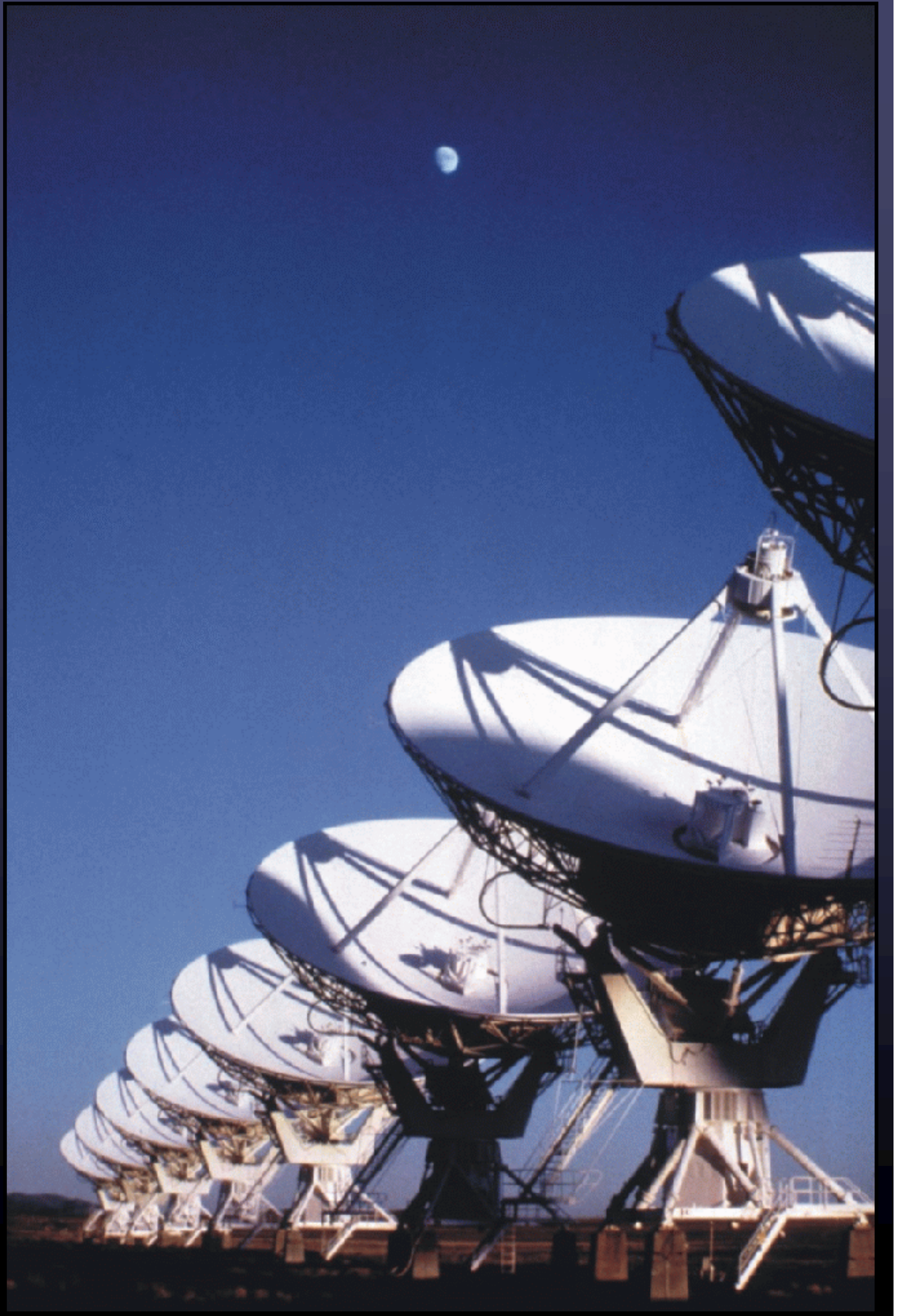
Radio Astronomy Received Power and other basics

Greg Taylor

University of New Mexico



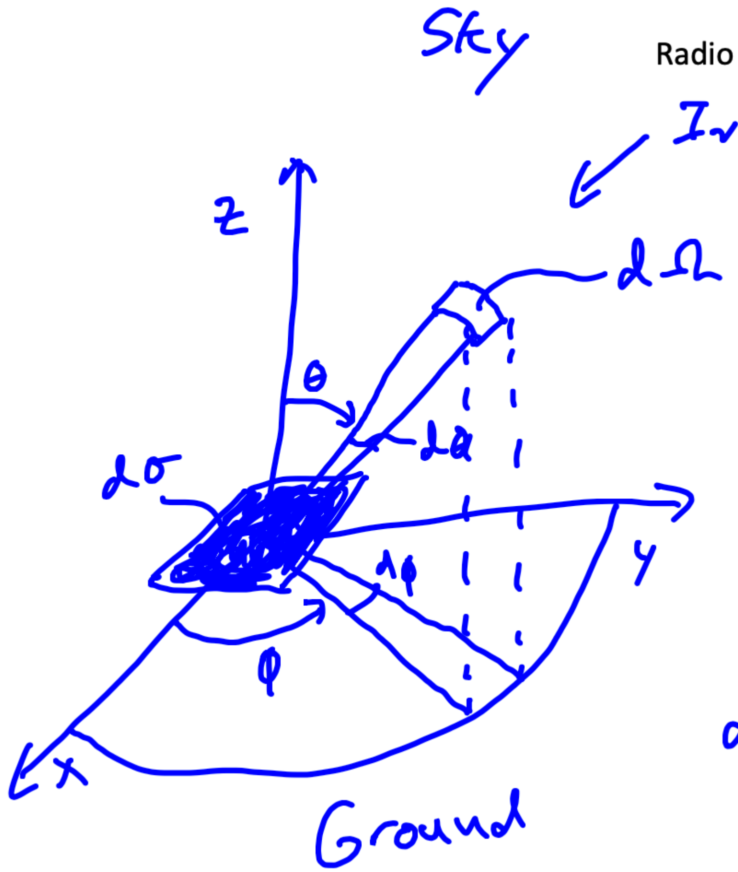
Astronomy 423 at UNM

Radio Astronomy

Announcements

- Observing proposals for VLA and LWA time due on Monday, Feb 1 by 4pm – send by e-mail to gbtaylor@unm.edu. Select projects in class Feb 3.

Radio Astronomy Notes



$\phi : 0 \rightarrow 2\pi$ azimuth

$\theta : 0 \rightarrow \frac{\pi}{2}$ altitude

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

W = Power received in watts

$$dW = I_\nu \cos\theta \, d\Omega \, d\sigma \, d\nu$$

dW = infinitesimal power in watts $\left(1\,W = 10^7 \frac{erg}{s}\right)$

$d\sigma$ = infinitesimal area of surface cm^2

$d\nu$ = infinitesimal bandwidth in Hz

$d\Omega$ = infinitesimal solid angle of source rad^2 or ster

I_ν = intensity (or brightness) $W\,cm^{-2}\,Hz^{-1}\,ster^{-1}$

flux $= \int_{\Omega_s, \nu} dW = \frac{\text{energy}}{s \cdot cm^2}$ Integrated intensity over the solid angle of the source & bandwidth

$S_\nu =$ flux density $= \frac{\text{energy}}{s \cdot cm^2 \cdot Hz} = \int_{\Omega_s} I_\nu(\theta, \phi) \cos\theta \, d\Omega$

$W = \int S_\nu \, d\sigma \, d\nu$

define $1 \text{ Jy} = 10^{-23} \text{ erg } s^{-1} cm^{-2} Hz^{-1}$

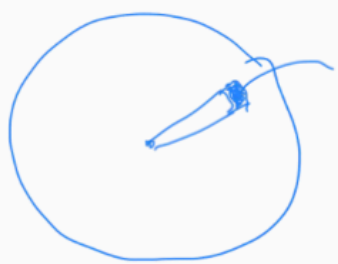
small because radio sources are far away

$S = \frac{W}{4\pi r^2}$

Geometry review

$s = r\alpha$ α in radians

"small angle formula"



$A = r^2 \Omega$

$A_{sphere} = 4\pi r^2$

$\Omega_{sky} = 4\pi$ steradians

Example: Consider a source with a uniform brightness of 10,000 Jy and a bandwidth of 1 GHz over the whole sky. How much power would a 100 m antenna collect?

$$W_\nu = \int_0^A S_\nu \, d\sigma = S_\nu \cdot \pi (50\,m)^2$$

$$= 10^4 \, Jy \cdot 7800 \, m^2 \cdot \frac{10^4 \, cm^2}{1 \, m^2} \cdot \frac{10^{-23} \, erg \, s^{-1} \, cm^{-2} \, Hz^{-1}}{1 \, Jy} \cdot \frac{10^{-7} \, W}{erg \, s^{-1}}$$

$$= 7.8 \times 10^{-19} \, W/Hz$$

$$W = \int W_\nu \, d\nu = 10^9 \, Hz \cdot 7.8 \times 10^{-19} \, \frac{W}{Hz}$$

$$= 7.8 \times 10^{-10} \, W \quad \text{not much!}$$

GBT power consumption $\sim 10^6$ W

efficiency $\sim 10^{-15}$ $\ll$ break even

Worksheet #1

- Download the worksheet from:

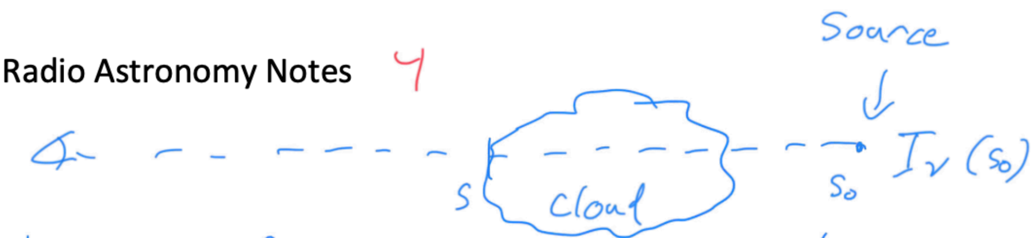
http://www.phys.unm.edu/~gbtaylor/astr423/WS1.pdf

Solve it in class.

Ask questions if you are stuck

Tell me when you have the answer.

Radiative Transfer



I_ν = Intensity – does not change with distance unless absorbed or emitted

$\frac{dI_\nu}{ds}$ = equation of transfer

loss: $dI_\nu = -\kappa_\nu I_\nu \, ds$ (κ_ν: opacity)

gain: $dI_\nu = \varepsilon_\nu \, ds$ (ε_ν: emissivity)

$$\frac{dI_\nu}{ds} = \varepsilon_\nu - \kappa_\nu I_\nu$$

Consider some basic cases

① Emission only $\kappa_\nu = 0$

$$\frac{dI_\nu}{ds} = \varepsilon_\nu \qquad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} \varepsilon_\nu(s)\,ds$$

② Absorption only $\varepsilon_\nu = 0$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu$$

$$\int \frac{dI_\nu}{I_\nu} = -\int \kappa_\nu \, ds$$

$$\ln I_\nu = -\int \kappa_\nu \, ds$$

$$I_\nu(s) = I_\nu(s_0)\, e^{-\int_{s_0}^{s} \kappa_\nu \, ds}$$

$\int_{s_0}^{s} \kappa_\nu \, ds = \tau$

$$I_\nu(s) = I_\nu(s_0)\, e^{-\tau}$$

τ = optical depth

② cont.

$$I_\nu(s) = I_\nu(s_0)\, e^{-\tau}$$

Consider different optical depths:

$\tau = 0$ $I_\nu(s) = I_\nu(s_0)$ (no absorption)

$\tau < 1$ "optically thin" – you can see through it

$\tau > 1$ "optically thick"

$\tau = 1$ intensity $\propto \frac{1}{e} \sim 0.37$

$\tau = 2$ intensity $\propto \frac{1}{e^2} \sim 0.14$

$\tau = 4.6$ $I \propto 0.01$ (−20 dB) 5 magnitudes

③ Thermodynamic Equilibrium

$$\frac{dI_\nu}{ds} = 0 \qquad I_\nu = B_\nu(T) \quad \text{Planck function}$$

$$0 = \varepsilon_\nu\, ds - \kappa_\nu I_\nu\, ds$$

$$I_\nu = \frac{\varepsilon_\nu}{\kappa_\nu} = B_\nu(T)$$

Thermal emission

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\,\frac{1}{e^{h\nu/kT} - 1}$$

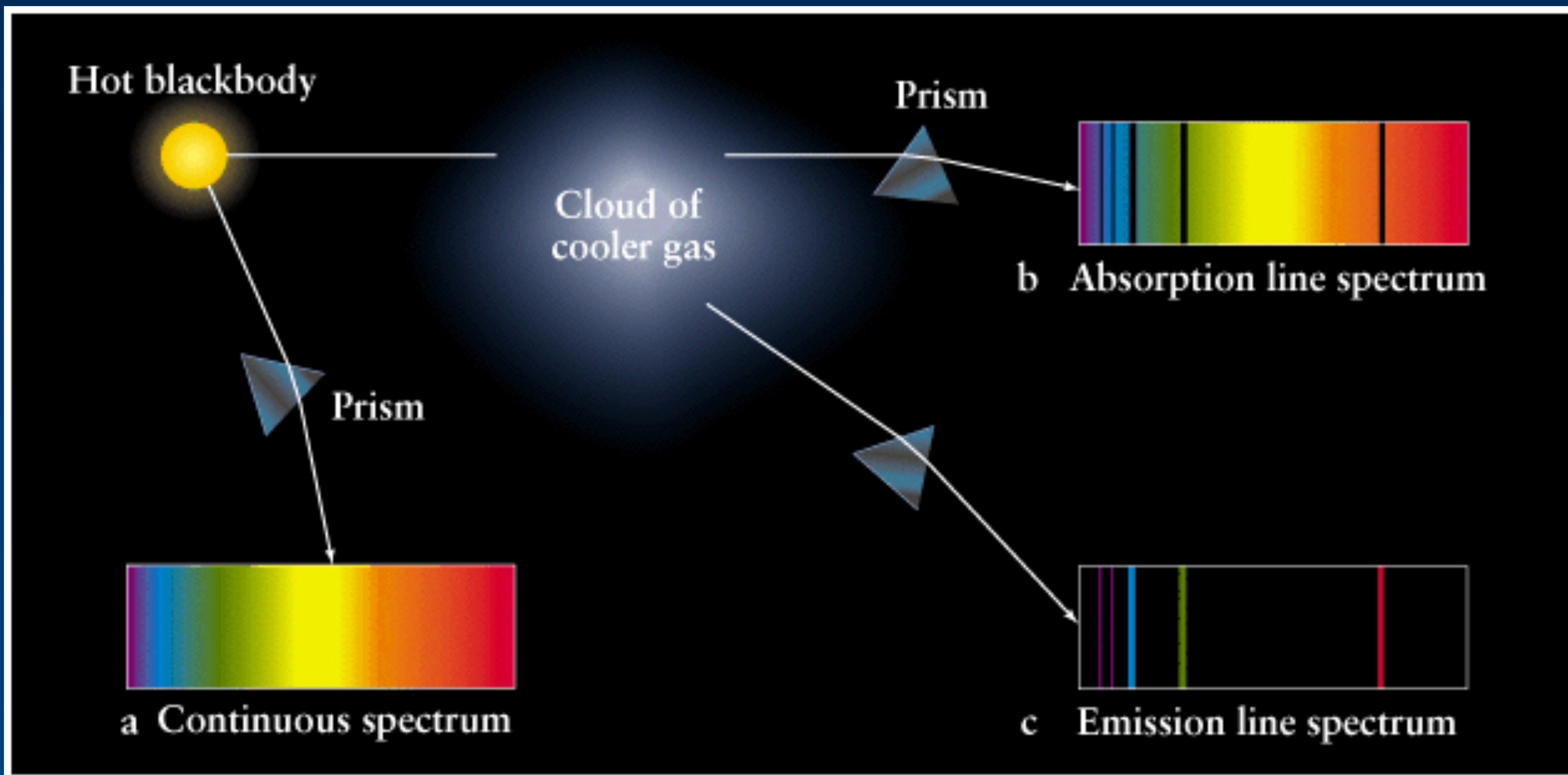
$E = h\nu$

T: Temperature in K

k: Boltzmann's const $1.38 \times 10^{-16}\ \frac{\text{erg}}{\text{K}}$

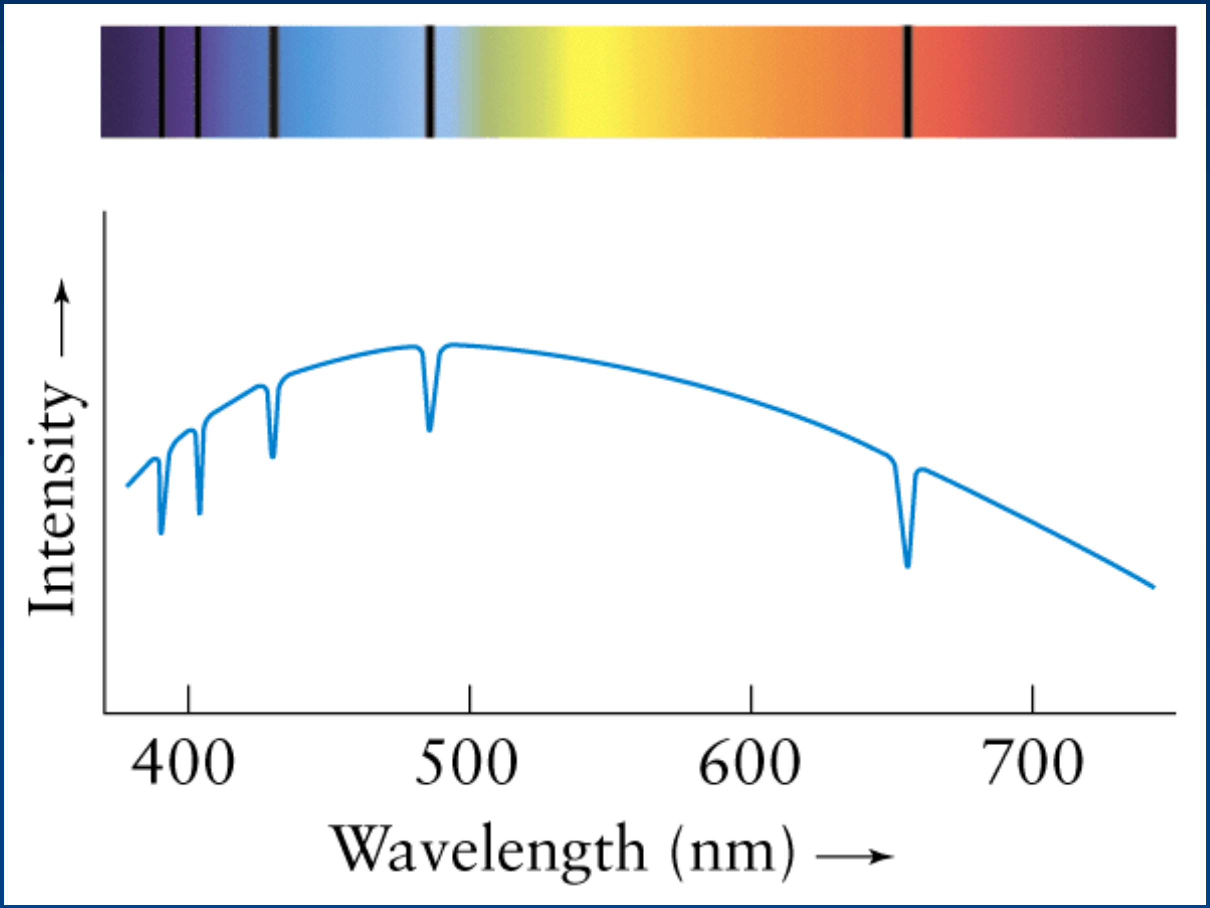
h: Planck's const 6.63×10^{-27} erg·s

Kirchoff's Laws Illustrated –



Note: two ways to show a spectrum:

1) as an image
2) as a plot of intensity vs wavelength (or frequency)
3) Example:

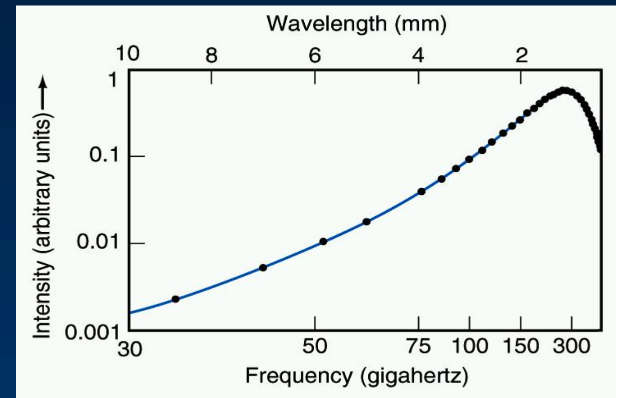


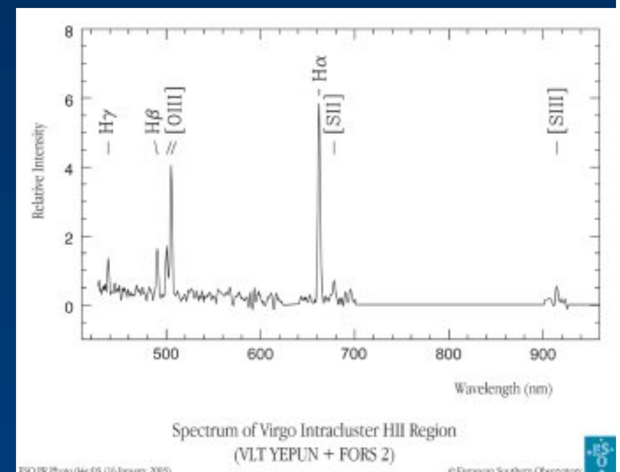
Astronomical and other examples:

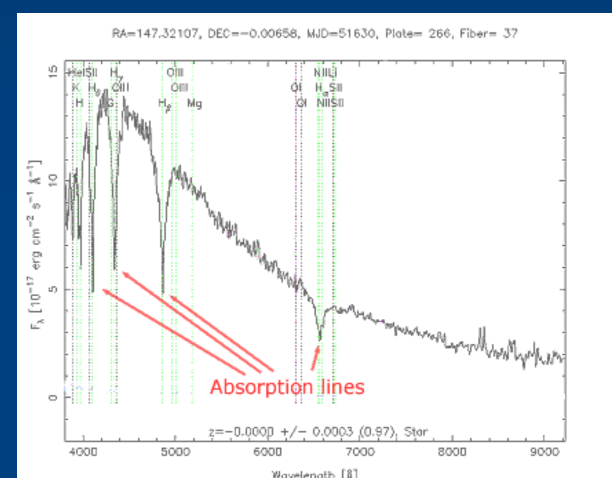
- Continuous: Incandescent lights, the Cosmic Microwave Background (CMB)

- Emission (bright) line: neon lights, hot interstellar gas -- HII regions, supernova remnants.

- Absorption (dark) line: stars (relatively cool atmospheres overlying hot interiors).

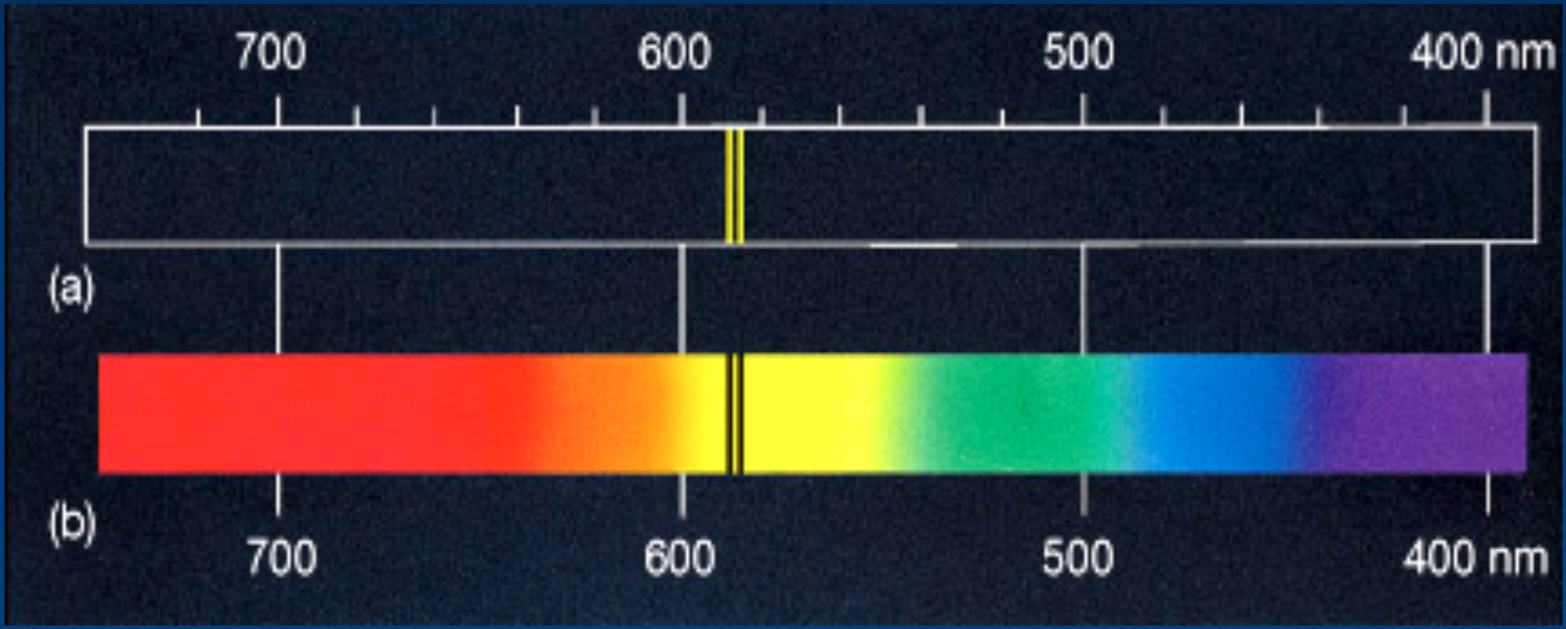






For a gas of a given element, absorption and emission lines occur at same wavelengths.

Sodium

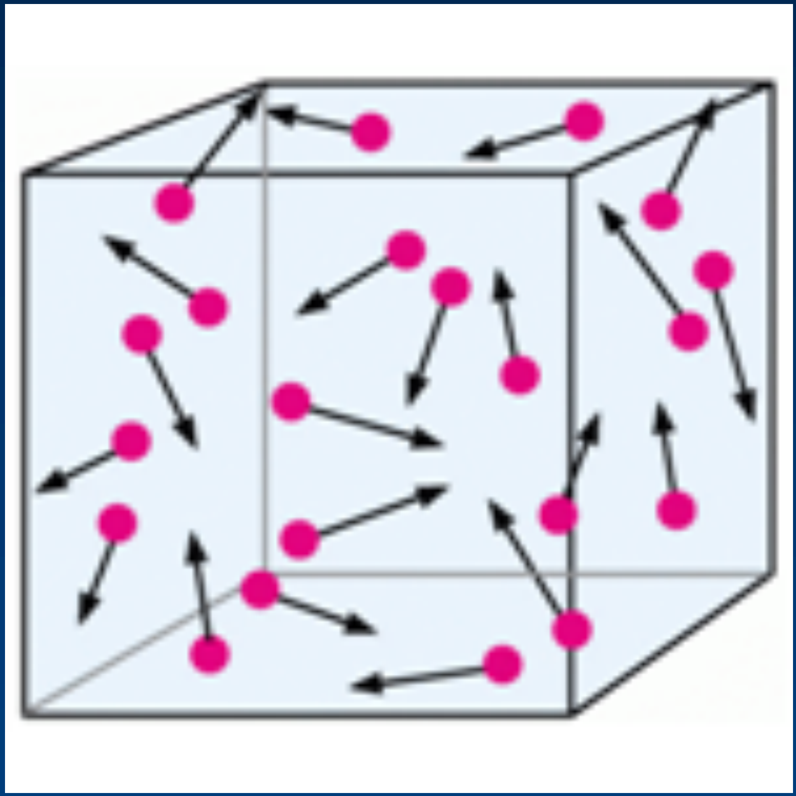


Temperature

- We have talked about "hot", "cold" – to understand what produces these spectra, we need understanding of temperature
- A measurement of the internal energy content of an object.
- Solids: Higher temperature means higher average vibrational energy per atom or molecule.
- Gases: Higher temperature means higher average kinetic energy (faster speeds) per atom or molecule.

How does temperature relate to random motion? For an ideal gas, if particles have mass m and typical speed, v, then

$$v = \sqrt{\frac{3kT}{m}}$$



k is Boltzmann's constant,
and has value 1.38 x 10^{-23} m^2 kg s^{-2} K^{-1}, (or Joules K^{-1}). We'll derive this in a later lecture.

Blackbody Radiation

- A blackbody is an object that absorbs all radiation, at all wavelengths: perfect absorber. No incident light is reflected
- As it absorbs radiation, it will heat up and radiate
- A blackbody will emit radiation at a broad range of wavelengths (continuous spectrum)

The spectrum of radiation the blackbody emits is entirely due to its temperature.

Intensity, or brightness, as a function of frequency (or wavelength) is given by Planck's Law:

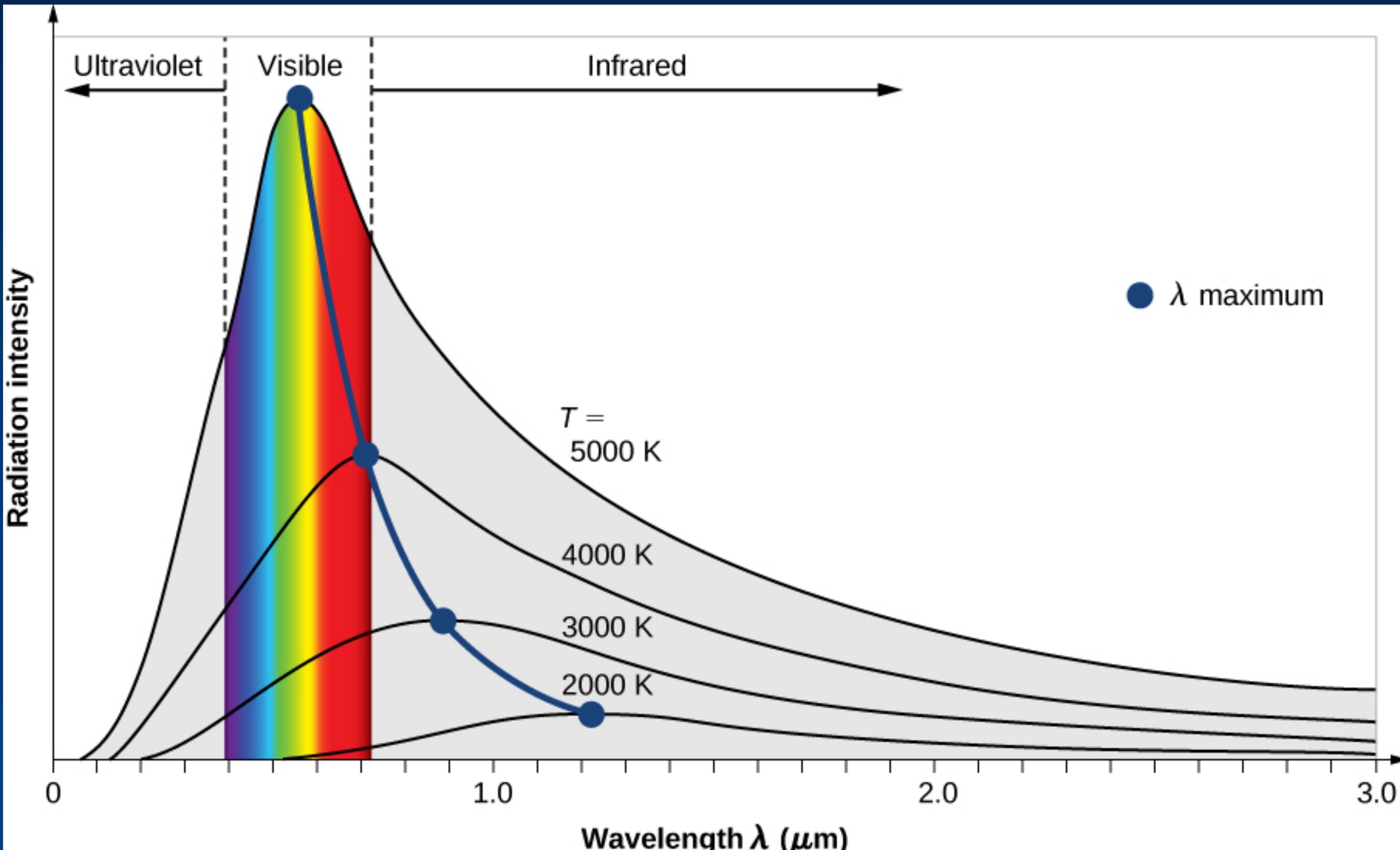
$$I_\nu = \frac{2h\nu^3}{c^2}\left[\frac{1}{e^{h\nu/kT}-1}\right] \quad \text{also} \quad I_\lambda = \frac{2hc^2}{\lambda^5}\left[\frac{1}{e^{hc/\lambda kT}-1}\right]$$

where k is Boltzmann constant $= 1.38 \times 10^{-23}$ J/K

and h is Planck's constant $= 6.6 \times 10^{-34}$ J s

Units of intensity: J s^{-1} m^{-2} $ster^{-1}$ Hz^{-1}

Example: 4 blackbody (Planck curves) for 4 different temperatures.



Wien's Law for a blackbody

- λ_{max} = 0.0029 (m K) / T
- λ_{max} is the wavelength of maximum emission of the object (in meters), and
- T is the temperature of the object (in Kelvins).

=> The hotter the blackbody, the shorter the wavelength of maximum emission

Hotter objects are bluer, cooler objects are redder.

Example 1: How hot is the Sun?

Measure λ_{max} to be about 500 nm, so

$T_{sun} = 0.0029 \text{ m K} / \lambda_{max} = 0.0029 \text{ m K} / 5.0 \times 10^{-7} \text{ m}$

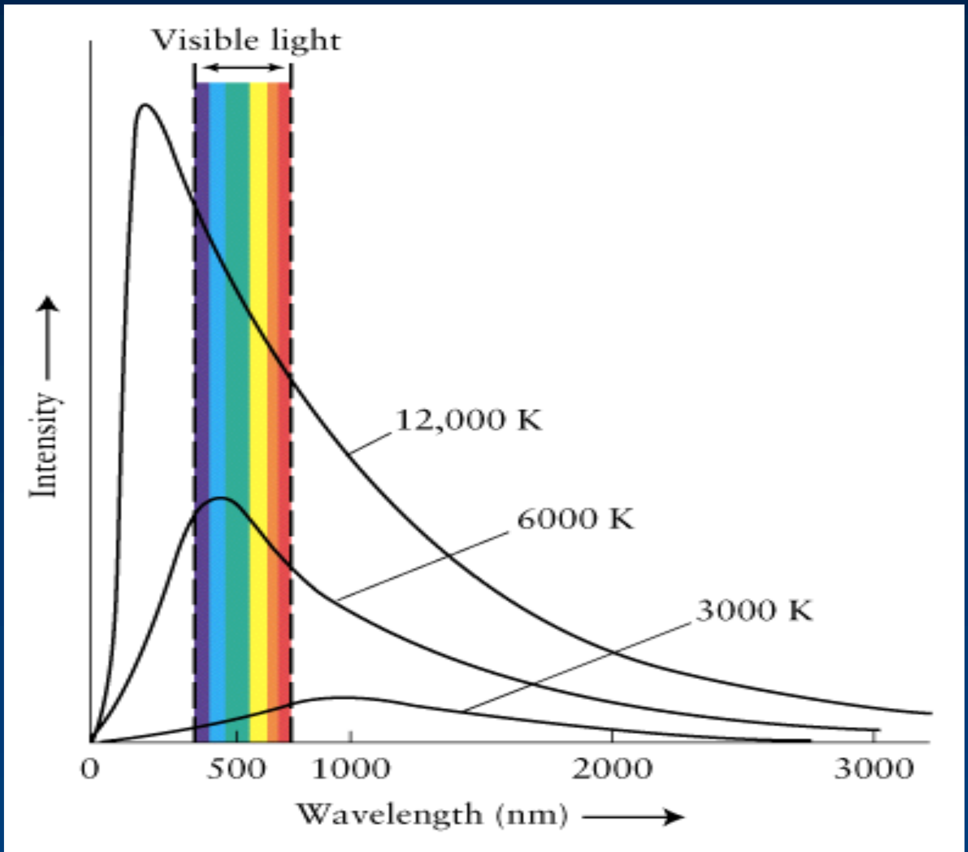
$= 5800 \text{ K}$

Example 2: At what wavelength would the spectrum peak for a star which is 5800/2 = 2900 K?

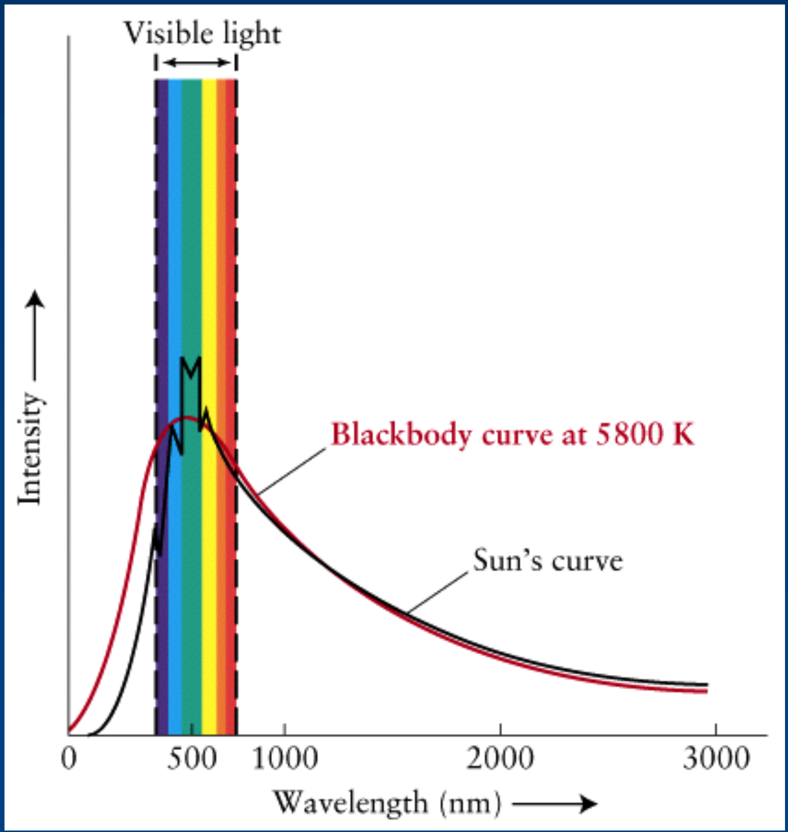
For a star with T= 5800 x 2 = 11,600 K?

What colors would these stars be?

Wavelengths of peaks of the curve illustrate Wien's Law.



The spectrum of the Sun is *almost* a blackbody curve.



Betelgeuse
surface temp
3500 K

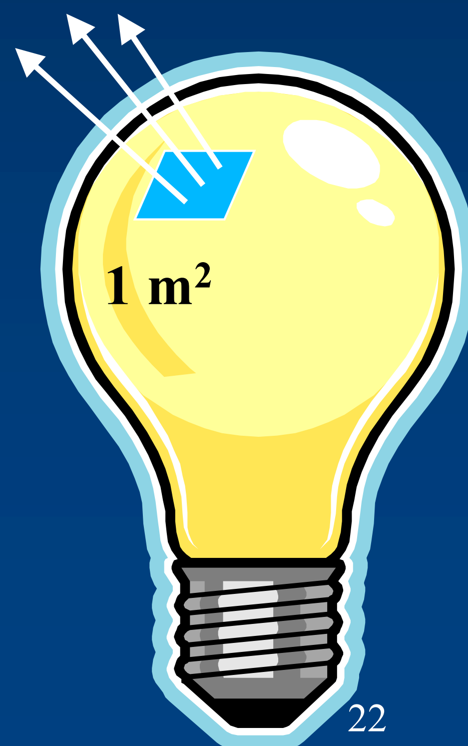
Rigel
surface temp
11,000 K

Stefan-Boltzmann Law for a blackbody

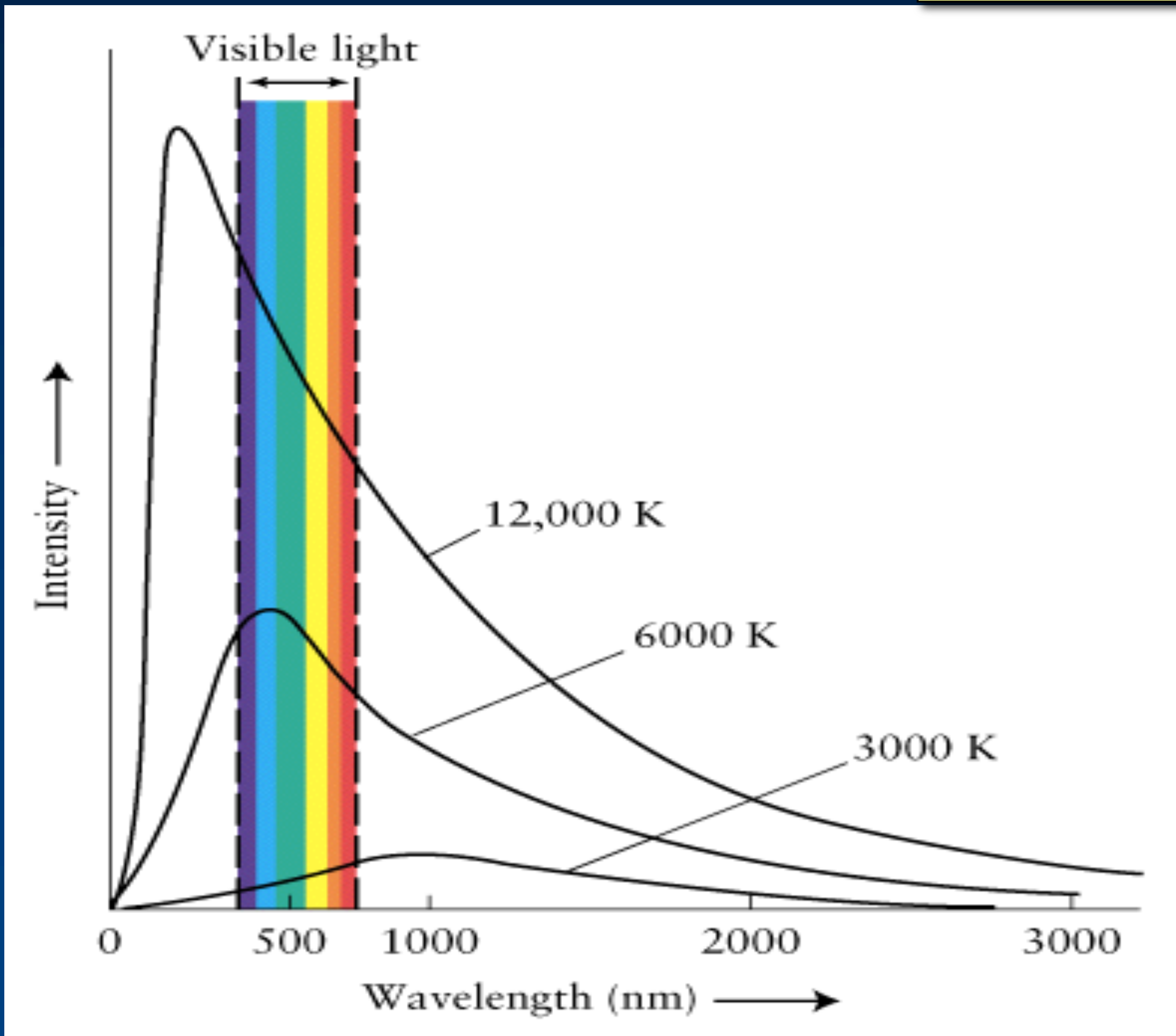
- $F = \sigma T^4$

- F is the emergent flux, in joules per square meter of surface per second ($J\ m^{-2}\ s^{-1}$, or $W\ m^{-2}$)
- σ is a constant = $5.67 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$
- T is the object's temperature, in K

The hotter the blackbody, the more radiation it gives off at all wavelengths



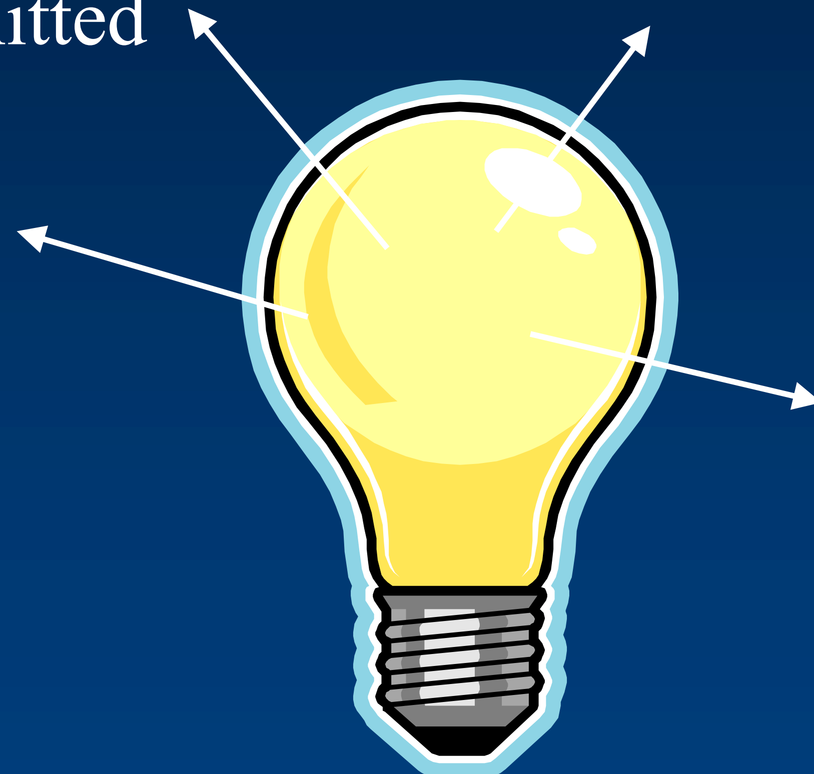
At any wavelength, a hotter body radiates more intensely



Example: If the temperature of the Sun were twice what it is now, how much more energy would the Sun produce every second?

Luminosity and Blackbody Radiation

Luminosity is radiation energy emitted per second from entire surface:



$L = F_{emergent}$ x (surface area)

Units of L are Watts (W, or J/s)

For sphere (stars),

$L = 4\pi R^2$ x $F_{emergent}$

For spherical blackbody (stars, approx.): $L = 4\pi R^2 \sigma T^4$

Spectrum of the Sun – what kind of spectrum is this?