Test 2 Review

Covers Telescopes, Stellar Atmospheres and Stellar Interiors (Chs 6, 8, 9 and 10)
All constants needed will be given on test.

Two equations from A2110/2115:

Small angle formula $$D = \frac{\alpha d}{206265}$$

Parallax $$d = 1/p$$

From Mechanics lectures, we've been using mean free path and time:

$$\lambda = \frac{1}{n\sigma} \qquad \tau = \frac{1}{n\sigma v}$$

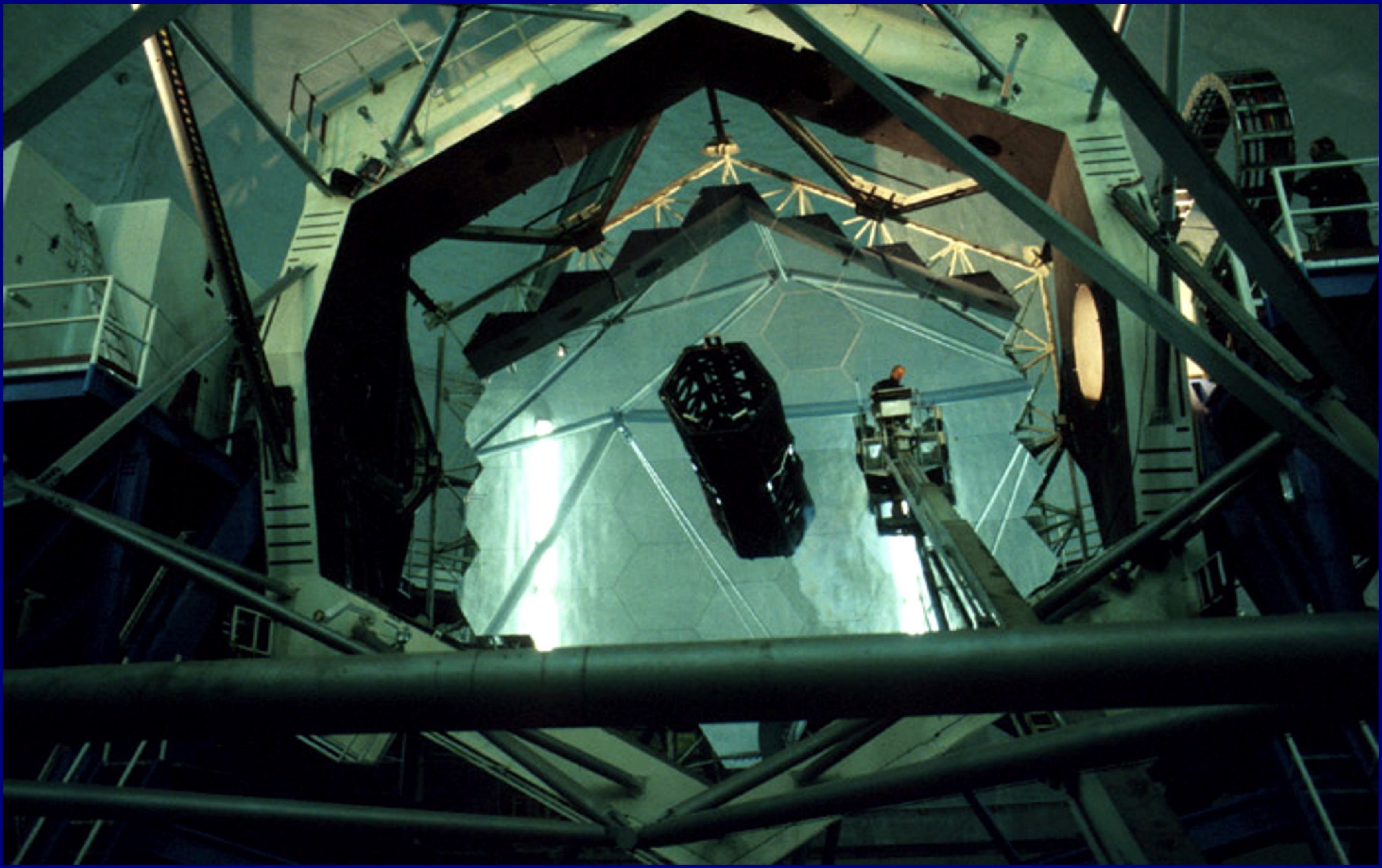
Stellar Interiors

Demo Statstar.py

You can still do this part of HW5 for credit

Mirror size

Mirror with larger area captures more light from a cosmic object. Can look at fainter objects with it.



Keck 10-m optical telescope.
30 m optical telescopes are now under construction!

Reasons for using telescopes, cont.

- **Magnification**: angular diameter as seen through telescope/angular diameter on sky
  - Typical magnifications 10 to 100 (depends on eyepiece)

- **Field of View:** how much of sky can you see at once? Typically many arcminutes – few degrees

- **Resolution**: The ability to distinguish two objects very close together. Angular resolution:

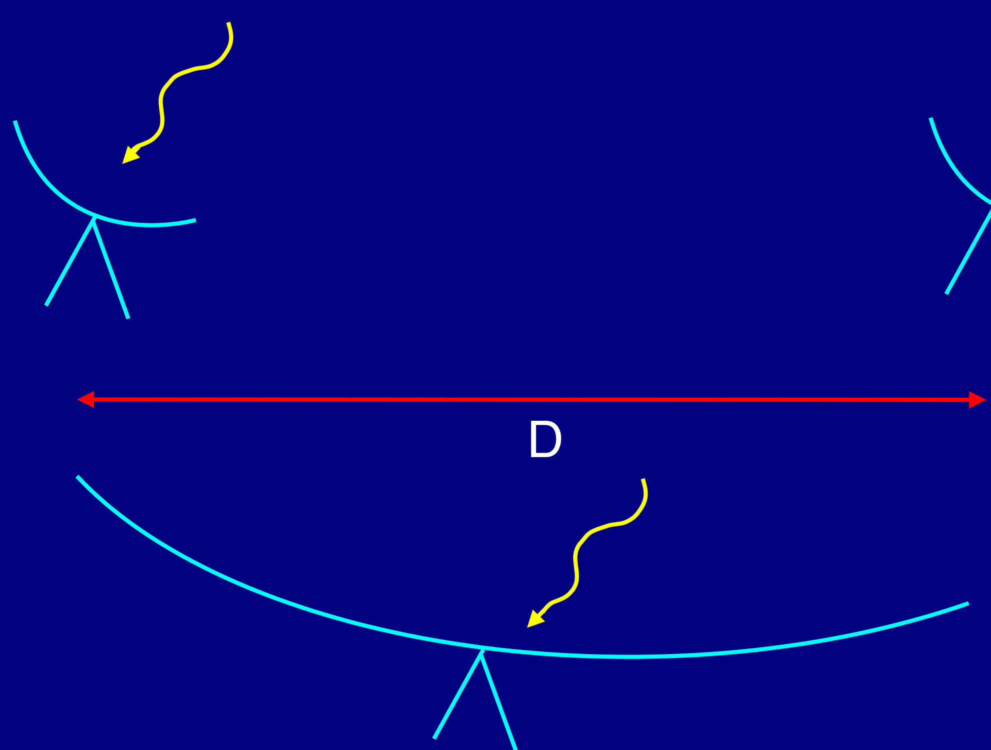
$$q = 2.5 \times 10^5 \; l/D$$

where q is angular resolution of telescope in arcsec, l is wavelength of light, D is diameter of telescope objective, in <u>same</u> distance units

- Example, for D=2.5 m, λ=500 nm, q = 0.05”

Interferometry

A technique to get improved angular resolution using an array of telescopes. Most common in radio, but also limited optical interferometry.



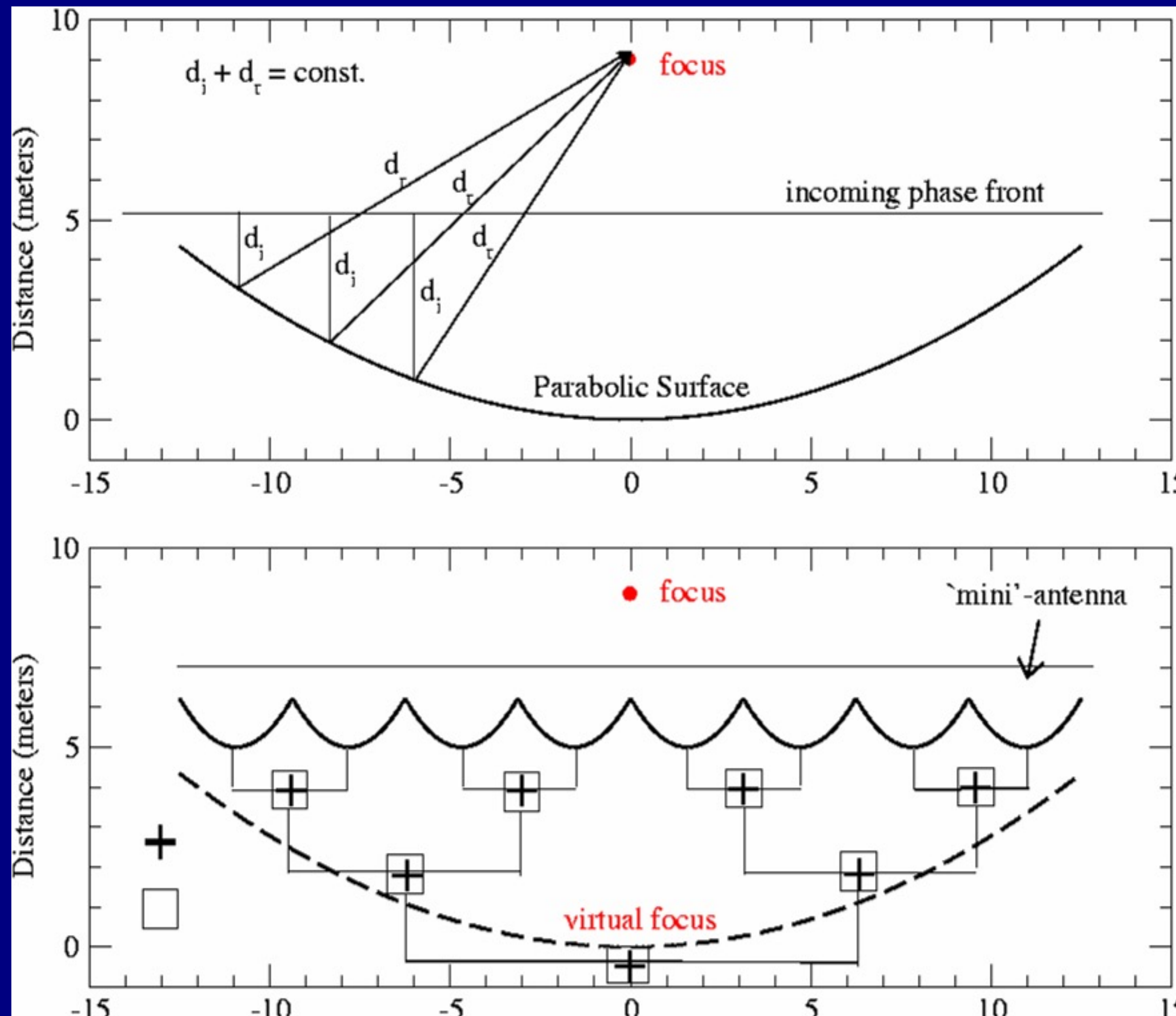
Consider two dishes with separation D vs. one dish of diameter D.
By combining the radio waves from the two dishes, the achieved angular resolution is the same as the large dish.

Aperture Synthesis – Basic Concept

If the source emission is unchanging, there is no need to collect all of the incoming rays at one time.

One could imagine sequentially combining pairs of signals. If we break the aperture into N sub-apertures, there will be N(N–1)/2 pairs to combine.

This approach is the basis of aperture synthesis.



Radiation

Planck radiation law

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$$

Wien and Stefan-Boltzmann laws

$$\lambda_{\text{max}} = \frac{0.0028979}{T}\ \text{m} \qquad F_e = \sigma T^4$$

Luminosity of spherical BB

$$L = 4\pi R^2 \sigma T^4$$

Incident flux at distance *r*

$$F_i = \frac{L}{4\pi r^2} = F_e \frac{R^2}{r^2}$$

Monochromatic incident flux

$$F_{i,\lambda} d\lambda = \frac{L_\lambda d\lambda}{4\pi R^2}$$

Ideal Gas Law

$$P = nkT$$

Maxwell Boltzmann velocity distribution

$$n_V dv = n \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT} 4\pi v^2 dv$$

Most probable speed $$v_{mp} = \sqrt{\frac{2kT}{m}}$$

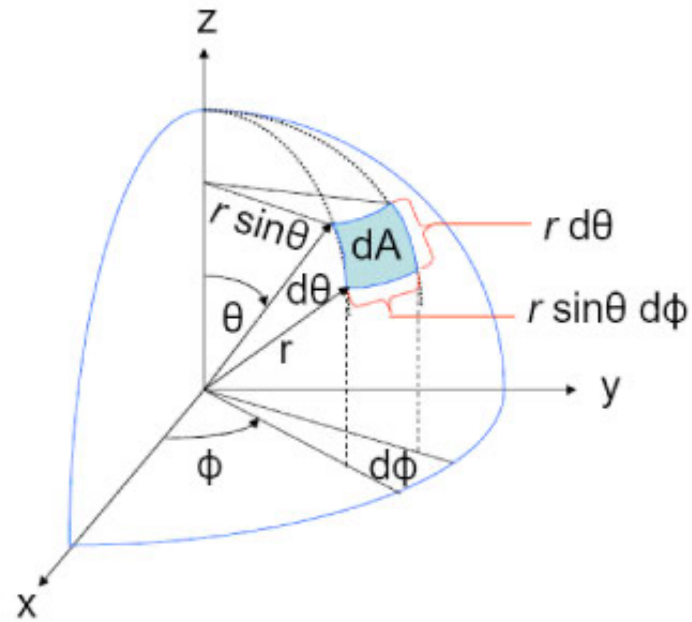
Root mean-square speed $$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Solid angle

- 2-D analog of an angle: the apex of a cone.

- 1-D angle gives you arc length $rd\theta = s$

- Solid angle gives you surface area $r^2 d\Omega = dA$



- Unit is steradian (sr), and there are 4π sr in a spherical surface.

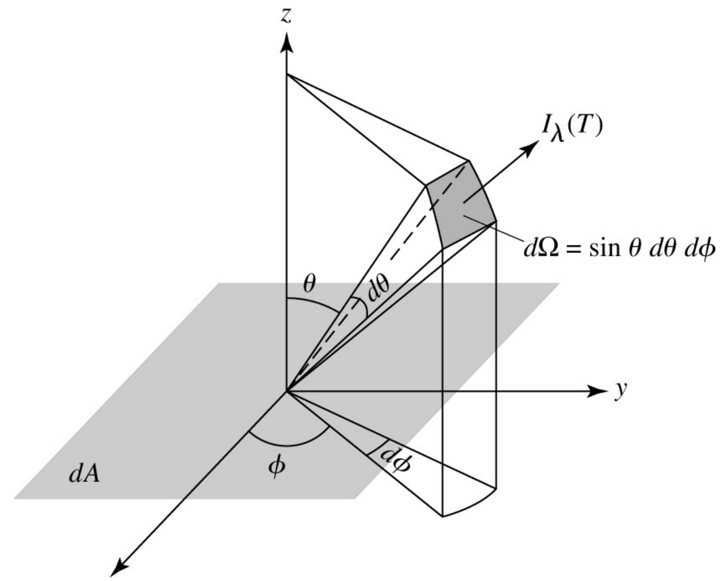
- A small element of area *dA* in spherical coordinates:
  - Side 1 has length $r d\theta$
  - Side 2 has length $r sin\theta d\varphi$
  - Area $dA = r^2 d\theta sin\theta d\varphi$ so $d\Omega = \sin\theta d\theta d\phi$

$$\int_\Omega d\Omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta d\theta d\phi = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi = 2 \times 2\pi$$

Power W = F 4π d^2 Δν

Radiative Transfer

$$F_\lambda d\lambda = \int_\Omega I_\lambda d\lambda \cos\theta d\Omega$$



$$I_\lambda d\lambda = \frac{E_\lambda d\lambda}{dt dA \cos\theta d\Omega}$$

Units of I_λ are $\mathrm{W\,m^{-2}\,m^{-1}\,sr^{-1}}$

$$\text{mean intensity} = \frac{\int I_\lambda d\Omega}{4\pi} = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} I_\lambda \sin\theta d\theta d\phi = < I_\lambda >$$

Energy density

$$u_\lambda d\lambda = \frac{1}{c}\int I_\lambda d\lambda d\Omega = \frac{4\pi}{c} < I_\lambda > d\lambda$$

Integrated (or Total) energy density $$u = \int_0^\infty u_\lambda d\lambda$$

For Blackbody radiation

$$u = aT^4 \qquad a = \frac{4\sigma}{c}$$

Radiation pressure

$$P_{rad} = \frac{1}{3}aT^4$$

Optical depth and intensity

$d\tau_\lambda = \kappa_\lambda \rho ds$ If κ_λ and ρ constant over s, then $\tau_\lambda = \kappa_\lambda \rho s$

$$I_\lambda = I_{0,\lambda} e^{-\kappa_\lambda \rho s} = I_{0,\lambda} e^{-\tau_\lambda}$$

Opacity per unit mass and per particle:

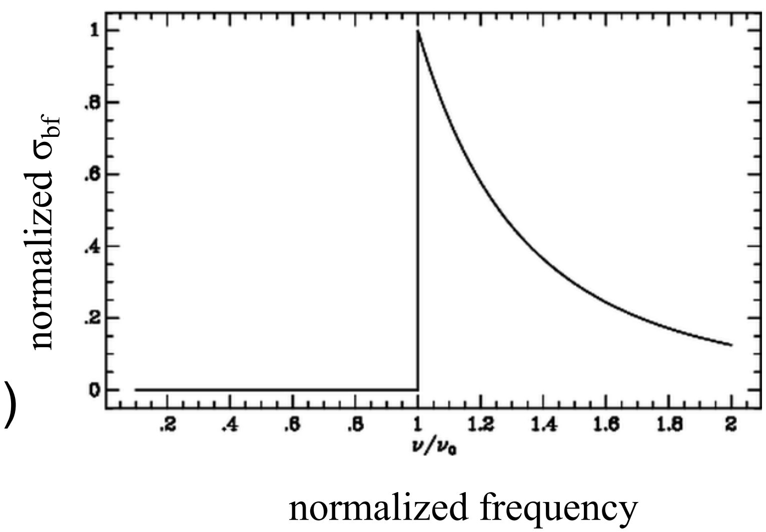
$$\kappa_\lambda \rho = \sigma_\lambda n$$

$$\tau_\lambda = \kappa_\lambda \rho s = n\sigma_\lambda s$$ or $$\tau_\lambda = \frac{s}{l}$$ where *l* is the mean free path

$\tau_\lambda \leq 1$ easy to travel distance *s* *optically thin*

$\tau_\lambda \gg 1$ many scatterings before reaching *s* *optically thick*

Sources of opacity. Only equations to learn are for bound-free opacity cross section for H:



$$\sigma_{bf} = 1.31 \times 10^{-19} \frac{1}{n^5} \left(\frac{\lambda}{500\,\mathrm{nm}} \right)^3 \ \mathrm{m}^2$$

(only for $\lambda \leq \dfrac{hc}{\chi_n}$; $\sigma_{bf} = 0$ for $\lambda > \dfrac{hc}{\chi_n}$)

and electron scattering cross section:

$$\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = 6.65 \times 10^{-25} \ \mathrm{cm}^2$$

More on optical depths:

Consider $\tau_\lambda = 1$

Then

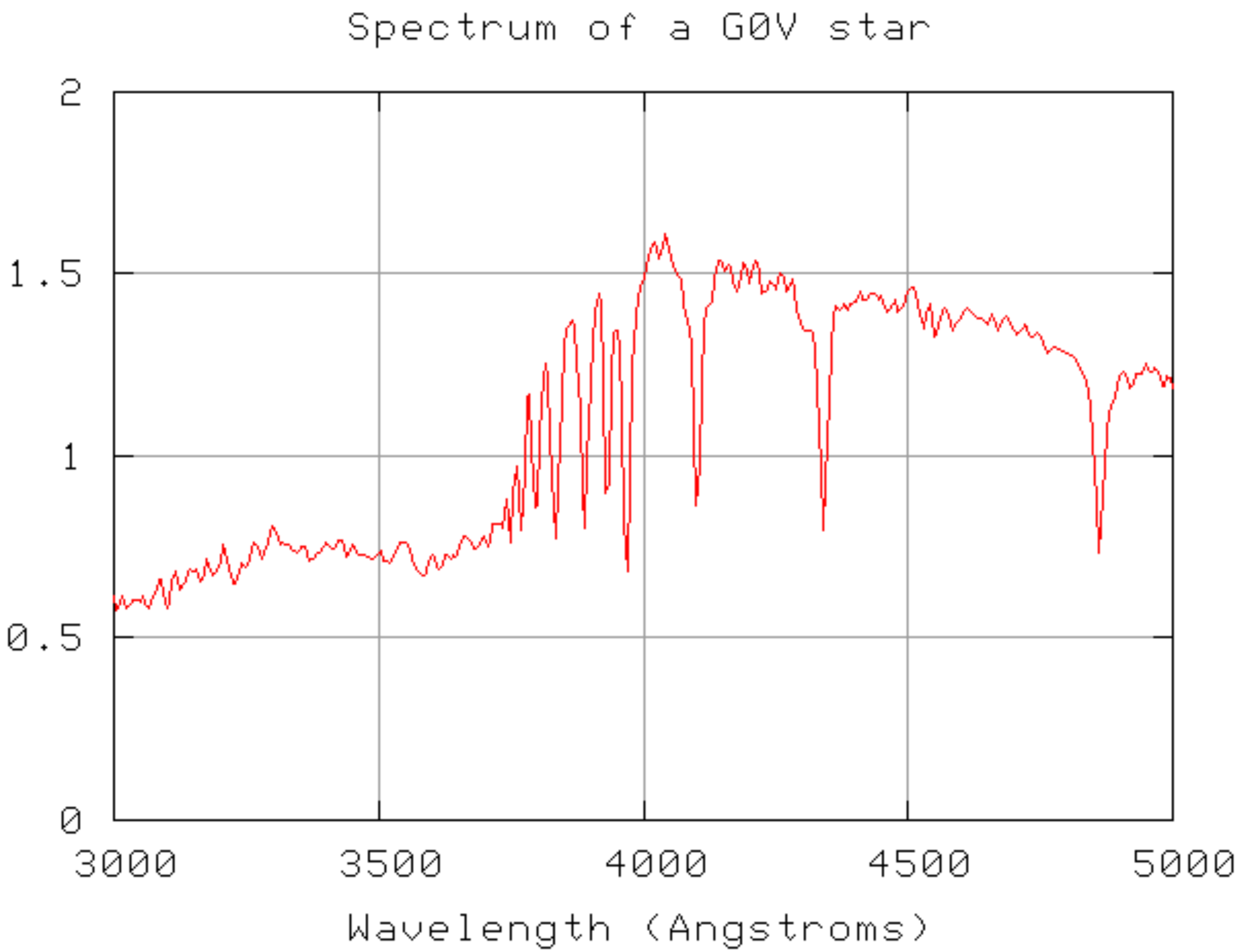
κ_λ is cross-section per mass, but σ_λ is cross-section per particle. Thus

$$\frac{1}{\kappa_\lambda \rho} = \frac{1}{n\sigma_\lambda} = l$$ mean free path!

Intensity falls by 1/e over one mean free path at λ

$$\tau_\lambda \gg 1 \ \text{optically thick}$$

$$\tau_\lambda \lesssim 1 \ \text{optically thin}$$



Sources of Opacity - How many can you find?

Sources of stellar opacity and emissivity (we won't write eqns for all of these – too complex!):

1) Bound-bound absorption

- When e^- makes upward transition in atom or ion. Subsequent downward transition either:
    - back to initial orbit (effectively a scattering process)
    - back to different orbit (true absorption process for original λ)
    - $\geq$ 2 transitions back to lower levels (true absorption, degradation of average photon energy)
- Call this $\kappa_{\lambda,bb}$. Recall mks units are $m^2\ kg^{-1}$. Is zero except at wavelengths capable of producing upward atomic transitions => absorption lines in stellar spectra. Depends on temperature, abundances, QM transition probabilities. No simple function.

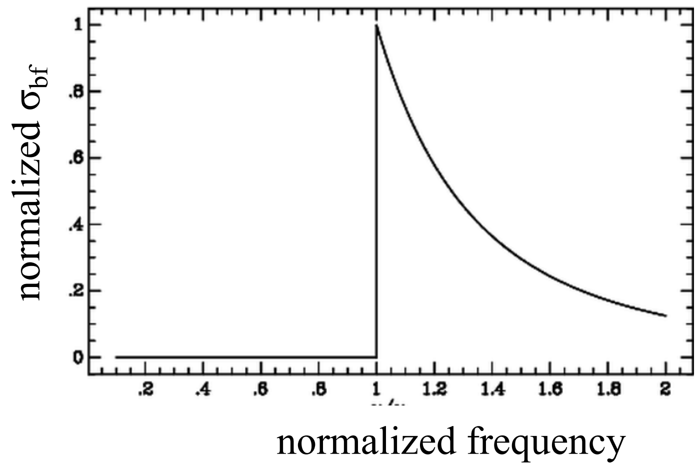
2) Bound-free absorption = photoionization

- $\kappa_{\lambda,bf}$ is a source of continuum opacity. Any photons with $\lambda < hc/\chi_n$ (where χ_n is the ionization potential of n^{th} orbital) can cause ionization. Inverse process: recombination - also degrades photon energies.

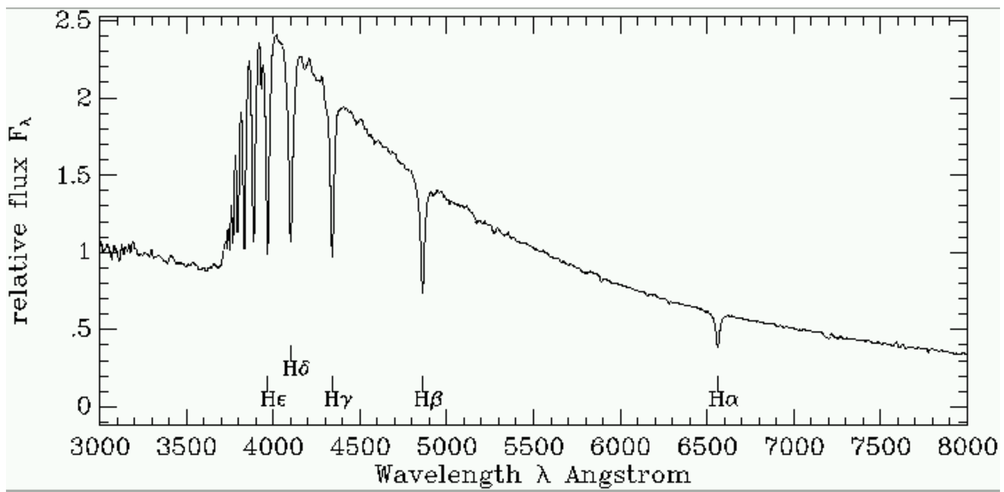
Example: H atom in level *n*

$$\sigma_{bf} = 1.31 \times 10^{-19} \frac{1}{n^5} \left(\frac{\lambda}{500\,\mathrm{nm}} \right)^3 \ \mathrm{m}^2$$

(only for $\lambda \leq \dfrac{hc}{\chi_n}$; $\sigma_{bf} = 0$ $\lambda > \dfrac{hc}{\chi_n}$

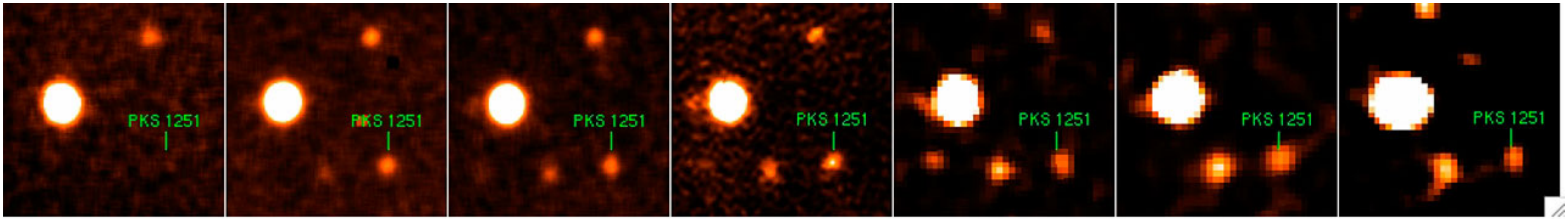


So for level *n*, $\kappa_{\lambda,\mathrm{bf}} = \sigma_{bf}$ times the number of atoms or ions in that level per kg

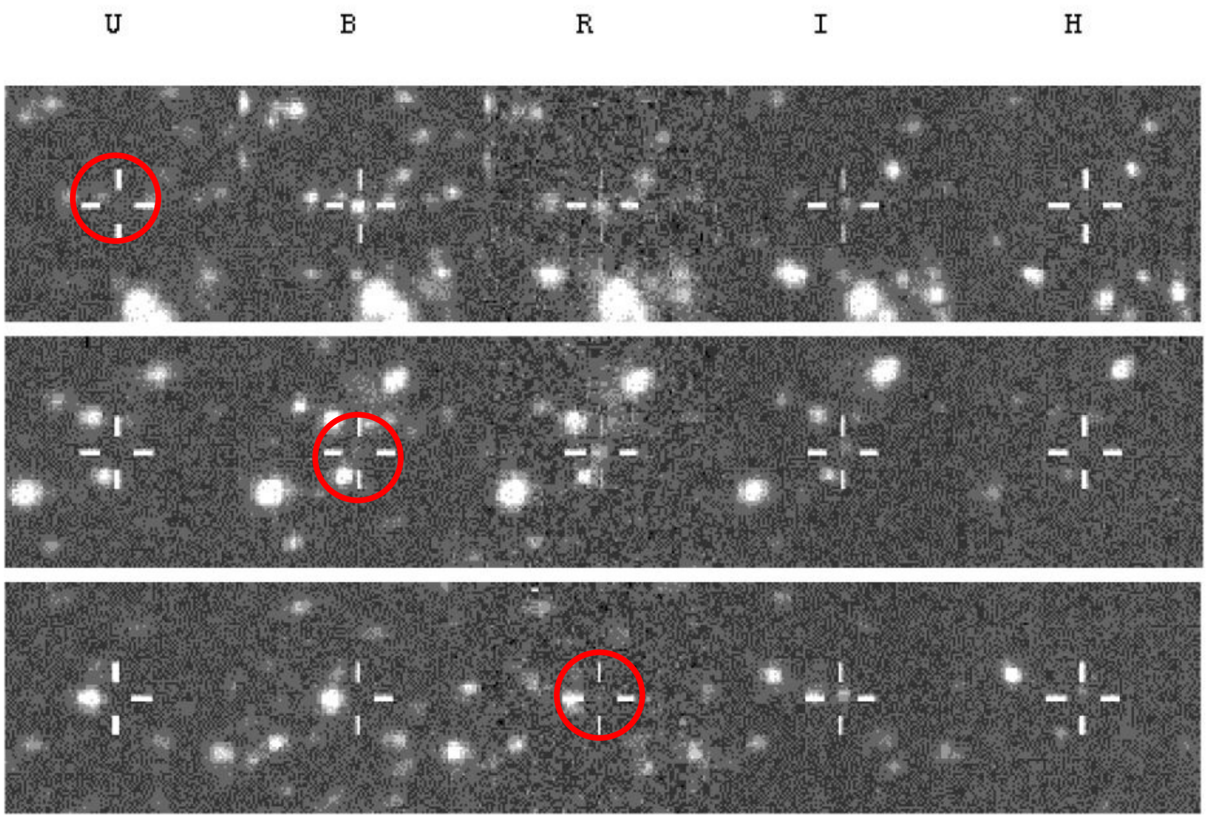


This causes the "Balmer jump". Photons lost to ionization of H from n=2 level. Requires E ≥ 13.6-10.2 = 3.4 eV, or $\lambda \leq 364.7$ nm. Because $\sigma_{b,f} \propto \lambda^3$, spectrum gets closer to blackbody again for shorter λ's.

Similar jump at E=13.6 eV for Lyman series, but in far UV (except at high redshifts!). Used to get redshifts and thus distances of faint galaxies.



visible → infrared



Which is the most distant object?

3) Free-free absorption

- $\kappa_{\lambda,ff}$: another source of continuum opacity. Free *e*$^-$ near ion absorbs photon and increases velocity.

Why won't isolated e$^-$ absorb photons?

- (converse: free-free emission, or *brehmsstrahlung,* e$^-$ loses energy passing by an ion, emits a photon)

4) Electron-scattering (Thomson scattering)

- κ_{es} : photon scatters off free e$^-$. Source of continuum opacity. Depends on the *Thomson cross section* of the *e*$^-$ (relatively small):

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-25}\ \mathrm{cm}^2$$

($r = e/m_e c^2$ often used as the classical 'radius' of an electron)

- But dominates at high-temperatures.

Main source of continuum opacity in stellar atmospheres of type:

F and cooler:	Photoionization of H^- ions.
	Any photon with $$\lambda \leq \frac{hc}{\chi} = \frac{hc}{0.754\,\mathrm{eV}} = 1640\,\mathrm{nm} \qquad \text{(IR)}$$
B, A:	Bound-free of H and free-free processes
O stars:	Electron scattering and bound-free processes of He
Interiors of stars:	Electron scattering

So when $\tau_\lambda = \frac{s}{l} \leq 1$, photons can easily escape the star from depth *s*.

More accurately (see C&O): $\tau_\lambda = \frac{2}{3}$ is the average point of origin of escaping photons.

$\Rightarrow$ we see into a star to a depth corresponding to $\tau_\lambda = \frac{2}{3}$

<u>Consequences:</u>

$$\tau_\lambda = \frac{2}{3} = \int_0^s \kappa_\lambda \rho ds$$

1) Absorption lines:

At the line cent κ_λ is highest => we don't see as deeply into atmosphere relative to neighboring λ's in line.

For λ's with no line, we see even deeper.

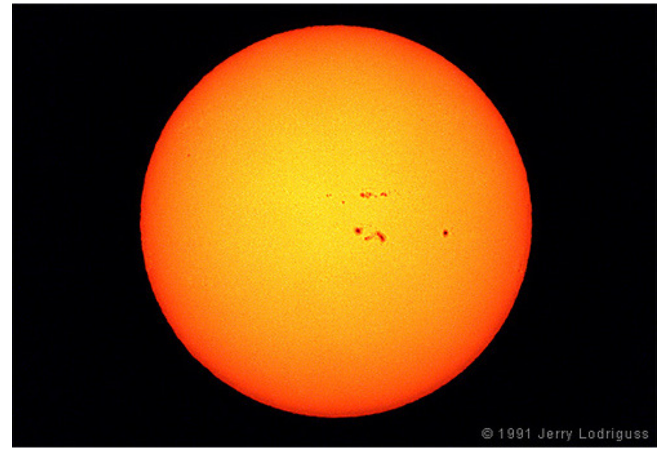
2. Limb darkening.

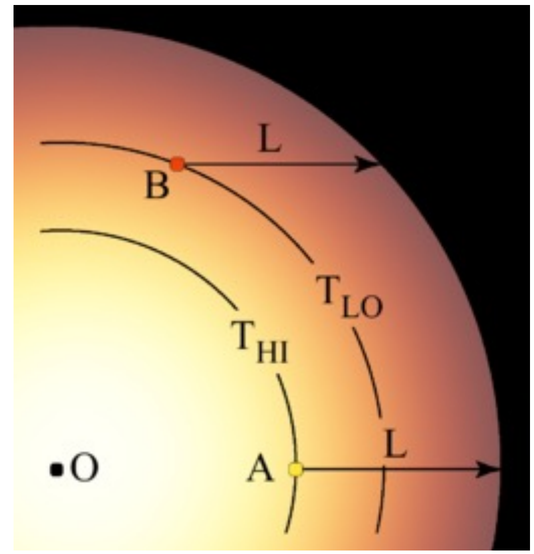
We see down to some $(\leftrightarrow \tau_\lambda = 2/3)$ across the disk of the Sun (L~ a few 100 km).

A depth L does not penetrate as deeply into the atmosphere at limb, as it does at the center.

If T drops with height R (=distance from center of Sun), blackbody radiation less intense at limb => darker.

What would we see if $T\uparrow$ height?





Broadening mechanisms

Natural broadening

$$\Delta\lambda = \frac{\lambda^2}{2\pi c}\left(\frac{1}{\Delta t_i} + \frac{1}{\Delta t_f}\right)$$

Doppler broadening

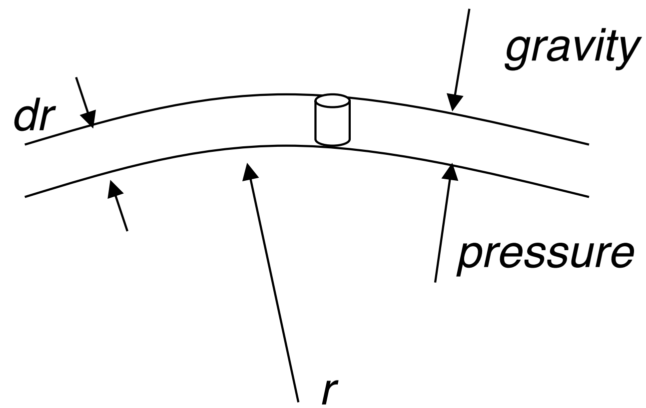
$$\Delta\lambda_{1/2} = \frac{2\lambda}{c}\sqrt{\frac{2kT\ln 2}{m}}$$

Pressure or collisional broadening

$$\Delta\lambda \sim \frac{\lambda^2}{\pi c} n\sigma \sqrt{\frac{2kT}{m}}$$

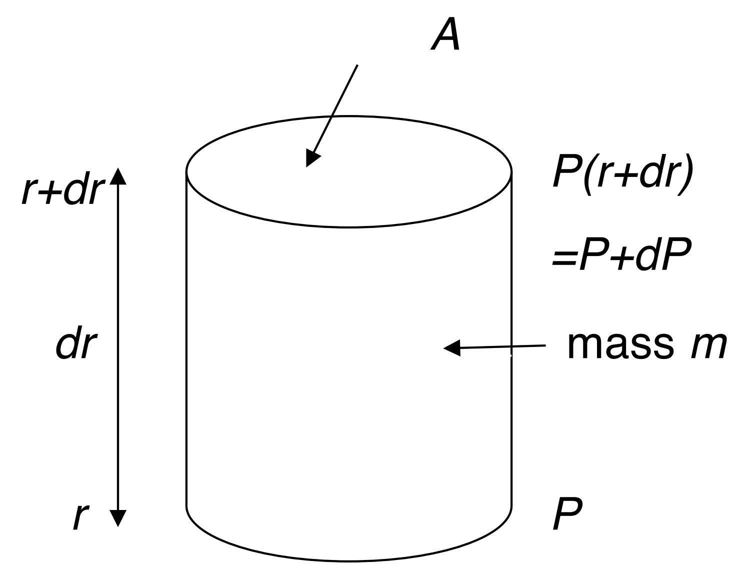
Skip Equivalent Width and Curve of Growth

Hydrostatic Equilibrium



Stability of star requires cylinder of gas is static -> no net forces.

At every *r*, gravity balanced by pressure.



Static => weight must be balanced by pressure difference over *dr*.

Or $AdP = -mg$.

Since $m = \rho V = \rho A dr$, then $AdP = -\rho g A dr$,

So
$$\boxed{\frac{dP}{dr} = -\rho g}$$

Equation of Hydrostatic Equilibrium (first of four fundamental differential equations of stellar structure)

g, the acceleration due to gravity, depends only on mass interior to *r*:

$$g(r) = \frac{GM_r}{r^2}$$

Three of the four differential equations of stellar structure

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\rho g$$

Mass conservation

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

Radiative temperature gradient

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$

Other equations relevant to stellar structure:

Total pressure $$P_{total} = \frac{\rho k T}{\mu m_H} + \frac{1}{3} a T^4$$

Mean molecular weight (skip approximate equations we had for neutral and ionized gas but be able to work it out for, e.g. ionized H, neutral He) $$\mu \equiv \frac{\bar{m}}{m_H}$$

Condition for convection

$$\frac{dT_0}{dr} < -\left(1 - \frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2} = \frac{dT}{dr}\Big|_{ad}$$

Convection is likely when:

1. $\overline{\kappa}$ is large (so $|\frac{dT}{dr}|$ is large)

2. $\frac{L_r}{4\pi r^2}$ is high, deep in cores of massive stars (so $|\frac{dT}{dr}|$ is large)

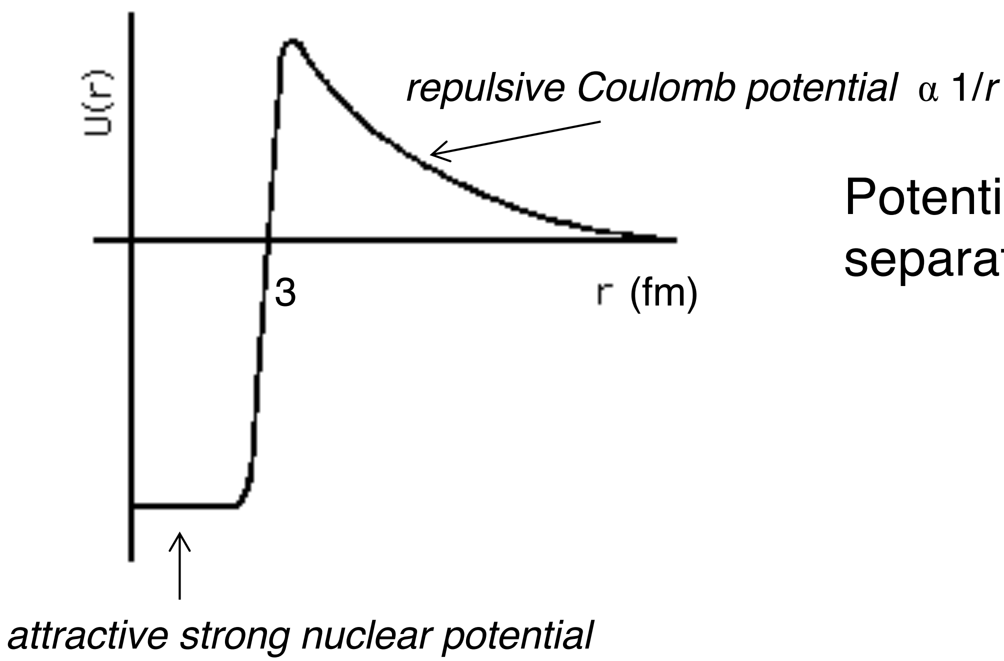
3. $g = \frac{GM_r}{r^2}$ is low (so $|\frac{dT}{dr}|_{ad}$ is low)

Fusion reactions

Under what conditions can fusion occur?

1) Nuclei can interact via the four fundamental forces, but only EM and strong nuclear force important here.

2) To fuse, two positively charged nuclei must overcome the Coulomb barrier (the long range force $\propto 1/r^2$) to reach separation distances where the strong force dominates (10^{-15} m, typical nuclear size)



Potential energy versus nuclei separation.

The height of the Coulomb barrier is given by:

$$\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r}$$

e = charge of electron = $1.6\text{x}10^{-19}$ C,

ε_0 = permittivity of free space = $8.85\text{x}10^{-12}$ $C^2N^{-1}m^{-2}$

Calculate potential energy required for fusion of two H nuclei for r = 1 fm.
Compare to the average kinetic energy of a particle ($3kT/2$) to find $T \sim 10^{10}$ K!

But T at center of Sun only $1.6\text{x}10^7$ K.

Quantum tunneling

According to Quantum Mechanics, there is a finite probability that a particle will penetrate the Coulomb barrier, due to the Heisenberg uncertainty in its position, even if it does not come close enough classically.

The probability for this tunneling for two like charges colliding at speed *v* depends on (Gamow 1928):

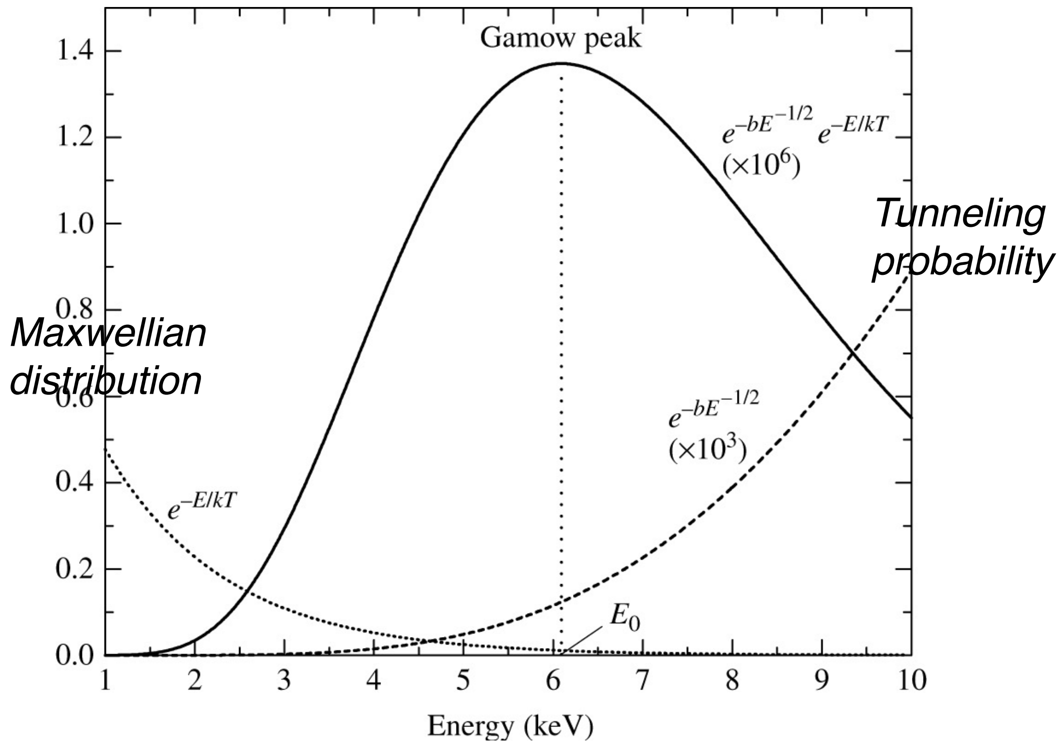
$$e^{-\frac{\pi Z_1 Z_2 e^2}{\epsilon_0 h v}}$$

Hence, this decreases with higher charge and increases with particle velocity *v* (thus energy of collision). But we also know that the velocity follows the Maxwell-Boltzmann distribution for an ideal gas. The fusion probability is therefore proportional to the product

$$e^{-\frac{\pi Z_1 Z_2 e^2}{\epsilon_0 h \nu}} (kT)^{-\frac{3}{2}} e^{-\frac{m v^2}{2kT}}$$

The Gamow peak

Fusion is most likely to occur in the energy window defined as the Gamow peak, which reflects the product of the Maxwell-Boltzmann distribution and tunneling probability. Area under Gamow peak determines reaction rate!



A higher electric charge means a greater repulsive force => higher E_{kin} and T required before reactions occur. For two protons, Gamow peak is at 10^6 keV which is, using $E=3kT/2$, about $T \sim 10^7$ K.

Simplified treatment – see C+O for complications.

Nuclei that are highly charged are also the more massive ones, so reactions between light elements occur at lower T's than reactions between heavy elements.

LWA-SV Field Trip

Saturday, October 29

9am depart UNM from PandA parking lot

10am arrive LWA-SV

11:30 depart site

12:30 pm return to UNM

Drivers Wanted: Greg (+3), Will (+3),