Radio Astronomy Amplifiers, Receivers

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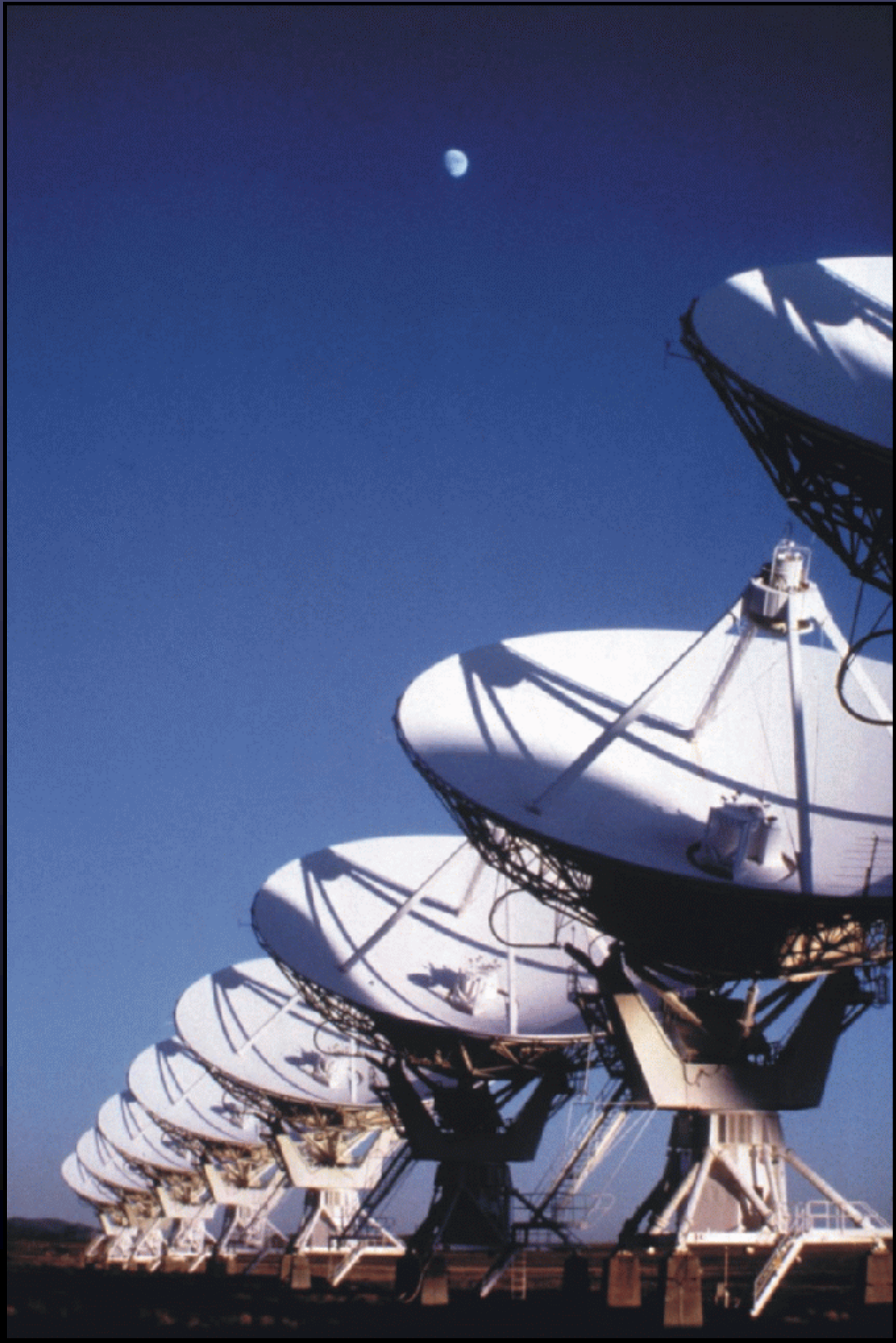


Announcements

- Class Time Allocation Meeting
- Goals: Identify 3 or 4 projects
- Assign people to projects
- Identify scheduler(s) for each project

- We will discuss and rank each proposal
- It is ok to change your ranking based on the discussion
- Most important is 1st, 2nd, 3rd ranking (win, place or show)

More useful is the limiting flux density ΔS

$$S_\nu = \int_\Omega B_\nu \cos\theta \, d\Omega$$

$$S_\nu = \frac{2kT}{\lambda^2} \Omega_a \qquad \frac{\Omega_a}{\lambda^2} = \frac{1}{A_e}$$

$$S_\nu = \frac{2k}{A_e} T$$

$$\Delta S = \frac{2k}{A_e} \Delta T$$

radiometer equation

single polarization

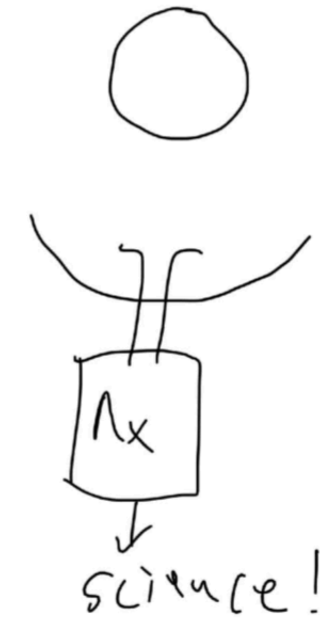
$$\boxed{\Delta S = \frac{2k}{A_e} \frac{T_{sys}}{\sqrt{\Delta\nu\,\tau}}}$$

Define Gain $= \frac{A_e}{2k}$ in $\frac{K}{Jy}$

$$SEFD = \frac{T_{sys}}{Gain} \text{ in } Jy$$

$$\Delta S = \frac{SEFD}{\sqrt{\Delta\nu\,\tau}}$$

Radio Astronomy Notes 5-1



Lets look at the moon, as all telescopes should be able to do ...

$$T_a \approx T_s \approx 225\,K$$

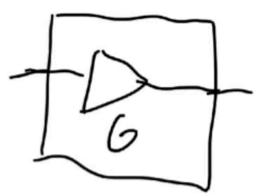
power received $W_\nu = kT_a = 4 \times 10^{-14}\ erg\ s^{-1} Hz^{-1}$

$W_\nu = 4 \times 10^{-21}\ W\ Hz^{-1}$

Electronic detecters are sensitive to signals as faint as 1 nanowatt $= 10^{-9}\ W$ so we need $G \gtrsim 10^{10}$ for detection

$G_{dB} = 10 \log (10^{10})\ dB$, so over 100 dB of gain!

becomes



which will add noise (T_{sys})
Why?

Fundamental noise limit, start with Heisenberg uncertainty principle

$$\Delta E \, \Delta t \geq \frac{\hbar}{2} \qquad \hbar = h/2\pi$$

$$\Delta E \, \Delta t \geq \frac{h}{4\pi}$$ and recall $E = h\nu$ so lets switch to ν

$$\Delta E = E_{photon} \cdot \Delta n$$ Δn = uncertainty in number of photons

$$\Delta E = h\nu \, \Delta n$$

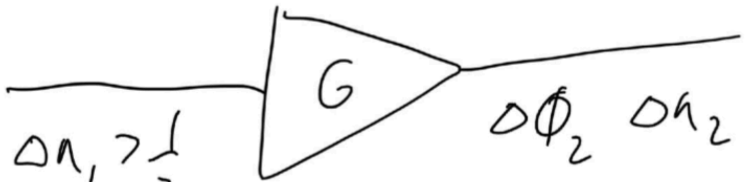
Change in phase, ϕ, is freq times change in time

$$\Delta\phi = 2\pi\nu \cdot \Delta t$$

$$\cancel{h}\cancel{\nu}\Delta n \cdot \frac{\Delta\phi}{2\cancel{\pi}\cancel{\nu}} \geq \frac{\cancel{h}}{4\cancel{\pi}}$$

$$\Delta n \, \Delta\phi \geq \frac{1}{2}$$ and in radio astronomy Δn can be large

Lets apply the fundamental limit to an amplifier



$\Delta\phi_1 \Delta n_1 > \frac{1}{2}$ $\quad\quad$ $\Delta\phi_2 \Delta n_2 > \frac{1}{2}$

and $\quad n_2 = n_1 G$

$\Delta n_2 = \Delta n_1 G \quad\quad \Delta n_1 = \Delta n_2 / G$

output phase $\phi_2 = \phi_1 + C$ $\quad$ shifted

$$\Delta\phi_2 = \Delta\phi_1$$

$\therefore \quad \Delta\phi_2 \Delta n_2 = \Delta\phi_1 \Delta n_1 G \quad$ and $\quad \Delta\phi_2 \Delta n_2 > \frac{1}{2}$

$\Delta\phi_1 \Delta n_1 \geq \frac{1}{2G}$ $\quad$ but this can't be true for any real gain $G >> 1$

In reality $\quad \Delta\phi_2 \Delta n_2 = \Delta\phi_1 \Delta n_1 (G + \Delta G)$ $\quad$ amplifier must add noise amplifier noise limits

$$kT_{out} = G\,(kT_a + \underbrace{kT_{rx}}_{h\nu})$$

$kT_{rx} = h\nu$

$T_{rx} = \frac{h\nu}{k}$ Quantum limit

Freq	λ	T_{min}
5 GHz	6 cm	0.2 K
300 GHz	1 mm	14 K
IR	10^{-6} m	14,000 K

amplifier noise goes up with frequency

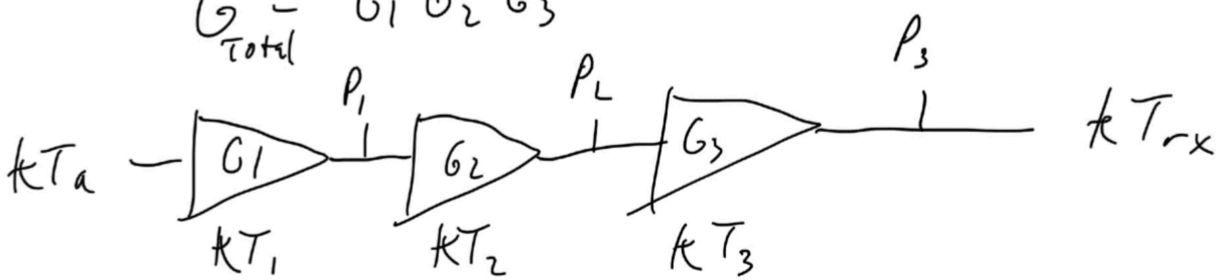
$T_{rx} = T_{min} + T_{ambient}$

Make $T_{ambient}$ small by cooling amplifier

Radio Astronomy Notes 5-3

Receivers

$$G_{Total} = G_1 G_2 G_3$$



$$P_1 = (kT_a + kT_1) G_1 \qquad \text{and} \qquad P_3 = k(T_a + T_{rx}) G_1 G_2 G_3$$

$$P_2 = (P_1 + kT_2) G_2 = (kT_a + kT_1) G_1 G_2 + kT_2 G_2$$

$$P_3 = (P_2 + kT_3) G_3 = (kT_a + kT_1) G_1 G_2 G_3 + kT_2 G_2 G_3 + kT_3 G_3$$

$$(\cancel{k}\cancel{T_a} + \cancel{k}T_{rx})G_1G_2G_3 = (\cancel{k}\cancel{T_a} + \cancel{k}T_1)G_1G_2G_3 + \cancel{k}T_2G_2G_3 + \cancel{k}T_3G_3$$

$$T_{rx}G_1G_2G_3 = T_1G_1G_2G_3 + T_2G_2G_3 + T_3G_3$$

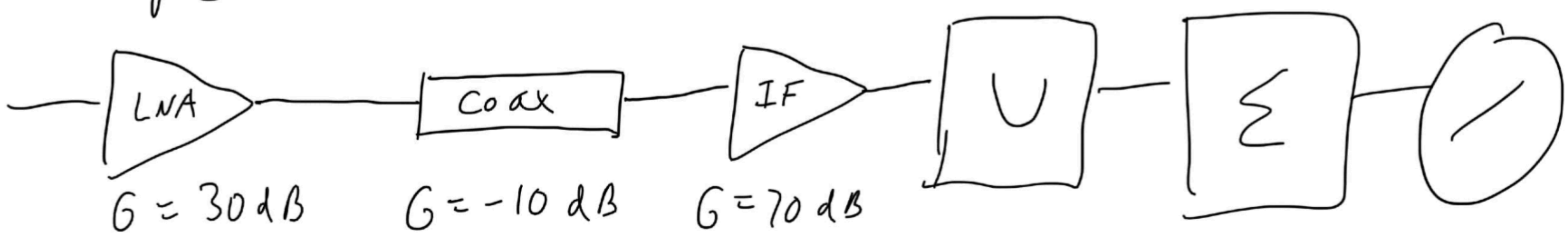
$$T_{rx} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1G_2}$$

which amplifier noise is most important?

why do we care?

Sensitivity $\quad \Delta T = \frac{T_{sys}}{\sqrt{\Delta\nu\tau}}$

LWA example



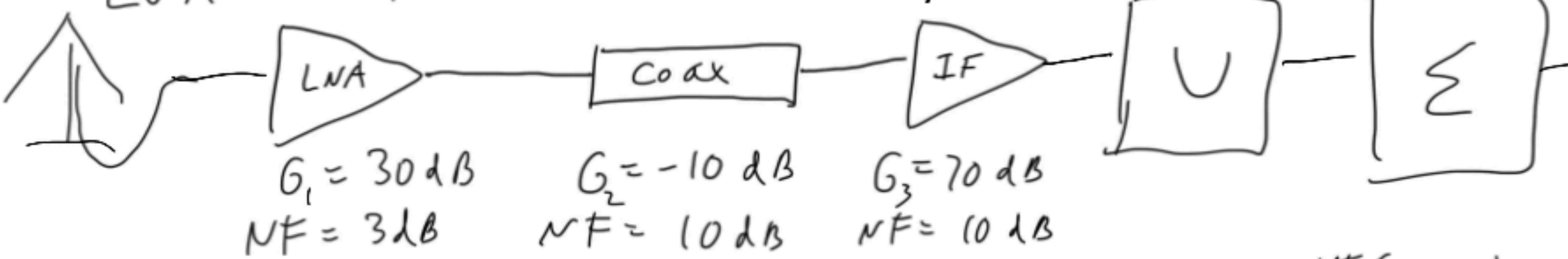
$G = 30\,dB$ $\quad$ $G = -10\,dB$ $\quad$ $G = 70\,dB$

$NF = 3\,dB$ $\quad$ $NF = 10\,dB$ $\quad$ $NF = 10\,dB$

$$NF = \frac{T_{noise}}{T_{ref}} + 1 \qquad T_{ref} = 290\,K$$

Radio Astronomy Notes

LWA Example



$$NF = \frac{T_{noise}}{T_{ref}} + 1 \qquad T_{ref} = 290\,K \qquad T = T_{ref}\left(10^{NF/10} - 1\right)$$

Worksheet #3

- Download the worksheet from:

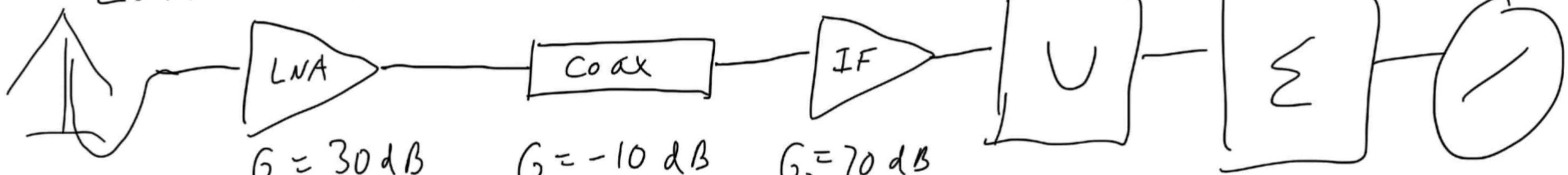
http://www.phys.unm.edu/~gbtaylor/astr423/WS3.pdf

Solve it in class.

Ask questions if you are stuck

Tell me when you have the answer.

LWA Example



$G_1 = 30\,dB$ $\quad G_2 = -10\,dB$ $\quad G_3 = 70\,dB$

$NF = 3\,dB$ $\quad NF = 10\,dB$ $\quad NF = 10\,dB$

$$NF = \frac{T_{noise}}{T_{ref}} + 1 \qquad T_{ref} = 290\,K \qquad T = T_{ref}\left(10^{NF/10} - 1\right)$$

$$G_{Total} = 30 - 10 + 70 = 90\,dB \qquad NF_{total} = 23\,dB$$

$$T_{rx} = T_{LNA} + \frac{T_{coax}}{G_1} + \frac{T_{IF}}{G_1 G_2}$$

$T_{LNA} = 290\,K$

$T_{coax} = 2610\,K$

$T_{IF} = 2610\,K$

$$T_{rx} = 290 + \frac{2610}{1000} + \frac{2610}{1000 \cdot 0.1}$$

$$= 290 + 2.6 + 26$$

$$= 319\,K$$

What if T_{LNA} doubled? $\quad T_{rx} = 609\,K$ (ouch)

What if T_{IF} doubled? $\quad T_{rx} = 345\,K$ (not so bad)

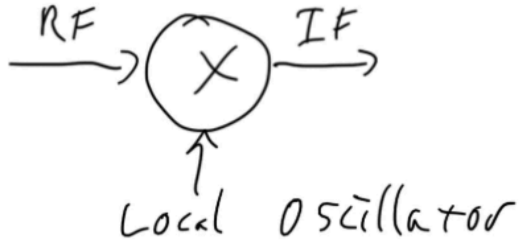
Heterodyne receiver

What if we want to transmit signals at a fixed frequency?

So called intermediate frequency

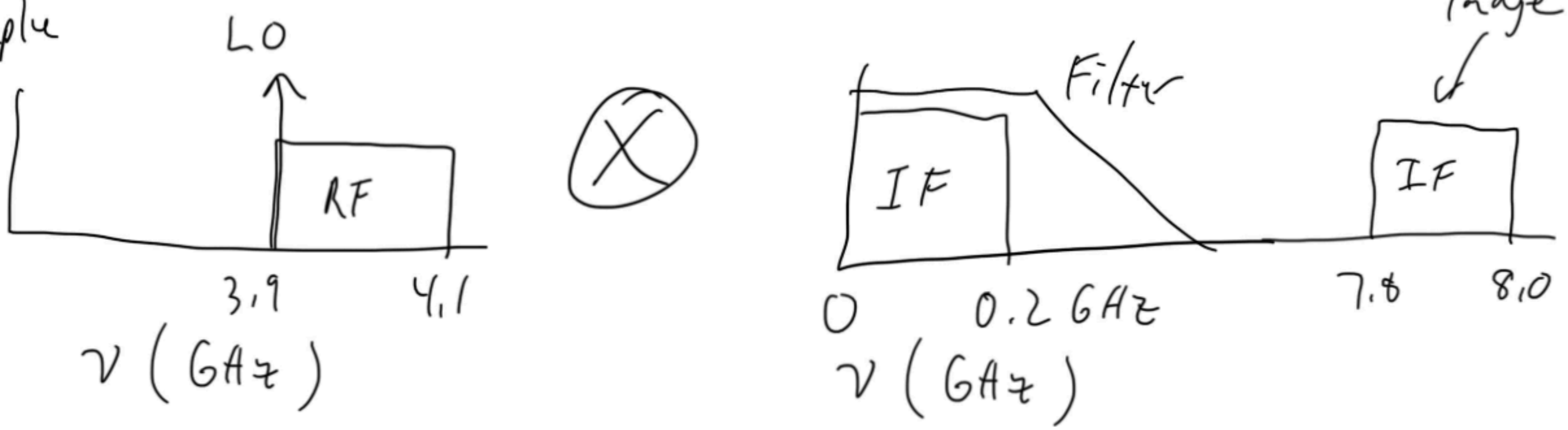
IF

Mixers: multiply two signals in time domain

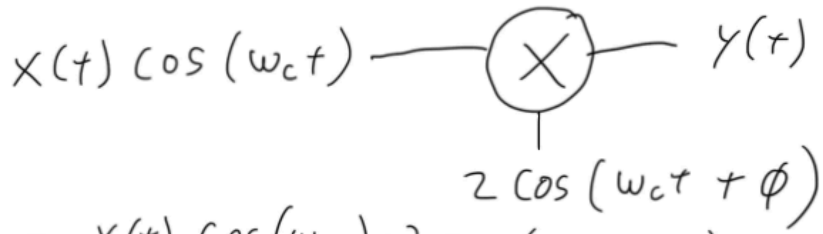


$$\text{IF: } \sin(\omega_{RF} t)\cos(\omega_{LO} t)$$
$$= \frac{1}{2}\left[\sin((\omega_{RF} - \omega_{LO})t) + \sin((\omega_{RF} + \omega_{LO})t)\right]$$

Baseband Transmission Example



What happens to phase of the RF signal?



$$y(t) = x(t)\cos(\omega_c t)\cdot 2\cos(\omega_c t + \phi)$$

$$= 2x(t)\cdot\frac{1}{2}\left[\cos(\omega_c t + \omega_c t + \phi) + \cos(\omega_c t + \phi - \omega_c t)\right]$$

filter this high frequency term out

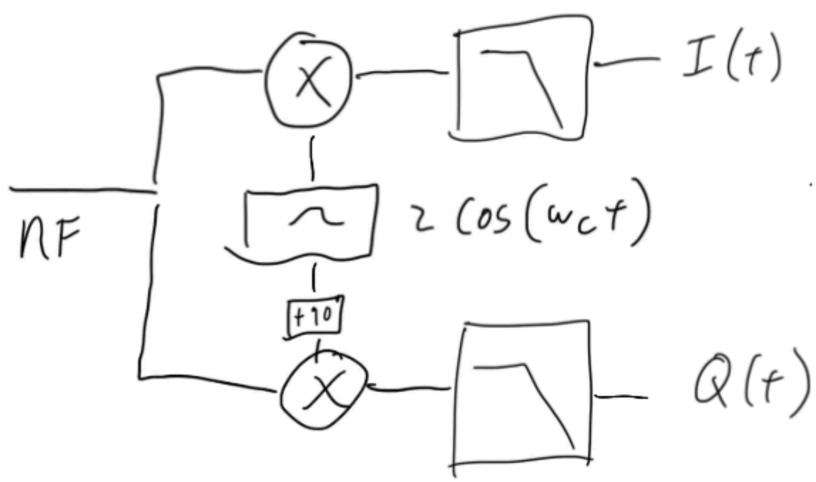
$$= x(t)\left[\cancel{\cos(2\omega_c t + \phi)} + \cos(\phi)\right]$$

$$= x(t)\cos(\phi)$$

downconverted signal depends on LO phase. what happens when $\phi = \frac{\pi}{2}$?

$y(t) = 0$???

Solution: use LO and 90° shifted LO to recover complete signal

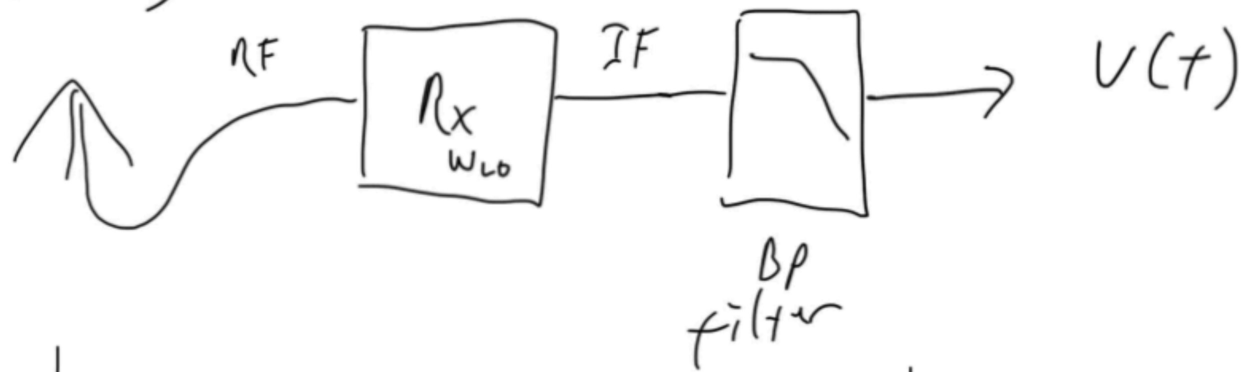


$$|RF| = \sqrt{I^2 + Q^2}$$

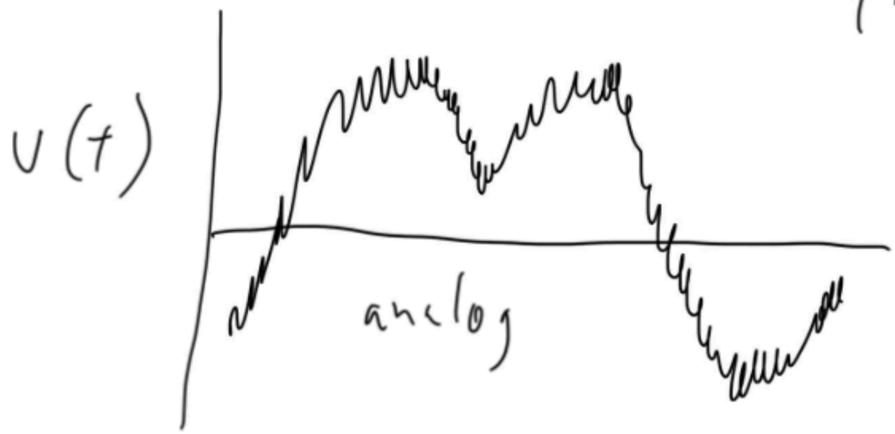
$$\phi_{RF} = \tan^{-1}\left(\frac{Q(t)}{I(t)}\right)$$

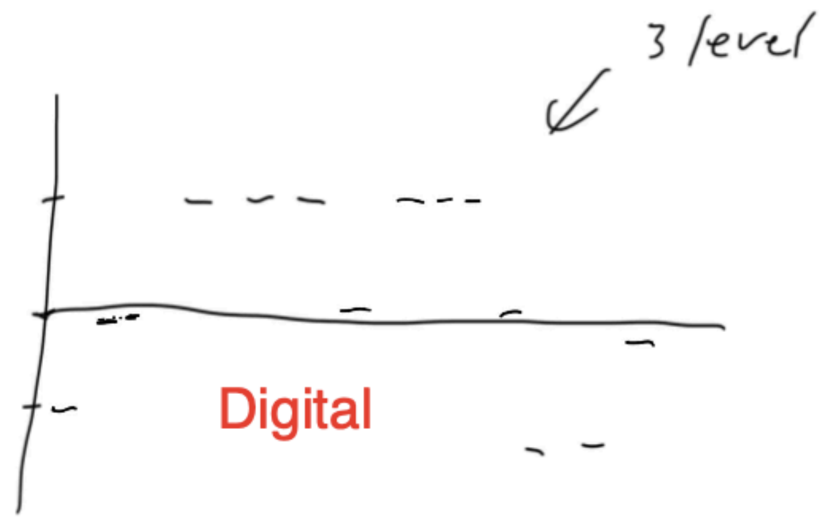
recovers amplitude & phase at baseband

Sampling



voltage time series





Some loss of information (η_s)
sample interval Δt

$\Delta t = \frac{1}{\Delta\nu}$

Nyquist sampling is
$\Delta t = \frac{1}{2\Delta\nu}$

Efficiency η_s

# bits	$1/(2\Delta\nu)$	$1/(4\Delta\nu)$
1	0.64	0.74
2	0.81	0.89
3	0.88	0.94
∞	1.00	1.00

Problems: RFI
: weak signals

Data rates

Suppose you want to record 64 MHz × 8 tunings (512 MHz) in two polarizations (RCP, LCP) from a VLBA antenna and store it for 4 hours

How much storage do you need?

$$\text{Data rate} = \underset{\text{Nyquist}}{2\Delta\nu}\ \underset{\text{bits}}{N}\ \underset{\text{pol}}{2}\ \underset{\text{tunings}}{8}\ \text{samples/sec}$$

$$= 2 \cdot 64\times10^6 \cdot 2 \cdot 2 \cdot 8$$

$$= 4096 \times 10^6 \text{ samples/sec}$$

$$= 4096 \text{ Mbps} \simeq 4 \text{ Gbps}$$

In 4 hours: 4096 Mbps · 3600 sec/hr · 4 hr = 74 TB fill up 7 × 12 TB drives

\$2800