# Test 1 Review

Covers start of semester up through stellar spectra (Chs 2-7).
All constants needed will be given on test.

Two equations from basic astronomy:

Small angle formula

$$D = \frac{\alpha d}{206265}$$

Parallax

$$d = 1/p$$

# Test #1

Covers material from first day of class, all the way through Stellar Spectra

Supporting reading chapters 2-5 and 7-8

Some questions are “concept” questions, some involve working with equations, calculations

Study your lecture notes, homeworks, worksheets, review supporting reading

Know equations/constants on the sheet just handed out.

Bring calculator, something to write with. Closed book, closed notes

Attempt every question – show what you know

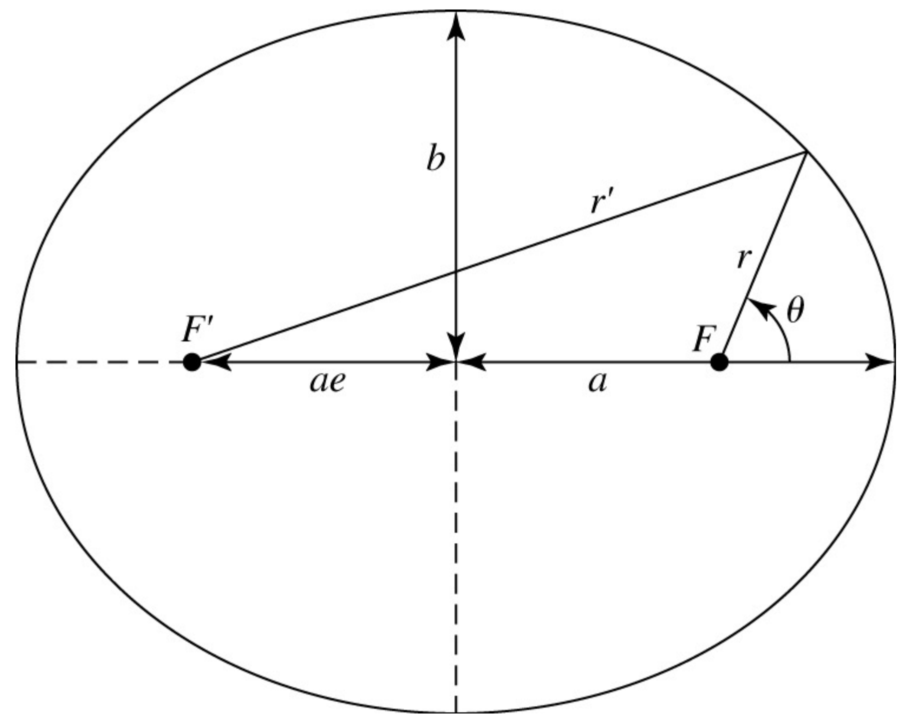
Don’t get bogged down on a question. There are two short answer problems at the back which should be quick.

## Ellipses

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \qquad (0 \leq e < 1)$$

$$A = \pi a b$$

$$b^2 = a^2(1-e^2)$$



## Mechanics

Newton II

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

For circular orbits, magnitude of *F* is $F = m\frac{v^2}{r}$

Torques and angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$ (=0 for central forces)

Gravity and gravitational acceleration

$$F = \frac{GMm}{r^2} \qquad F = mg \Rightarrow g = \frac{GM}{r^2}$$

Energy of mass m in orbit around a fixed mass M (approximately true for, e.g. satellite orbiting Earth)

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

(Kinetic) (Potential)

Escape velocity

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

But in general, both objects orbit center of mass, so for objects 1 and 2, total energy is

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{Gm_1 m_2}{r^2}$$

| | | |
|---|---|---|
| Total energy *E* | < 0 | circular or elliptical (bound) |
| | = 0 | parabolic (marginal) |
| | > 0 | hyperbolic (unbound) |

Bound orbits: ellipses with C of M at one focus (cf. Kepler's 1st law). Two bodies in orbit equivalent to reduced mass orbiting fixed mass M (sum of masses) at distance r (= sum of separations of each mass from C of M)

$$E = \frac{1}{2}\mu v^2 - \frac{GM\mu}{r} \qquad \vec{L} = \mu\vec{r} \times \vec{v} = \vec{r} \times \vec{p}$$

Kepler's 2nd law is conservation of angular momentum

$$\frac{dA}{dt} = \frac{1}{2}\frac{L}{\mu} = const$$

$$L = \mu\sqrt{GMa(1-e^2)}$$

Newton's generalization of Kepler's 3rd law:

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)}a^3$$

Note, still true that $P^2 = \dfrac{a^3}{m_1 + m_2}$

if $P$ in yrs, $a$ in AU and $m$'s in Msun

Virial theorem (for an ensemble of objects in equilibrium, but also true for the time average of an elliptical orbit)

$$2 < KE > + < U > = 0$$

Tidal force

$$\frac{dF}{dr} = -\frac{2GMm}{r^3}$$

Roche limit

$$d \simeq 2.5(\frac{\rho_p}{\rho_s})^{1/3} R$$

Mean free path and time

$$\lambda = \frac{1}{n\sigma} \qquad \tau = \frac{1}{n\sigma v}$$

**Luminosity**

$$L = \int_A \boldsymbol{F_e} \quad dA$$

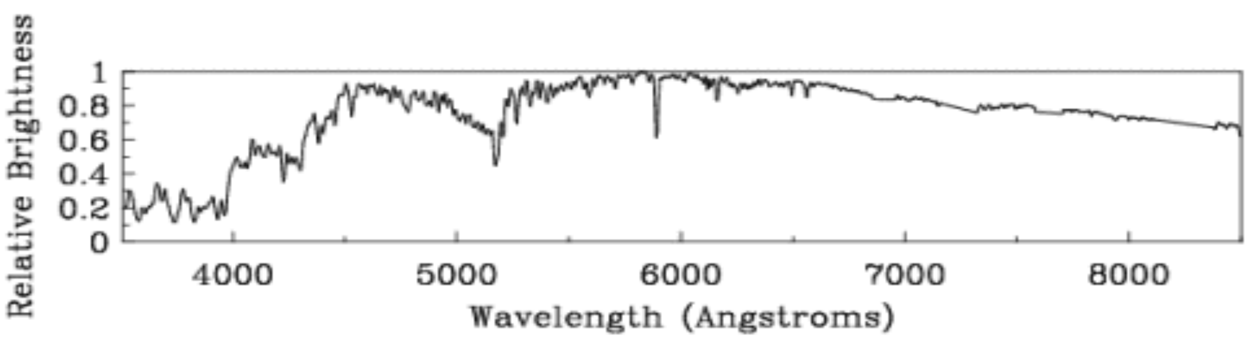
If $F_e$ = constant over $A$, for a spherical blackbody:

$$\boxed{L = 4\pi R^2 \sigma T^4}$$

Since stars are not perfect blackbodies, define an 'effective' temperature, $T_e$

$$L = 4\pi R^2 \sigma T_e^4$$

(the temperature of a blackbody of the same luminosity)



**Kirchhoff's laws:**

1. A hot, dense gas or hot solid object produces a continuous spectrum with no dark spectral lines.
2. A hot, diffuse gas produces bright spectral lines (emission lines).
3. A cool, diffuse gas in front of a source of a continuous spectrum produces dark spectral lines (absorption lines) in the continuous spectrum.

Empirical! What is the *physical* basis?

First law: This was the topic of the last lecture… Continuous spectrum of BB radiation ( $B_\lambda(T)$ or $B_\nu(T)$ emitted at any T>0 K.

**More on wave-particle duality:**

de Broglie proposal (also true for *massive* particles):

they also exhibit wavelike behavior with a characteristic wavelength given by their momentum

*de Broglie relation*

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Confirmed in electron double-slit experiment.

**Heisenberg's uncertainty principle:**

We cannot say with 100% certainty *where* a particle is and *what* its energy is.

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

*"Nature is intrinsically fuzzy"*

Often you will see this form for making estimates: $\Delta x \Delta p_x \approx \hbar$

Or, in terms of energy and time: $\Delta E \Delta t \approx \hbar$

The last statement means that spectral lines cannot be perfectly sharp. This is called *natural broadening.*

**Radiation**

Planck radiation law

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Wien and Stefan-Boltzmann laws

$$\lambda_{\max} = \frac{0.0028979}{T} \text{ m} \qquad F_e = \sigma T^4$$

Luminosity of spherical BB

$$L = 4\pi R^2 \sigma T^4$$

Incident flux at distance *r*

$$F_i = \frac{L}{4\pi r^2} = F_e \frac{R^2}{r^2}$$

Monochromatic incident flux

$$F_{i,\lambda} d\lambda = \frac{L_\lambda d\lambda}{4\pi R^2} = \pi B_\lambda \frac{R^2}{r^2} d\lambda$$

**Quantum mechanics**

Photon energy

$$E = h\nu = \frac{hc}{\lambda}$$

de Broglie relation

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Bohr H energy levels and transition wavelengths

$$E = -\frac{m_e e^4}{32\pi^2\epsilon^2\hbar^2}\frac{1}{n^2} = -13.6\mathrm{eV}\frac{1}{n^2}$$

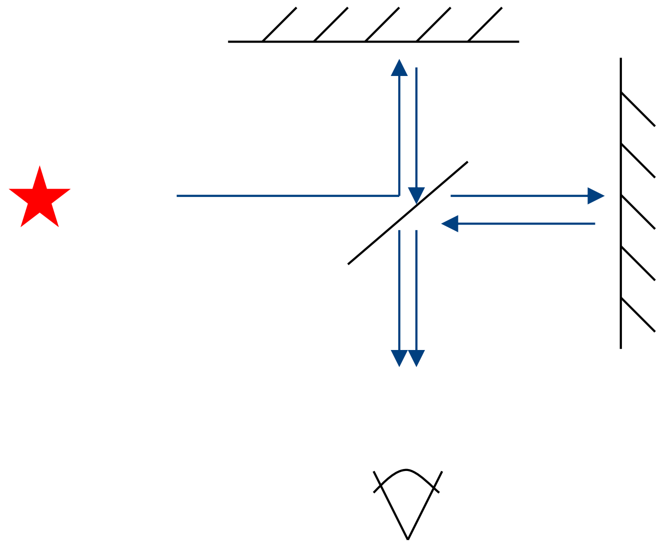
$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right)$$

Heisenberg uncertainty relations

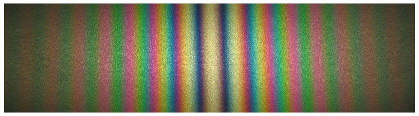
$$\Delta x \Delta p_x \approx \hbar \qquad \Delta E \Delta t \approx \hbar$$

**Michelson-Morley experiment:**

Difference measurement employing interference of two light beams:



Observe fringe pattern of interfering light, rotate device 90 degrees and count number of fringes that shift due to changing time difference between paths.



The difference in path lengths would lead to interference fringe maxima at certain positions.  But hard to place elements accurately enough to measure true path lengths.  So instead rotate the apparatus 90 degrees and measure shift in position of fringes as the path lengths change.

Null result: T's are always the same whether parallel or perpendicular to Earth's motion.  Ether does not exist.

## Special Relativity

Time dilation

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} \equiv \gamma \Delta t' \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \geq 1$$

Proper time is measured in primed frame

Length contraction

$$L' = \frac{L}{\gamma}$$

Object's length is *L* in frame where object is at rest

Lorentz coordinate and velocity transformations

$$x = \gamma(x' + ut') \qquad x' = \gamma(x - ut)$$

$$y = y' \qquad y' = y$$

$$z = z' \qquad z' = z$$

$$t = \gamma(t' + \frac{ux'}{c^2}) \qquad t' = \gamma(t - \frac{ux}{c^2})$$

$$v'_x = \frac{v_x - u}{1 - {uv_x}/{c^2}} \qquad v'_y = \frac{v_y\sqrt{1 - {u^2}/{c^2}}}{1 - {uv_x}/{c^2}} \qquad v'_z = \frac{v_z\sqrt{1 - {u^2}/{c^2}}}{1 - {uv_x}/{c^2}}$$

To get inverse velocity transforms, just replace u with –u.

Relativistic Doppler shift

$$\nu_{obs} = \frac{\nu_{rest}\sqrt{1 - u^2/c^2}}{1 + v_r/c}$$

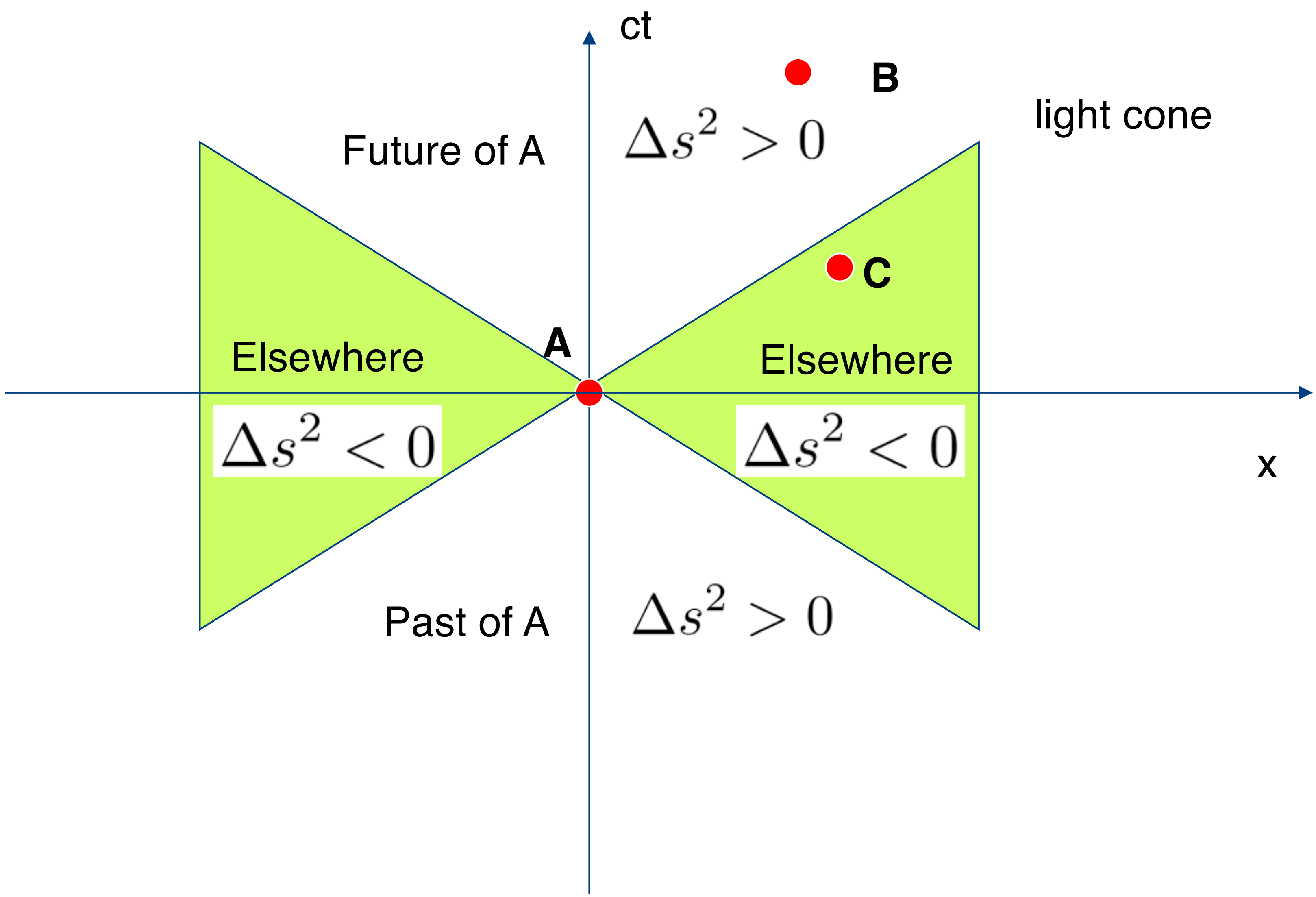
$$v_r = u \cos\theta$$

$\theta$ is angle between direction to source and direction of source's motion

Momentum and energy

$$\vec{p} = \gamma m \vec{v} \qquad E = \gamma m c^2 \qquad E^2 = p^2 c^2 + m^2 c^4$$

## Past, future and elsewhere

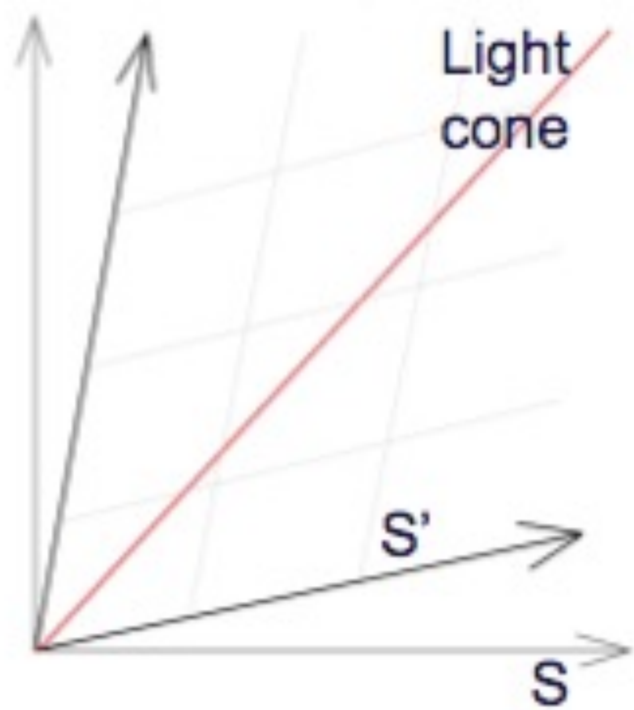


**Spacetime diagram and the Lorentz transformations**

Changing from one reference frame to another via the Lorentz transformations will:

a) Affect the time coordinate (time dilation)
b) Affect the space coordinates (length contraction)

This is leading to a distortion of the spacetime diagram.



**Binary stars**

Visual binaries

$$\frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{a_2}{a_1} \qquad P^2 = \frac{4\pi^2}{G(m_1 + m_2)}a^3$$

(in principle, both masses can be found, if distance and inclination known. Assume *i*=0 deg for this test).

Astrometric binaries

$$P^2 = \frac{4\pi^2 a_1^3}{G m_2^3}(m_1 + m_2)^2$$

Spectroscopic binaries

Double-lined

$$\frac{m_1}{m_2} = \frac{v_{2r}^{max}}{v_{1r}^{max}} \qquad m_1 + m_2 = \frac{P}{2\pi G}\frac{(v_{1r}^{max} + v_{2r}^{max})^3}{sin^3 i}$$

Single-lined

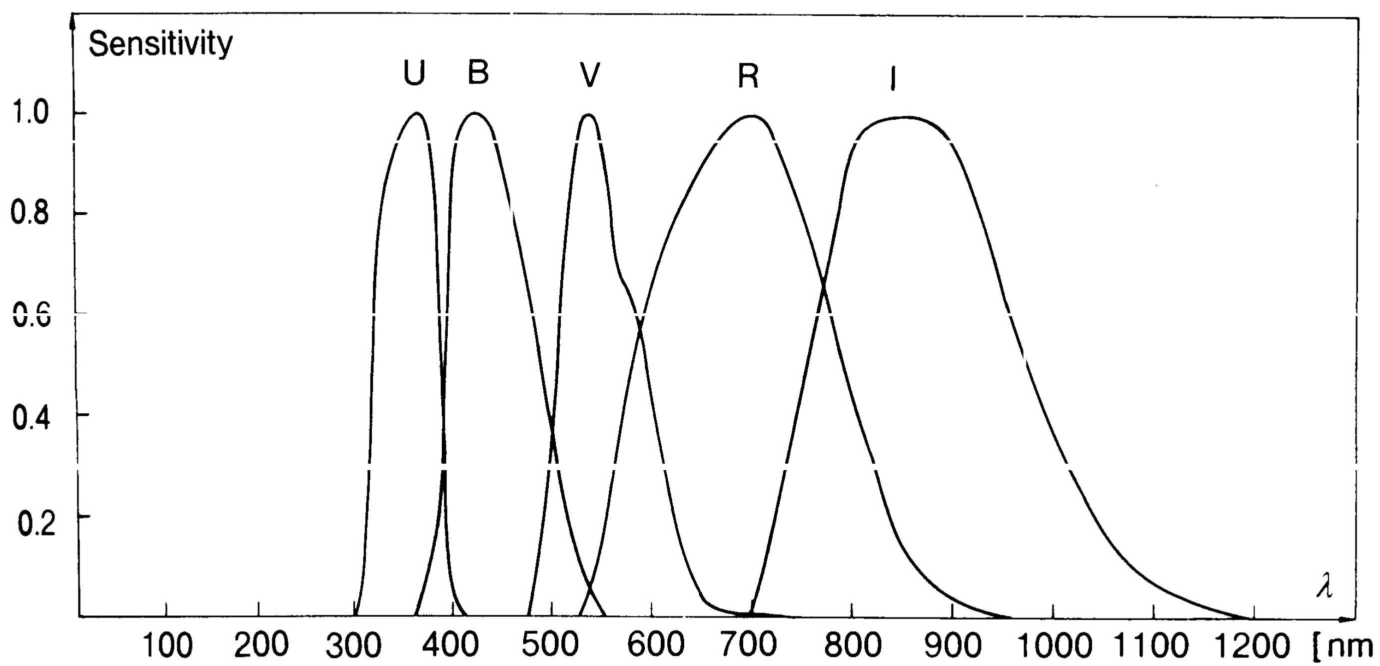
$$\Rightarrow \frac{m_2^3}{(m_1 + m_2)^2}\sin^3 i = \frac{P}{2\pi G}\ (v_{1r}^{max})^3$$

Also, sort the following table out on your own from the physics and geometry involved in each type of event:

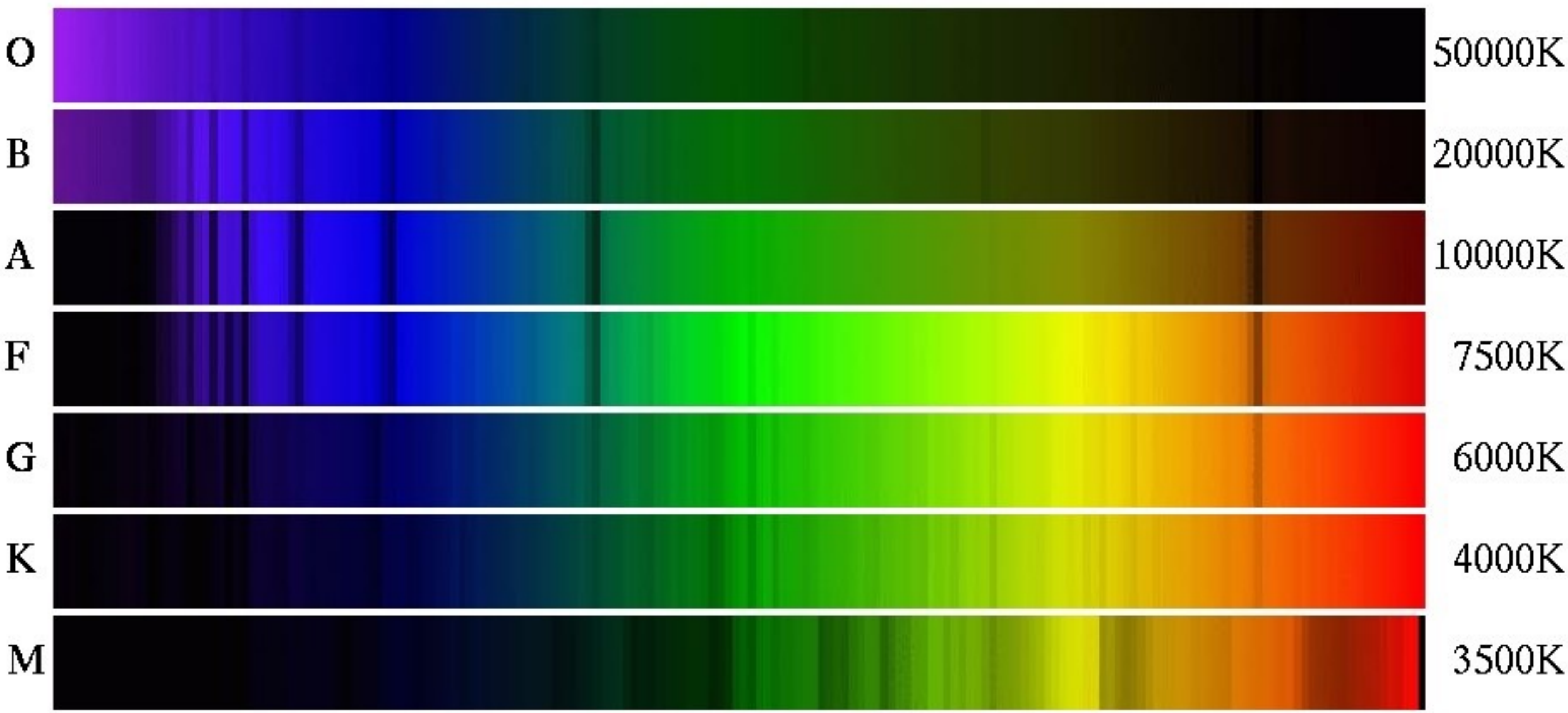
| **Type of binary** | **Observations performed (or needed)** | **Parameters determined** |
|---|---|---|
| Visual | a) Apparent magnitudes and $\pi$<br>*b)* $P$, $a$, and $\pi$<br>c) Motion relative to CM | Stellar luminosities<br>Semi-major axis ($a$)<br>Mass sum ($M+m$) |
| Spectroscopic | a) Single velocity curve<br>b) Double velocity curve | Mass function f($M,m$)<br>Mass ratio ($M/m$) |
| Eclipsing | a) Shape of light curve eclipses<br>b) Relative times between eclipses<br>c) Light loss at eclipse minima | Orbital inclination ($i$)<br>Relative stellar radii ($R_{l,s}/a$)<br>Orbital eccentricity ($e$)<br>Surface temperature ratio ($T_l/T_s$) |
| Eclipsing/spectroscopic | a) Light and velocity curves<br>b) Spectroscopic parallax + apparent magnitude | Absolute dimensions ($a$, $r_s$, $r_{l,}$)<br>$e$ and $i$<br>Distance to binary<br>Stellar luminosities<br>Surface temperatures ($T_l$, $T_s$) |

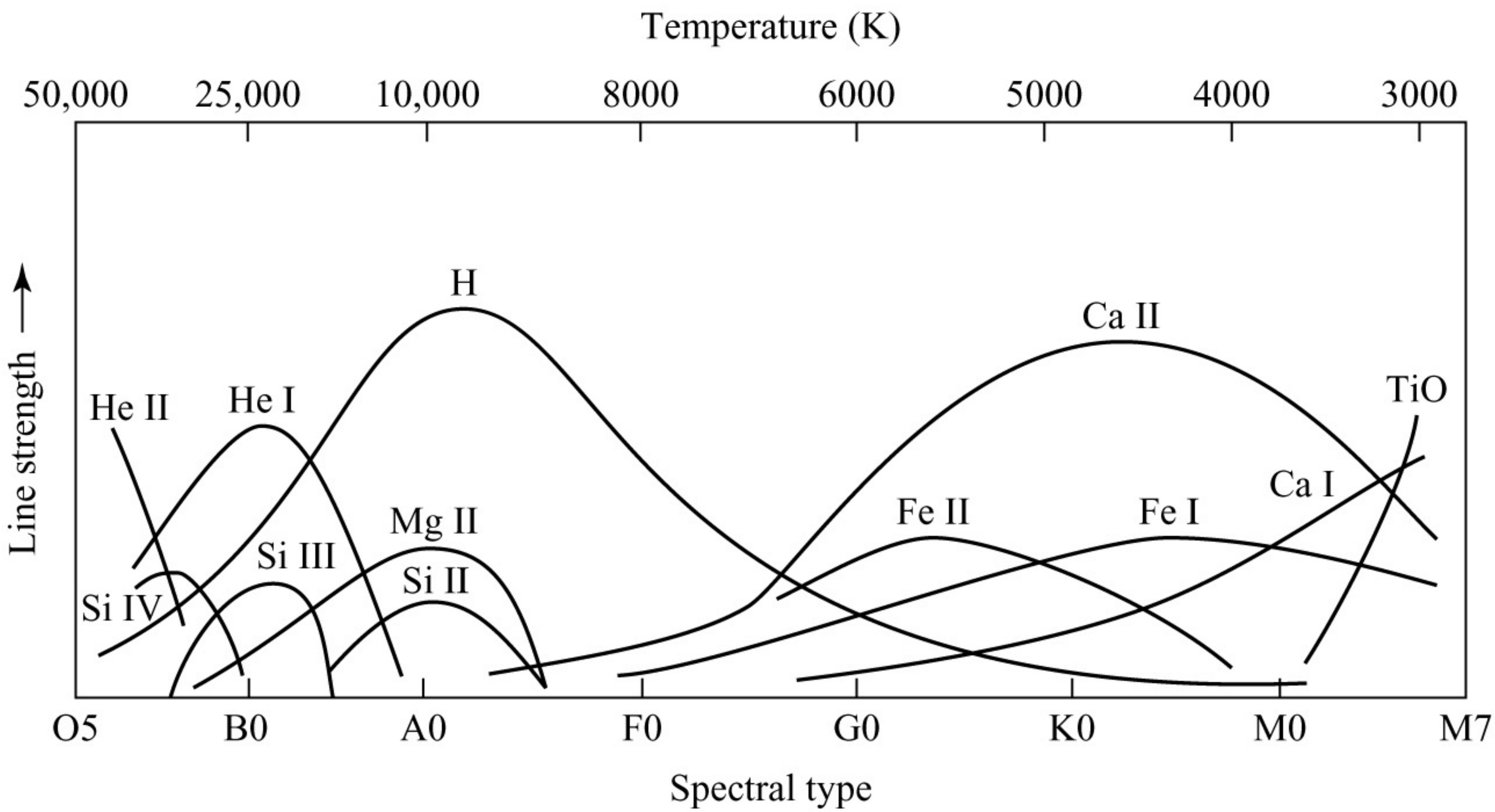
## UVBRI system

| Filter name | Effective wavelength (nm) | 0-magnitude flux (Jy) |
|---|---|---|
| U | 360 | 1880 |
| B | 440 | 4400 |
| V | 550 | 3880 |
| R | 700 | 3010 |
| I | 880 | 2430 |



# Stellar Spectra




Temperature (K)
50,000
25,000
10,000
8000
6000
5000
4000
3000
Line strength
H
Ca II
TiO
He II
He I
Ca I
Fe II
Fe I
Mg II
Si III
Si II
Si IV
O5
B0
A0
F0
G0
K0
M0
M7
Spectral type