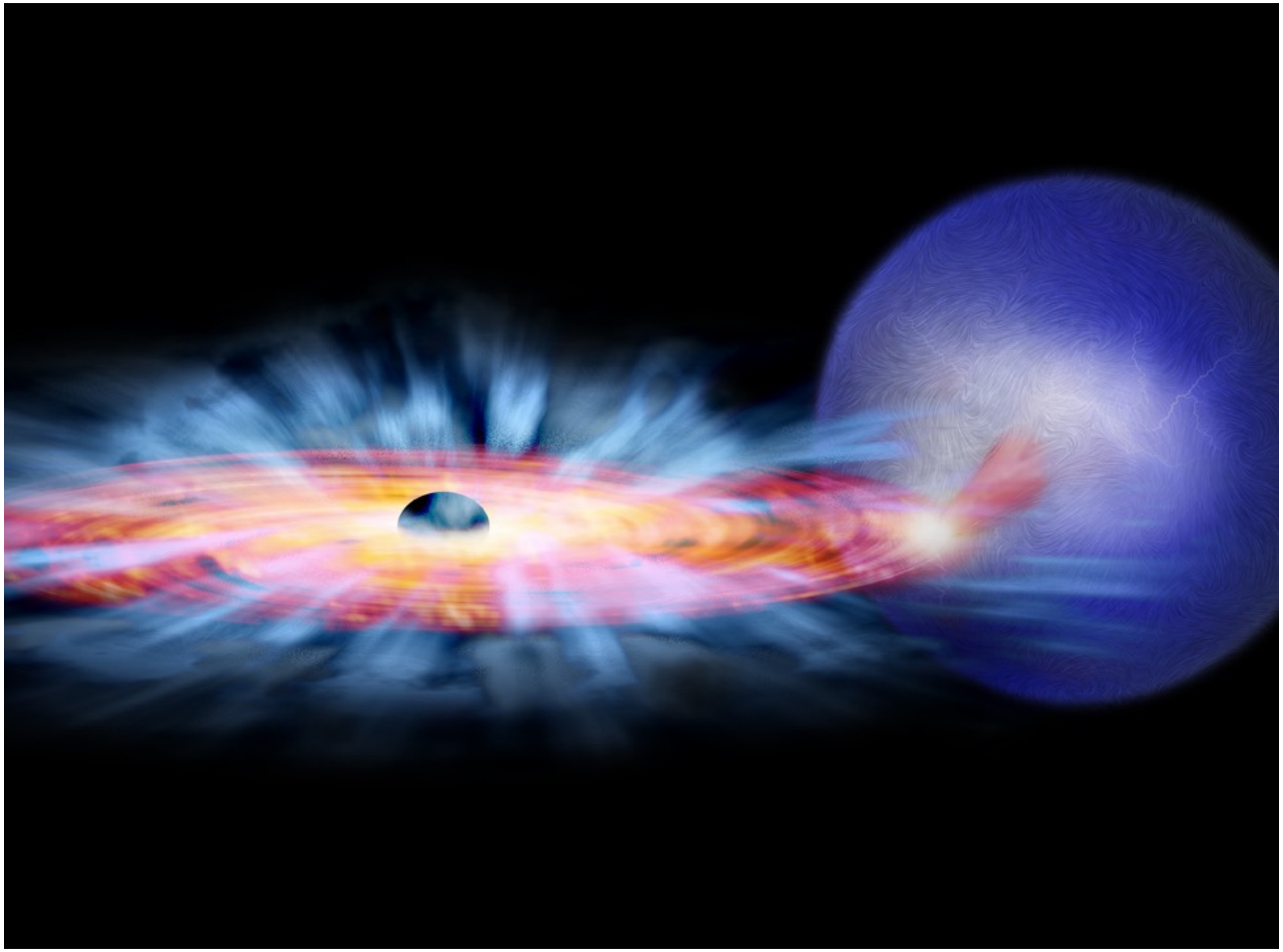
Astronomy 421



Black Holes

Outline

General Relativity refresher

Gravitational redshift

The Schwarzschild Metric

The Kerr Metric for rotating black holes

Black holes

Black hole candidates

The equivalence principle

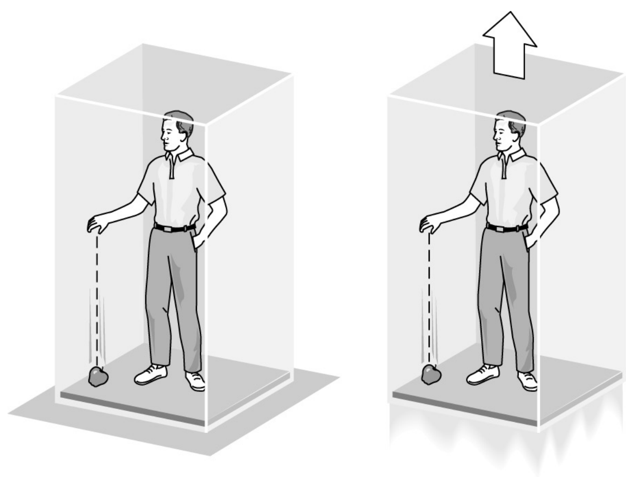
Special relativity: reference frames moving at constant velocity.

General relativity: accelerating reference frames and equivalence gravity.

<u>Equivalence Principle of general relativity:</u>

The effects of gravity are equivalent to the effects of acceleration.

Lab on Earth



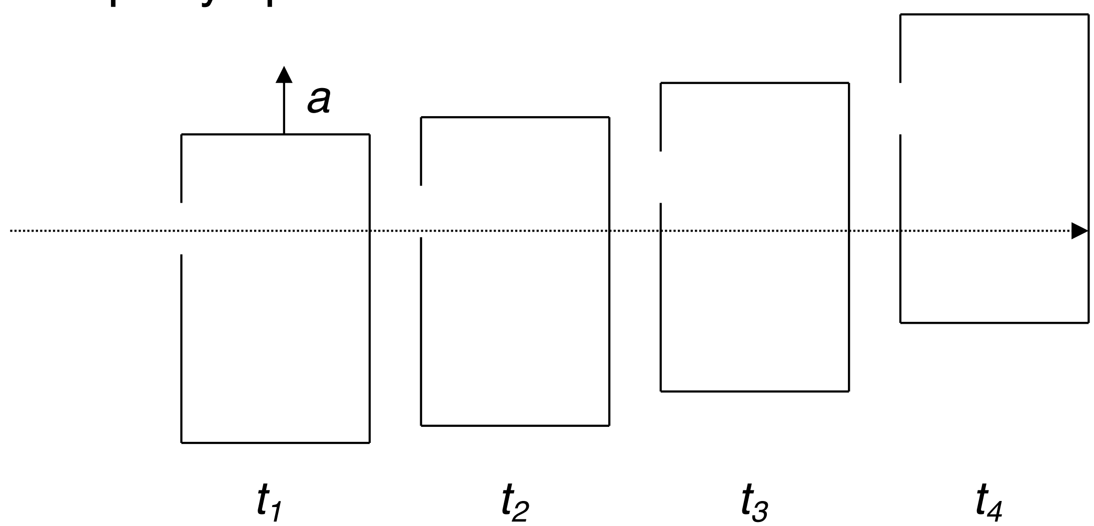
(a) (b)

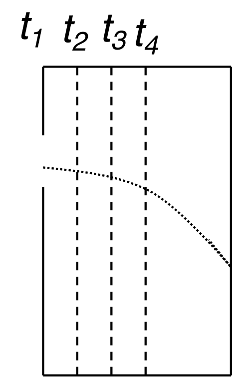
Lab accelerating in free space with upward acceleration g

In a local sense it is impossible to distinguish between the effects of a gravity with an acceleration *g,* and the effects of being far from any gravity in an upward-accelerated frame with *g*.

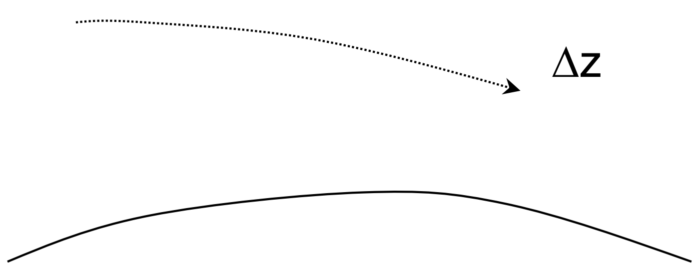
Consequence: *gravitational deflection of light*.

Light beam moving in a straight line through a compartment that is undergoing uniform acceleration in free space. Position of light beam shown at equally spaced time intervals.



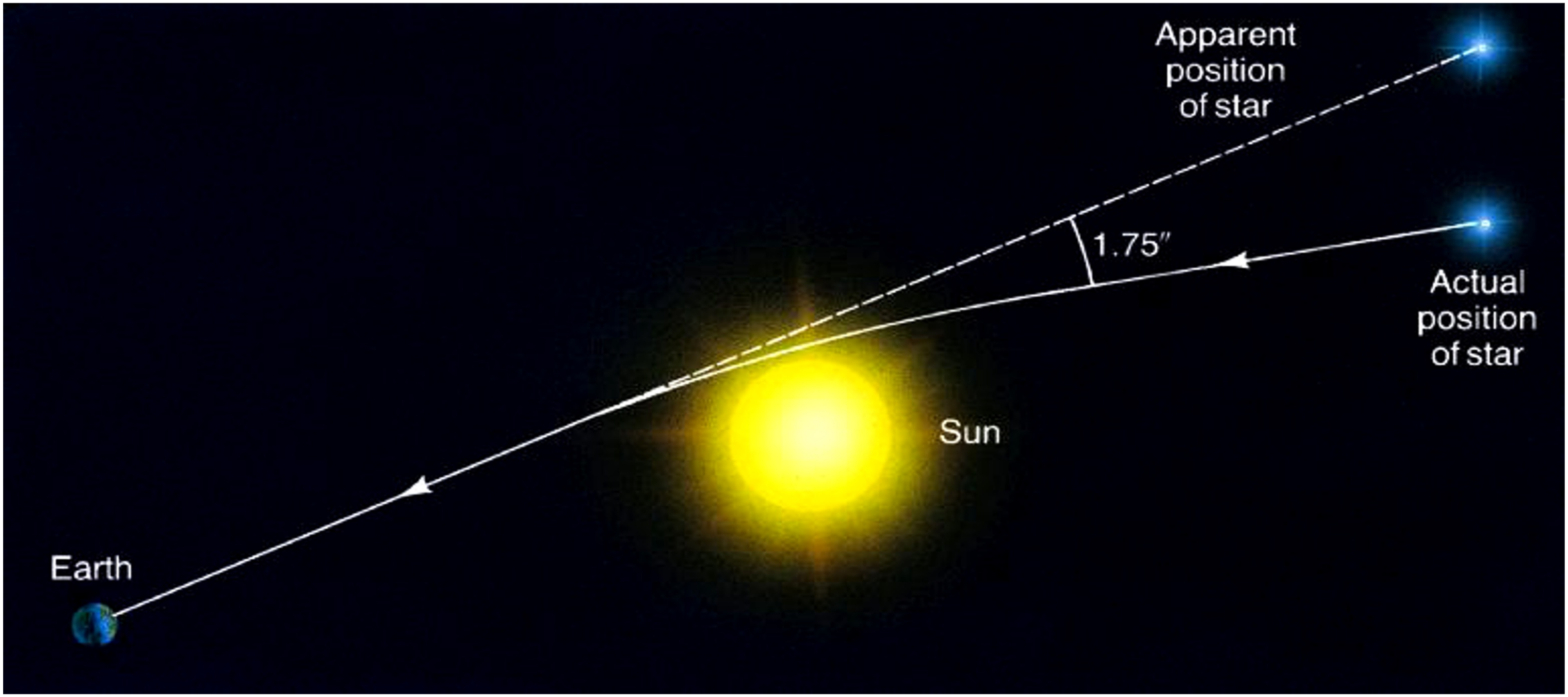


In the reference frame of the compartment, light travels in a parabolic path.

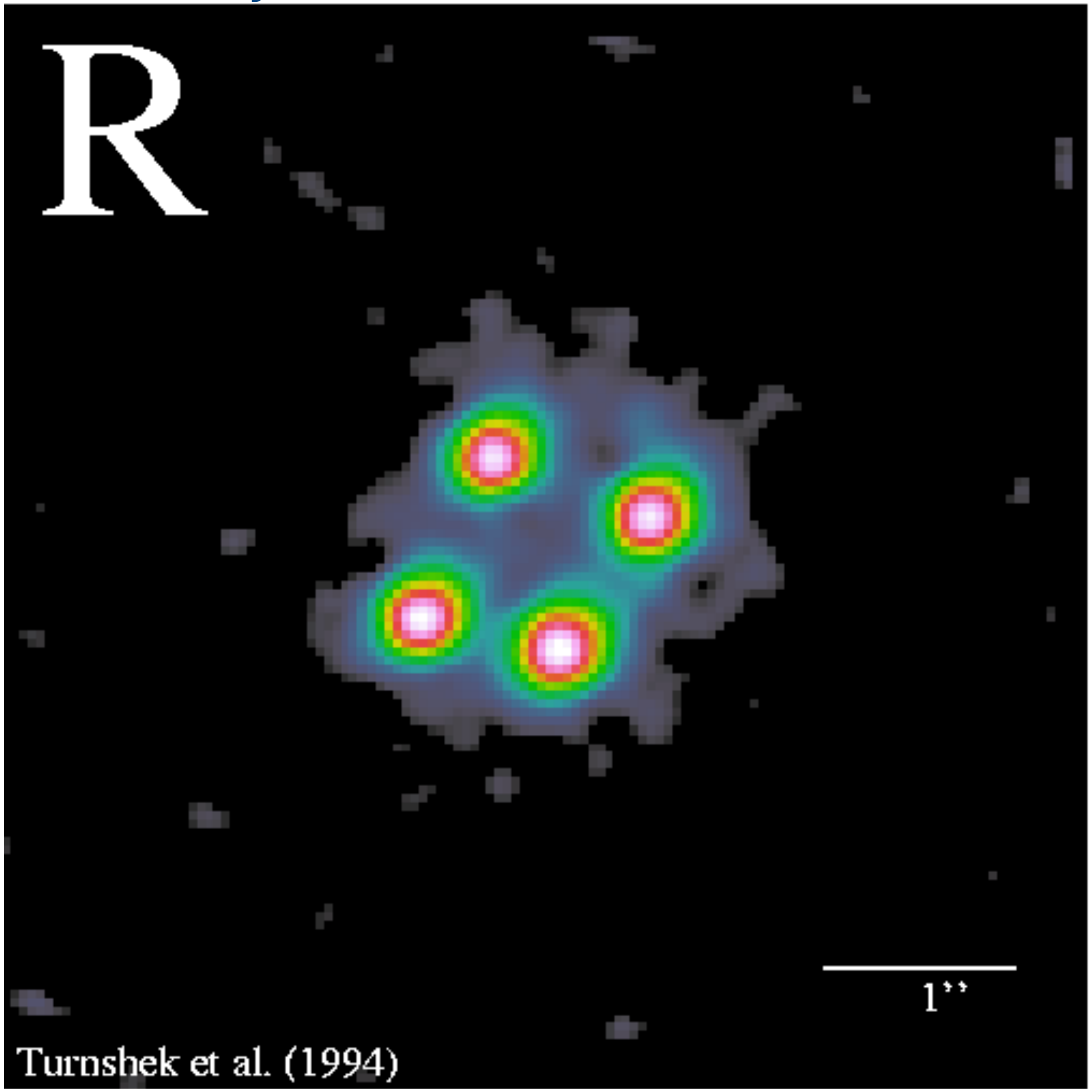


Same thing must happen in a gravitational field.

Observed! In 1919 eclipse by Eddington



Gravitational lensing of a single background quasar into 4 objects



1413+117 the
“cloverleaf” quasar
A ‘quad’ lens

Gravitational lensing. The gravity of a foreground cluster of galaxies distorts the images of background galaxies into arc shapes.

Saturn-mass
black hole

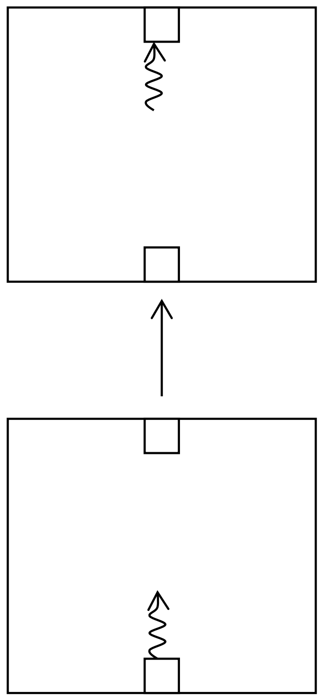
Another consequence: *gravitational redshift.*

Consider an accelerating elevator in free space. Acceleration = g.

$v=v_1$

$t=t_1$

Light received when elevator receding at v_1. Frequency is redshifted.



$v=0$

$t=0$

Light emitted when elevator at rest.

Frequency of emitted photon ν_0.

For outside observer: if photon travels a height h in time t_1, then $v_1=gt_1=g(h/c)$

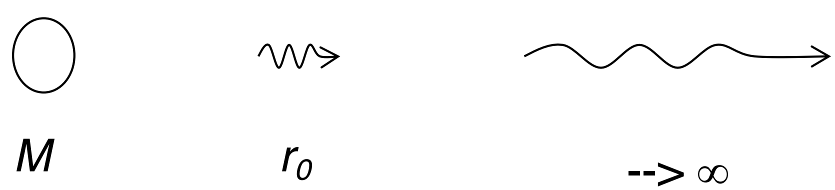
Thus gravity is producing a *redshift*

$$\frac{\Delta\nu}{\nu_0} = -\frac{v_1}{c} = -\frac{gh}{c^2} = \frac{GM\,h}{r^2\,c^2}$$

Since acceleration is equivalent to gravity, photons redshift as they move away from gravity source.

Exact GR expression for redshift at an infinite distance away:

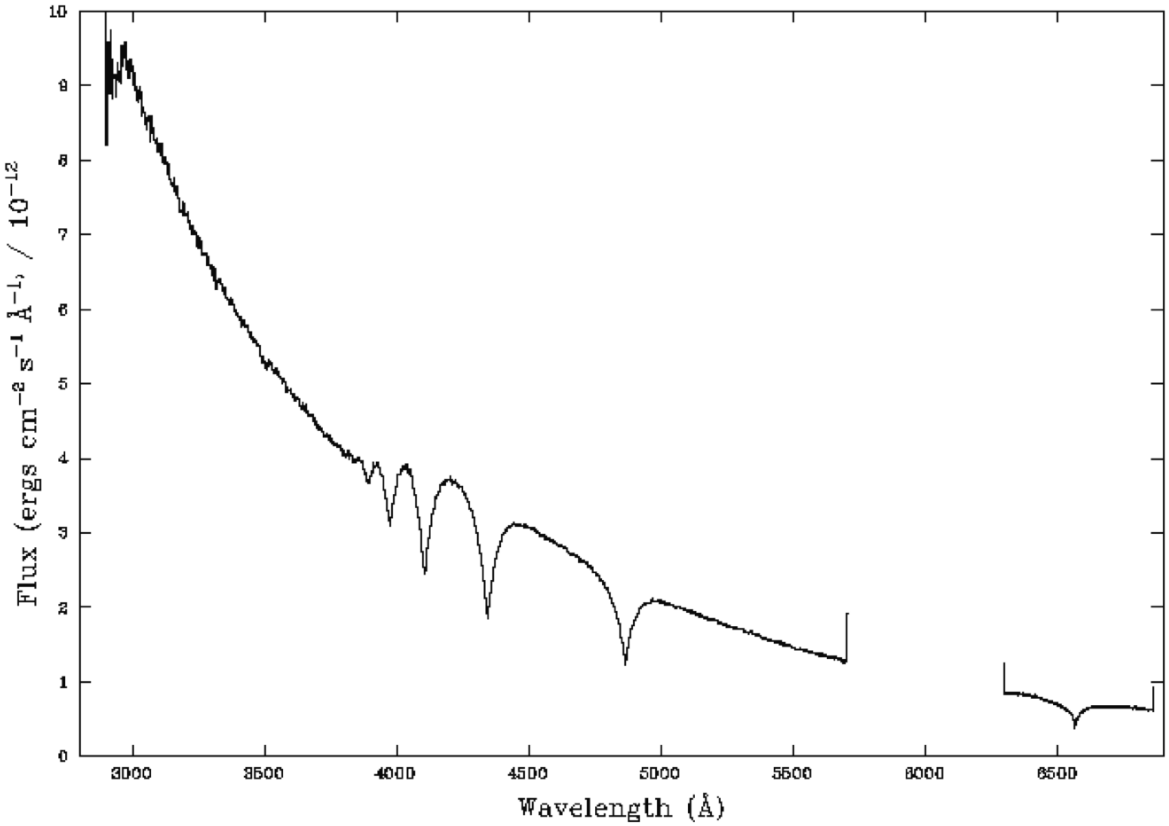
$$\frac{\nu_\infty}{\nu_0} = \left(1 - \frac{2GM}{r_0 c^2}\right)^{1/2}$$



For photon going to ∞ from r_0.

Gravitational Redshift from Sirius B

$$\frac{\nu_\infty}{\nu_0} = \left(1 - \frac{2GM}{r_0 c^2}\right)^{1/2}$$



HST spectroscopy yields a gravitational redshift of 80.65 ± 0.77 km s^{-1}. Joyce et al. (2018)

Worksheet: From this and the known radius of Sirius B of 5615 km, derive the mass of Sirius B and compare it to the dynamical mass of 1.018 +/- 0.011 Msun measured by Bond et al. (2017)

Another consequence: *gravitational time dilation.*

Period of photon serves as a clock, $\Delta t = 1/\nu$. Seen from ∞, clock at r_0 runs slower than clock at ∞.

$$\frac{\Delta t_0}{\Delta t_\infty} = \frac{\nu_\infty}{\nu_0} = \left(1 - \frac{2GM}{r_0 c^2}\right)^{1/2}$$

Gravitational time dilation

Question:

Suppose we start with two atomic clocks and take one up a high mountain for a week. Which is true?

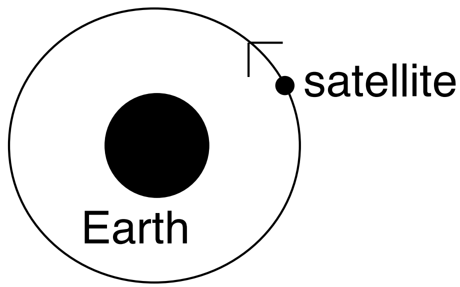
A: The two clocks will show the same amount of time has passed.

B: The mountain clock will be slightly ahead (fast)

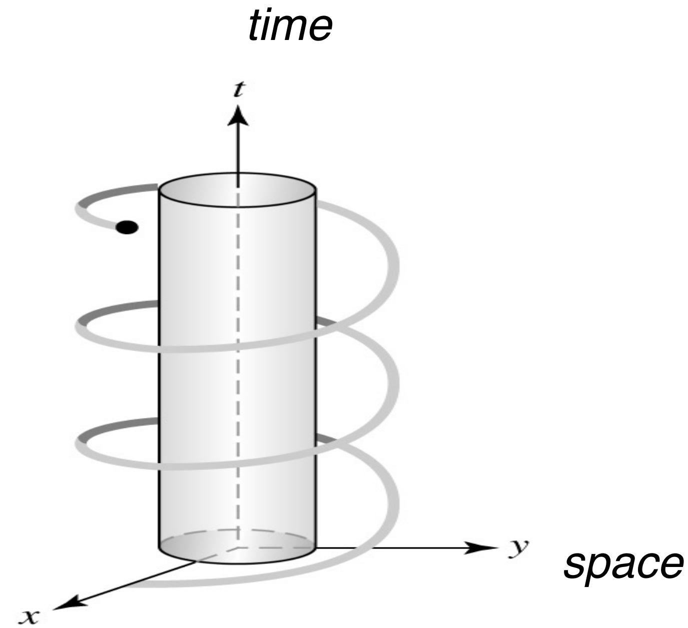
C: The mountain clock will be slightly behind (slow)

Reminder: Spacetime and light cones

Relate spatial position and time:



Spatial diagram: at a given time



(c)

Spacetime diagram of satellite in its orbit around the Earth, showing two spatial dimensions. These paths are called *worldlines.*

Measure of spacetime *distances* is the *interval.* For two events separated by Δx, Δy, Δz, Δt, interval Δs is

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

Recall that in special relativity, $(\Delta s)^2$ is invariant under Lorentz transformation.

Proper time between events: time interval in frame where $\Delta x = \Delta y = \Delta z = 0$. Thus the proper time is

$$\Delta \tau = \frac{\Delta s}{c}$$

Proper distance between events: distance measured in frame where $\Delta t = 0$.

$$\Delta \mathcal{L} = \sqrt{-(\Delta s)^2}$$

A *metric* is a differential distance formula. For 3D space:

$$(dl)^2 = (dx)^2 + (dy)^2 + (dz)^2$$

$$\Delta l = \int_{l_1}^{l_2} \sqrt{(dl)^2}$$

(integral along an arbitrary path from l_1 to l_2)

For "flat" spacetime (far from any source of gravity)

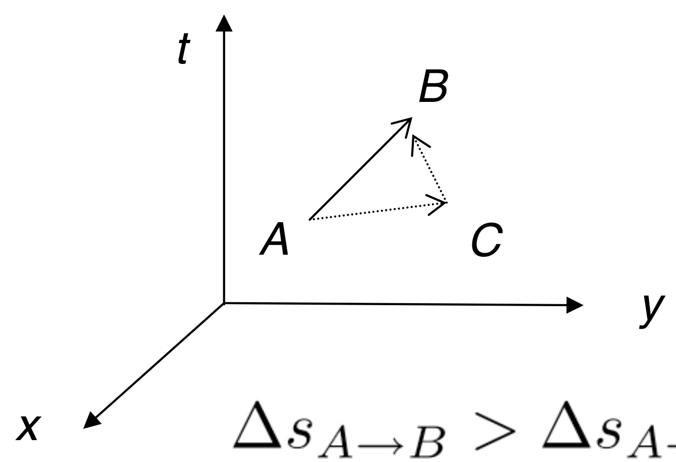
$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$

$$\Delta s = \int_{A}^{B} \sqrt{(ds)^2}$$

along a worldline connecting A and B.

In 3D space, if no force is acting, objects travel in a straight line (Newton's 1st law), corresponding to minimum Δl.

In flat spacetime, a straight, timelike worldline between *A* and *B* has *maximum* Δs of any worldline between *A* and *B.*



$$\Delta s_{A\to B} > \Delta s_{A\to C\to B}$$

Geodesic: straightest possible worldline, with maximal Δs.

Freely falling objects (or light) follow geodesics in spacetime. In flat spacetime, freely falling objects travel in a straight line in space (Newton I) , and spacetime. If gravity is present, Einstein showed that straightest possible worldline is *curved.* What is metric for curved spacetime?

Flat metric in spherical coordinates:

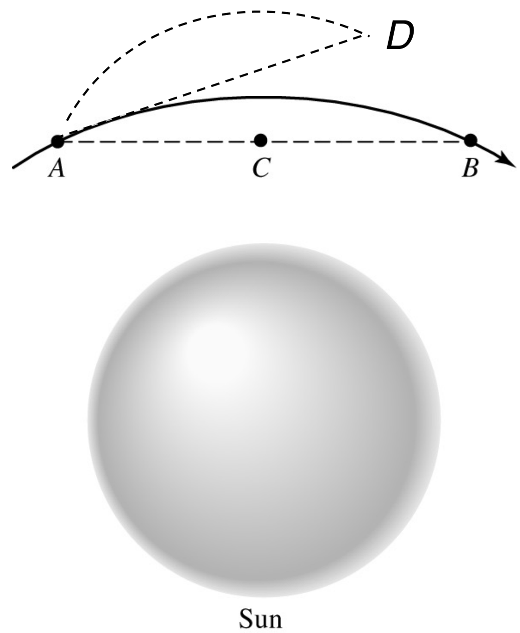
$$(ds)^2 = (cdt)^2 - (dr)^2 - (rd\theta)^2 - (r \sin\theta d\phi)^2$$

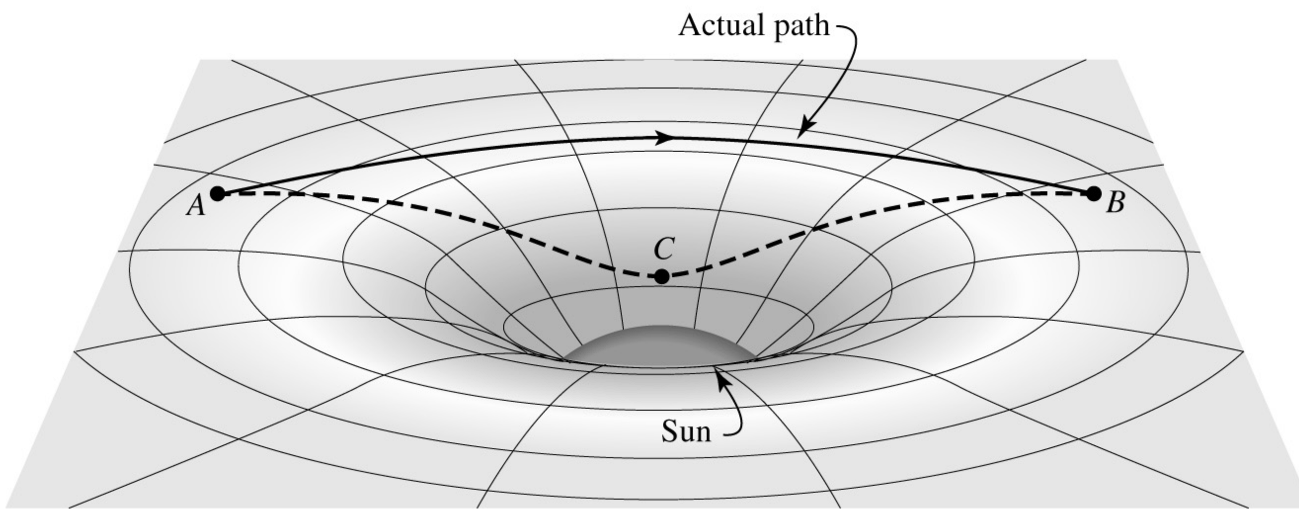
Gravitation is no longer a force in GR - manifested as a curvature of spacetime.

The curvature of spacetime is determined by the distribution of mass, momentum and energy in space time, as described by Einstein's field equation.

In a vacuum outside a spherically symmetric mass distribution, the line element is described by the *Schwarzschild metric.*

Curved light paths can now be seen as a consequence of this.





Can fit more rulers along A →C→B than along A→B. So photon on path A →C→B finds it longer. Also time runs slower. So A →C→B is not shortest path through spacetime. Δs not maximized. So geodesic must be curved.

Schwarzschild Metric

The Schwarzschild metric is a metric at distance *r* from mass *M*. Not valid inside *M*, and defined for vacuum only.

$$(ds)^2 = (cdt\sqrt{1 - 2GM/rc^2})^2 - (\frac{dr}{\sqrt{1 - 2GM/rc^2}})^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2$$

This metric is needed to calculate any meaningful physical quantities.

Note that only *dt* and *dr* are modified from flat metric. Also note that *dt*, *dr*, $d\theta$ and $d\varphi$ are the incremental coordinates used by an observer at infinity where spacetime is flat. That is, at $r \to \infty$ (or M $\to$ 0), the Schwarzschild metric reverts to flat spacetime metric.

ds still invariant.

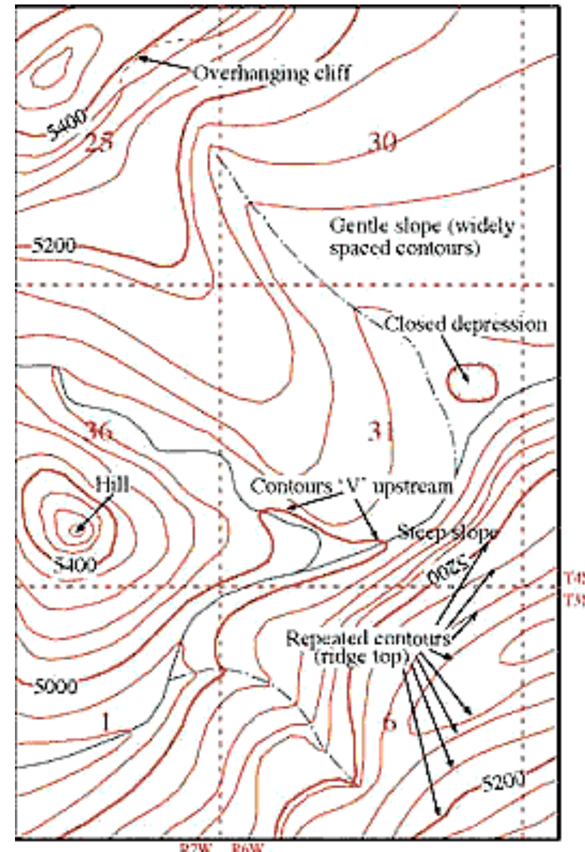
Curvature of space

Radial distance between two points measured simultaneously $(dt = d\theta = d\phi = 0)$ is the *proper distance.*

$$d\mathcal{L} = \sqrt{-(ds)^2} = \frac{dr}{\sqrt{1 - 2GM/rc^2}}$$

$$d\mathcal{L} > dr$$

$\Rightarrow$ a proper distance $d\mathcal{L}$ measured at r is greater than the coordinate distance measured at ∞ .



Analogy with a topographical map. You can fit more of a flat coordinate unit distance *dr* on a hillside. The actual distance is greater than the map coordinate distance. Distance projected onto flat space is shorter than it is on "curved" hillside => distances look compressed. Now add a dimension to this!

Courtesy German Wine Institute

Black holes

Consider a star of mass *M* , collapsed to radius less than

$$R_s = \frac{2GM}{c^2} \text{ Schwarzschild radius}$$

$$\text{Recall } d\tau = \frac{ds}{c} = dt\sqrt{1 - \frac{2GM}{rc^2}}$$

$$\text{at } r = R_s \; dt = \frac{d\tau}{\sqrt{1 - Rs/Rs}} \to \infty$$

That is, a clock measuring proper time approaching R_S will appear to a distant observer to be going slower and slower. Would take infinite time to reach R_s. Also gravitational redshift: photons approach infinite wavelength.

Likewise for lengths, when $dt = d\theta = d\phi = 0$

$$dr = d\mathcal{L}\sqrt{1 - R_s/R_s} = 0$$

(like a vertical cliff in topographic analogy).

So approaching R_S, clocks approaching stop and rulers approach infinitely short lengths as seen by a distant observer. All objects approach motionlessness.

Also consider escape velocity

$$v = \sqrt{\frac{2GM}{r}}$$

If v $\geq$ c, nothing can escape from radius r. Set

$$c = \sqrt{\frac{2GM}{r}} \qquad \Rightarrow r = \frac{2GM}{c^2} = R_s$$

Nothing is able to get out from R_s. An object smaller than this will appear black – hence a black hole.

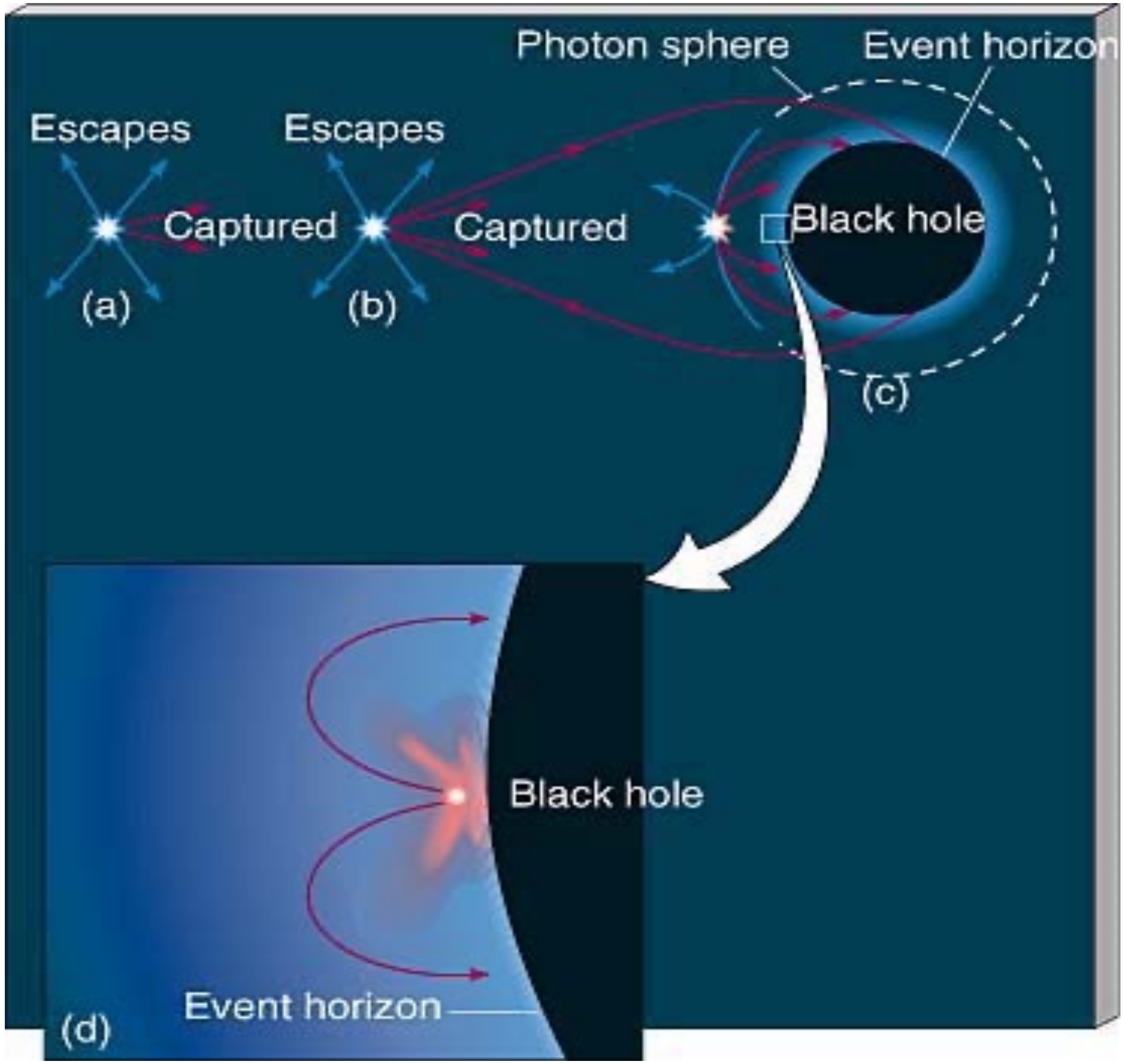
A black hole is any mass contained within its own Schwarzschild radius

$$R_s\ [\mathrm{km}] = 3M\ [\mathrm{M}_\odot]$$

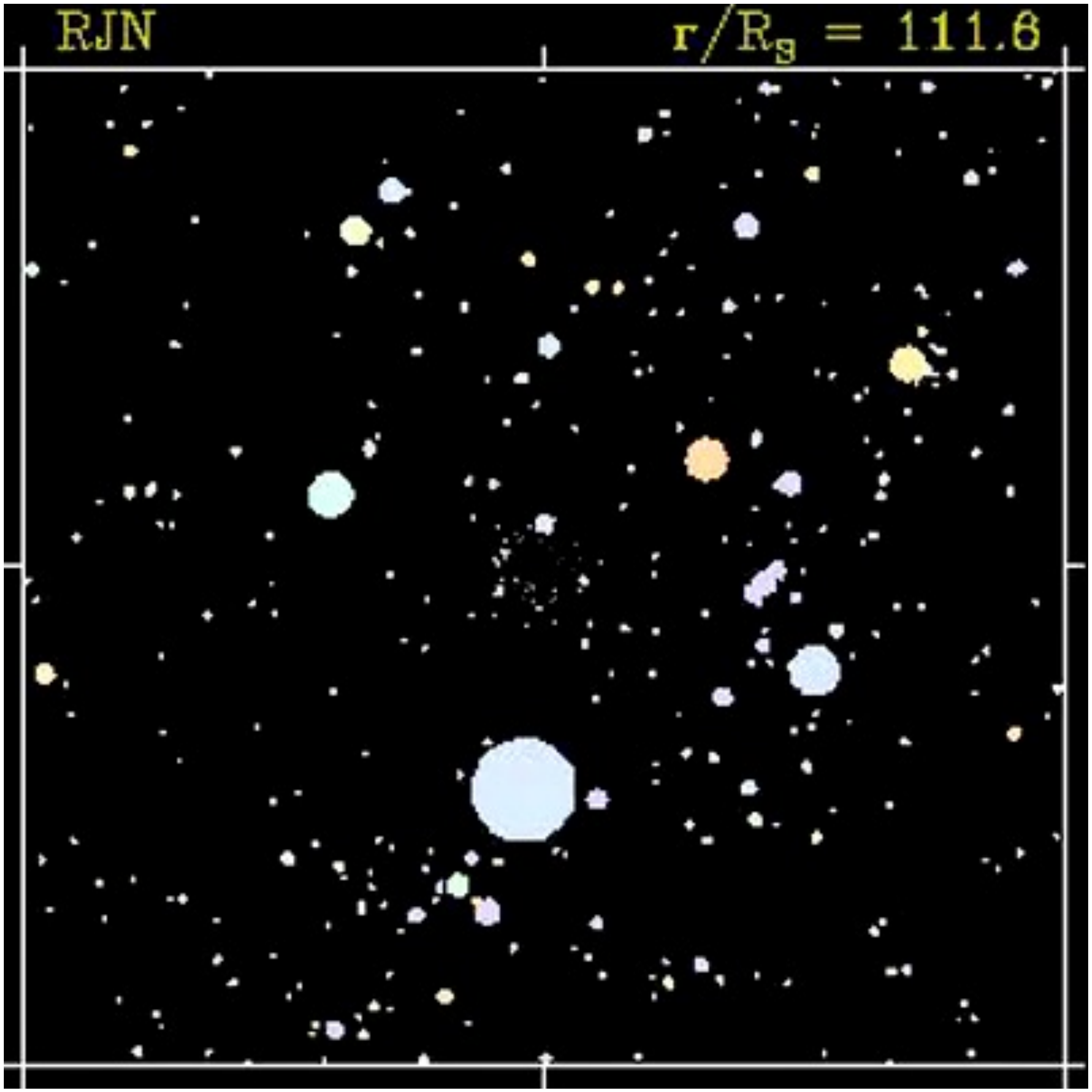
- Observer falling into a black hole also experiences huge tidal forces.
- The *Event Horizon* is a spherical, mathematical surface with $r=R_s$. Need not coincide with any physical surface. Information from the outside can propagate inwards through the horizon, but it cannot propagate outwards
- Inside the event horizon, at $r < R_s$, the dt^2 and dr^2 terms change sign – timelike intervals become spacelike and are not allowed.

=> While impossible to stay at one point in time at the outside, it is equally impossible to stay at one point in radius on the inside. Any object passing through the event horizon is doomed to continue inwards until a singularity at $r=0$.

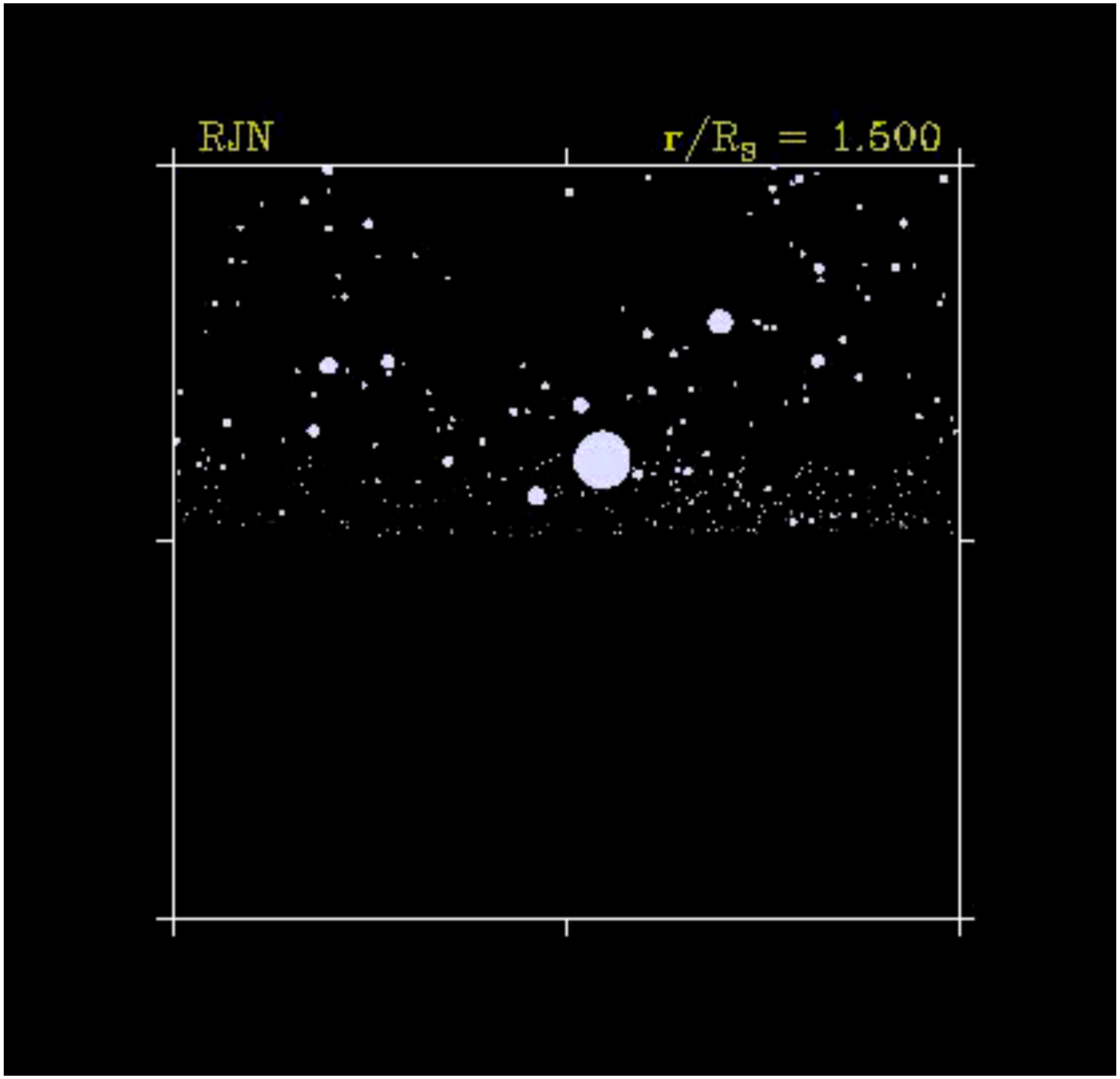
Curvature at event horizon is so great that space "folds in on itself", i.e. anything crossing it is trapped.

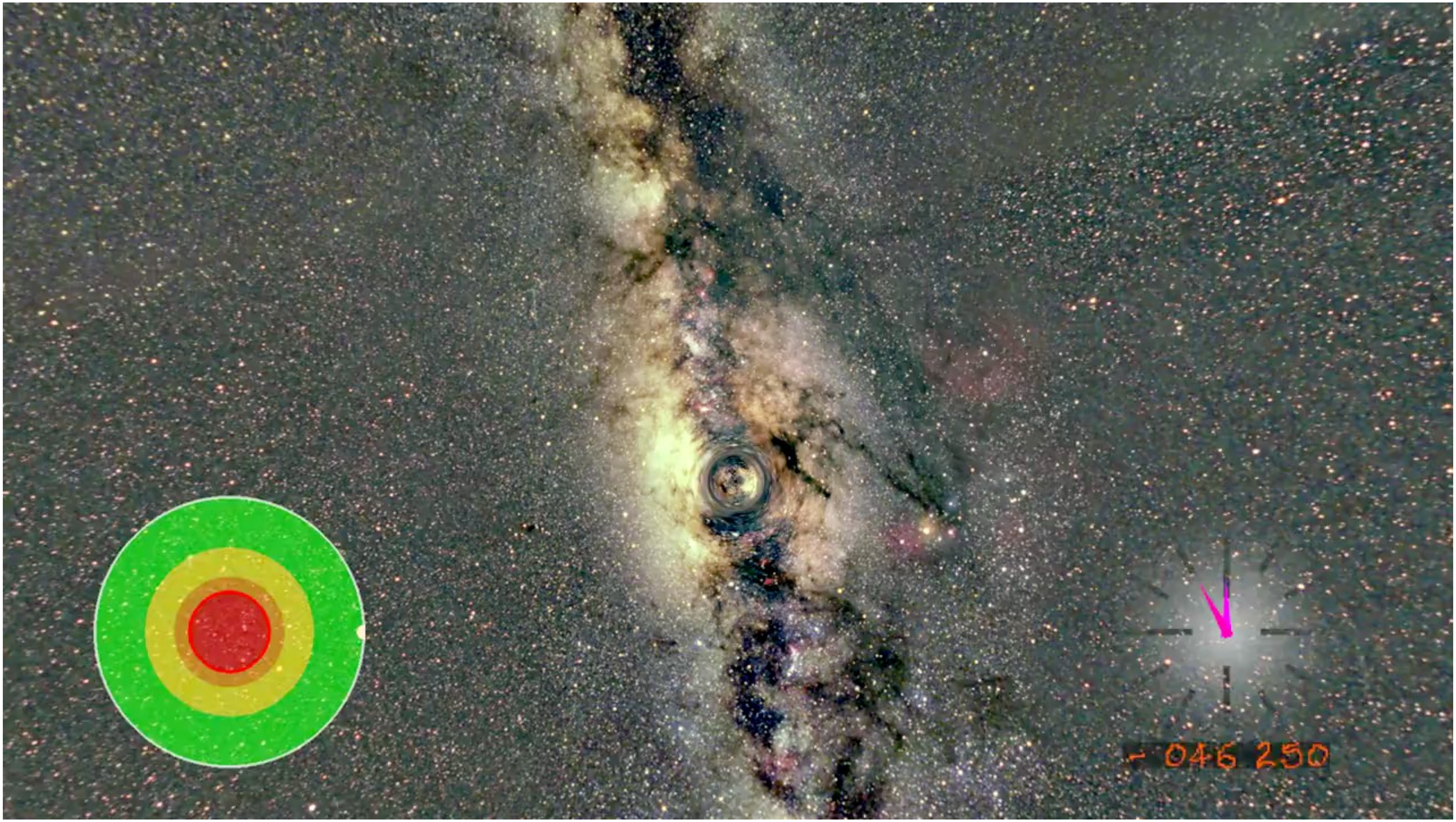


Approaching a Black Hole:



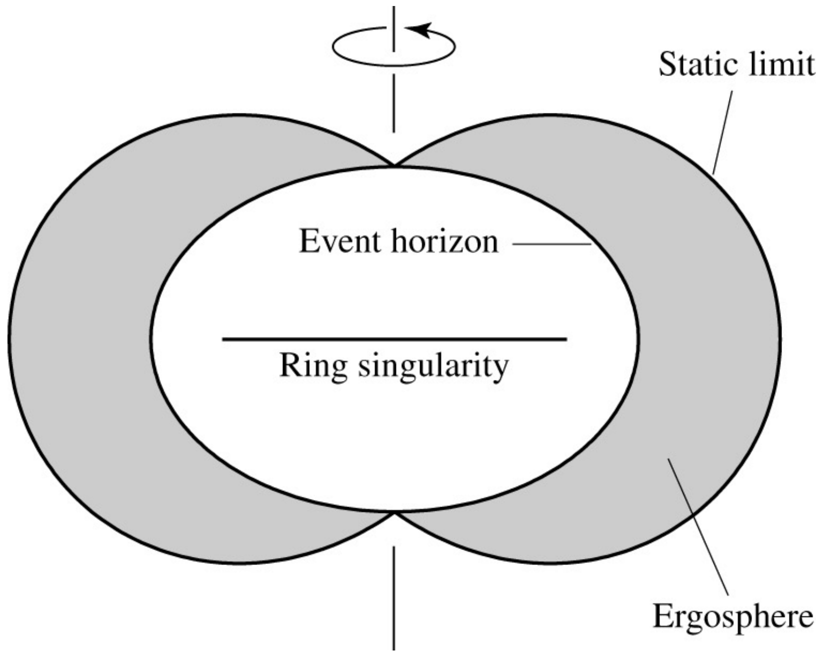
Circling a Black Hole at the Photon Sphere:





Simulation by Andrew Hamilton

A black hole with mass and angular momentum is a *Kerr black hole.* For a Kerr black hole the event horizon is closer to the center, but there is a static limit further out.



Inside the static limit it is impossible to remain at rest - everything forced to rotate with black hole – frame dragging

Maximal black hole: Spin energy = 0.29 Mc^2

Innermost Stable Circular Orbit (ISCO) is closer to event horizon for rotating BH -> can extract more energy from matter falling in

no Hair Theorem for Black Holes

Properties of a black hole:

- Mass
- Spin (angular momentum)
- Charge (tends to be zero

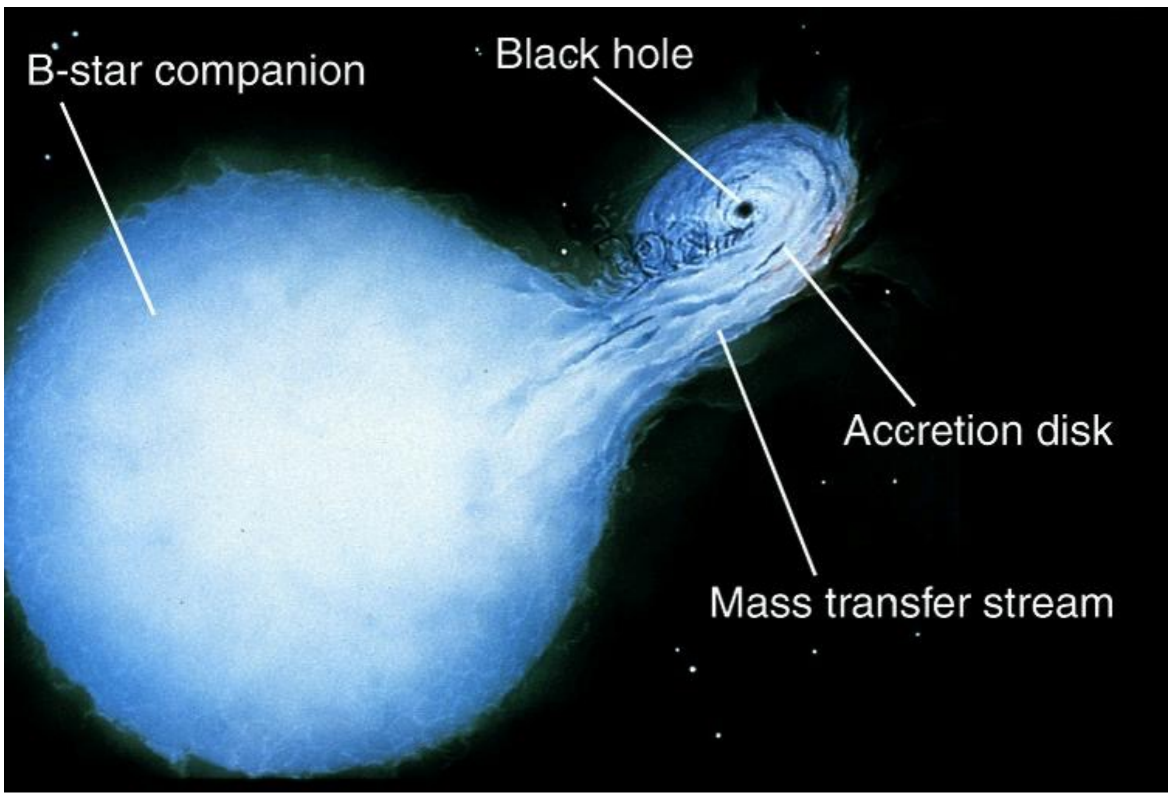
What about Entropy? Do black holes violate Thermodynamics?

Do Black Holes Really Exist? Good Candidate: Cygnus X-1

- Binary system: 30 M_{Sun} star with unseen companion.

- Binary orbit => companion > 7 M_{Sun}.

- X-rays => million degree gas falling into black hole.

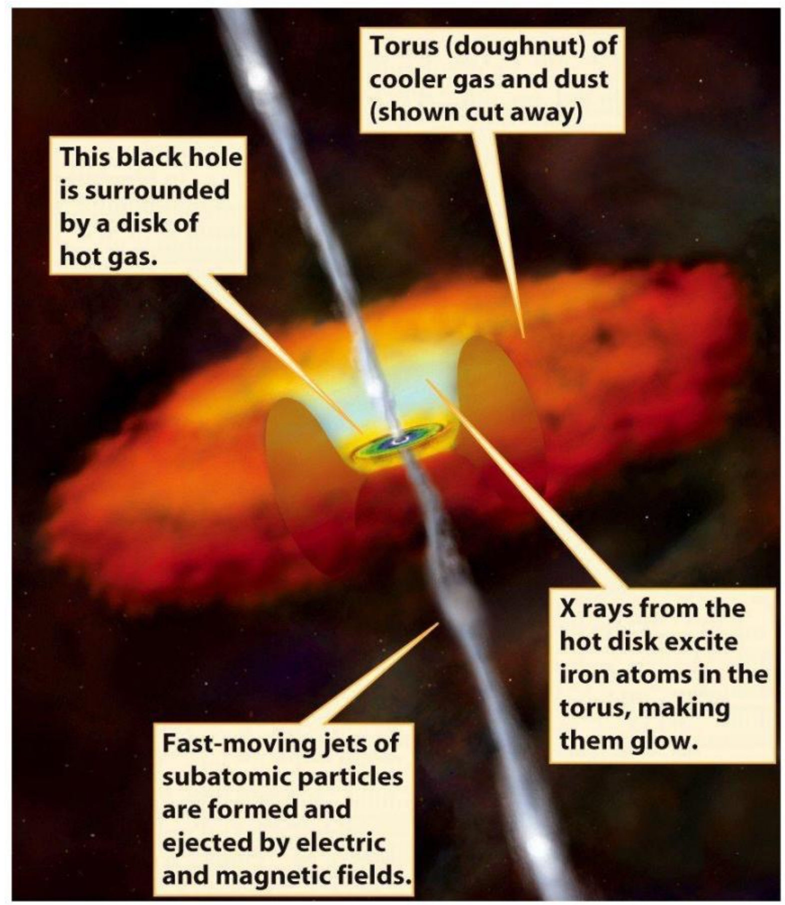


Supermassive black holes

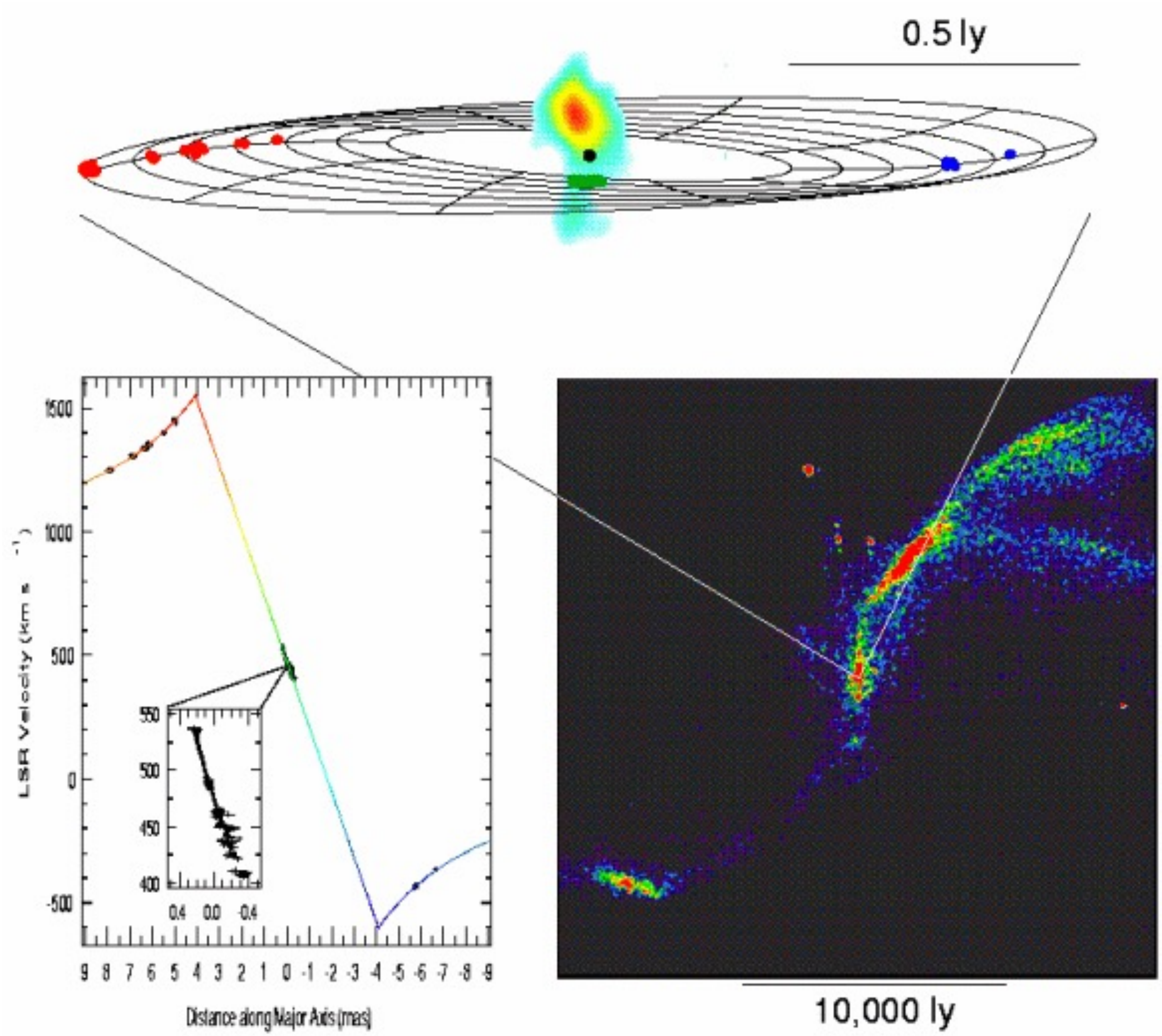
We find supermassive black holes at the centers of essentially all large galaxies.

Evidence comes from high orbital speeds of nearby gas or stars. Masses range from $10^6\ M_{\odot}$ to $10^9\ M_{\odot}$.

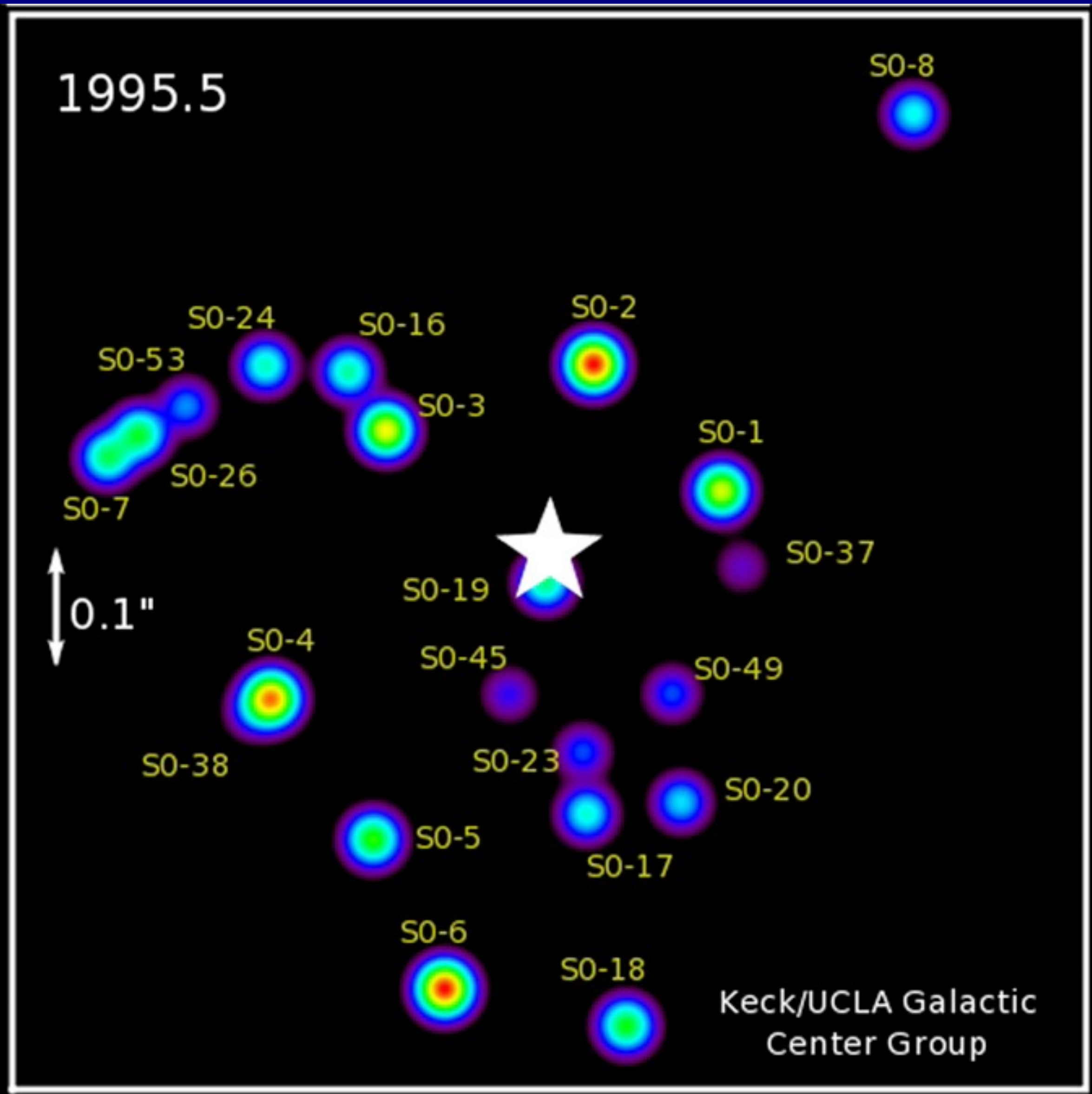
These objects show themselves by their accretion disks or jets.



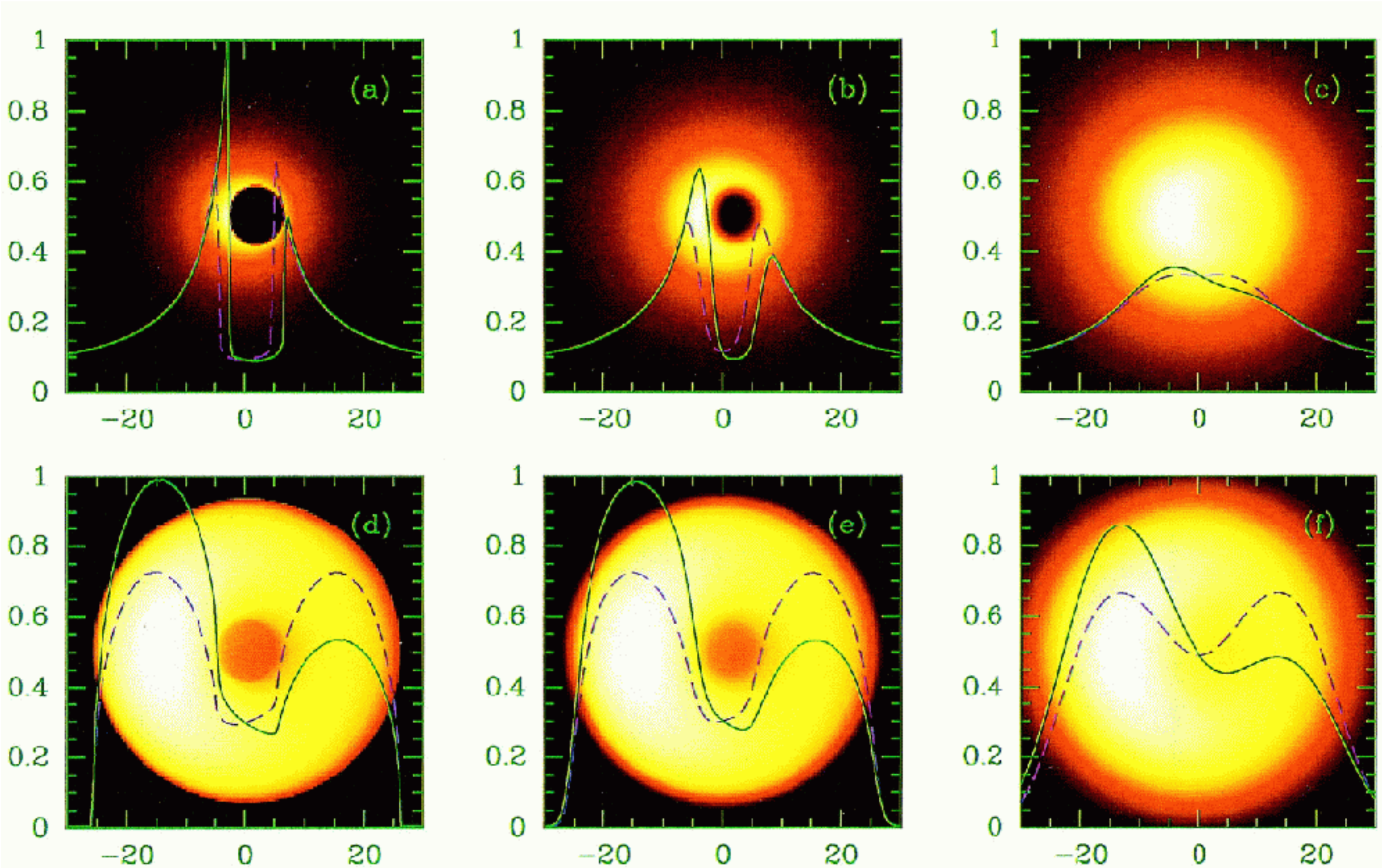
Best evidence: NGC4258

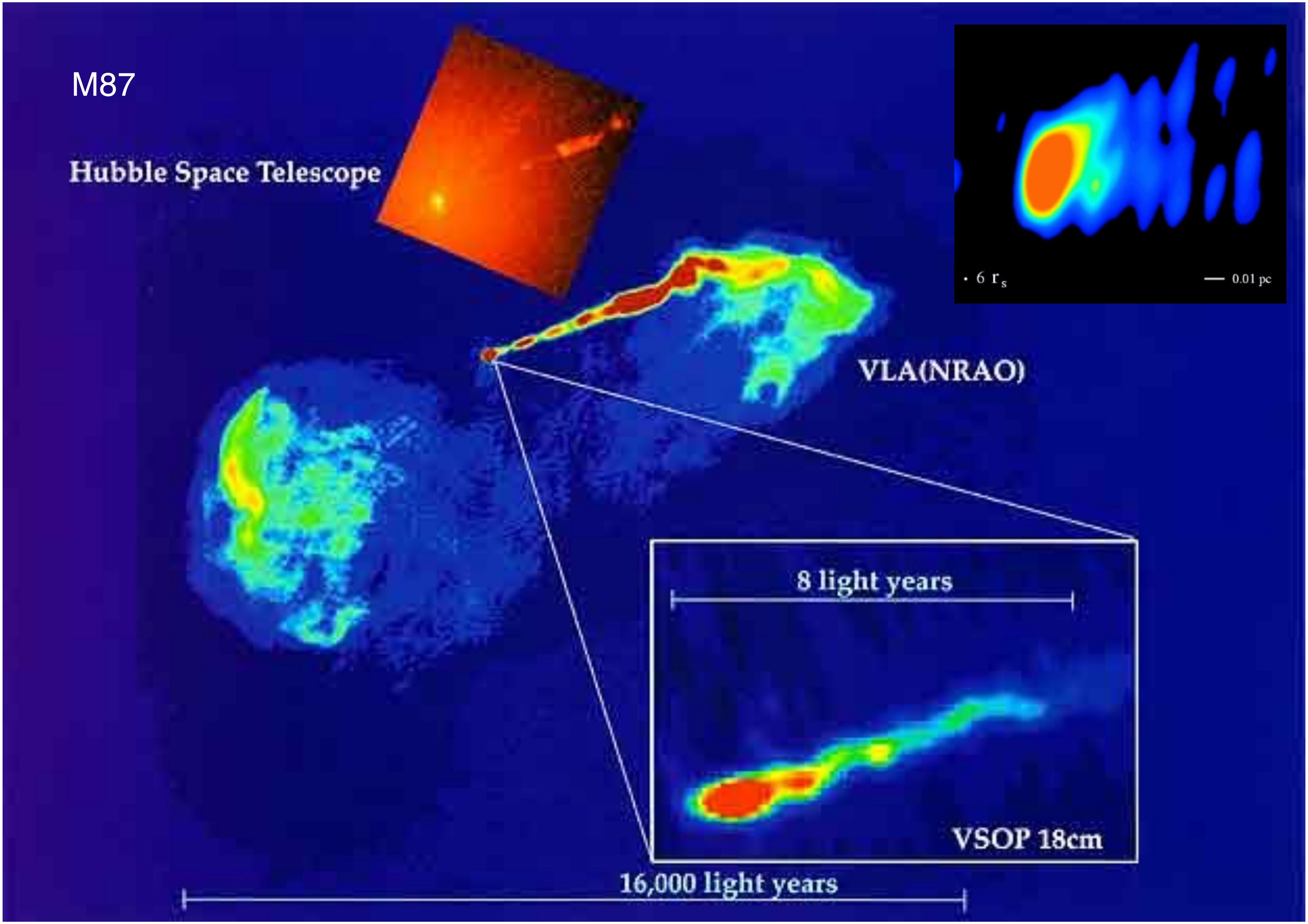


Supermassive (3 million solar mass) Black Hole at the Galactic Center

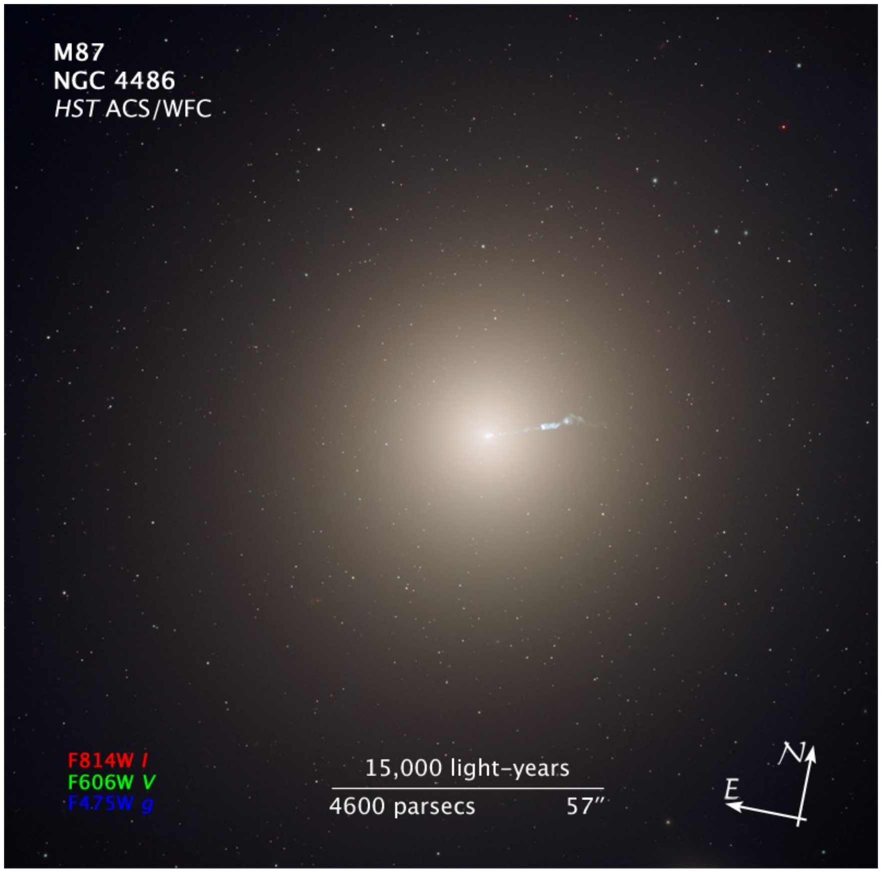


Shadow of a Black Hole

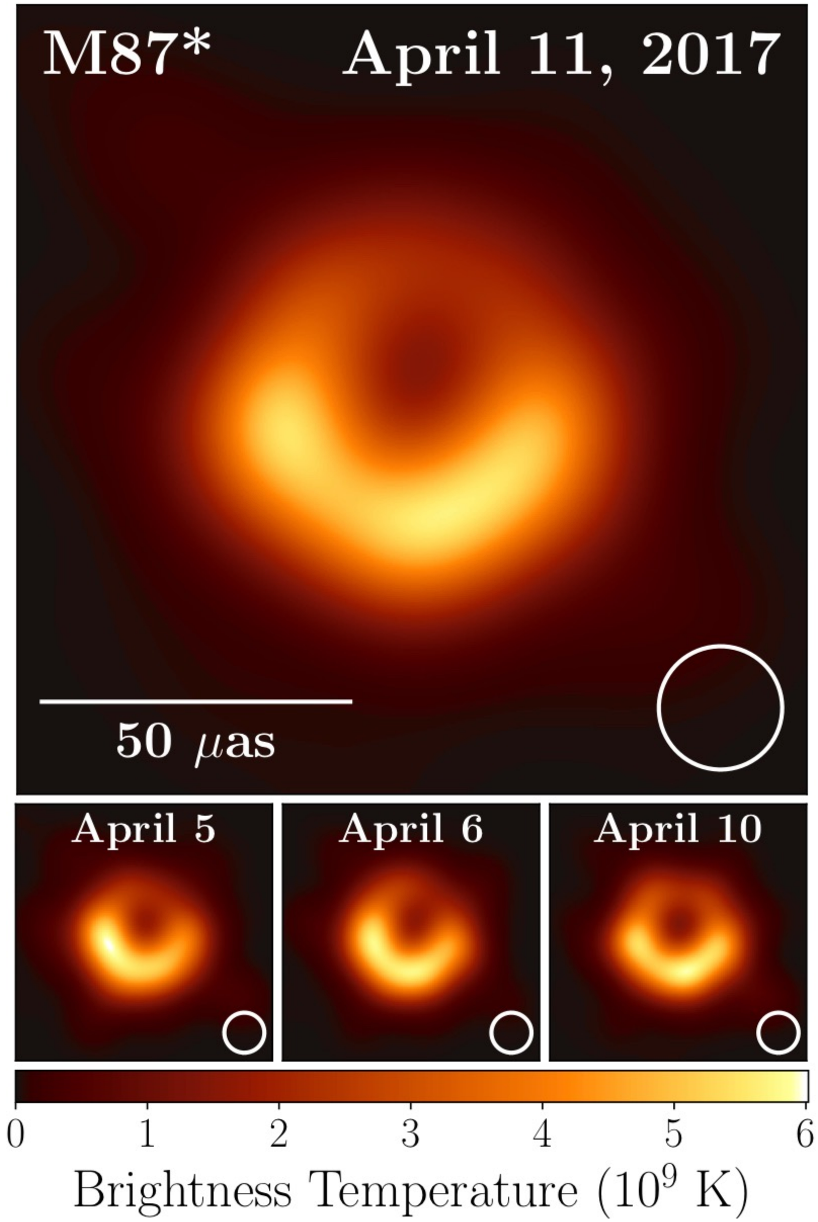



M87
Hubble Space Telescope
VLA(NRAO)
6 r_s
0.01 pc
8 light years
VSOP 18cm
16,000 light years

Shadow of a Black Hole



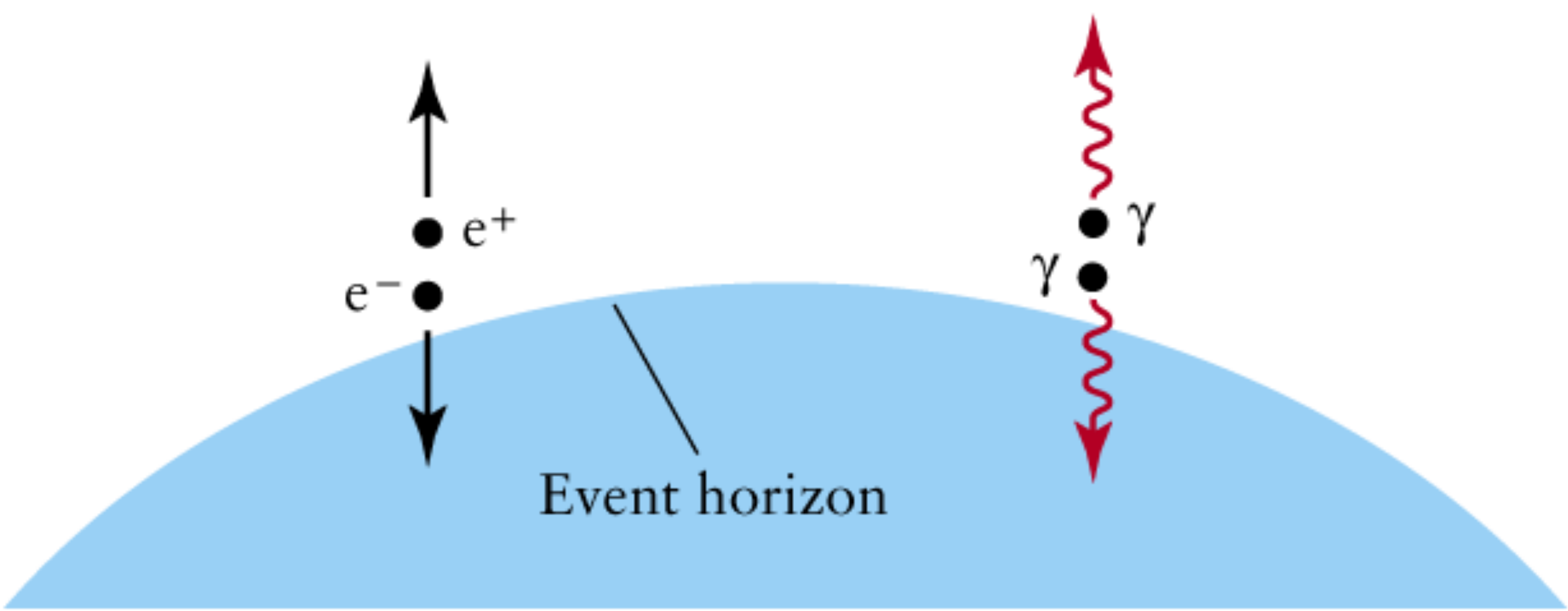
Optical image of the host galaxy



Radio image from Event Horizon Telescope

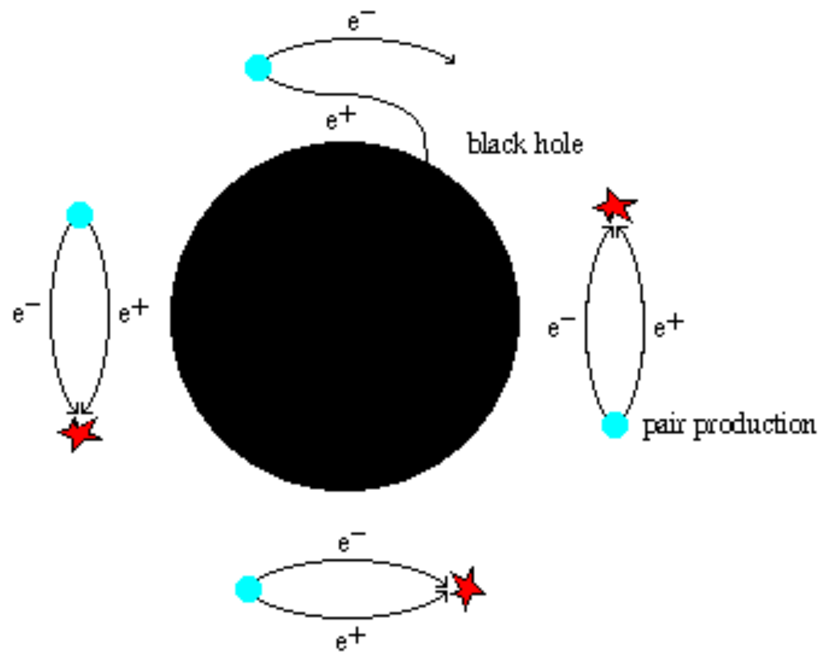
Do black holes exist forever?

- Maybe not: there may be something called *Hawking radiation.*
- Virtual pairs of particles pop in and out of existence. If they are near the event horizon, one might get caught, the other escape, cause the black hole to evaporate. Relevant only for mini-black holes.



Hawking Radiation

the strong gravitational field around a black hole causes pair production



$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

$T \sim 10^{-7}\, M_{Sun}/M \;\; K$

if a pair is produced outside the event horizon, then one member will fall back into the black hole, but the other member will escape and the black hole loses mass

- BHs slowly lose mass and energy by Hawking Radiation

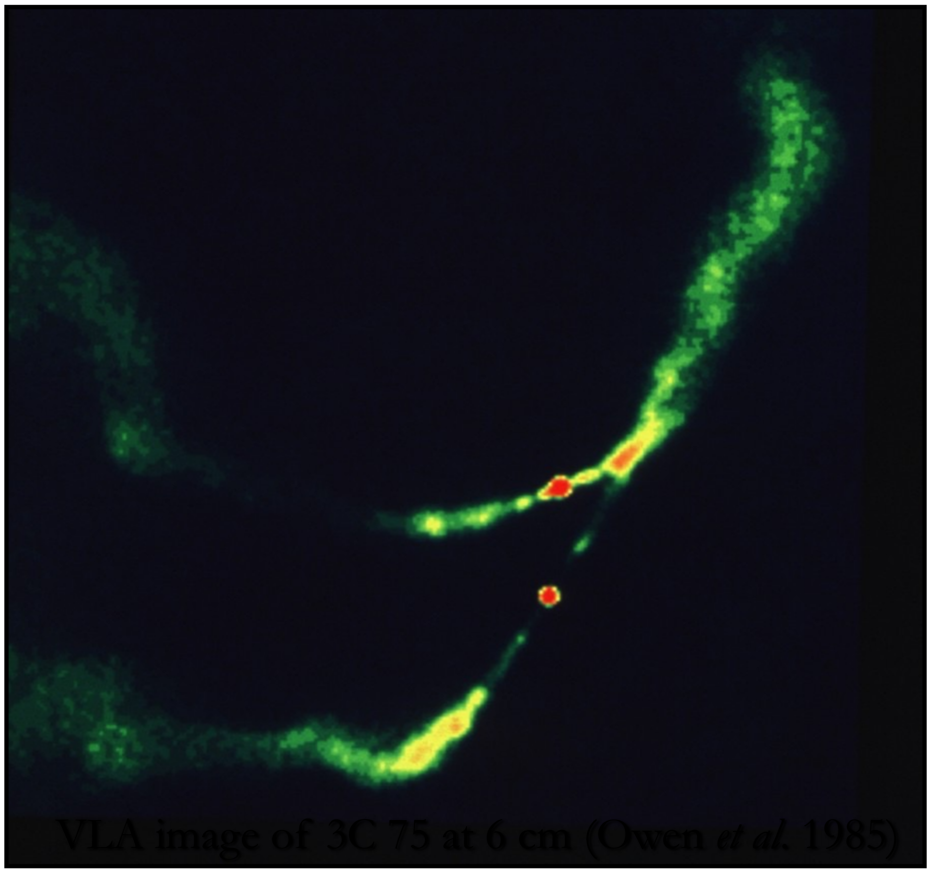
$$t_{evap} \approx (\frac{M}{M_\odot})^3 x\ 2x10^{67} yrs$$

BHs of mass < 10^{11} kg would have evaporated by now.

Supermassive Binary Black Holes

3C 75

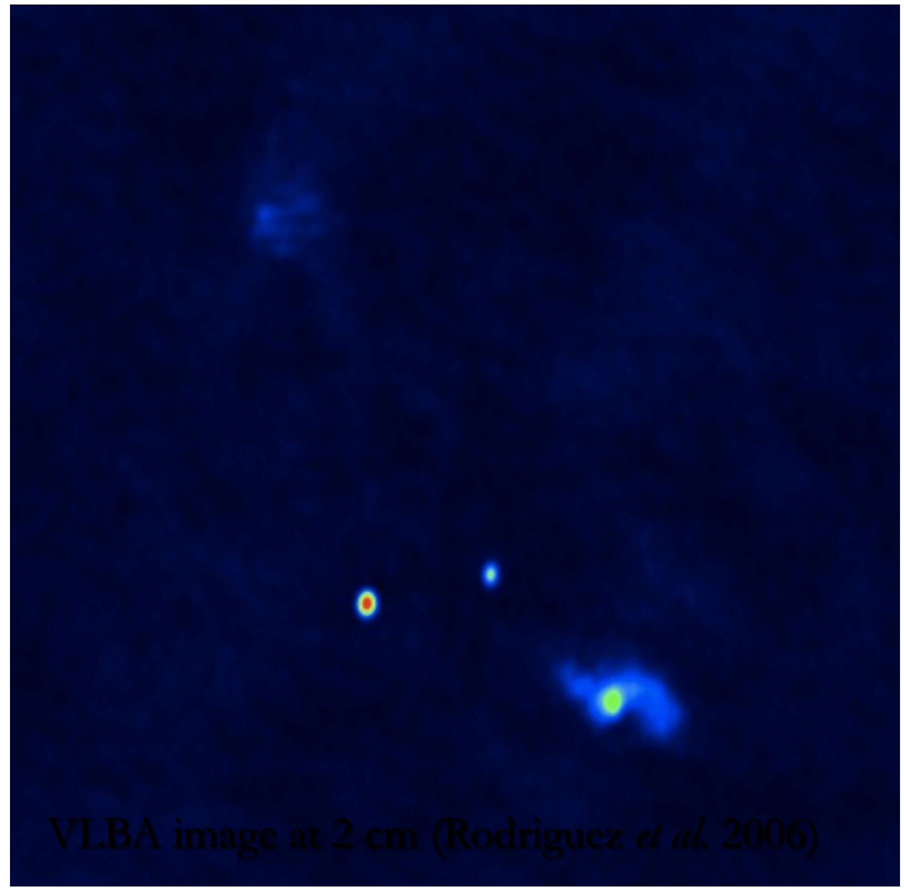
7 kpc separation



VLA image of 3C 75 at 6 cm (Owen *et al.* 1985)

0402+379

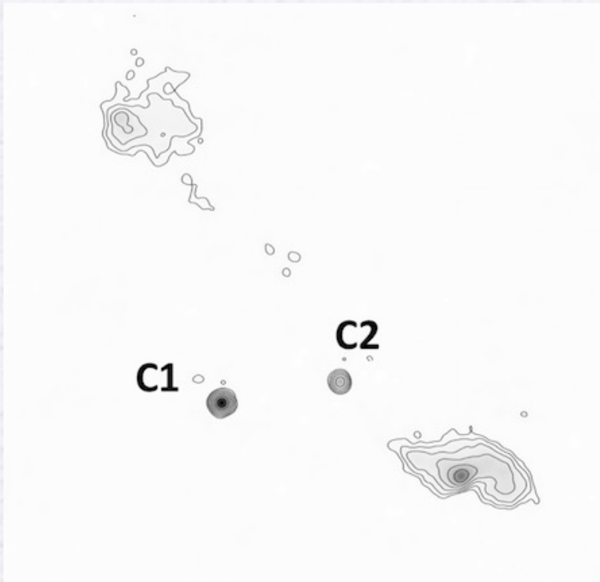
7 pc separation



VLBA image at 2 cm (Rodriguez *et al.* 2006)

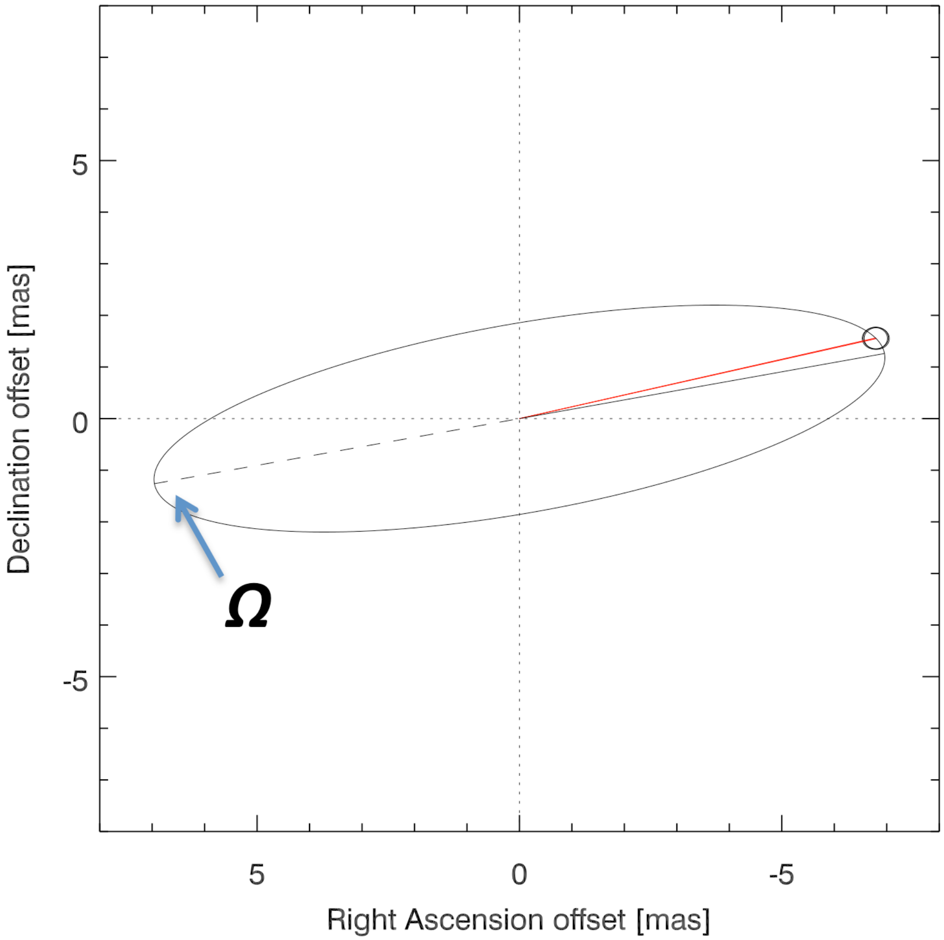
Constraining the Orbit in 0402+379

Supermassive binary black hole system 0402+379

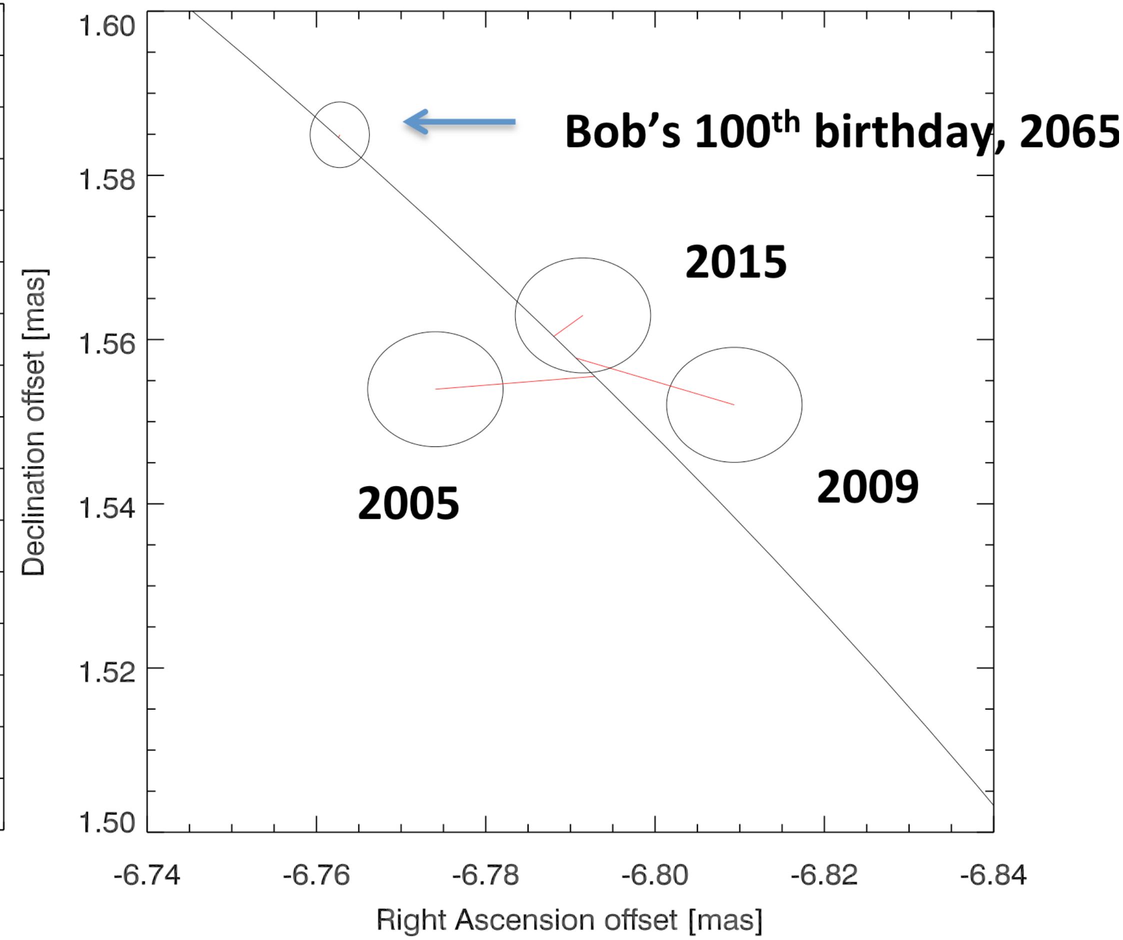


Bansal et al. 2017

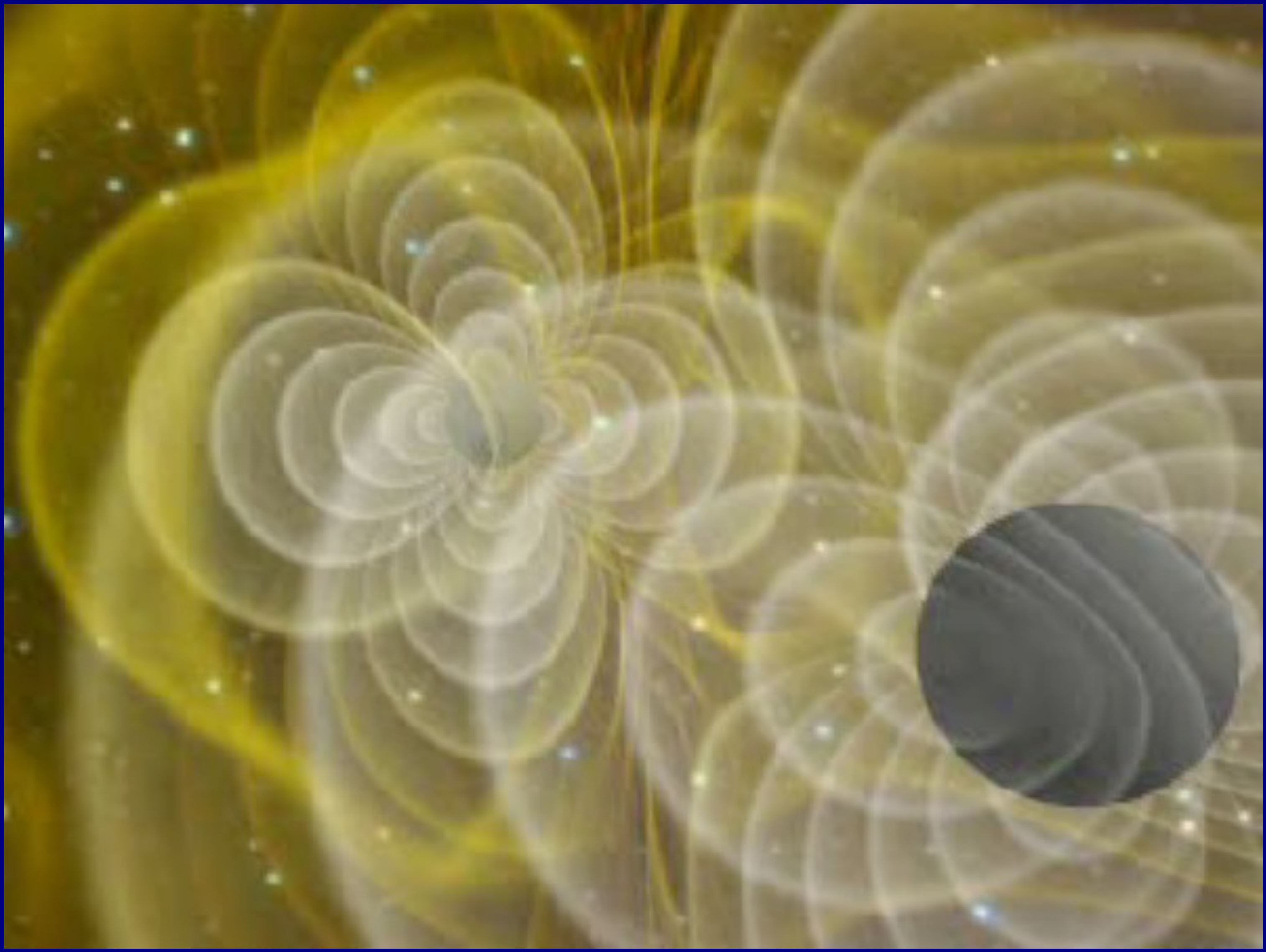
Orbital Fitting



Circular orbit at an inclination



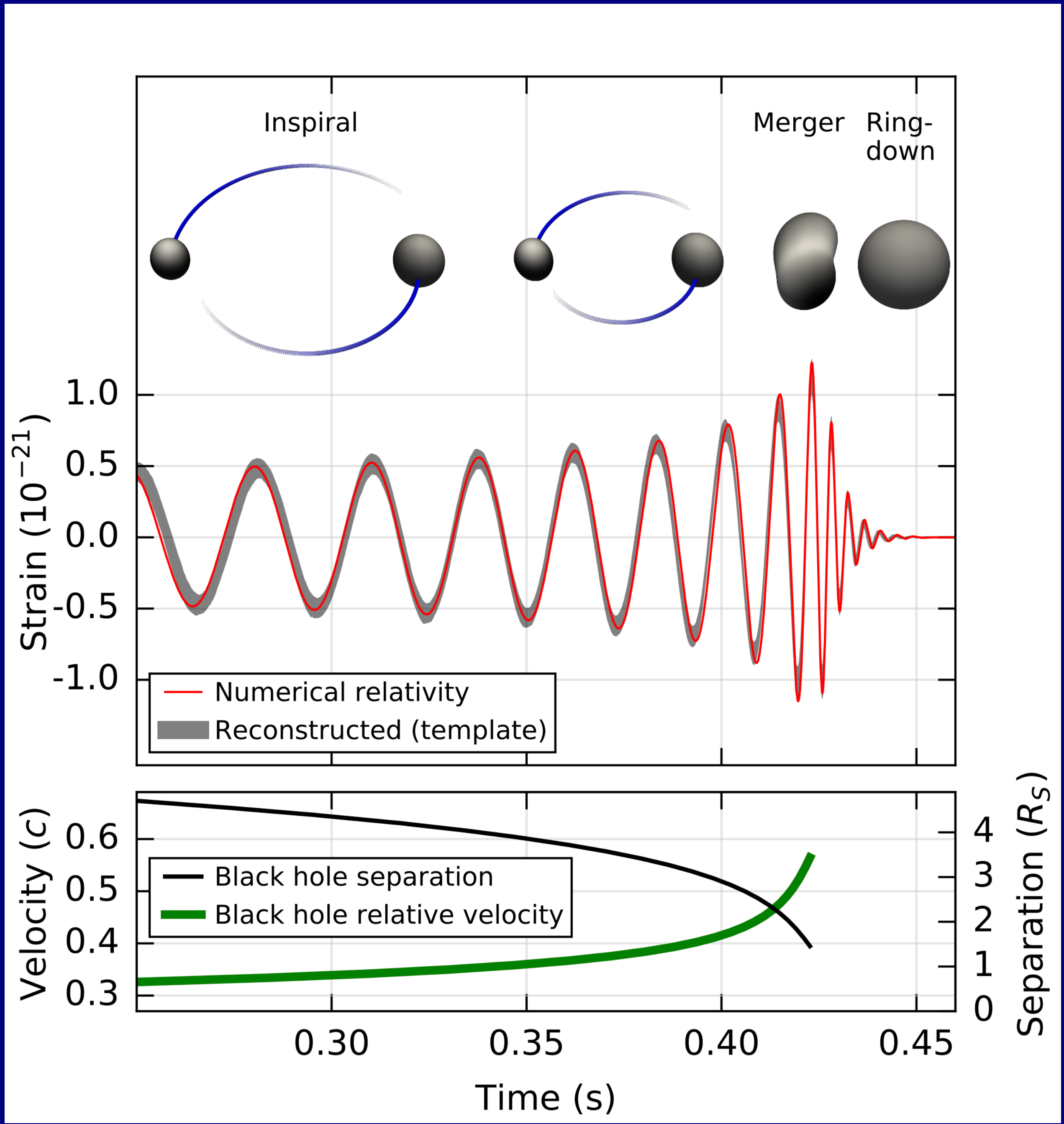
Close-up view



Gravitational Waves

LIGO (Laser Interferometric Gravity-Wave Observatory)

Gravitational Waves



Gravitational Waves