Radio Astronomy Noise Temperature

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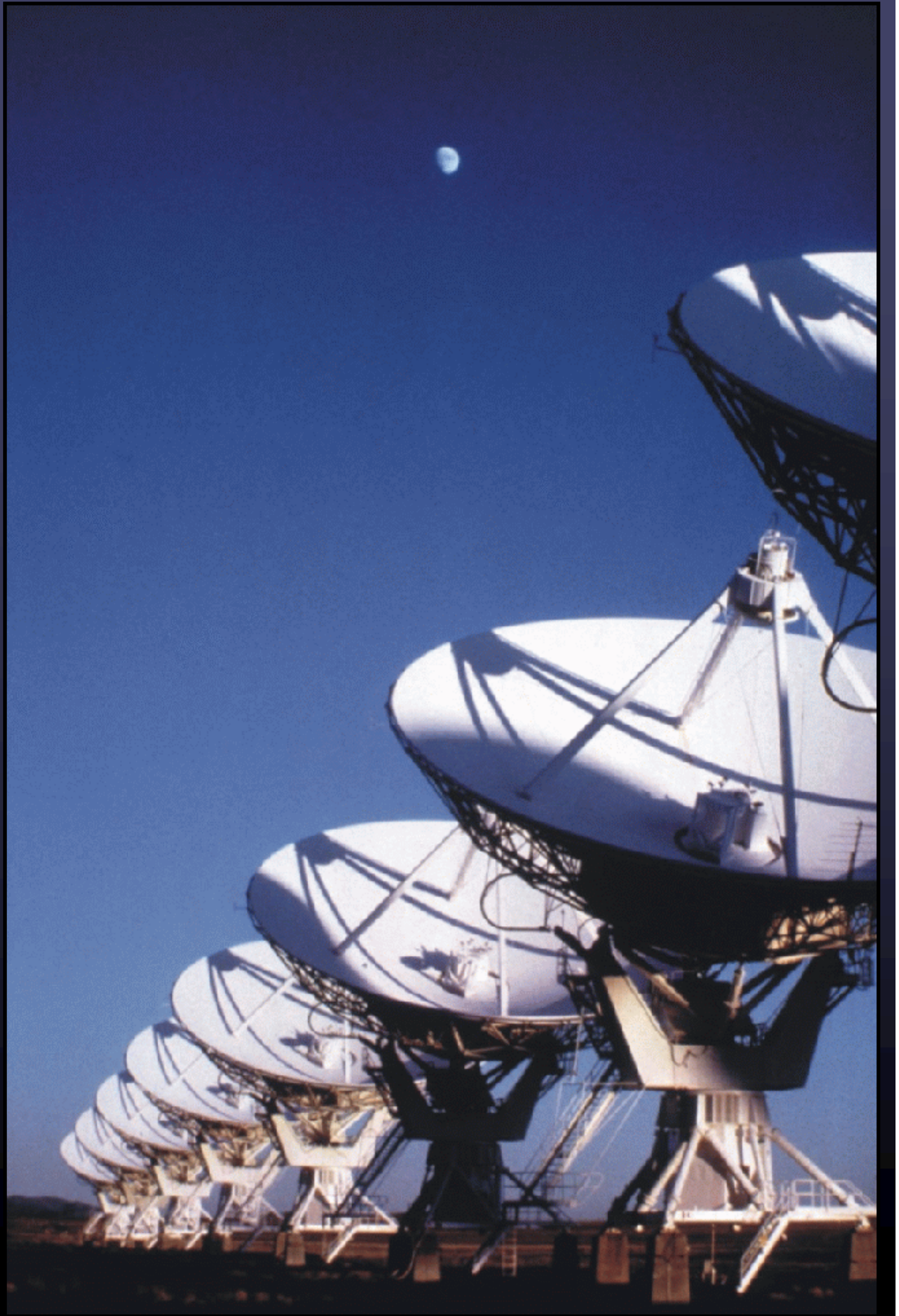
Astronomy 423 at UNM

Radio Astronomy



Announcements

- Observing proposals for VLA and LWA time due TODAY by 4pm – send by e-mail to gbtaylor@unm.edu. Select projects in class Feb 3.
- Homework 2 is assigned and due Feb 10.

Useful Links

Start with the Syllabus for the course:

http://www.phys.unm.edu/~gbtaylor/astr423/RA_Syllabus.html

Planck's Law: $B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$

a) low freq limit (Rayleigh-Jeans case)

$h\nu \ll kT \qquad e^{h\nu/kT} \sim 1 + \frac{h\nu}{kT} + \cdots$

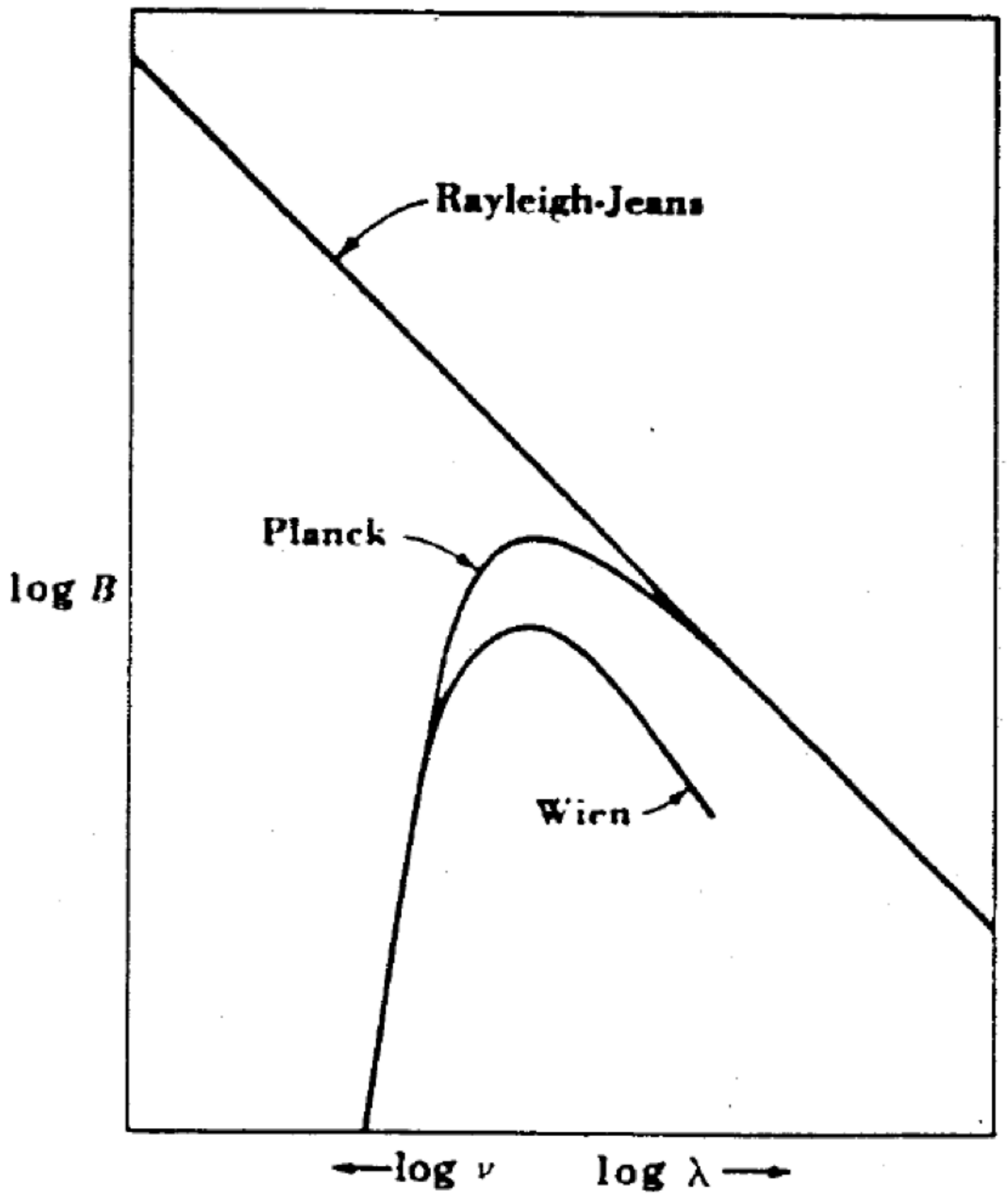
$$B_{\nu\,RJ} = \frac{2h\nu^3}{c^2} \cdot \frac{1}{\cancel{1} + \frac{h\nu}{kT} \cancel{-1}} = \frac{2\cancel{h}\nu^{\cancel{3}\,2}}{c^2} \cdot \frac{kT}{\cancel{h\nu}}$$

$$= \frac{2\nu^2}{c^2} kT = \frac{2kT}{\lambda^2}$$

b) high frequency limit (Wien's Law)

$h\nu \gg kT \qquad e^{h\nu/kT} \gg 1$

$$B_{\nu W} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT}} = \frac{2h\nu^3}{c^2} e^{-h\nu/kT}$$


Rayleigh-Jeans
Planck
log B
Wien
←log ν
log λ →

Radio Astronomy Notes 4-2

The RJ approximation is very useful for radio astronomy

$$S_\nu = \int_\Omega B_\nu \, d\Omega$$ for Thermal Equilibrium

note: $d\Omega = \frac{\pi \theta^2}{4} \sim \theta^2$

$$S_\nu = \int_\Omega \frac{2kT_b}{\lambda^2} \, d\Omega$$

$$S_\nu = \frac{2kT_b}{\lambda^2} \theta^2$$

$$T_b = \frac{S_\nu \lambda^2}{2k\theta^2}$$ is the brightness Temperature

Worksheet #2

- Download the worksheet from:

http://www.phys.unm.edu/~gbtaylor/astr423/WS2.pdf

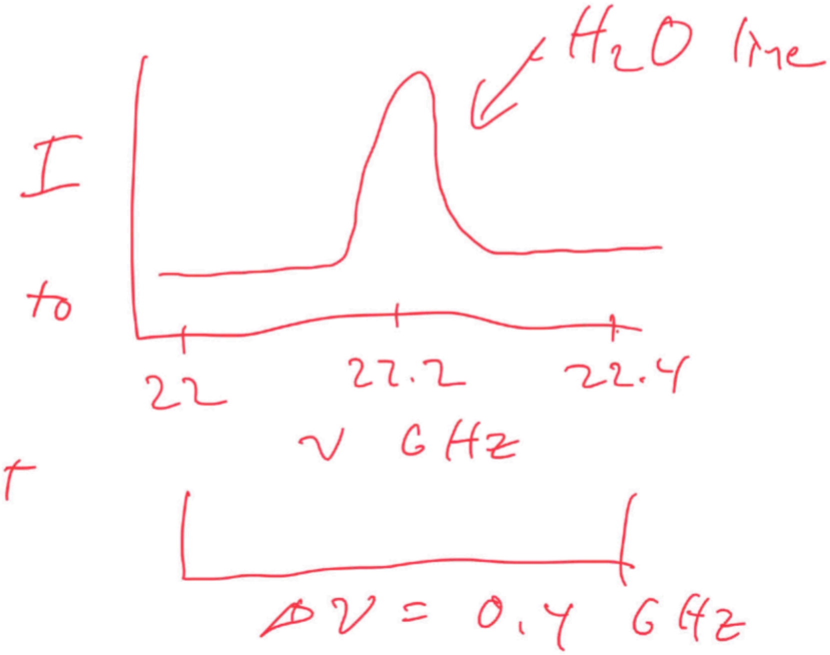
Solve it in class.

Ask questions if you are stuck

Tell me when you have the answer.

Lets talk about bandwidth

The range of frequencies
Some instrument is sensitive to



Bandwidth $\propto$ Information content

Bandwidth helps determine
sensitivity

Nyquist Theorem! $\Delta\nu \, \Delta t = 1$

$$\Delta\nu = \frac{1}{\Delta t}$$

$\Delta\nu = 400$ MHz $\qquad \Delta t = 2.5 \times 10^{-9}$ sec

$= 2.5$ nanosec

400×10^6 independent intervals in 1 second

$$400 \times 10^6 \ \frac{\text{bits}}{\text{s}} = 400 \text{ Mpbs}$$

Temperature and Noise

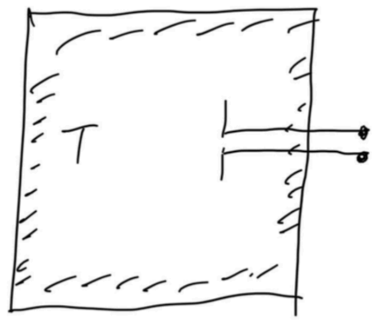
We've been talking about radiation from an astronomical source and the power received over some detector area (like a 100 m antenna) and some bandwidth. For blackbody objects the power can be related to the brightness temperature: $W \sim T_b$

We can also think about this power generating a signal at the detector:

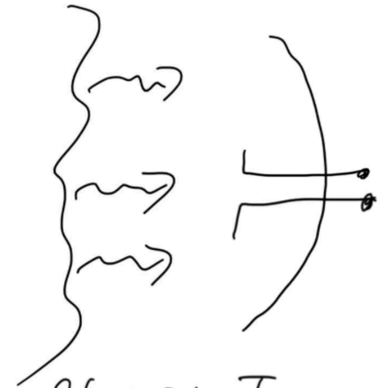
Electrical Power ~ Detector Temperature

Ultimately there will be a a number of elements that contribute power (or temperature): receiver, cables, sky …

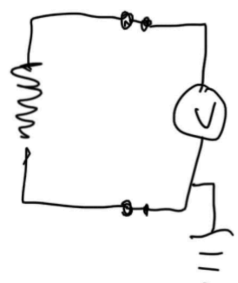
Temperature of e^-s bouncing around



antenna in a box of T



Sky at T



Measure voltage

Thermal noise = Johnson Noise

$$W_\nu = \langle I V \rangle = \frac{V^2}{R} \qquad V^2 = RkT$$

from Brownian motion

$$W_\nu = kT$$

Example: $T = 300\,K$

$$W_\nu = 5\times10^{-14}\ \mathrm{erg\ s^{-1}\ Hz^{-1}} = 5\times10^{-21}\ \frac{W}{Hz}$$

Engineers talk about dBm = noise power relative to 1 mW

$$W = kT\Delta\nu \qquad 300\,K \qquad \Delta\nu = 10\,kHz$$

$$= 5\times10^{-21}\ \frac{\cancel{W}}{\cancel{Hz}}\cdot 10^4\ \cancel{Hz} = \frac{5\times10^{-17}\ \cancel{W}}{0.001\ \cancel{W}} = 5\times10^{-14} = -133\ dBm$$

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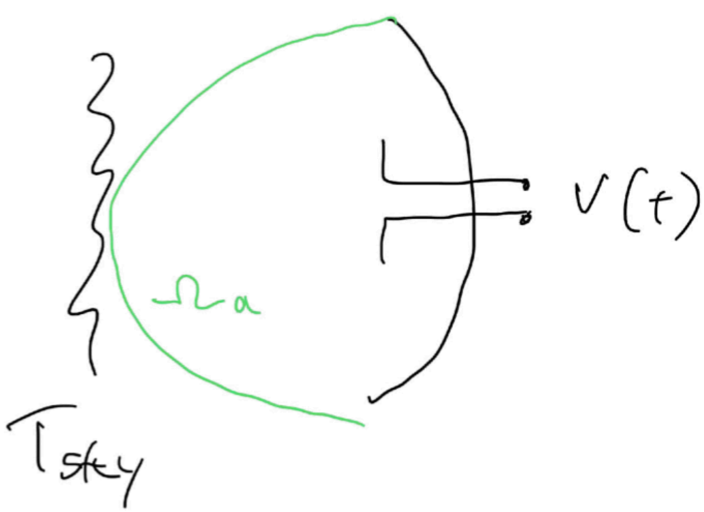
Caution! Not all circuit elements can be characterized by thermal noise

1 mW transmitter $\Rightarrow 7\times10^{19}$ K (non-thermal process)

Consider an antenna looking at a uniform sky at temperature T



Lets calculate the power received by the antenna with effective area A_e beam Ω_a

assume lossless antenna

$$dW = I_\nu \cos\theta \, d\Omega \, d\sigma \, d\nu$$

$$W_\nu = \int I_\nu \cos\theta \, d\Omega \, d\sigma \qquad \int d\sigma = A_e$$

$$= A_e \int I_\nu \cos\theta \, d\Omega$$

$$= \frac{A_e}{2} B_\nu(T) \underbrace{\int_\Omega \cos\theta \, d\Omega}_{= \Omega_a}$$

one polarization $\rightarrow$ 2

$$W_\nu = \frac{A_e}{2} B_\nu(T) \Omega_a \qquad B_\nu = \frac{2\nu^2}{c^2} k T_s$$

$$= \frac{A_e}{\cancel{2}} \frac{\cancel{2} k T_s}{\lambda^2} \Omega_a \qquad = \frac{2 k T_s}{\lambda^2}$$

$$W_\nu = \underbrace{\frac{A_e \Omega_a}{\lambda^2}}_{=1} k T_s \qquad \frac{A_e \Omega_a}{\lambda^2} = 1$$

For uniform brightness and no loss

$$T_a = T_b \qquad A_e = \frac{\lambda^2}{\Omega_a}$$

Radio Astronomy Notes 4-6

Special cases:

(1) If absorbing material is present along the line-of-sight

$T_a \neq T_s$

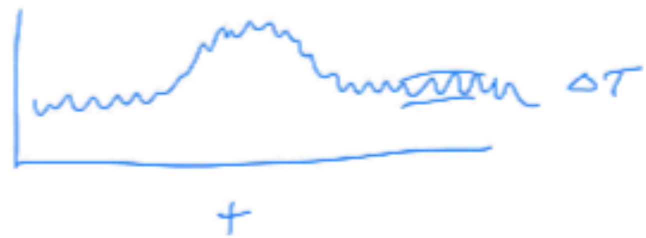
(2) If source is smaller than the beam

$T_a \neq T_s$

What is the minimum temperature you can measure?

$$\Delta T = \frac{T_{sys}}{\sqrt{N}}$$

$T_a \propto V(t)$



$N = \frac{\tau}{\Delta t}$ and recall $\Delta t \Delta \nu = 1$

$$\Delta T = \frac{T_{sys}}{\sqrt{\Delta \nu \tau}}$$

τ = integration time

$\Delta \nu$ = bandwidth

More useful is the limiting flux density ΔS

$$S_\nu = \int_\Omega B_\nu \cos\theta \, d\Omega$$

$$S_\nu = \frac{2kT}{\lambda^2} \Omega_a \qquad \frac{\Omega_a}{\lambda^2} = \frac{1}{A_e}$$

$$S_\nu = \frac{2k}{A_e} T$$

$$\Delta S = \frac{2k}{A_e} \Delta T$$

radiometer equation

$$\boxed{\Delta S = \frac{2k}{A_e} \frac{T_{sys}}{\sqrt{\Delta\nu \tau}}}$$

single polarization

Define Gain $= \frac{A_e}{2k}$ in $\frac{K}{Jy}$

$\text{SEFD} = \frac{T_{sys}}{\text{Gain}}$ in Jy

$$\Delta S = \frac{\text{SEFD}}{\sqrt{\Delta\nu \tau}}$$

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$\tau = 10$ minutes

Example: VLA at 6 GHz (C-Band) $\Delta\nu = 2$ GHz

SEFD = 310 Jy

$$\Delta S = \frac{SEFD}{\sqrt{2 \cdot N \cdot 2\times10^9 \cdot 600}}$$

N = # baselines

= 351 for 27 antennas

= 10 mJy (exposure calculator says 8.1 mJy)

Gain of VLA antenna? $D = 25\,m = 2500$ cm

$$\text{Gain} = \frac{A_e}{2k} = \frac{\pi D^2/4}{2 \cdot 1.38\times10^{-16}} \cdot \frac{10^{-23}}{1\ \text{Jy}}$$

$$\text{Gain} = 0.18\ \frac{K}{Jy} \qquad SEFD = \frac{T_{sys}}{\text{Gain}}$$

$T_{sys} = 55\ K$