



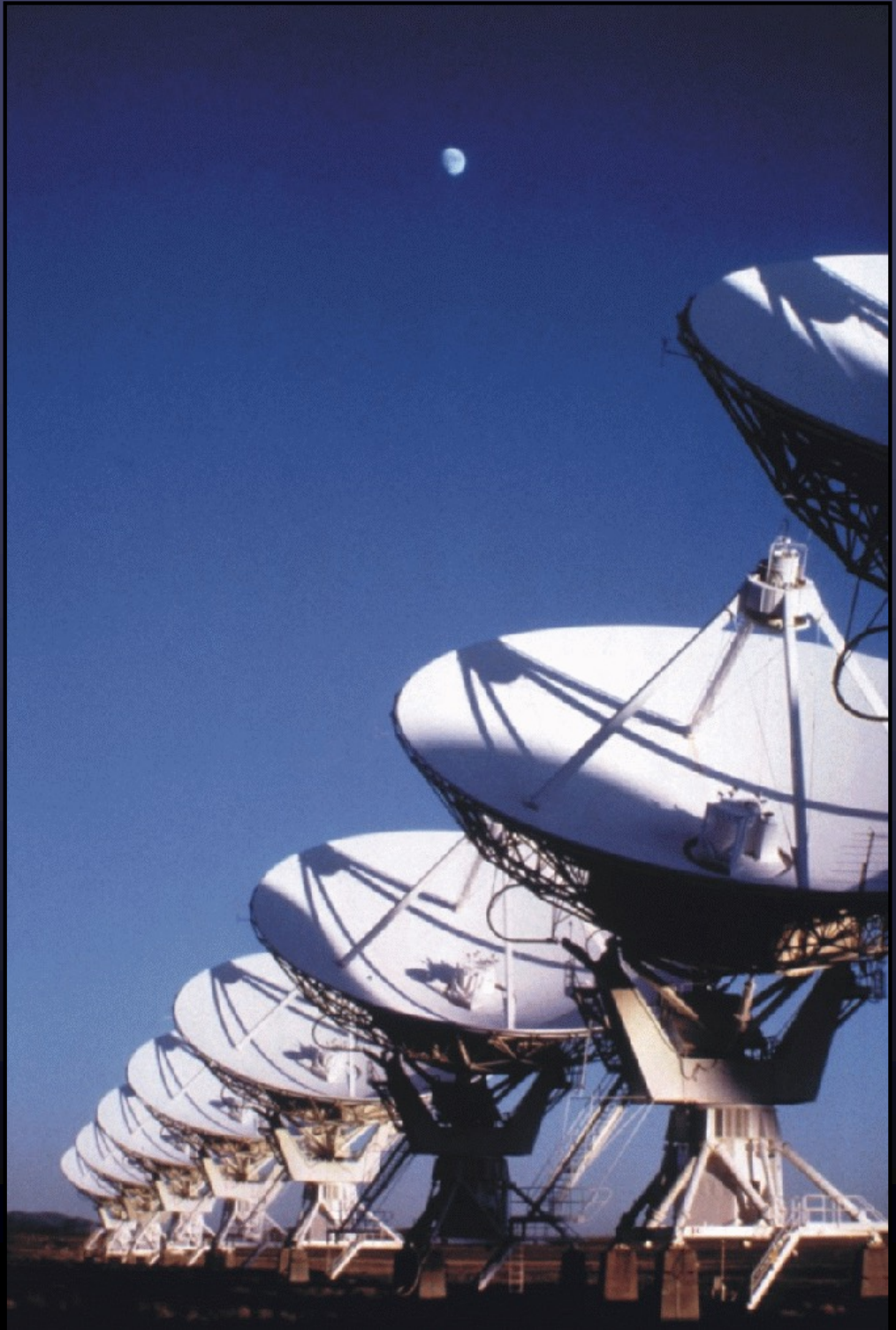
Radio Astronomy Noise Temperature

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Astronomy 423 at UNM

Radio Astronomy



Announcements

- Observing proposals for LWA time due TODAY by 4pm – send by e-mail to gbtaylor@unm.edu.
- Homework 2 is assigned and due Feb 12.



Useful Links

Start with the Syllabus for the course:

<https://leo.phys.unm.edu/~gibtaylor/astr423/>

Astro-ph: <https://arxiv.org/archive/astro-ph>

ADS: <https://ui.adsabs.harvard.edu/classic-form>

NASA extragalactic database: <https://ned.ipac.caltech.edu>

LWA Observing Page:

<https://lwalab.phys.unm.edu/OpScreen/index.html>



Radio Astronomy Notes 4-1

Planck's Law: $B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$

a) low freq limit (Rayleigh-Jeans case)

$h\nu \ll kT$ $e^{h\nu/kT} \sim 1 + \frac{h\nu}{kT} + \dots$

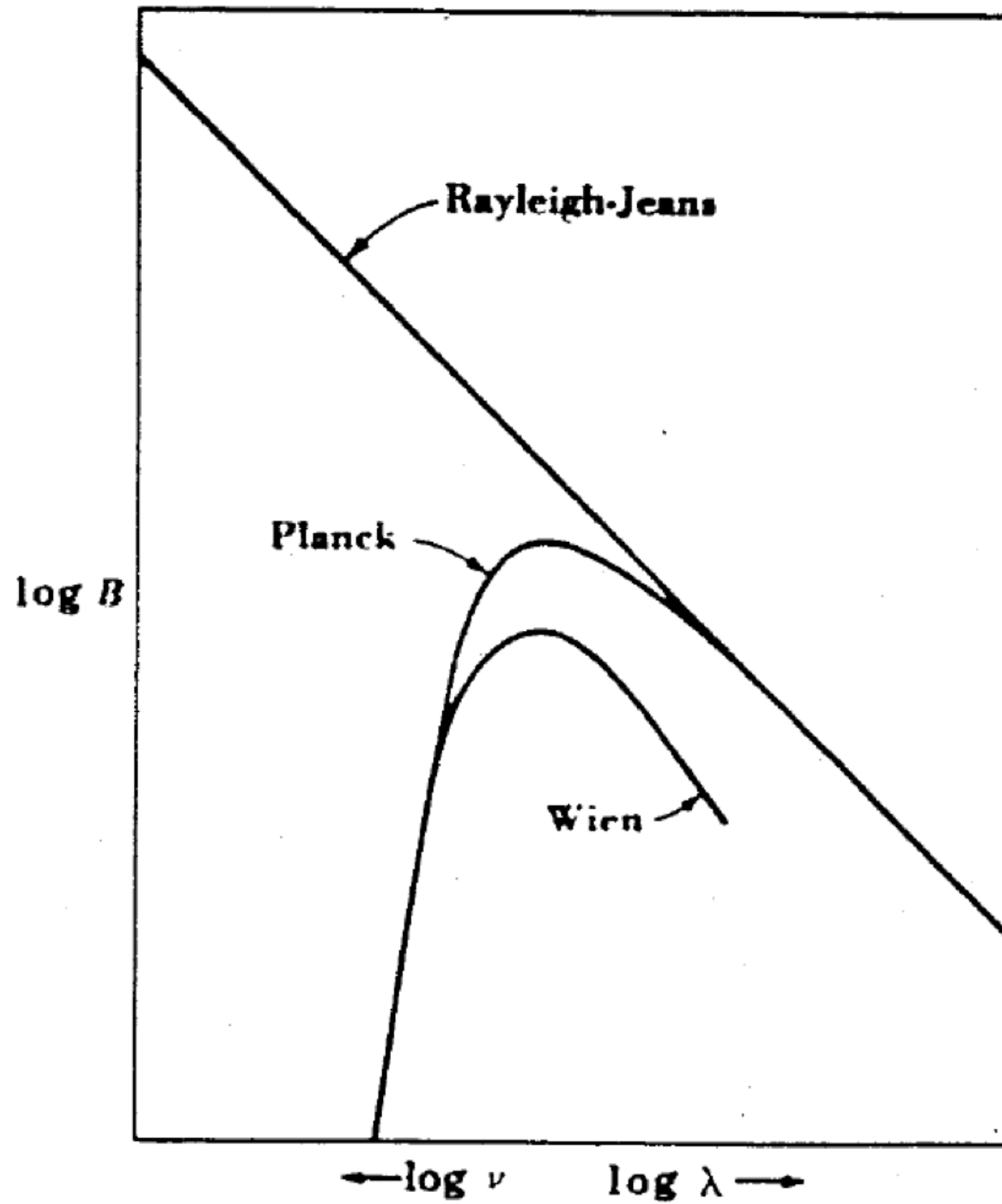
$$B_{\nu RJ} = \frac{2h\nu^3}{c^2} \cdot \frac{1}{1 + \frac{h\nu}{kT}} = \frac{2h\nu^3}{c^2} \cdot \frac{kT}{h\nu}$$

$$= \frac{2\nu^2}{c^2} kT = \frac{2kT}{\lambda^2}$$

b) high frequency limit (Wien's Law)

$h\nu \gg kT$ $e^{h\nu/kT} \gg 1$

$$B_{\nu W} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT}} = \frac{2h\nu^3}{c^2} e^{-h\nu/kT}$$



Radio Astronomy Notes 4-2

The RT approximation is very useful for radio astronomy

$$S_\nu = \int_{\Omega} B_\nu d\Omega \quad \text{for Thermal Equilibrium}$$

note: $d\Omega = \frac{\pi \theta^2}{4} \sim \theta^2$

$$S_\nu = \int_{\Omega} \frac{2kT_b}{\lambda^2} d\Omega$$

$$S_\nu = \frac{2kT_b}{\lambda^2} \theta^2$$

$$T_g = \frac{S_\nu \lambda^2}{2k \theta^2} \quad \text{is the brightness Temperature}$$

Worksheet #2

Problem: Objects with high brightness temperatures are easy to detect. Calculate the brightness temperature for the entire radio galaxy 3C286, a favorite VLA calibrator, which is 14.7 Jy at 20 cm and has a size about $1''$, and for the core which has a size of about 1 mas and flux density about 1.5 Jy.



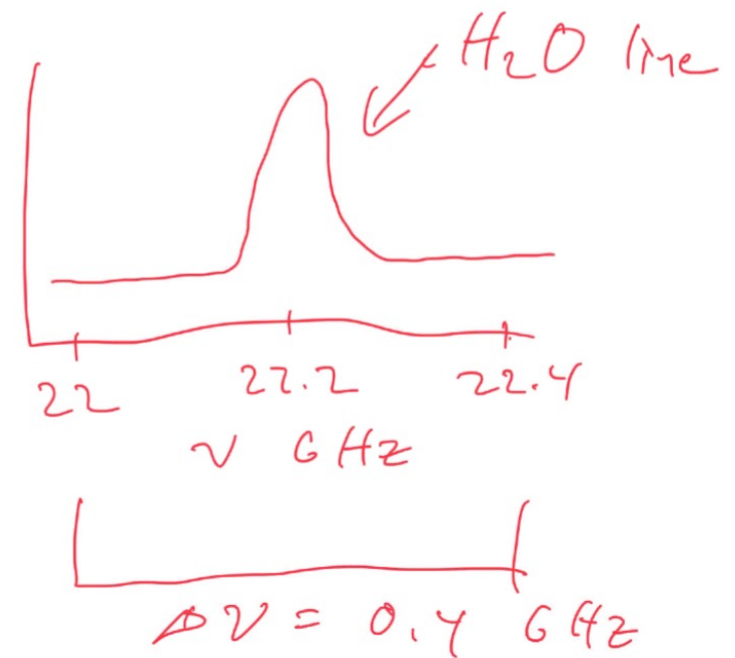
Radio Astronomy Notes 4-3

Lets talk about bandwidth

The range of frequencies
Some instrument is sensitive to

Bandwidth \propto Information content

Bandwidth helps determine
sensitivity



Nyquist Theorem: $\Delta\nu \Delta t = 1$

$$\Delta\nu = \frac{1}{\Delta t}$$

$$\Delta\nu = 400 \text{ MHz} \quad \Delta t = 2.5 \times 10^{-9} \text{ sec}$$

$$= 2.5 \text{ nanosec}$$

400×10^6 independent intervals in 1 second

$$\frac{400 \times 10^6 \text{ bits}}{5} = 400 \text{ Mbps}$$

Temperature and Noise

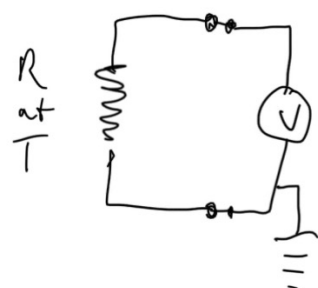
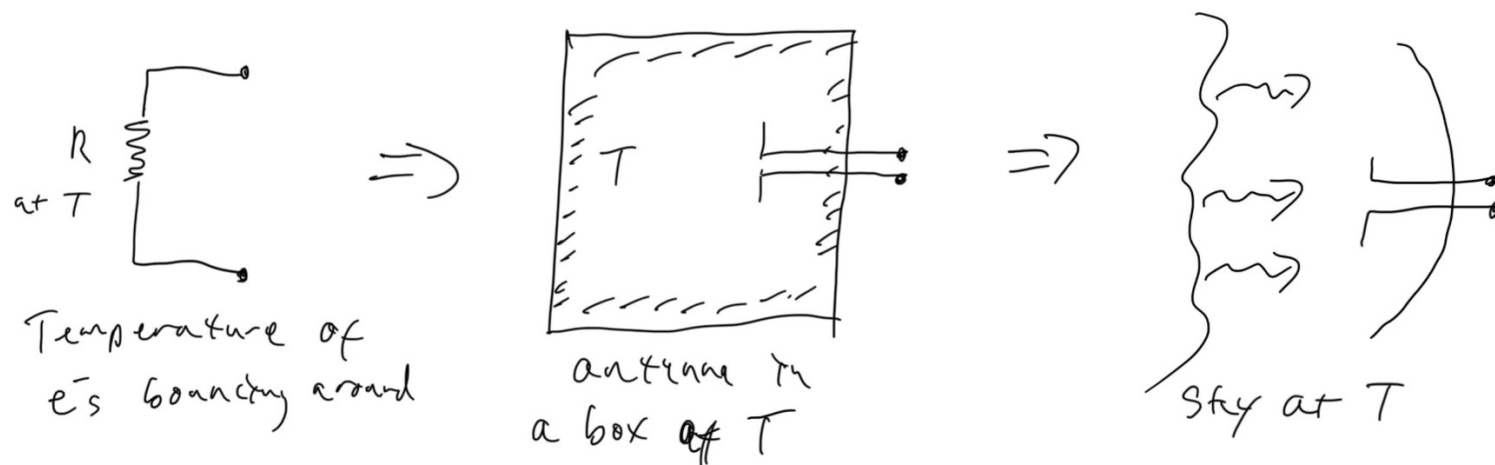
We've been talking about radiation from an astronomical source and the power received over some detector area (like a 100 m antenna) and some bandwidth. For blackbody objects the power can be related to the brightness temperature: $W \sim T_b$

We can also think about this power generating a signal at the detector:

Electrical Power \sim Detector Temperature

Ultimately there will be a number of elements that contribute power (or temperature): receiver, cables, sky ...





Measure voltage

Thermal noise = Johnson Noise

$$W_v = \langle I V \rangle = \frac{V^2}{R}$$

$$V^2 = R k T \quad \text{from Brownian motion}$$

$$W_v = k T$$

Example: $T = 300 \text{ K}$

$$W_v = 5 \times 10^{-14} \text{ erg s}^{-1} \text{ Hz}^{-1} = 5 \times 10^{-21} \frac{\text{W}}{\text{Hz}}$$

Engineers talk about dBm = noise power relative to 1 mW

$$W = k T \Delta \nu \quad 300 \text{ K} \quad \Delta \nu = 10 \text{ kHz}$$

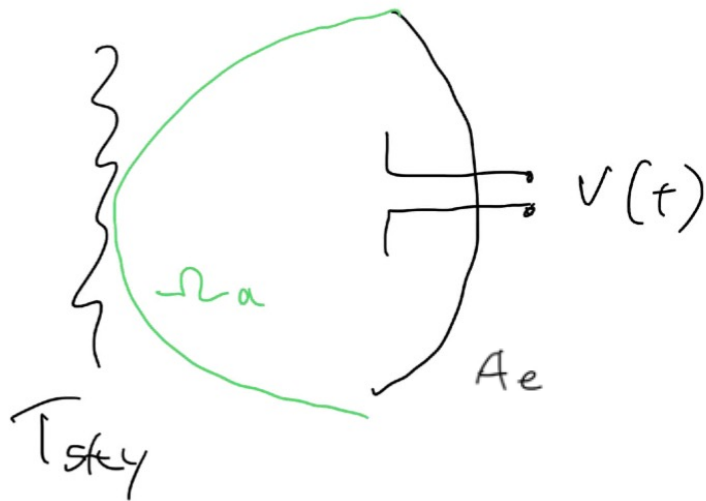
$$= 5 \times 10^{-21} \frac{\text{W}}{\text{Hz}} \cdot 10^4 \text{ Hz} = \frac{5 \times 10^{-17} \text{ W}}{0.001 \text{ W}} = 5 \times 10^{-14} = -133 \text{ dBm}$$

Radio Astronomy Notes 4-5

Caution! Not all circuit elements can be characterized by thermal noise

1 mW transmitter $\Rightarrow 7 \times 10^{19} \text{ Hz}$ (non-thermal process)

Consider an antenna looking at a uniform sky at temperature T



Lets calculate the power received by the antenna with effective area A_e beam Ω_a
assume lossless antenna

$$dW = I_\nu \cos \theta \, d\Omega \, d\nu$$

$$W_\nu = \int I_\nu \cos \theta \, d\Omega \, d\nu$$

$$\int d\Omega = A_e$$

$$= A_e \int I_\nu \cos \theta \, d\Omega$$

$$= \frac{A_e}{2} B_\nu(T) \int \cos \theta \, d\Omega = \Omega_a$$

one polarization \rightarrow

$$W_\nu = \frac{A_e}{2} B_\nu(T) \Omega_a$$

$$B_\nu = \frac{2\nu^2}{c^2} k T_b$$

$$= \frac{A_e}{2} \frac{2 k T_b}{\lambda^2} \Omega_a$$

$$= \frac{2 k T_b}{\lambda^2}$$

$$W_\nu = \frac{A_e \Omega_a}{\lambda^2} k T_b$$

$$\frac{A_e \Omega_a}{\lambda^2} = 1$$

$$A_e = \frac{\lambda^2}{\Omega_a}$$

For uniform brightness and no loss

$$T_a = T_b$$

Radio Astronomy Notes 4-6

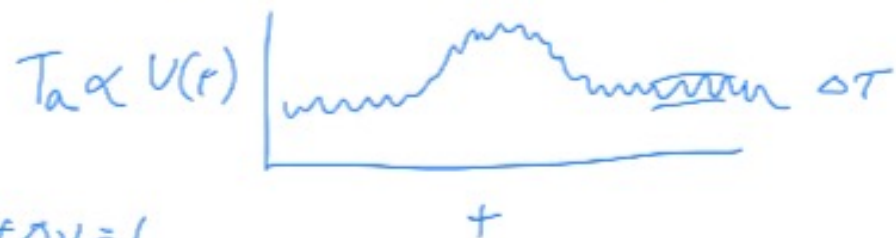
Spectral cases:

(1) If absorbing material is present along the
line-of-sight
 $T_a \neq T_s$

(2) If source is smaller than the beam
 $T_a \neq T_b$

What is the minimum temperature you can measure?

$$\Delta T = \frac{T_{sys}}{\sqrt{N}}$$



$N = \frac{\tau}{\Delta t}$ and recall $\Delta t \Delta \nu = 1$

$$\Delta T = \frac{T_{sys}}{\sqrt{\Delta \nu \tau}}$$

τ = integration time
 $\Delta \nu$ = bandwidth

More useful is the limiting flux density ΔS

$$S_\nu = \int_{\Omega} B_\nu \cos \theta \, d\Omega$$

$$S_\nu = \frac{2kT}{\lambda^2} \Omega_a$$

$$S_\nu = \frac{2k}{A_e} T$$

$$\Delta S = \frac{2k}{A_e} \Delta T$$

$$\Delta S = \frac{2k}{A_e} \frac{T_{sys}}{\sqrt{\Delta\nu \tau}}$$

$$\Delta S = \frac{SEFD}{\sqrt{\Delta\nu \tau}}$$

$$\frac{A_e}{\lambda^2} = \frac{1}{A_e}$$

radiometer equation

single polarization

$$\text{Defining Gain} = \frac{A_e}{2k} \text{ in } \frac{K}{Jy}$$

$$SEFD = \frac{T_{sys}}{\text{Gain}} \text{ in } Jy$$

Radio Astronomy Notes 4-7 $\tau = 10 \text{ minutes}$

Example: VLA at 6 GHz (C-Band) $\Delta\nu = 26 \text{ Hz}$

$$\text{SEFD} = 310 \text{ Jy}$$

$$\Delta S = \frac{\text{SEFD}}{\sqrt{2 \cdot N \cdot 2 \times 10^9 \cdot 600}}$$

$$N = \# \text{ baselines} \\ = 351 \text{ for 27 antennas}$$

$$= 10 \text{ mJy} \text{ (exposure calculator says 8.1 mJy)}$$

Gain of VLA antenna? $D = 25 \text{ m} = 2500 \text{ cm}$

$$\text{Gain} = \frac{A_e}{2k} = \frac{\pi D^2 / 4}{2 \cdot 1.38 \times 10^{-16}} \cdot \frac{10^{-23}}{1 \text{ Jy}}$$

$$\text{Gain} = 0.18 \frac{\text{K}}{\text{Jy}}$$

$$\text{SEFD} = \frac{T_{\text{sys}}}{\text{Gain}}$$

$$T_{\text{sys}} = 55 \text{ K}$$