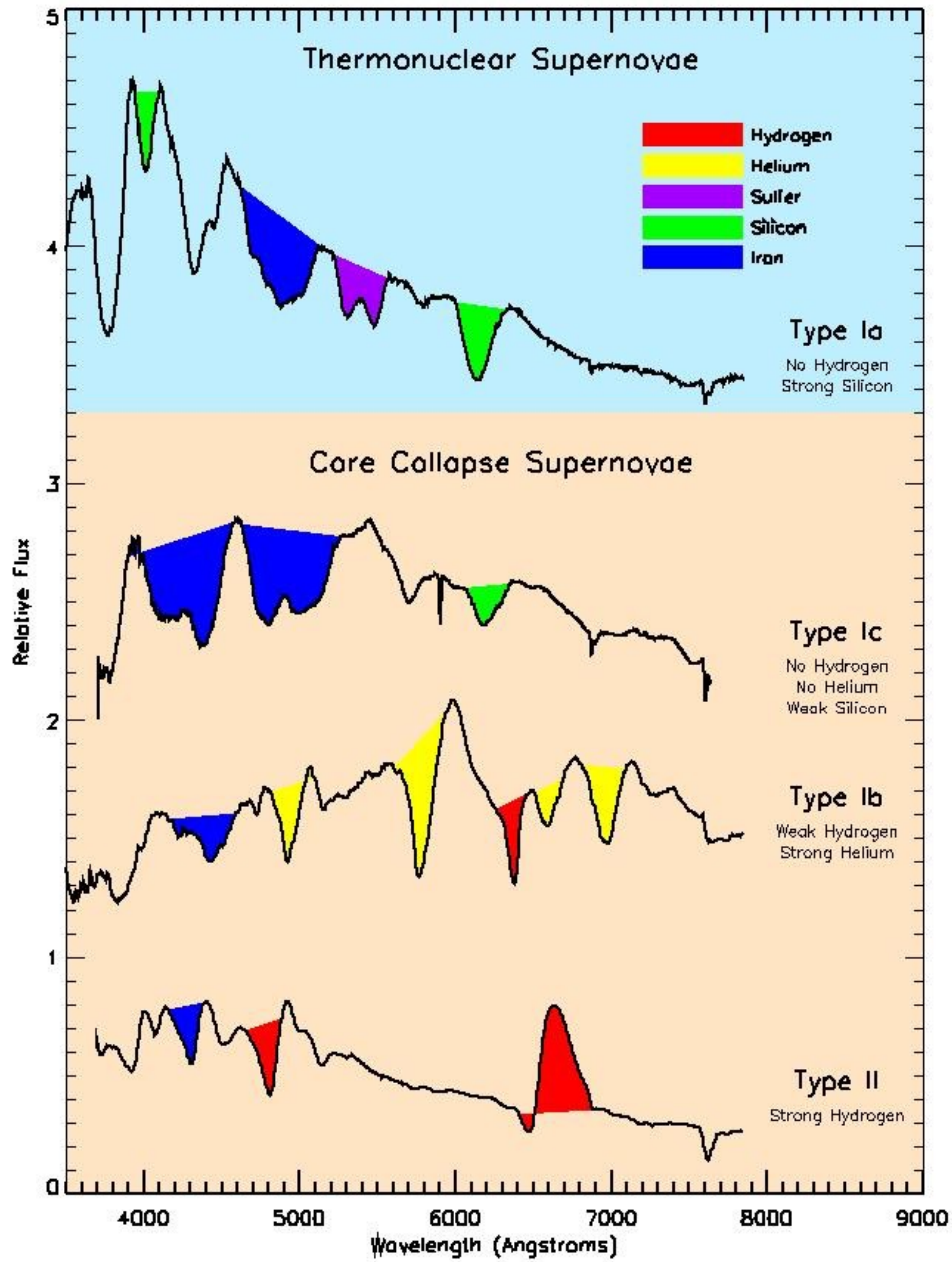
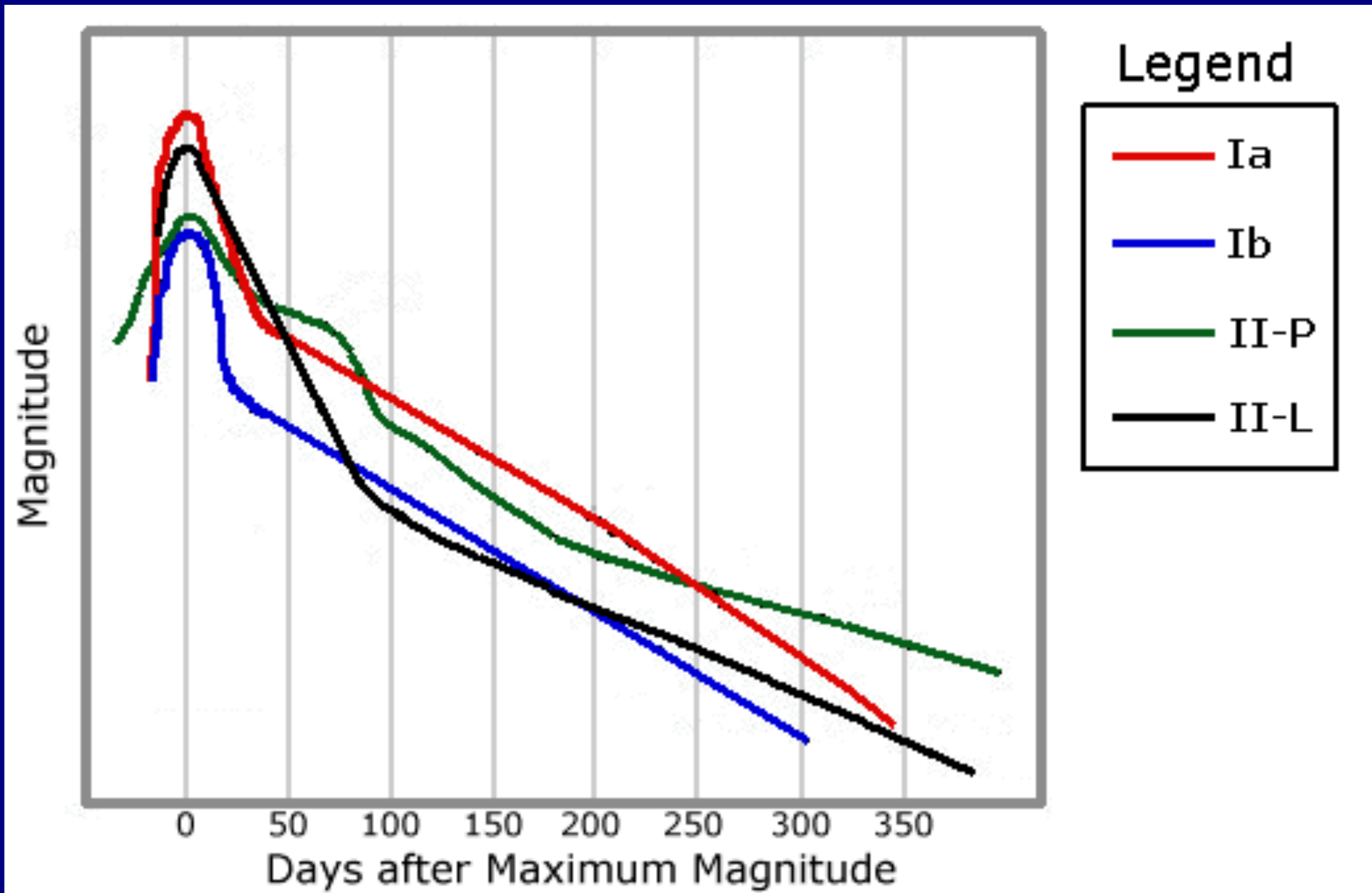
Supernova Spectra



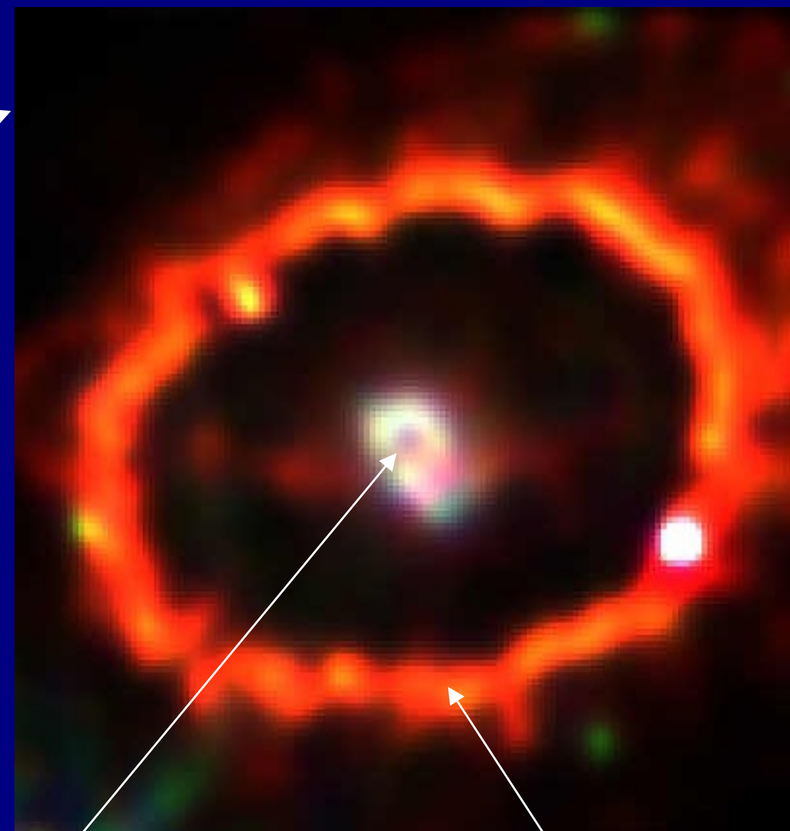
Supernova light curves



SN 1987A evolved quickly!

1994

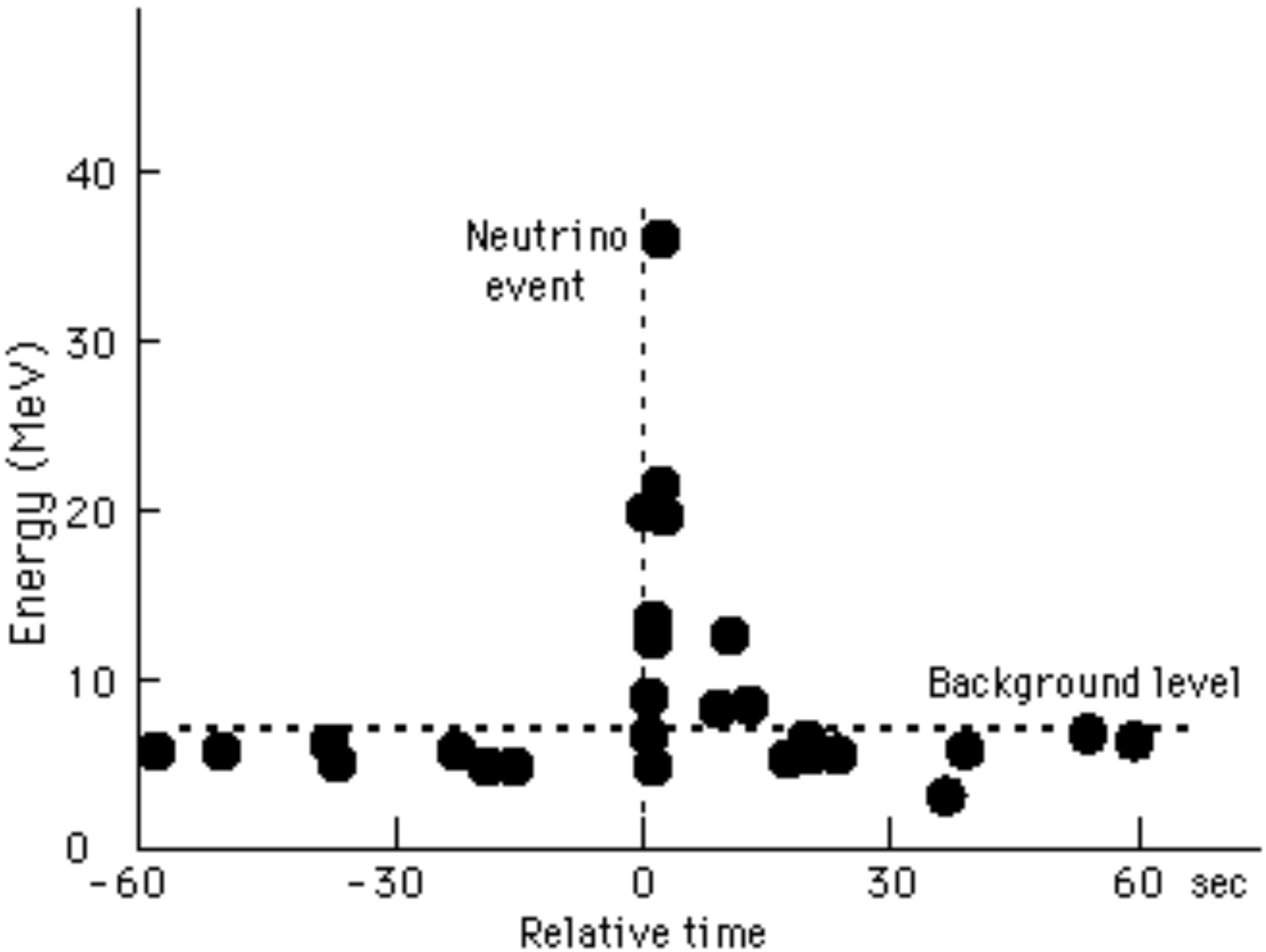
1998



Expanding debris from star. Speed almost 3000 km/sec!

Light from supernova hitting ring of gas, probably a shell from earlier mass loss event.

Neutrinos from SN1987A



Kamiokande II: 11
IMB: 8
Baksan: 5

All in 13 seconds
Arriving ~2 hours
Before optical light

Making elements beyond iron – neutron capture

If stellar nucleosynthesis creates elements with atomic weights up to iron, where do heavier elements come from?

In supernovae, we create plenty of free neutrons, while in normal stellar evolution, some reactions also create them. This makes possible:

$$ {}^{A}_{Z}X + n \rightarrow {}^{A+1}_{Z}X + \gamma \quad \text{(no Coulomb barrier)} $$

New nucleus is either stable (=> new isotope) or unstable and undergoes "β-decay"

$$ n \rightarrow p + e^{-} + \bar{\nu}_e $$

If unstable, two limits:

1) If β-decay time << n-capture timescale, β-decay occurs first:

$$ {}^{A}_{Z}X \rightarrow {}^{A+1}_{Z+1}X + e^{-} + \overline{\nu_e} + \gamma $$

"s process". Happens during stellar evolution.

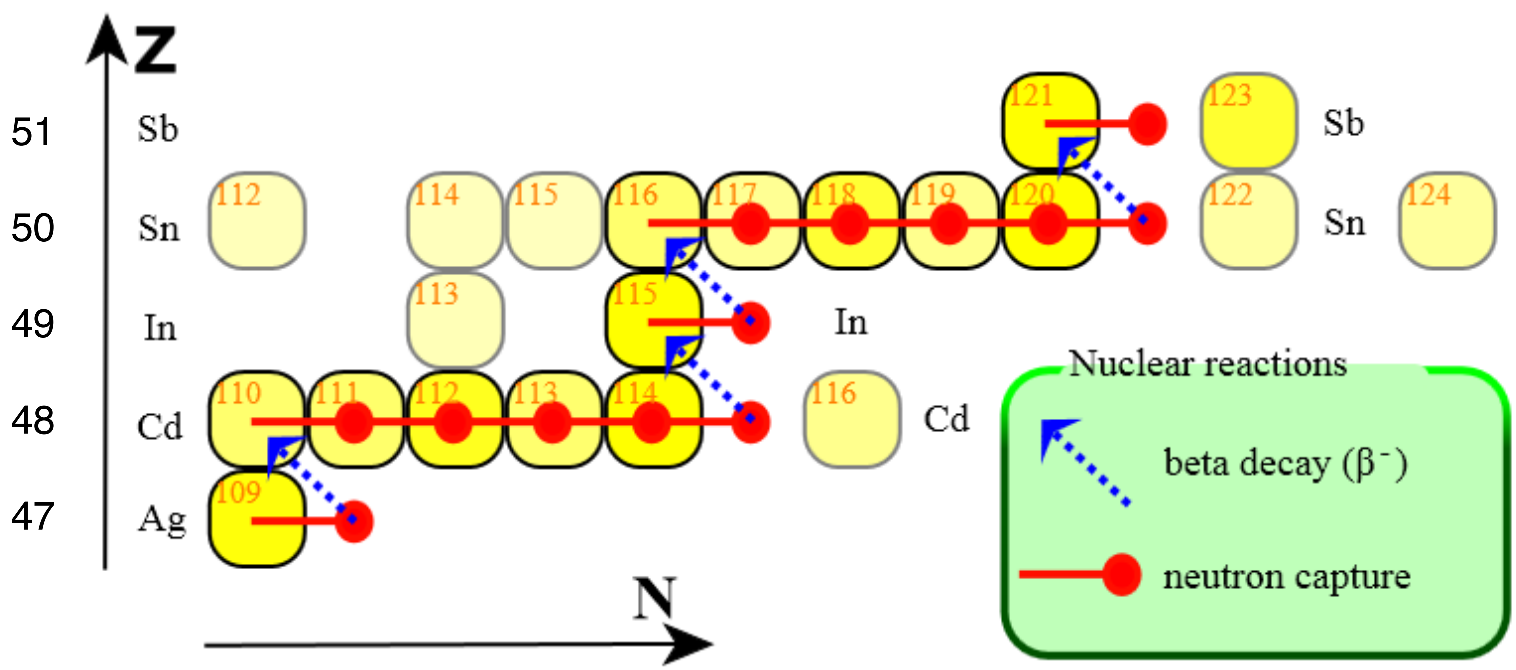
2) If n-capture timescale << β-decay time, add more n's until isotope with short decay time reached, and then it will undergo β-decay:

$$^{A+i}_{Z}X \rightarrow \, ^{A+i}_{Z+1}X + e^- + \bar{\nu}_e$$

until a stable isotope is reached

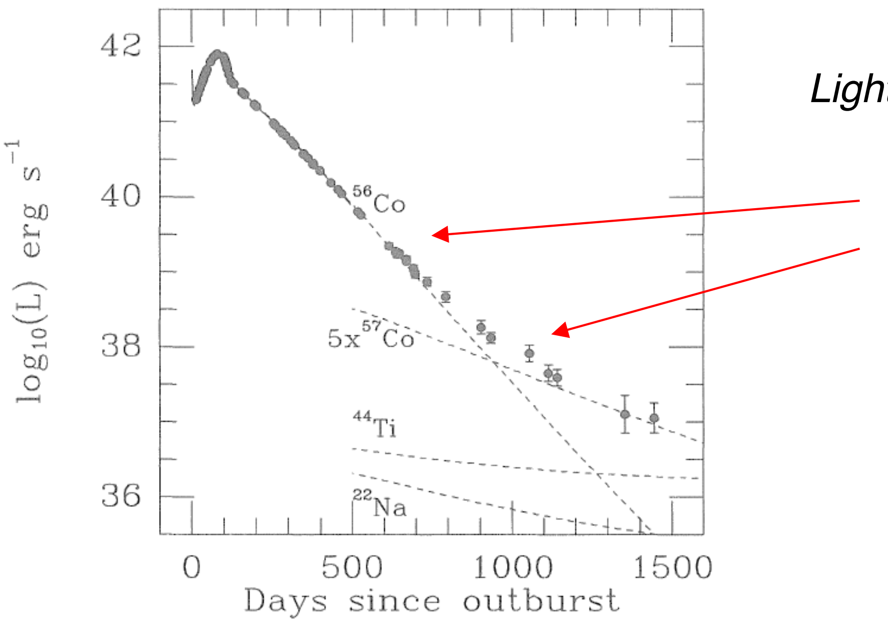
"r process" => n-rich but stable isotopes. Happens in supernovae.

Example of some elements and isotopes produced by s-process:



but, e.g. $^{122}_{50}Sn$, $^{124}_{50}Sn$ can only be made by r-process

Supernova light curves and radioactive decay



Light curve of SN 1987a

Eventually, envelope cooled and dimmed, so that light was dominated by exponential decay of other radioactive isotopes

Radioactive elements produced by fusion in the shock front. Decay:

$$^{56}_{28}Ni \rightarrow \, ^{56}_{27}Co + e^+ + \nu_e + \gamma$$ Half life: 6 days

$$^{56}_{27}Co \rightarrow \, ^{56}_{26}Fe + e^+ + \nu_e + \gamma$$ Half life: 78 days

$$^{57}_{27}Co \rightarrow \, ^{57}_{26}Fe + e^+ + \nu_e + \gamma$$ Half life: 272 days

etc. Shape of light curve depends on abundances, so we can identify responsible elements => can work out how much produced.

Astronomy 421

Lecture 22: End states of stars - White Dwarfs

Outline – White Dwarf Stars

Sirius B

White Dwarf Properties

Electron degeneracy condition, pressure, relativistic pressure

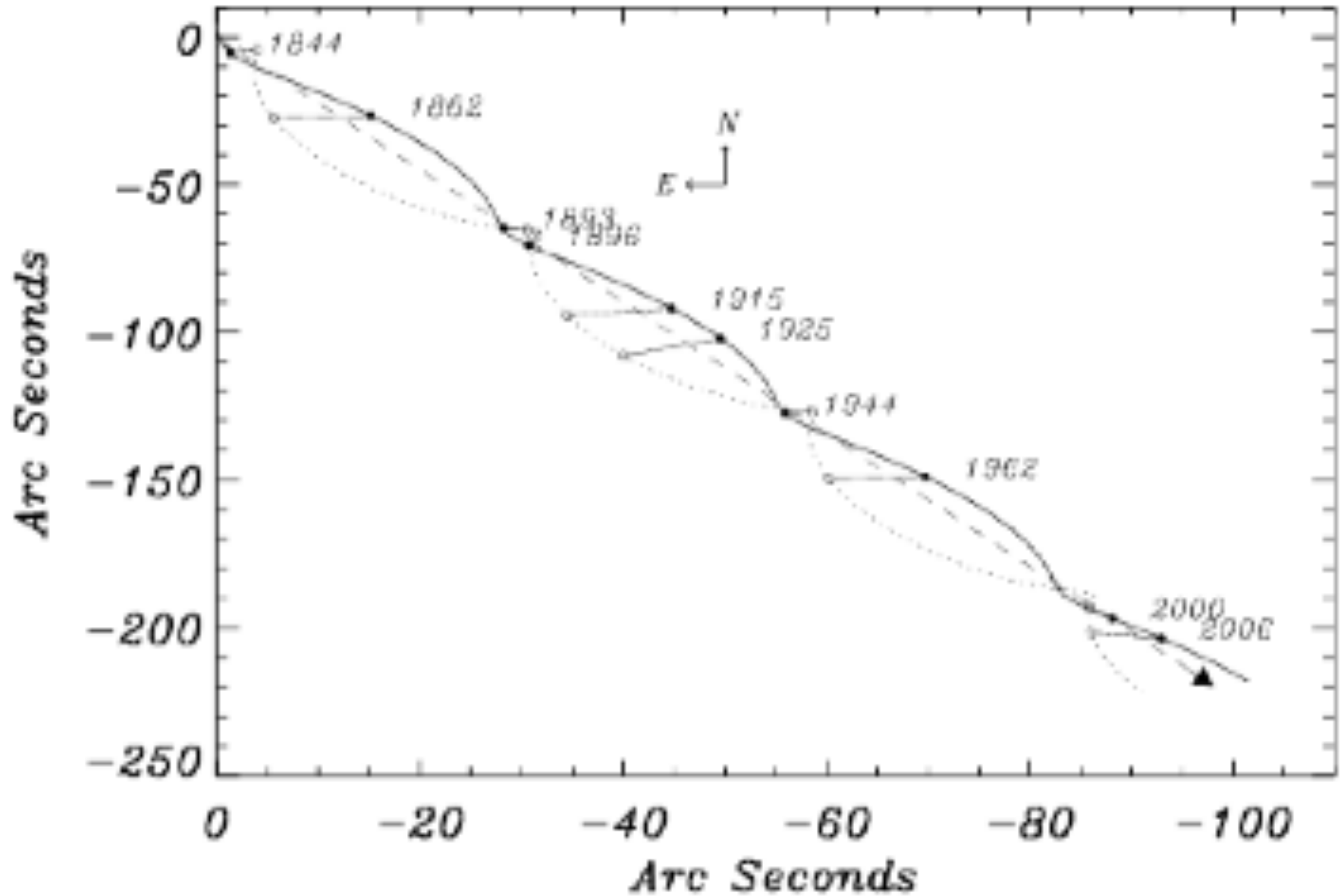
Mass-volume relation

Chandrasekhar limit

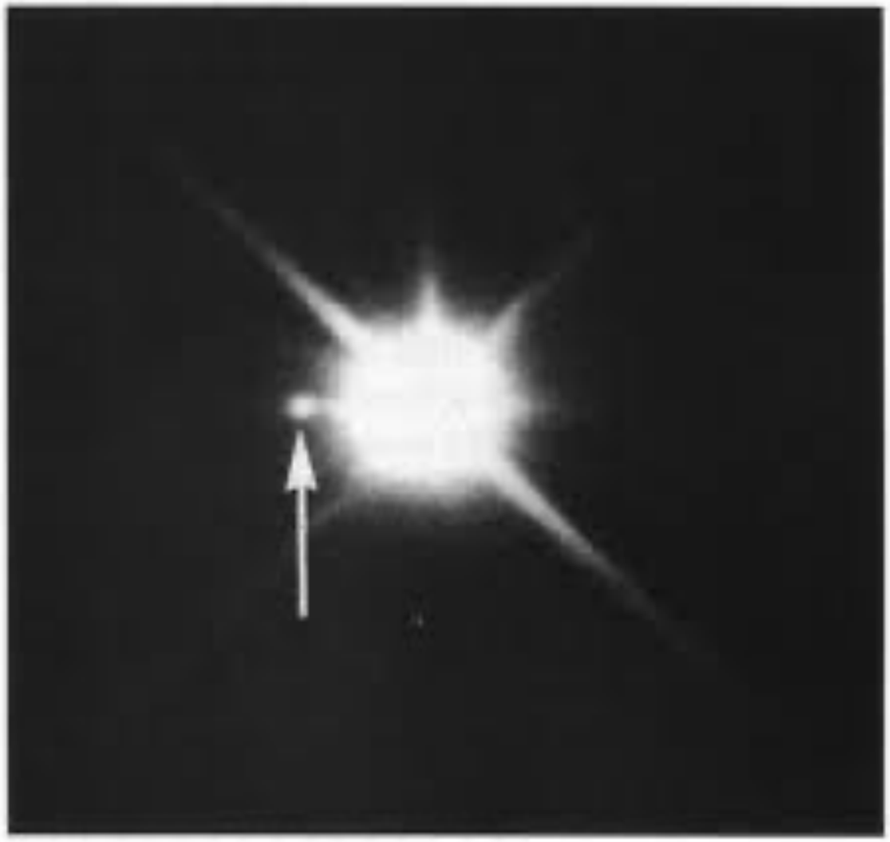
White dwarf cooling and evolution

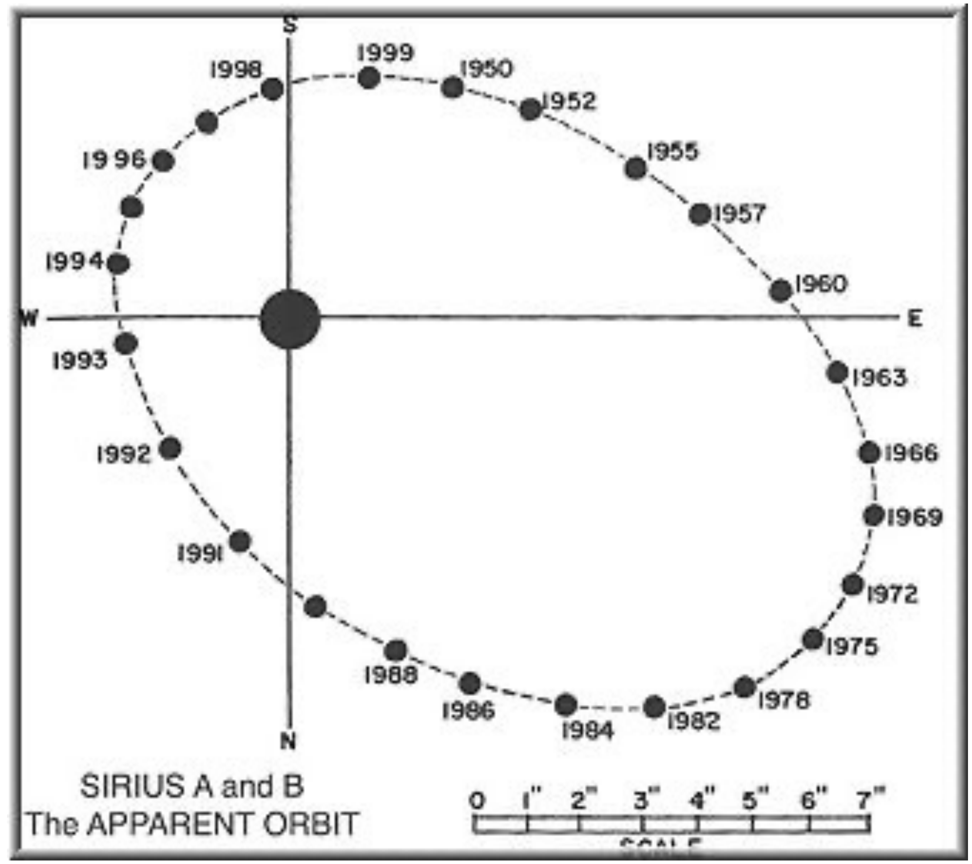
Discovery of Sirius B aka “the pup”

Trajectory of Sirius A is not a straight line:

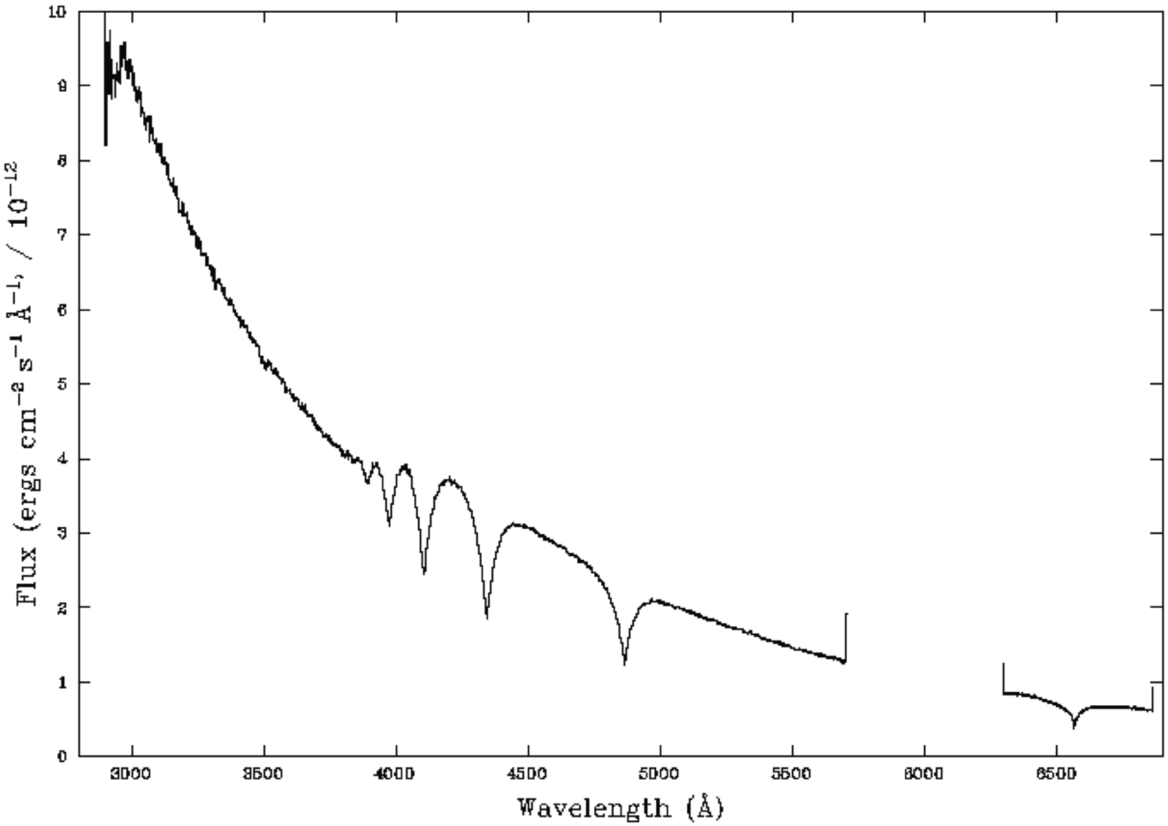


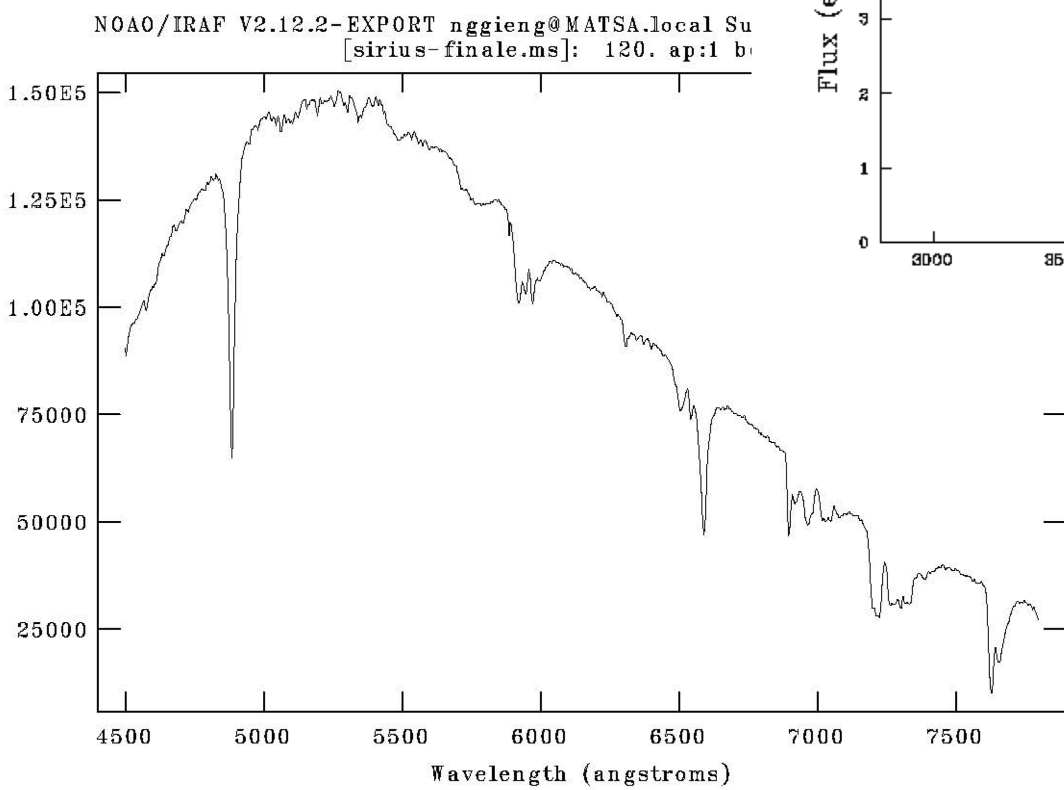
Discovery of Sirius B aka “the pup”



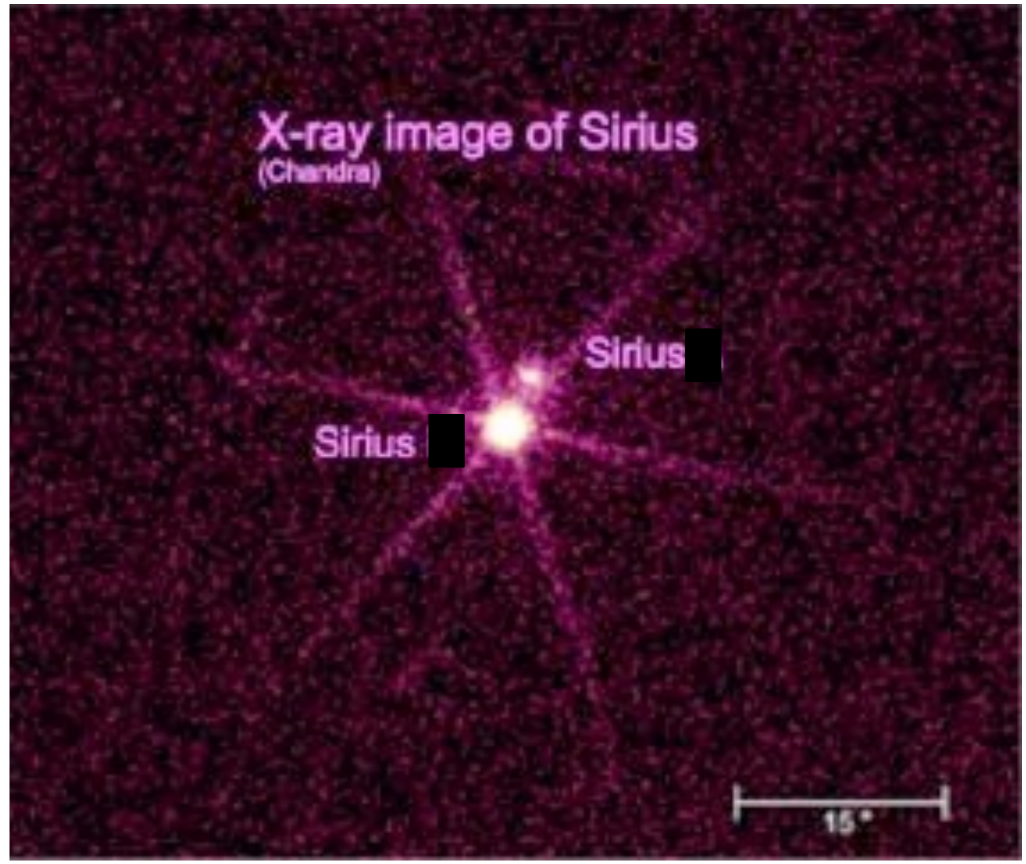


Spectrum of Sirius B and A

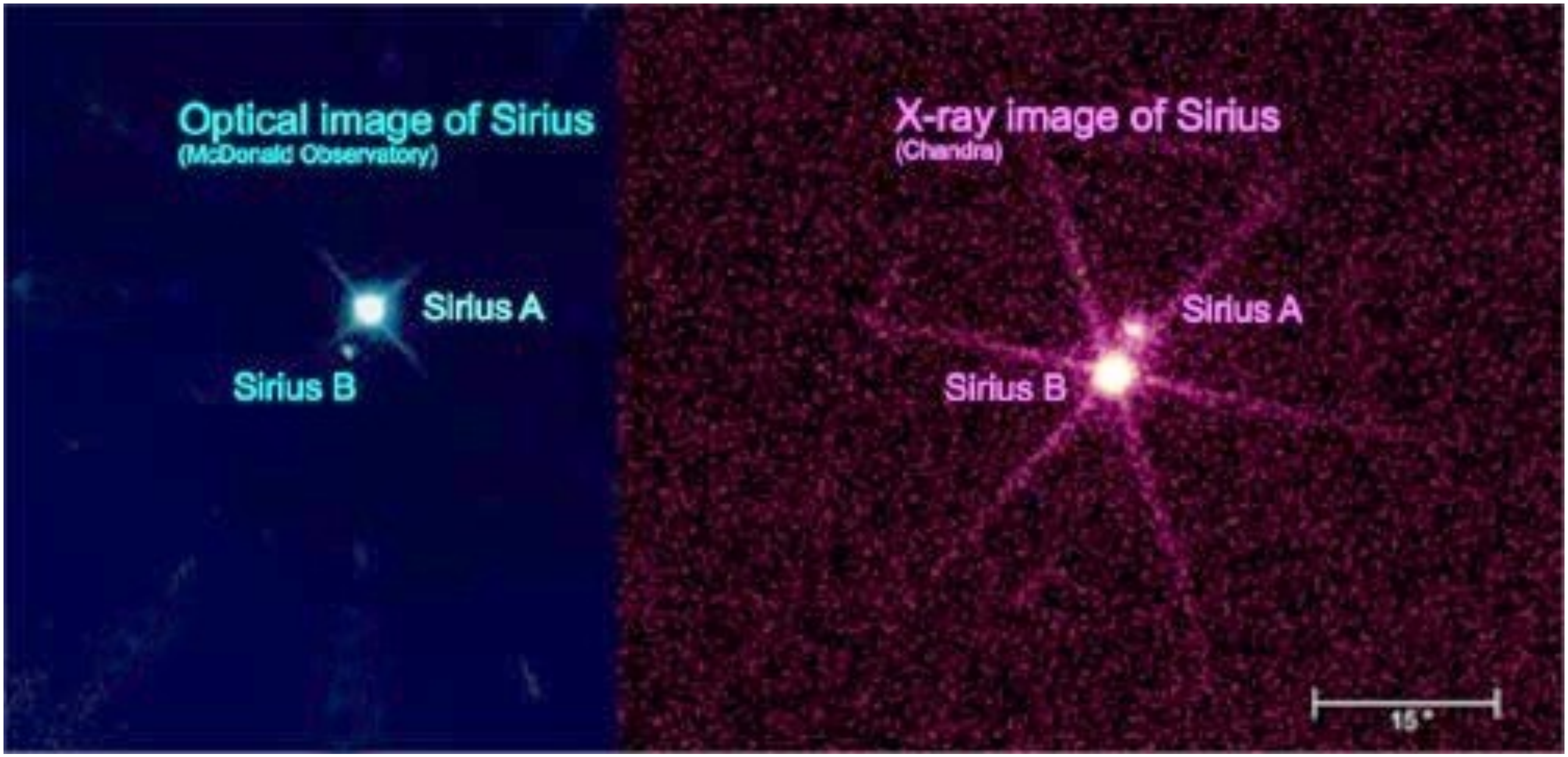




X-ray image of Sirius B aka “the pup”



Discovery of Sirius B aka “the pup”



End states of stars

Possibilities:

1. Violent explosion, no remnant (Type Ia SN)

2. White dwarf
   - $M \sim 0.6 M_\odot$
   - $R \sim R_\oplus$
   - $\rho \sim 10^9$ kg m^{-3}

3. Neutron star
   - $M \sim 1.4 M_\odot$
   - $R \sim 10$ km
   - $\rho \sim 10^{17}$ kg m^{-3}

4. Black hole
   - $M \geq 3 M_\odot$

White dwarfs

- Most famous one is Sirius B
- Visual binary, yields mass $M \sim 1.05\ M_\odot$
- From distance and spectrum, we get *L, Teff*.
  This implies $R \sim 5{,}500$ km < $R_\oplus$ (range 10^3- 10^4 km for WDs)
- T_{eff} = 27,000 K (range 5,000-80,000 K for WDs)
- central temperature T_c = several x 10^7 K
- Should be isothermal except for very outer layers – efficient energy transport by electron conduction equalizes *T*.

C, O interior, not fusing.

Residual H, He in atmospheres of most

=> absorption lines, typically VERY broad. Why?

If there is no fusion, what supports a WD?

Degenerate matter

If energy is removed from a gas, the temperature as well as the pressure drops. The gas thus compresses, but only to a limit:

available volume per particle ~ "actual volume" of particle.

Quick and dirty derivation of the energy of such gas: if there are *N* particles in volume *V*, and a particle size is *x*, this says:

$$\frac{V}{N} \sim x^3$$

Recall the uncertainty principle: if a particle is confined to a region of size x, its momentum is roughly:

$$p \sim \frac{\hbar}{x}$$

That is, its momentum, and therefore energy and pressure, become large if confined to a small volume.

Then its kinetic energy (non-relativistic) is:

$$E = \frac{\hbar^2}{2m}\left(\frac{N}{V}\right)^{2/3} = \frac{\hbar^2}{2m} n^{2/3}$$

A more complicated derivation (see C+O 16.1) leads to a very similar result, called the Fermi energy

$$\boxed{E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}}$$

For fully ionized gas of element ${}^{A}_{Z}X$

$$n_e = \frac{\# e^-}{\text{nucleus}} \frac{\#\,\text{nuclei}}{\text{volume}} = \frac{Z}{A}\frac{\rho}{m_H}$$

Thus, for electrons:

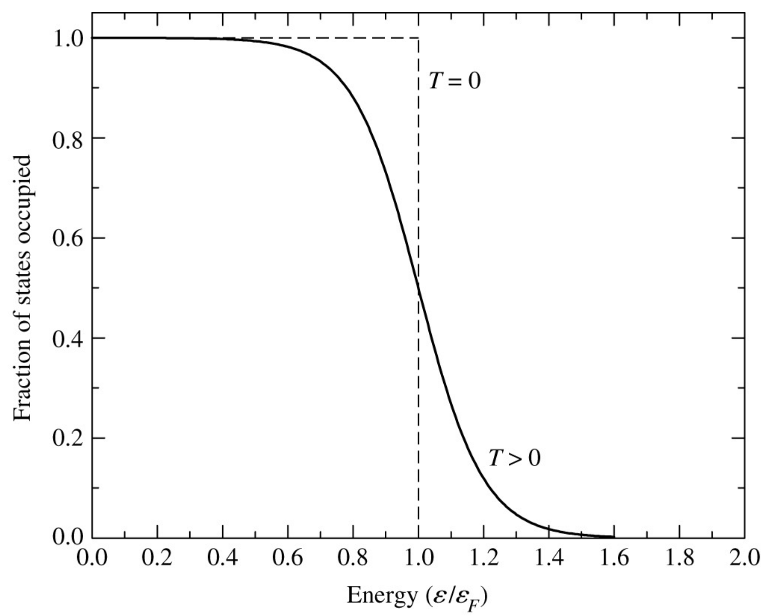
$$E_F = \frac{\hbar^2}{2m_e}\left[3\pi^2\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{2/3}$$

The Fermi Energy

The highest energy of an electron in a collection of electrons at a temperature of absolute zero.

In classical statistical mechanics, temperature of a system is the measure of its average kinetic energy.

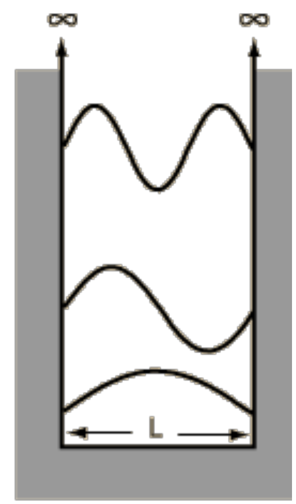
In quantum statistical mechanics, Fermi energy corresponds to last filled level at absolute zero, and the corresponding temperature is the Fermi temperature.



Fraction of energy states occupied by fermions.

Degenerate matter

Classically, in a gas at *T*=0 K, all particles would have zero energy. But if they are confined to a volume, then from the Heisenberg Uncertainty Principle they cannot. Also, the Pauli Exclusion Principle forbids more than two fermions in a system of particles from having the same energy. So electrons crowd into lowest unoccupied states.



Simpler version of C+O's derivation: consider the 1-D infinite square well.

The energy levels are given by:

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$$

Two particles can be in *n*=1, two in *n*=2, etc. So if *N* particles, highest level filled is *N/*2. So set *n* = *N*/2 in above, to get:

$$E_n = N^2 \frac{\pi^2 \hbar^2}{8mL^2}$$

For a volume of side L, the number density of particles is $(N/L)^3$.

So the minimum energy is:

$$E_n = \frac{\pi^2 \hbar^2}{8m} n^{2/3}$$

The exact expression given in C+O 16.1 is:

$$E_F = \frac{\pi^2 \hbar^2}{2m} (3\pi^2 n)^{2/3}$$ Fermi Energy

Note that the energy is bigger for electrons than protons or neutrons, so consider electrons. For a fully ionized gas of element ${}^{A}_{Z}X$

$$n_e = \frac{\# e^-}{\text{nucleus}} \frac{\#\,\text{nuclei}}{\text{volume}} = \frac{Z}{A} \frac{\rho}{m_H}$$

$$E_F = \frac{\hbar^2}{2m_e} \left[3\pi^2 \left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{2/3}$$

If the temperature is low enough, electrons start arranging themselves in their lowest possible energy states. Approximately, gas gets close to degeneracy when typical electron energy, *3kT/2* , gets close to E_F. Since E_F is bigger for electrons, they will become degenerate at a lower temperature than the protons or neutrons.

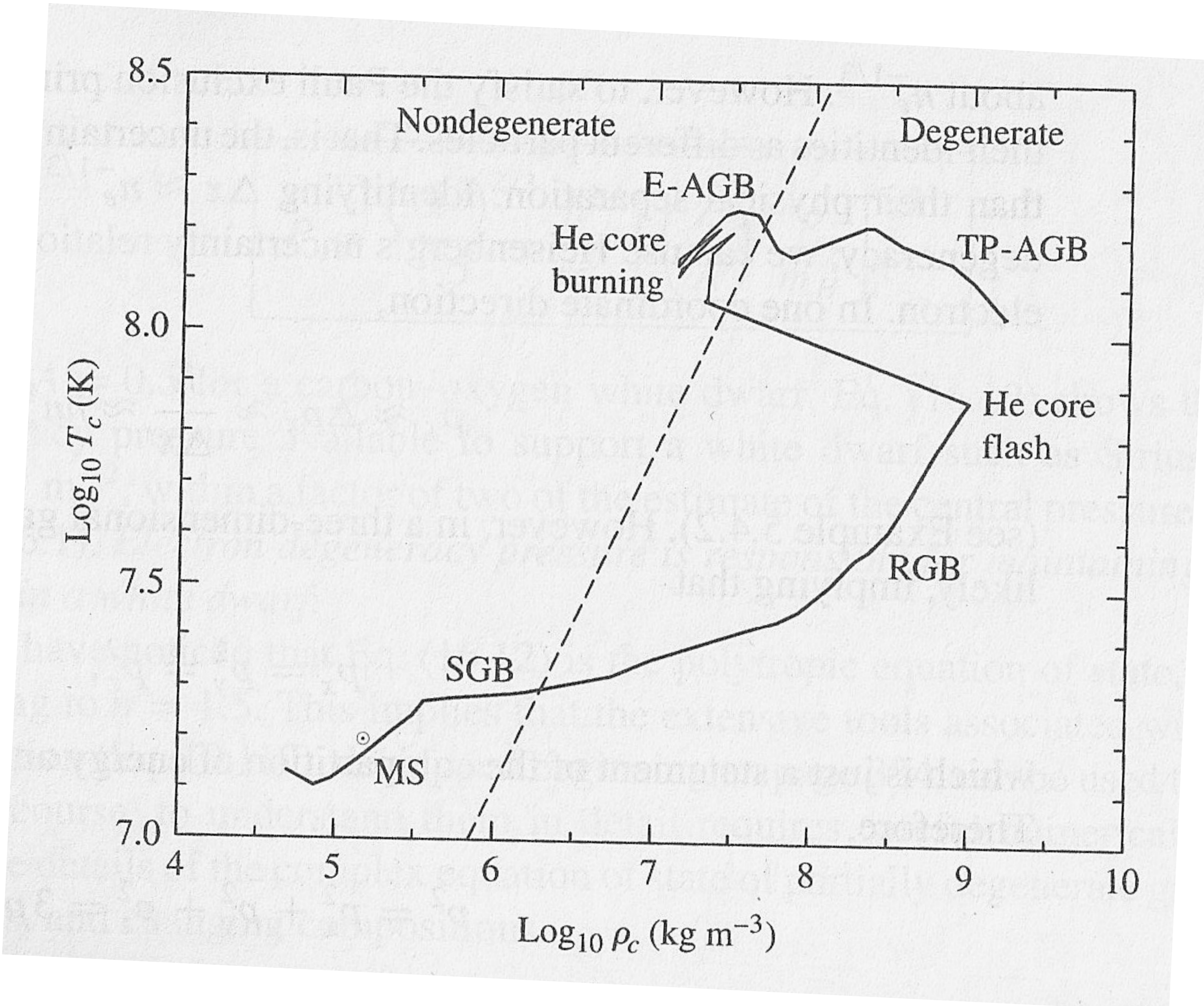
Thus, electrons degenerate when:

$$E = \frac{3kT}{2} < \frac{\hbar^2}{2m_e} \left[3\pi^2 \left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{2/3}$$

$$\boxed{\frac{T}{\rho^{2/3}} < \frac{\hbar^2}{3m_e k} \left[\frac{3\pi^2}{m_H} \left(\frac{Z}{A} \right) \right]^{2/3}}$$

For *Z/A*=0.5, this is 1261 K m^2 $kg^{-2/3}$.

State of Matter in the Core of the Sun



Degeneracy pressure

From chapter 10, for particles in a box with number density *n*, speed *v*, momentum *p,* the pressure is:

$$P = \frac{1}{3} n v p$$

If particles are confined to a volume $(\Delta x)^3$ $\Delta x = n^{-1/3}$

Heisenberg's uncertainty principle then says:

$$p \sim \frac{\hbar}{\Delta x} = \hbar n^{1/3}$$

If speeds are non-relativistic:

$$v = \frac{p}{m} = \frac{\hbar}{m} n^{1/3}$$

For e^-'s with full ionization,

$$n_e = \frac{Z}{A} \frac{\rho}{m_H}$$

So, using the expression for the pressure:

$$P = \frac{1}{3} n_e v p \approx \frac{1}{3} \frac{\hbar^2}{m_e} \left[\left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{5/3}$$

Exact calculation gives:

$$\boxed{P = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} \left[\left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{5/3}}$$

Electron degeneracy pressure (non-relativistic)

<u>Key point:</u> $P \propto \rho^{5/3}$ - independent of *T* !

P non-zero even if classically *T* -> 0. Any energy generation which raises *T* will not increase *P*. Recall He flash. Runaway fusion until $T/\rho^{2/3}$ high enough.

Worksheet 13. Show that mass times volume = constant for a White Dwarf

Mass-Volume relationship for degenerate stars

The interior pressure within a star is of order $G\frac{M^2}{R^4} \sim G\rho^2 R^2$

Set this equal to the degeneracy pressure, which is proportional to $\rho^{5/3}$

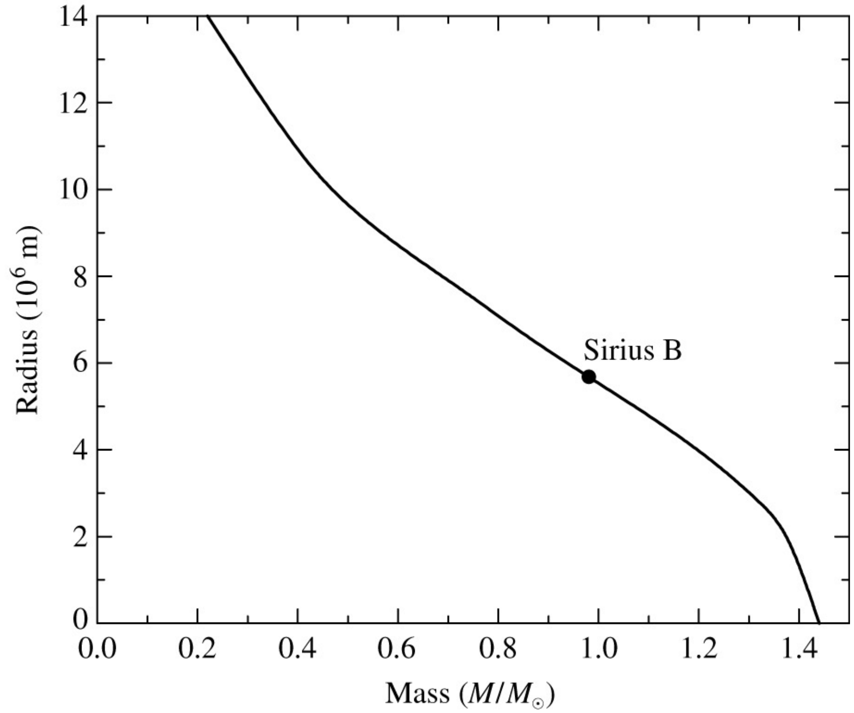
to find that

*Mass * Volume = constant*

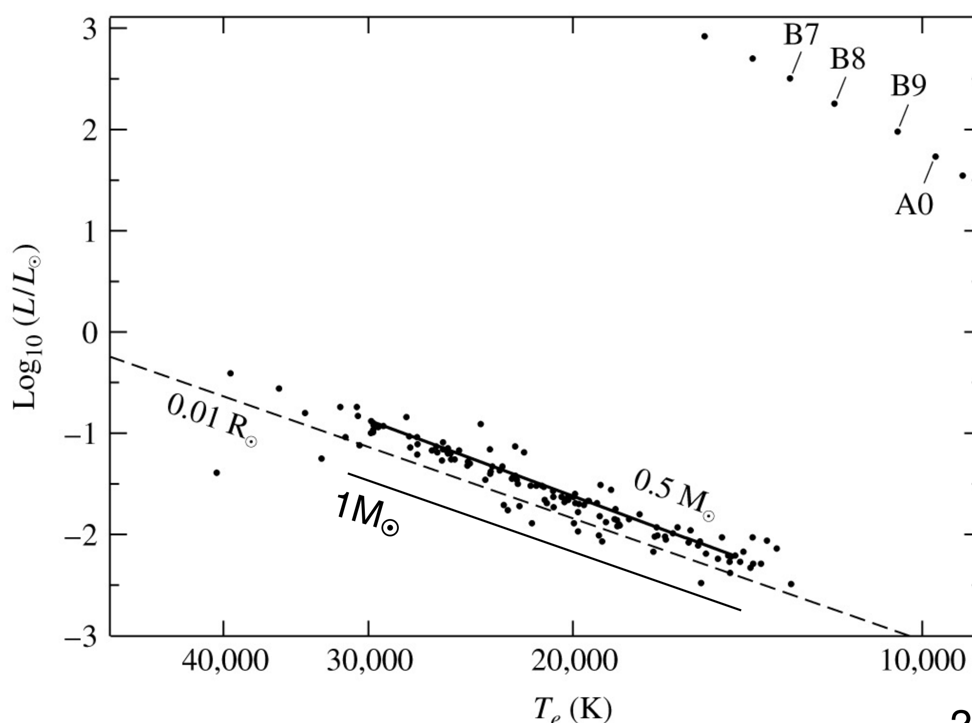
(and for a $1M_{\odot}$ WD, find a radius of about 3000 km, close to what is derived from observations).

Thus more massive WDs are more compact.

Strange implication: More massive WD's have *smaller R* to provide needed P_{degen}.



Location on H-R diagram:



The Chandrasekhar Limit

So by adding more mass, could we shrink the star to a zero volume?

There is a limit to the amount of matter that can be supported by electron degeneracy pressure, leading to a maximum mass for a White Dwarf.

When does contraction due to gravity overcome pressure from degeneracy? Let's consider how things scale:

$$F_G \sim \frac{GM^2}{R^2}$$

The "pressure" due to gravity:

$$P_G \sim \frac{F_G}{4\pi R^2} \propto \frac{M^2}{R^4}$$

For non-relativistic degeneracy

$$P_{non-rel} \propto \rho^{5/3} \propto \frac{M^{5/3}}{R^5}$$

Relativistic degeneracy pressure

Repeat derivation with *v=c* (instead of *v=p/m)*.

$$P = \frac{(3\pi^2)^{1/3}}{4}\hbar c\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{4/3}$$

Needed when *P* very high, then particle separations small, so speed very high according to Heisenberg's UP.

If $P_G > P_{non\text{-}rel}$, causing R to decrease, then $P_{non\text{-}rel}$ increases, and rises faster than P_G. Can find a new equilibrium at smaller R, higher P's.

For the relativistic case:

$$P_{rel} \propto \rho^{4/3} \propto \frac{M^{4/3}}{R^4}$$

If $P_G > P_{rel}$, R decreases, P_{rel} increases by same amount as P_G, so P_{rel} can't increase to overcome P_G. The decrease of R continues => collapse!

If WD is massive enough, i.e. has small enough volume, then electrons start to become relativistic according to Uncertainty Principle, and the gas then has relativistic degeneracy. This is an unstable equilibrium.

So maximum possible WD mass corresponds to when $P_{grav} \sim P_{rel\ degen.}$

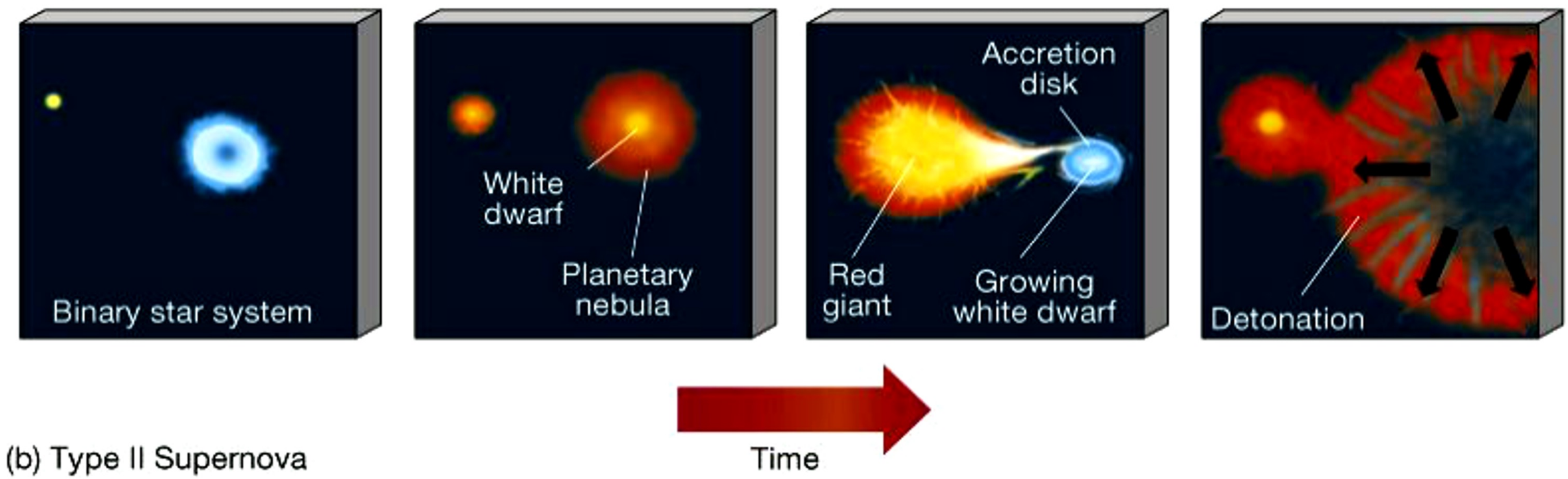
For Z/A=0.5, the limit is

$$\boxed{M_{ch} = 1.44 M_\odot}$$

Chandrasekhar limit

Type Ia supernovae

If enough mass dumped onto WD by binary companion to push it over Chandrasekhar limit, starts collapsing until hot enough for C,O fusion. Proceeds rapidly through WD, explosion, no remnant.



Evolution of White Dwarfs:

No fusion, no energy generation => WD cool and becomes dimmer.
Energy carried by electron conduction.

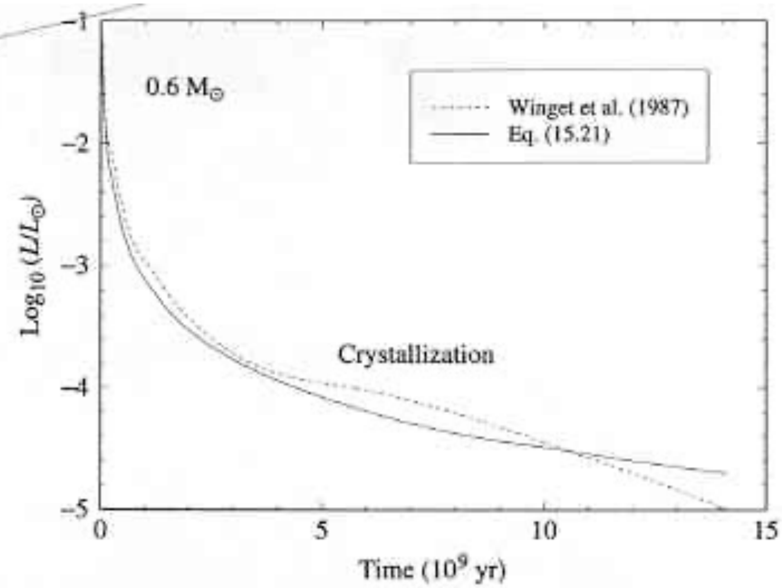


Figure 15.9 Theoretical cooling curves for 0.6 M$_\odot$ white-dwarf models. [The solid line is from Eq. (15.21), and the dashed line is from Winget et al., *Ap. J. Lett.*, *315*, L77, 1987.]

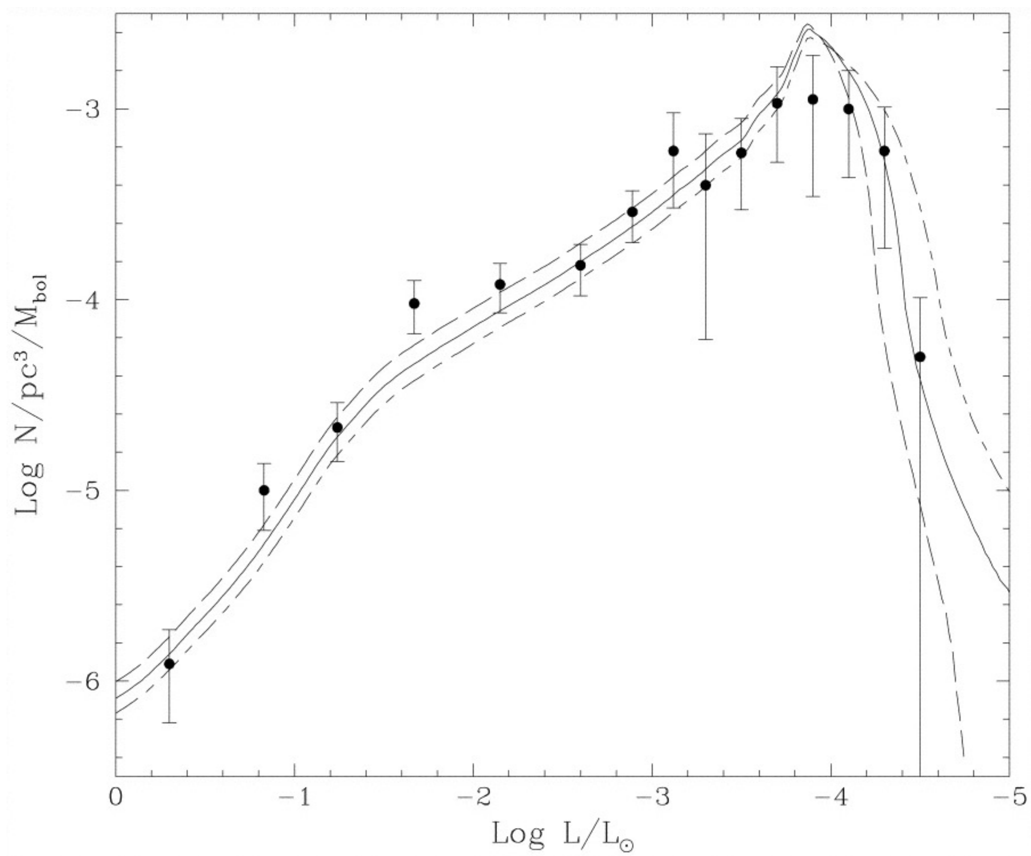
C+O:

$$L = L_0(1 + \frac{5}{2}\frac{t}{\tau_0})^{\frac{-7}{5}}$$

$$T = T_0(1 + \frac{5}{2}\frac{t}{\tau_0})^{\frac{-2}{5}}$$

Cooling ions will settle into lattice, maintained by Coulomb repulsion of nuclei. Since pressure comes from degeneracy, all this happens at constant radius.

White Dwarf Luminosity Function for the disk of the Milky Way, and various predictions for it based on theoretical cooling curves (and assuming a constant birthrate and a WD mass distribution):



Since *L*, *T* related to age since WD was a fully fledged star, can use the numbers of WDs at different *L*, *T* to constrain MW's past stellar content. Lack of really cool, dim WDs with log ($L/L_\odot$) < - 4.5 implies first WDs formed 9 Gyrs ago. This is then about the age of the disk.