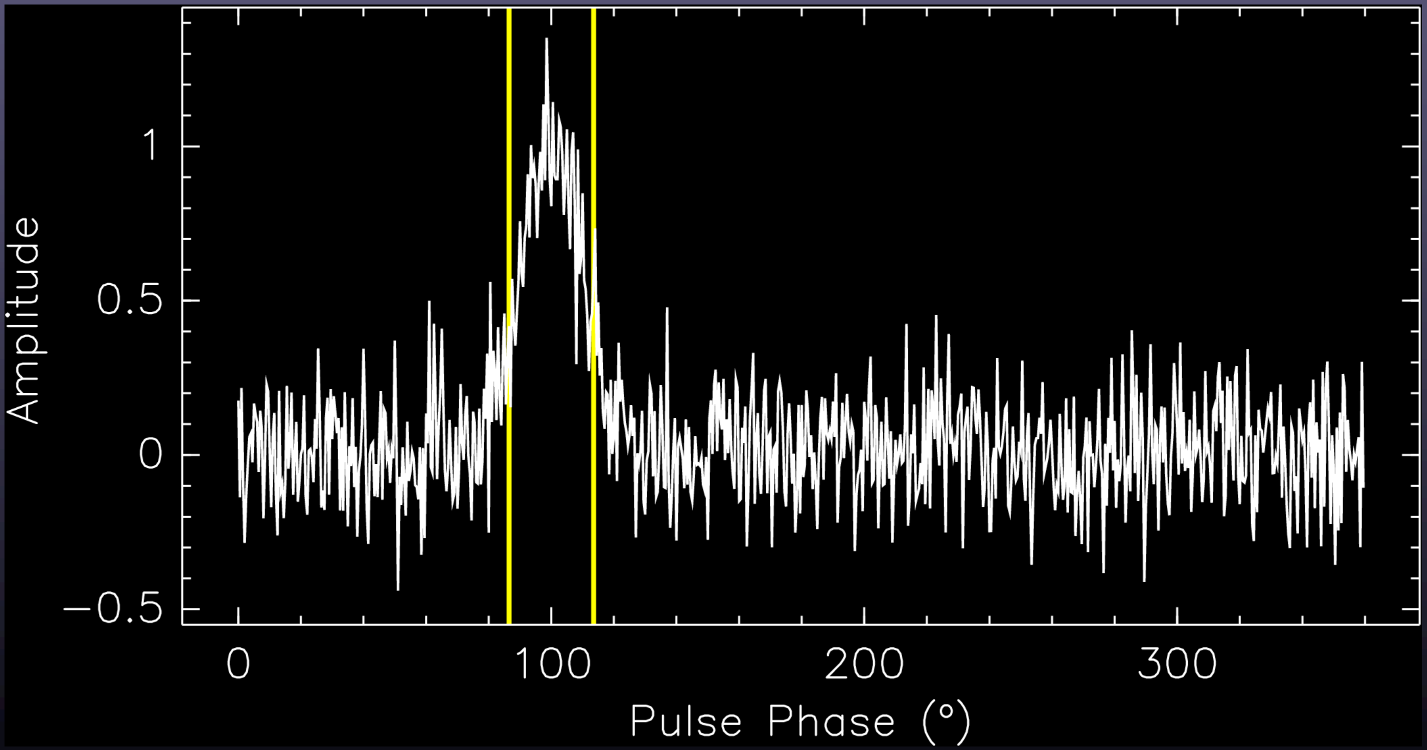


The Delay Model

- τ is the difference between the geometric delays of antenna j and antenna i. It can be + or - .
- The *delay center* moves across the sky
  - τ is changing constantly
- Fringes at the delay center are stopped.
  - Long time integrations can be done
  - Wide bandwidths can be used
- Simple delay models incorporate:
  - Antenna locations
  - Source position
  - Earth orientation
- VLBI delay models must include much more!

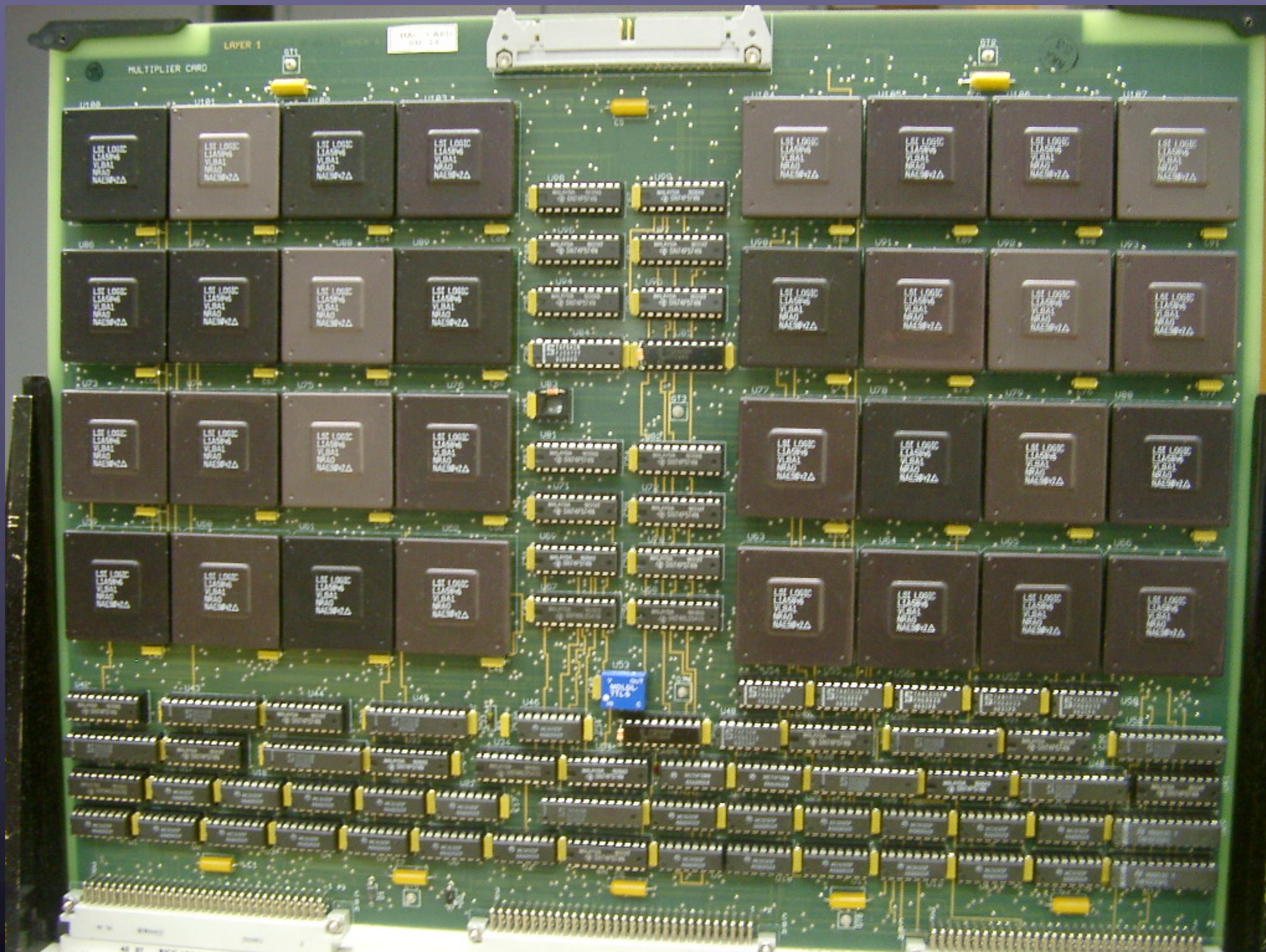
Pulsar Gating

- Pulsars emit regular pulses with small duty cycle
- Period in range 1 ms to 8 s; $\Delta t \ll P_{\rm pulsar} < T$
- Blanking during off-pulse improves sensitivity
- Propagation delay is frequency dependent

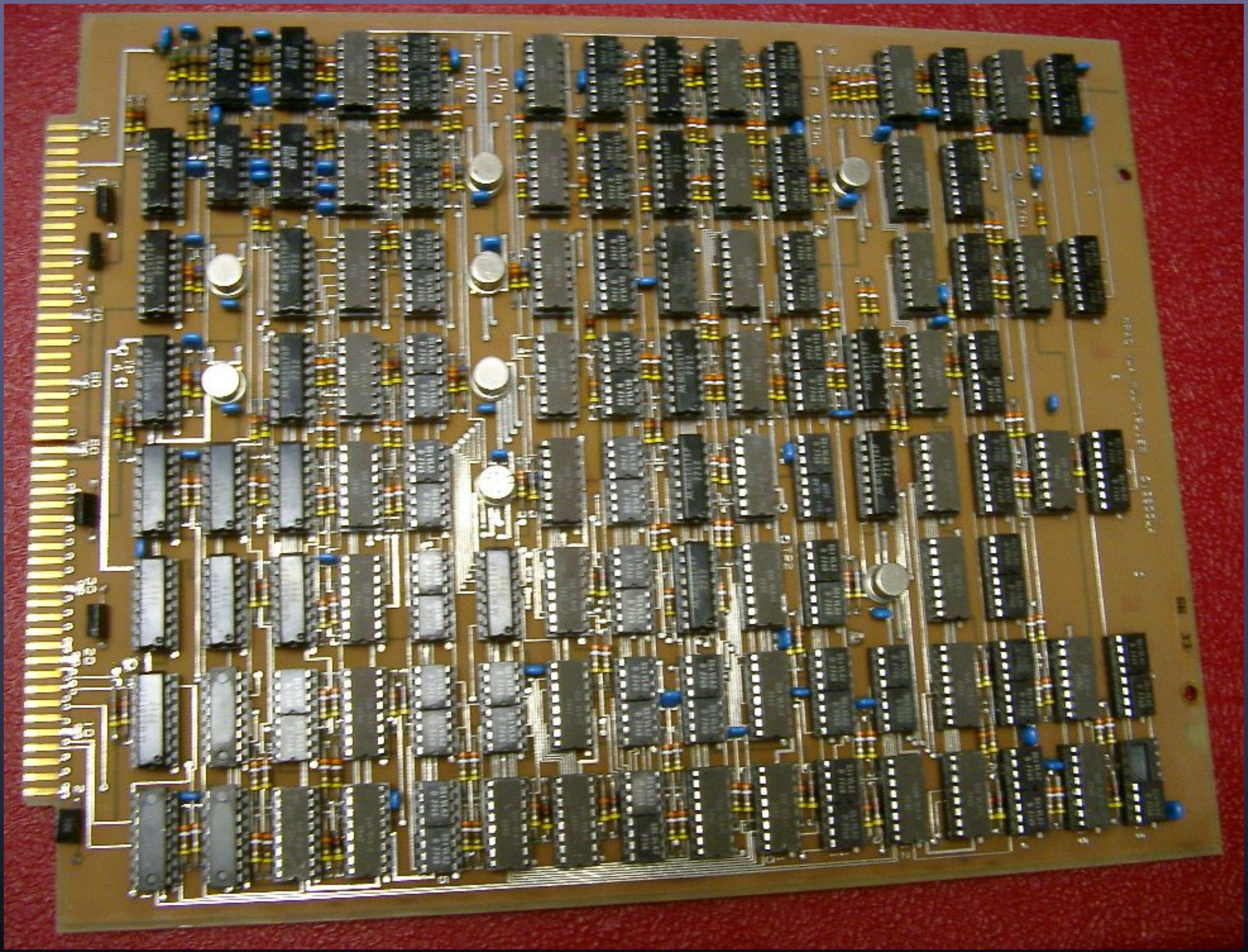


The [old] VLBA Correlator

VLBA Multiply Accumulate (MAC) Card

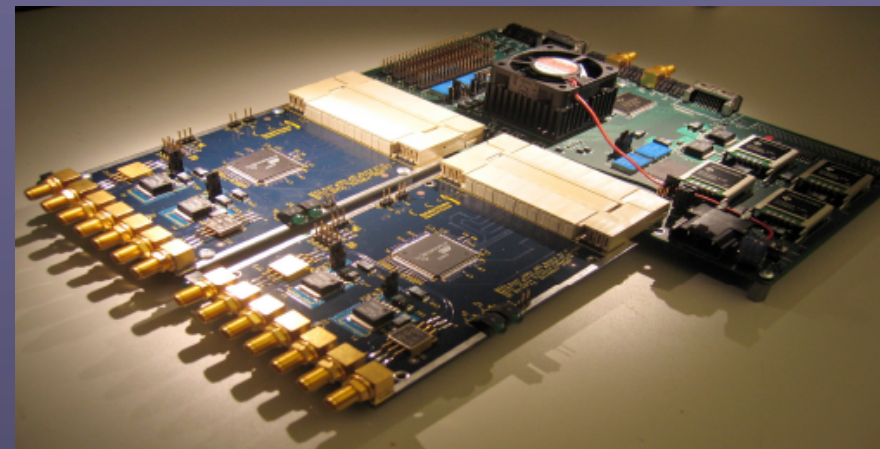


[Old] VLA MAC Card

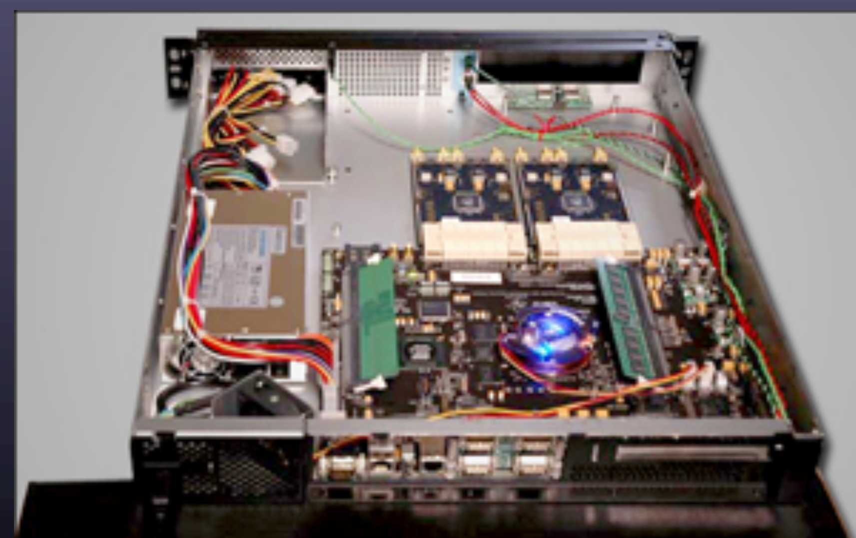


BEE2-based Correlator

- BEE2: FPGA-based, scalable, modular, upgradeable signal processing system for radio astronomy developed at Berkeley
- ROACH2 boards at LWA-SV
- Being used for several projects
  - 300-station FX correlator for EOR telescope (HERA)
  - 288-station correlator for LWA-OVRO
- Modest hardware cost ($15k/ROACH2 + switch)
  - LWA-SV uses 16 ROACH2 + 7 GPU servers
- Real effort is in the FPGA “software”



IBOB: Internet BreakOut Board



ROACH board

The VLA WIDAR Correlator

- XF architecture duplicated 64 times, or “FXF”
  - Four 2GHz basebands per polarization (3 bit sampling)
  - Digital filterbank makes 16 subbands per baseband
  - 16,384 channels/baseline at full sensitivity
  - 4 million channels with less bandwidth!
- Initially will support 32 stations with plans for 48
- 2 stations at 25% bandwidth or 4 stations at 6.25% bandwidth can replace 1 station input
- Correlator efficiency is about 95%
  - Compare to 81% for VLA
- VLBI and LWA ready

Basic Correlator Stages for the LWA

1. Correlate LWA1 beams with single dipoles at LWA1 and LWA-SV (success!)
2. Correlate LWA1 and LWA-SV using LSL and supercorrelator.py
3. Digitize VLA dishes and correlate with LWA1 and LWA-SV (works!) on LWAUCF with LSL supercorrelator.py
4. Correlate ~10 LWA stations (the “swarm”) use DifX software correlator
5. Correlate ~50 stations (the “hive”) go to GPU based correlator?

Current LWA Correlator

eLWA Correlator Status

System	Status	Last Updated
Dispatcher	Running	1 minute(s)
LWA-SV	Running	1 minute(s)
LWA1	Running	1 minute(s)

Project	Observation Date	Raw Size	Status
20B-296, 38964945	2021/02/09 07:24 UTC	13.347 TB	completed for 6 days, 14:39:24
20B-252, 38964902	2021/02/14 19:12 UTC	4.595 TB	completed for 6 days, 13:42:12
20B-296, 39179170	2021/02/15 08:53 UTC	6.019 TB	completed for 4 days, 13:09:36

Last retrieved 0 minutes ago

Disk Usage

Node	Mount Point	Usage	Free
lwaucf1	/data/local	11%	16T
lwaucf2	/data/local	21%	14T
lwaucf3	/data/local	35%	12T
All	/home	38%	2.2T
All	/data/network	88%	18T

Last retrieved 0 hour(s), 3 minute(s) ago.

Further Reading

- http://www.nrao.edu/whatisra/mechanisms.shtml
- http://www.nrao.edu/whatisra/
- www.nrao.edu

- Synthesis Imaging in Radio Astronomy
- ASP Vol 180, eds Taylor, Carilli & Perley