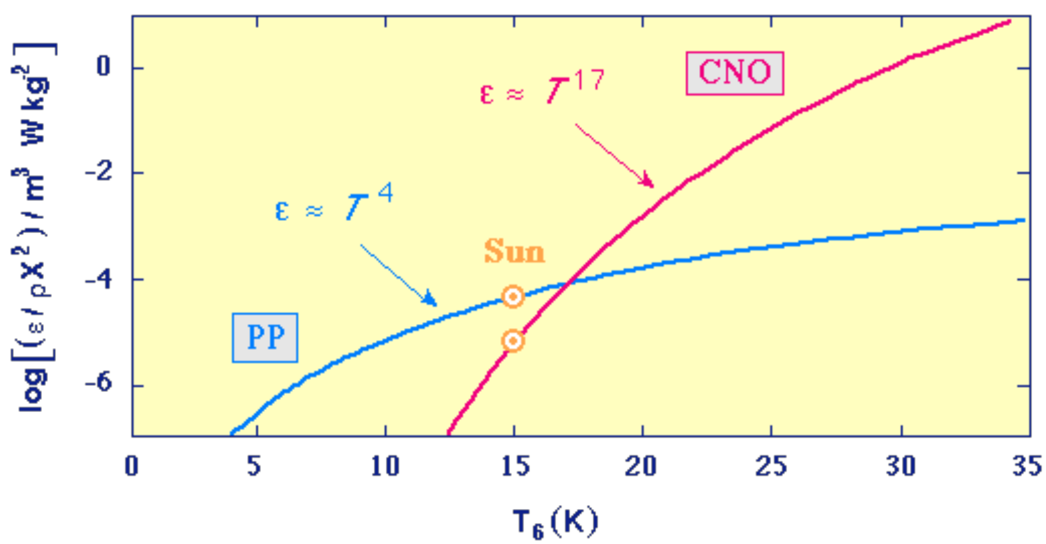
If we equate the rate of energy production in the PP chain and the CNO cycle, we can find a *T* at which they produce the same rate of energy production.

This occurs around T~$1.7\text{x}10^7$ K.

Below this temperature the PP chain dominates, and above it the CNO cycle dominates.

This temperature limit occurs in stars slightly more massive than the Sun, around 1.2-1.5 solar masses.



Triple alpha process:

He $\rightarrow$ C in post-MS stars. Simplest reaction should be fusion of two He nuclei. But, there is no stable configuration with A=8. For example, ^{8}Be has a lifetime of about 10^{-16} s!

However, a third He nucleus can be added before ^{8}Be decay, forming ^{12}C by the triple alpha process. This makes it essentially a three-body interaction.

$$I \quad {}^4_2He + {}^4_2\,He \ \rightarrow \ {}^8_4Be + \gamma$$

$$II \quad {}^8_4Be + {}^4_2\,He \ \rightarrow \ {}^{12}_6C + \gamma$$

$$\epsilon \propto Y^3 \rho^2 T^{41}$$ (Y is mass fraction of He)

This requires T > 10^8 K. Occurs in cores after H exhausted, and they have compressed and heated up.

Other reactions at higher T's produce O, Ne, Na, Mg, Si, P and S.

Common reactions include:

$$X +^4_2 He \;\rightarrow\; X'$$

$$^{12}_{6}C +^4_2 He \;\rightarrow\; ^{16}_{8}O + \gamma$$

$$^{16}_{8}O +^4 He \;\rightarrow\; ^{20}_{10}Ne + \gamma$$

$$^{12}_{6}C + ^{12}_{6}C \;\rightarrow\; ^{24}_{12}Mg + \gamma$$

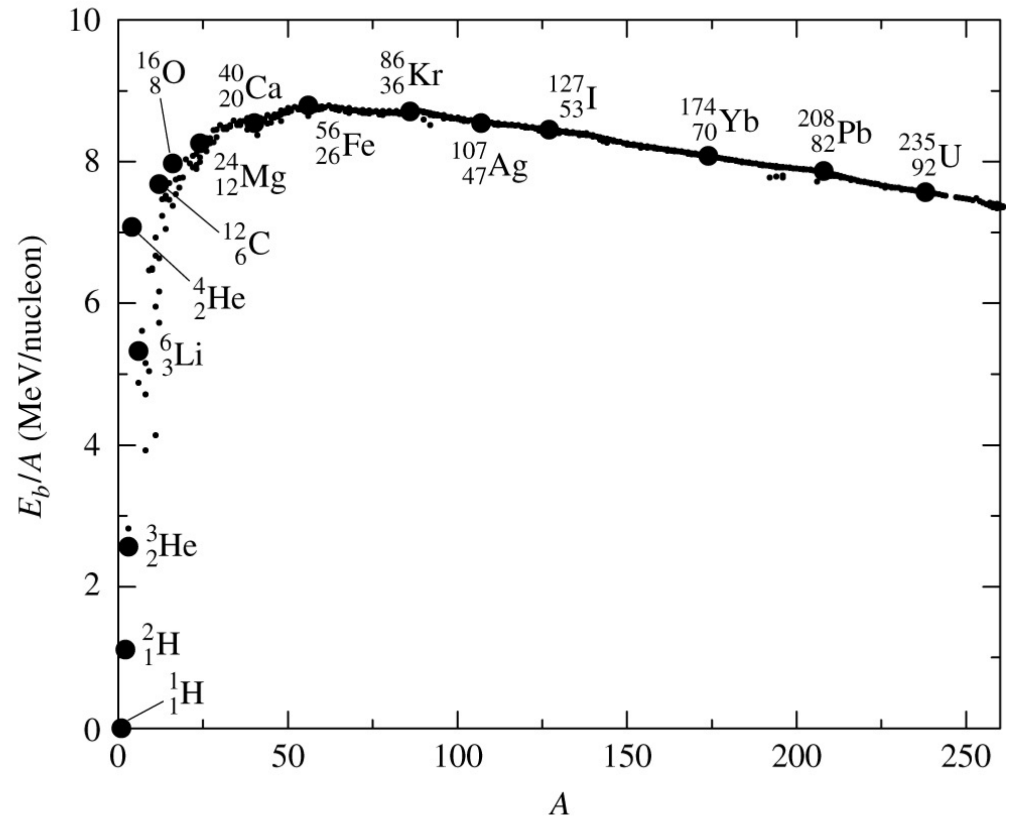
$$^{12}_{6}C + ^{12}_{6}C \;\rightarrow\; ^{23}_{12}Mg + n$$

note: some reactions make free neutrons, important later

$$^{16}_{8}O + ^{16}_{8}O \;\rightarrow\; ^{32}_{16}S + \gamma$$

(=> elements with A/4 = integer are abundant)

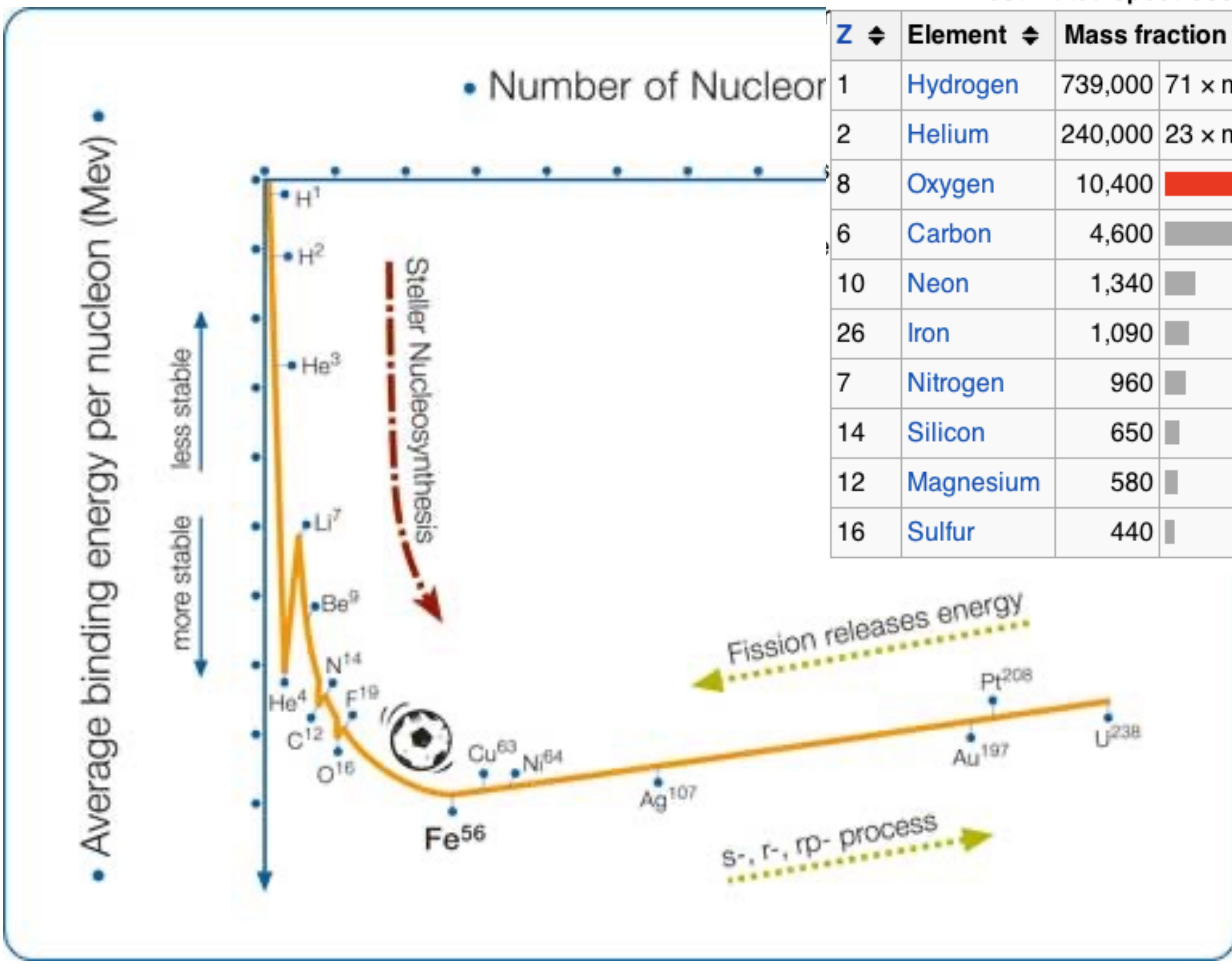
The general trend is an increase of binding energy per nucleon with atomic mass up to A=56 (Fe), then a slow decline.



Fe most strongly bound nucleus, adding more protons will cause the additional Coulomb force to become more important than the strong nuclear force.

Must put energy in to make nuclei larger than Fe, thus no energy generation from fusion beyond Fe. Fission of heavy nuclei into lighter ones can release energy (down to Fe).

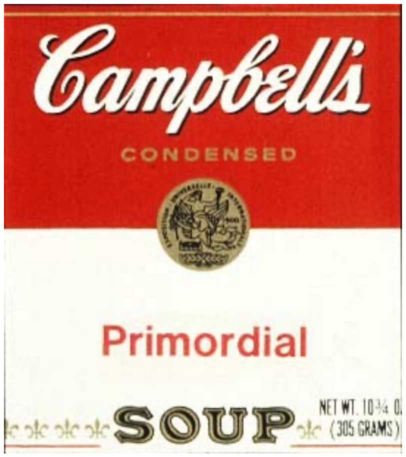
Binding Energy per nucleon



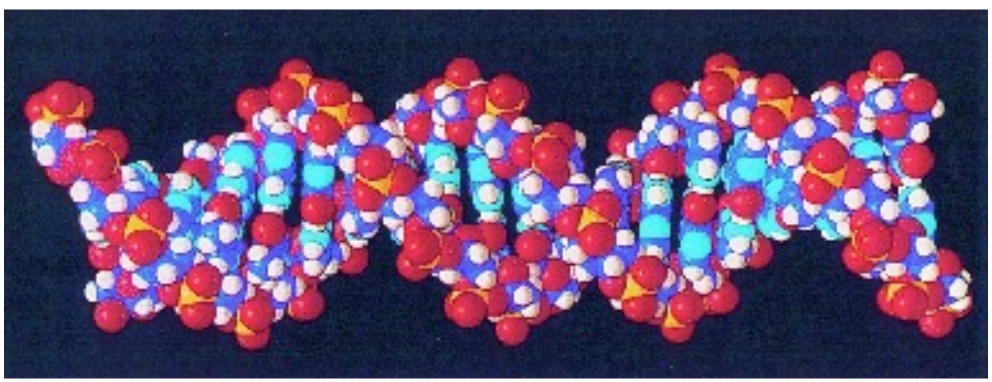
Ten most common elements in the Milky Way Galaxy estimated spectroscopically[1]

Z	Element	Mass fraction in parts per million	
1	Hydrogen	739,000	71 × mass of Oxygen (red bar)
2	Helium	240,000	23 × mass of Oxygen (red bar)
8	Oxygen	10,400	
6	Carbon	4,600	
10	Neon	1,340	
26	Iron	1,090	
7	Nitrogen	960	
14	Silicon	650	
12	Magnesium	580	
16	Sulfur	440	

Cosmic Abundances



- All life (as we know it) is made of carbon based molecular chains
- Only 30 complex molecules comprised of only five (5) basic elements
- Urey-Miller experiment in 1953 showed that we could build amino acids



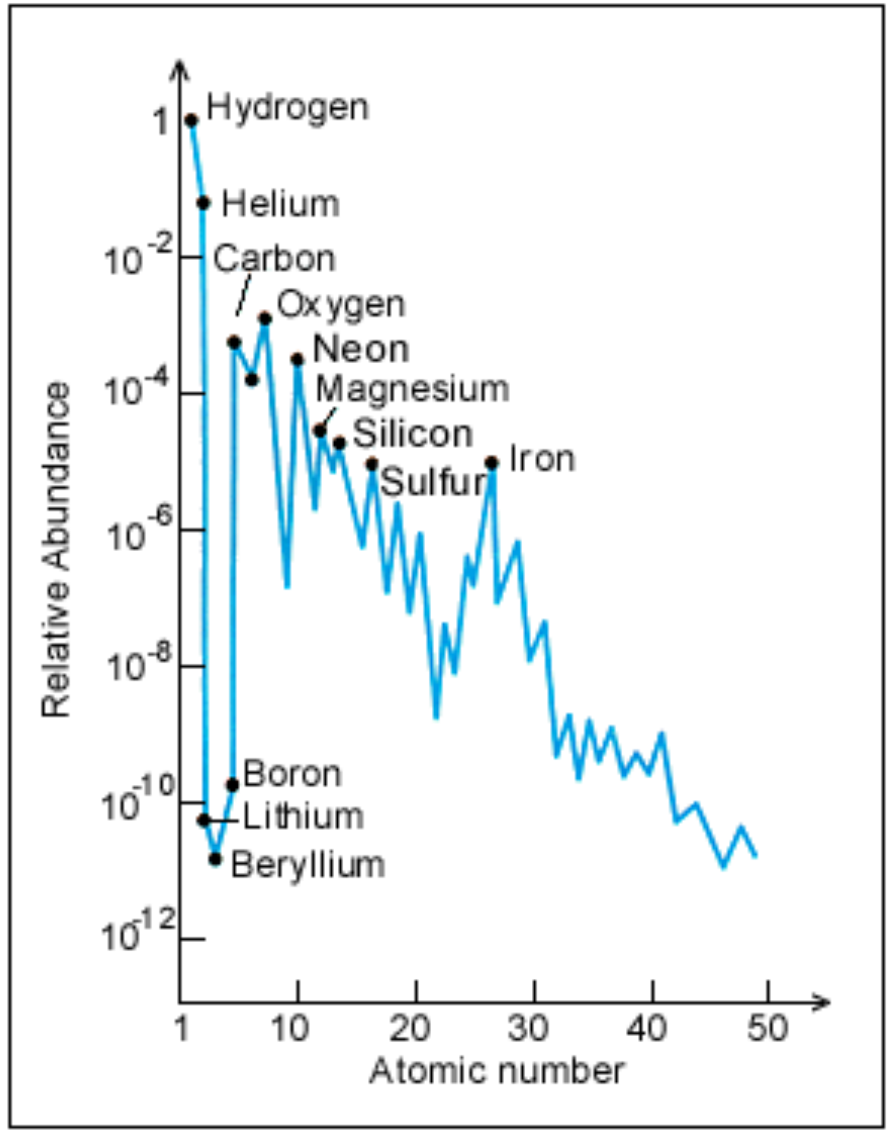
DNA molecule

C = carbon
H = hydrogen
N = nitrogen
O = oxygen
P = phosphorous

Sun's photospheric abundances reflect abundances of pre-solar nebula, enriched by stellar winds and supernovae of previous generations.

Note peaks where A/4 = integer (or Z/2 = integer). Note drop after Fe, where steady fusion no longer creates energy.

Li abundance low because it is destroyed in stars. 33^{rd} most abundant element.



Summary: stellar model building

$$\frac{dP}{dr} = -\rho g$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT_0}{dr} < -\left(1 - \frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2} = \frac{dT}{dr}|_{ad}$$

Also have ideal gas law and equations for specify fusion energy generation rate and tables for opacity. Solve these DE numerically in narrow spherical shells, subject to boundary conditions:

$$M_r \longrightarrow 0,\ \ L_r \longrightarrow 0 \quad \text{as} \ \ r \longrightarrow 0$$

$$T, \rho, P \longrightarrow 0 \quad \text{as} \ \ r \longrightarrow r_{star}$$

Now:

Stellar evolution: read chapter 12 (skip chapter 11 on Sun)

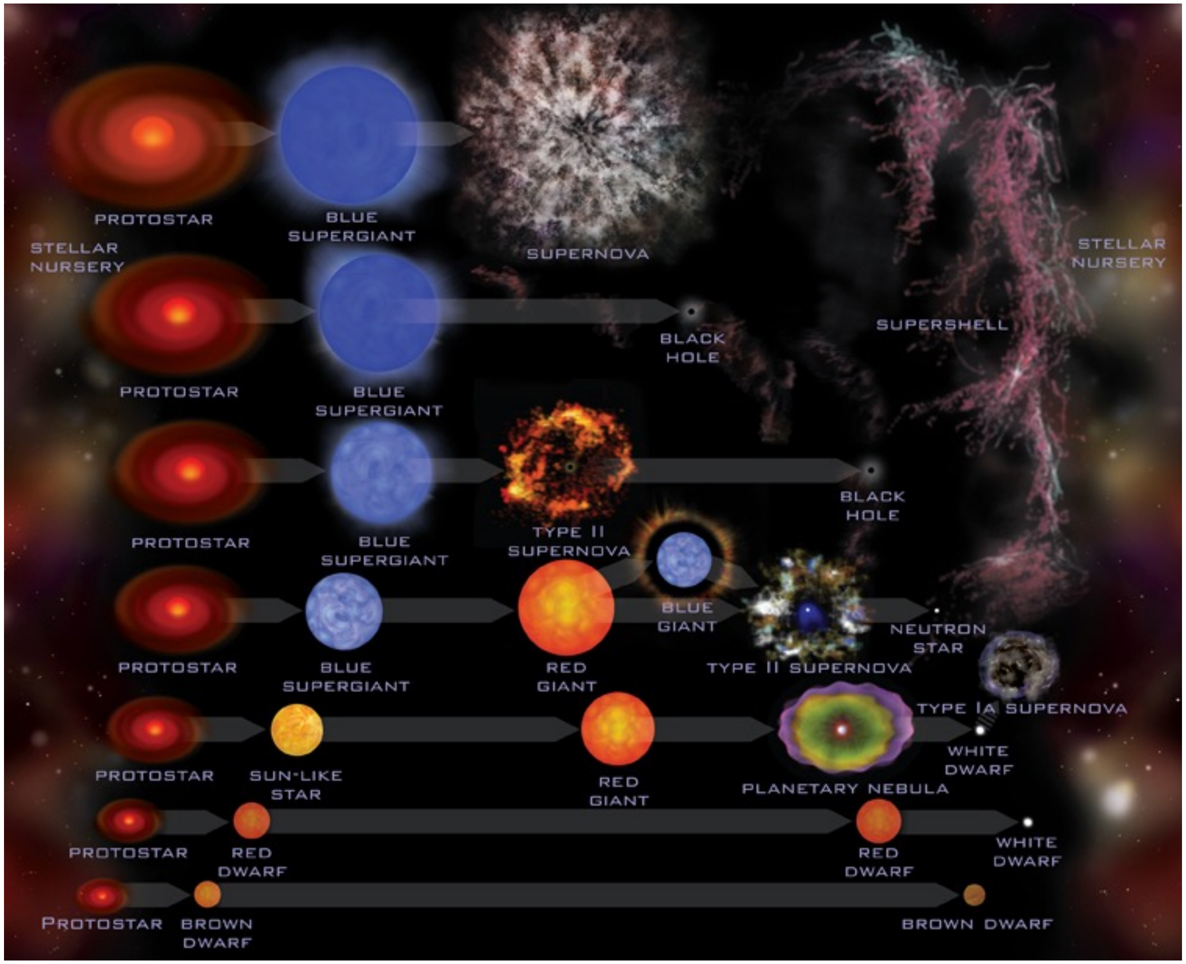
Review 2 on Tuesday Oct 25

EXAM 2 on Thursday Oct 27

Covers chapters 6, 9 and 10.

Calculator allowed, equations provided

Astronomy 421



Lecture 17: Stellar Evolution

Outline

Width of MS – evolution and metallicity

MS scaling laws

Evolution of “low-mass” stars

Evolution of “high-mass” stars, including supernovae

Clusters

Stellar evolution

Stars have many properties:

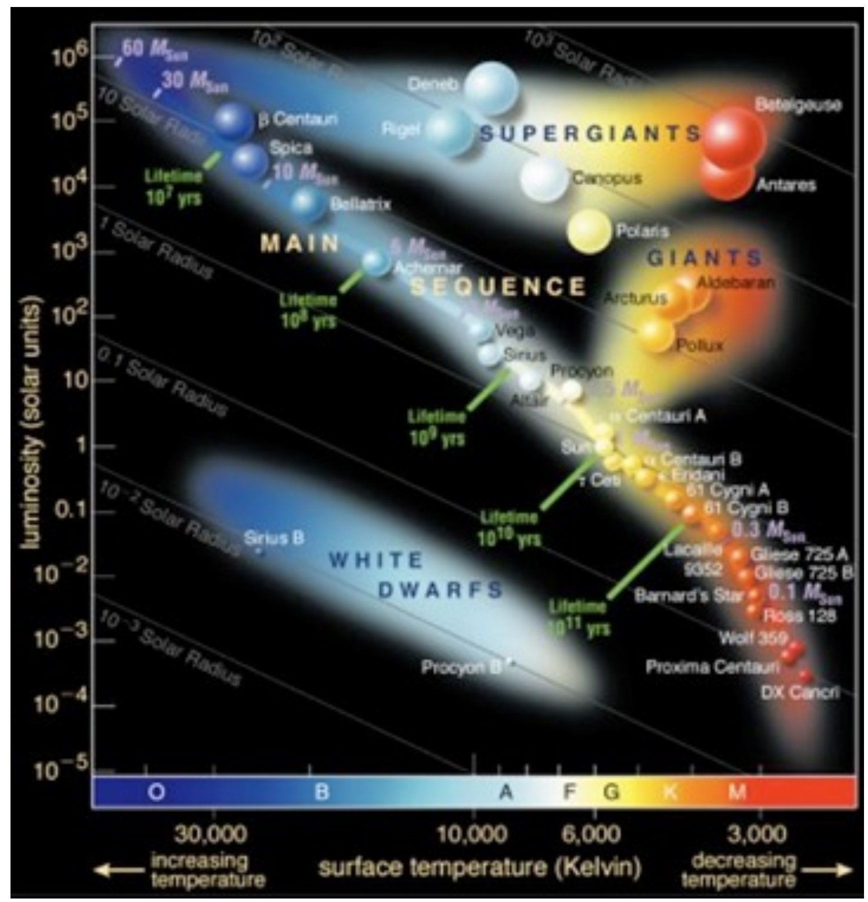
mass, luminosity, radius, chemical composition, surface temperature, core temperature, core density etc..

However, only two properties, the mass and initial chemical composition, dictate the other properties as well as the star's evolution (at least for isolated stars)

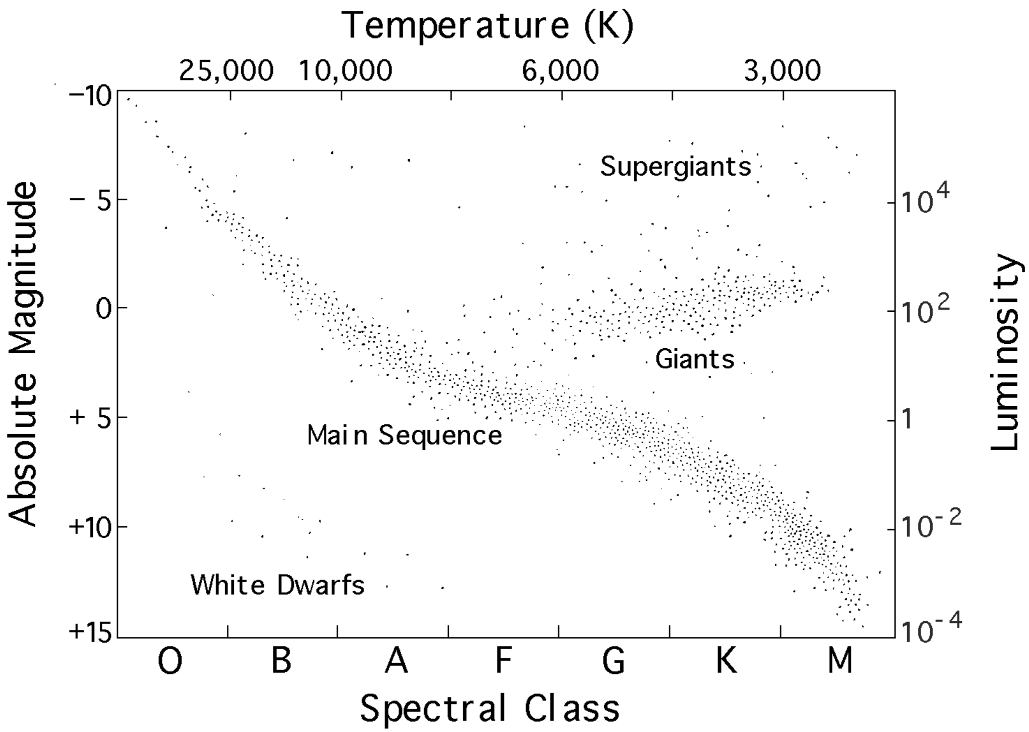
This is the Vogt-Russell "theorem".

There is only one way to make a star with a given mass and chemical composition.

=> If we have a protostar with a given mass and composition, we can calculate how that star will evolve over its entire life.



MS evolution and MS width:



Why does MS have width?

During MS H burns into He. After 50% of H has been used, the number of particles in core has decreased by a factor 0.73 (assuming original 10% He). What are the implications?

$$P = \frac{\rho k T}{\mu m_H}$$

As H => He, μ increases in core.

$\Rightarrow$ *P* drops => core contracts

$\Rightarrow$ ρ, *T* rise => *P* rises and new equilibrium established

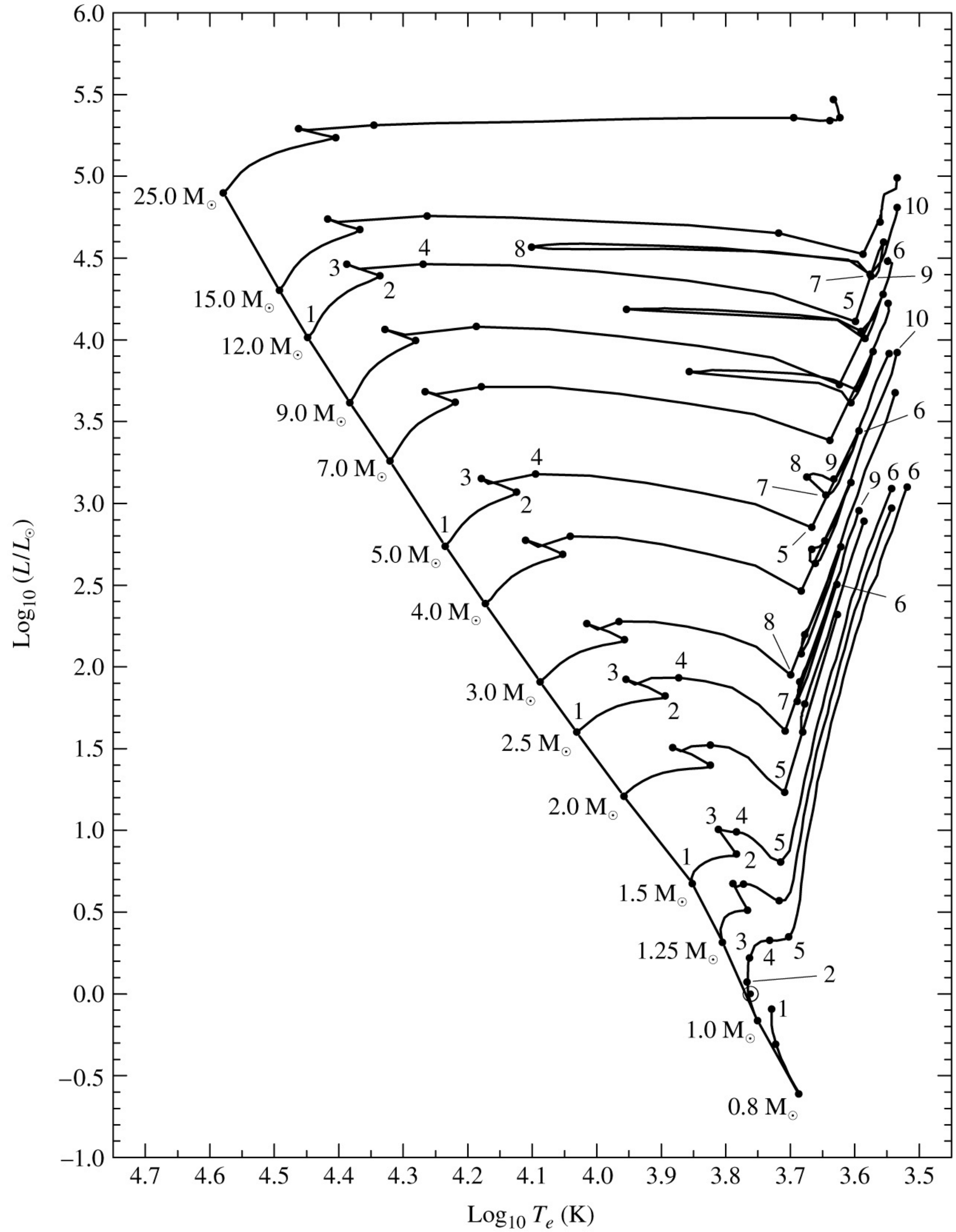
$\Rightarrow$ $\epsilon_{pp} \propto \rho X^2 T^4$ rises => *L* rises

Since its arrival on the MS, the Sun has become 30% more luminous. Stars of a given mass but different ages contribute to MS width.

Fig 13.1 from C+O.

ZAMS or Zero Age Main Sequence is at point 1.

Exhaustion of core H at point 2.



Metallicity and MS width:

Lower metal abundances

$\Rightarrow$ fewer free e^- in atmosphere (most e^-'s come from metals because of lower Ionization Potentials, despite lower abundances)

$\Rightarrow$ less H^- forms

$\Rightarrow$ less continuum opacity

$\Rightarrow$ see deeper down

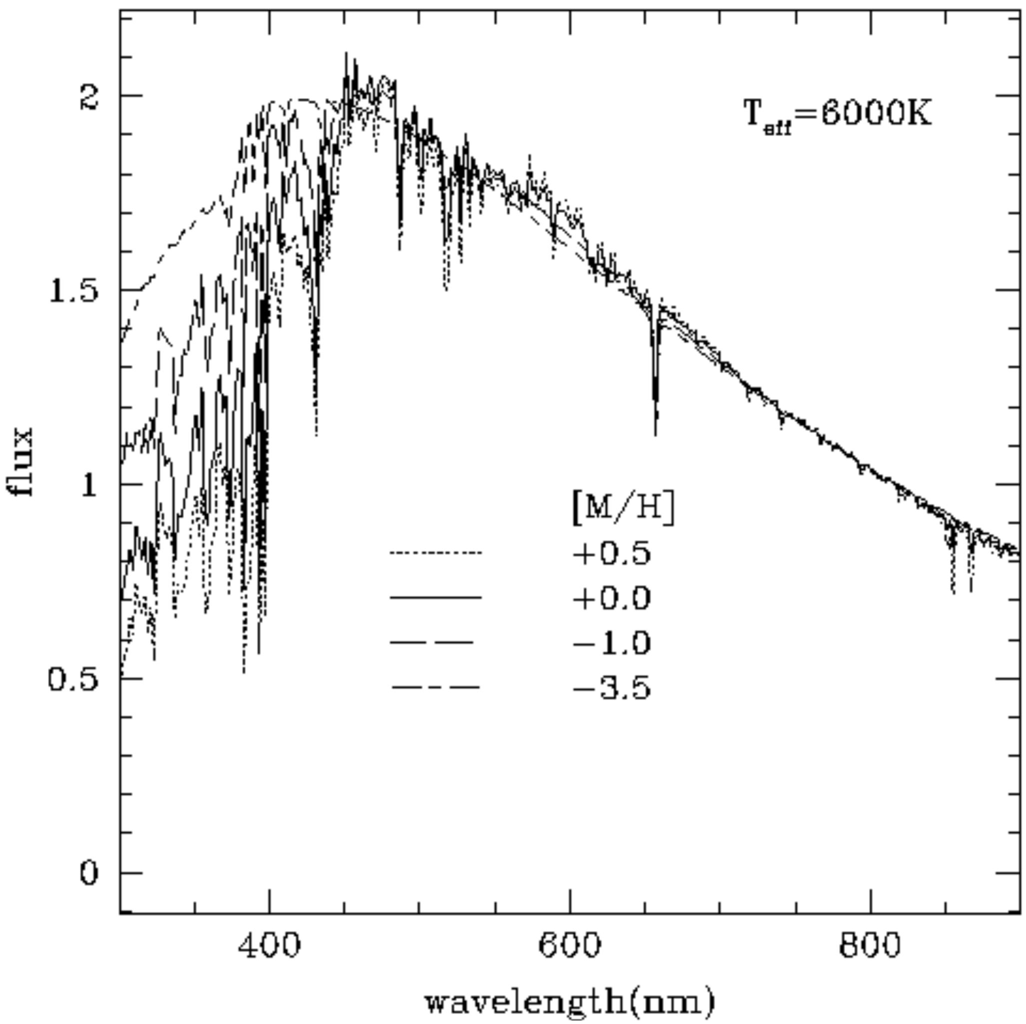
$\Rightarrow$ see higher atmospheric temperature.

Also, many metal absorption lines in blue, UV. "Line blanketing" - decrease in continuum due to many overlapping absorption lines.

Lower metals => more blue, UV, lower *B-V*.

Width on MS is due to both evolution as well as differences in chemical composition.

Examples of model stellar atmospheres with different “metallicities” showing effect of line blanketing on blue/UV light.



Main sequence relations:

M - L: $$\log(\frac{L}{L_\odot}) = \beta + \alpha \log(\frac{M}{M_\odot})$$

M	β	α
$M < 0.5M_\odot$	-0.15	2.85
$0.5M_\odot < M < 2.5M_\odot$	0.07	3.60
$M > 2.5M_\odot$	0.48	2.91

M - R:

$$\log(\frac{R}{R_\odot}) = \log(\frac{M}{M_\odot}) + 0.10 \quad (M \leq 0.4M_\odot)$$

$$\log(\frac{R}{R_\odot}) = 0.73 \log(\frac{M}{M_\odot}) \quad (M \geq 0.4M_\odot)$$

M - t_{MS}:

$$t_{\rm MS} \propto \frac{M_{core}}{L} \propto \frac{M}{L} \propto M^{1-\alpha}$$

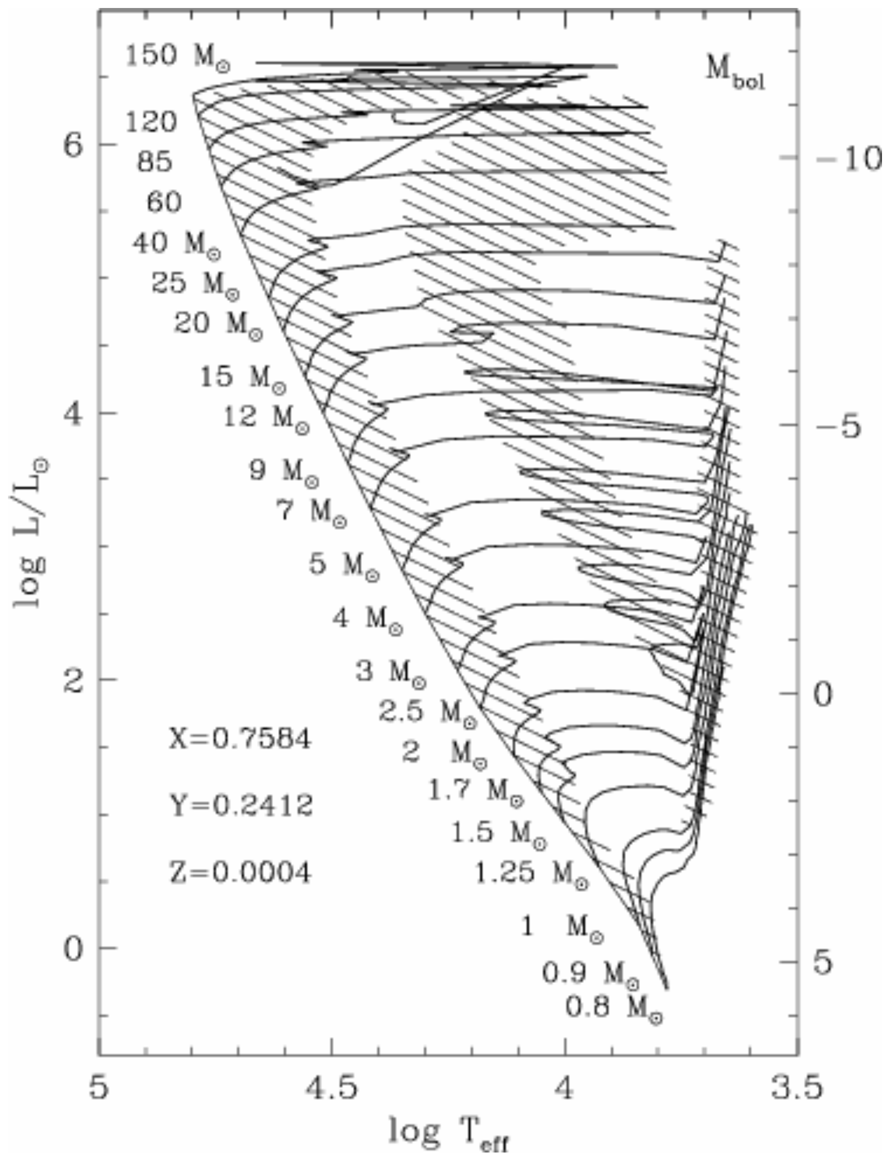
$$t_{\rm MS} \simeq t_0 \left(\frac{M}{M_\odot}\right)^{1-\alpha}$$

M	t_0 (10^{10} years)
$M < 0.5M_\odot$	1.55
$0.5M_\odot < M < 2.5M_\odot$	0.93
$M > 2.5M_\odot$	0.36

Geneva tracks

Right: *Geneva* tracks. Stellar modeling used to create evolutionary models of stars.

Done with complex computer code, based on physics we saw in Chapter 10.



Conditions for convection:

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$

dT/dr increases when opacity or radiative flux increases. At some point convection takes over, being a more efficient way of transporting energy. This happens in layers with very high opacity, or when a large fraction of energy is released within a small volume.

Low mass stars: cores are fusing at lower rates by the p-p chain at a lower temperature. High opacity in outer layers due to, e.g. H^-. Thus they have radiative cores and convective envelopes.

High mass stars: high fusion rate from CNO cycle leads to large core luminosity and steep temperature gradient. Opacity lower because most gas ionized. Hence convective cores and radiative envelopes.

HII regions

- Hot stars emit vast amounts of UV radiation as MS stars => ionizes surrounding gas => HII regions.
- Balance between ionization of H and recombination. We see Balmer lines (reddish because of Balmer -alpha line).
- In large star formation regions, the UV radiation from massive stars may be strong enough to stop formation of light stars before they have fully collapsed.

The part in balance between ionization and recombination is called a *Strömgren sphere.* Size can be estimated, assuming no photons escape so that all end up ionizing an H atom.

Thus the total ionization rate equals production rate of ionizing photons by central star, N.

Recombination rate per H ion , R_p proportional to the interaction probability (determined from quantum mechanics) α and the density of free electrons, n_e.

Total recombinaton rate R_{tot} = recombination rate per H ion times number of H ions. $R_p = \alpha n_e$,

$$R_{tot} = R_p n_p \frac{4\pi r^3}{3} = \alpha n_e^2 \frac{4\pi r^3}{3}$$

If equilibrium, $R_{tot} = N$. Thus:

$$\Rightarrow r = \left(\frac{3N}{4\pi\alpha n_e^2}\right)^{1/3}$$

Example sizes of Strömgren spheres:

Assume typical values $\alpha \simeq 3 \times 10^{-13}\ \mathrm{cm}^3\,\mathrm{s}^{-1}$ $n_e \simeq 10\ \mathrm{cm}^{-3}$

G2V star $N = 10^{39}\,\mathrm{s}^{-1}, \quad r \simeq 2 \times 10^{16}\,\mathrm{cm} \ \simeq 0.007\ \mathrm{pc}$

B0V star $N = 4 \times 10^{46}\,\mathrm{s}^{-1}, \quad r \simeq 6 \times 10^{18}\,\mathrm{cm} \ \simeq 2\ \mathrm{pc}$

O5V star $N = 3 \times 10^{49}\,\mathrm{s}^{-1}, \quad r \simeq 6 \times 10^{19}\,\mathrm{cm} \ \simeq 20\ \mathrm{pc}$

Massive O stars can evaporate gas cloud in which they were formed.

In practice, the HII region will be overpressured wrt surrounding gas and thus expand, further destroying stellar birth place.

How bright can a star be?

There is a physical limit to how bright a star can be. Recall total pressure is the sum of ideal gas pressure and radiation pressure:

$$P = \frac{\rho k T}{\mu m_H} + \frac{1}{3} a T^4$$

For high T, low ρ, radiation pressure can dominate. In this case, the pressure gradient is:

$$\frac{dP_{rad}}{dr} = -\frac{\overline{\kappa}\rho}{c} F_{rad}$$

$$\Rightarrow \quad \frac{dP}{dr} = -\frac{\overline{\kappa}\rho}{c}\frac{L}{4\pi r^2}$$

To remain in hydrostatic equilibrium:

$$\frac{dP}{dr} = -g\rho = -G\frac{M\rho}{r^2}$$

This leads to maximum radiative luminosity a star can have and be stable, the *Eddington Limit*

$$\boxed{L_{edd} = \frac{4\pi Gc}{\overline{\kappa}}M}$$

Observations: massive stars tend to have L near L_{edd}. What happens if $L > L_{edd}$?

If $L > L_{edd}$, pressure gradient greater than hydrostatic equilibrium case => radiation drives wind.

For massive stars, opacity is from e^- scattering (why?)

$$\overline{\kappa} = \overline{\kappa_{es}} = 0.2(1+X)\ \mathrm{m^2\,kg^{-1}}$$

For X=0.7

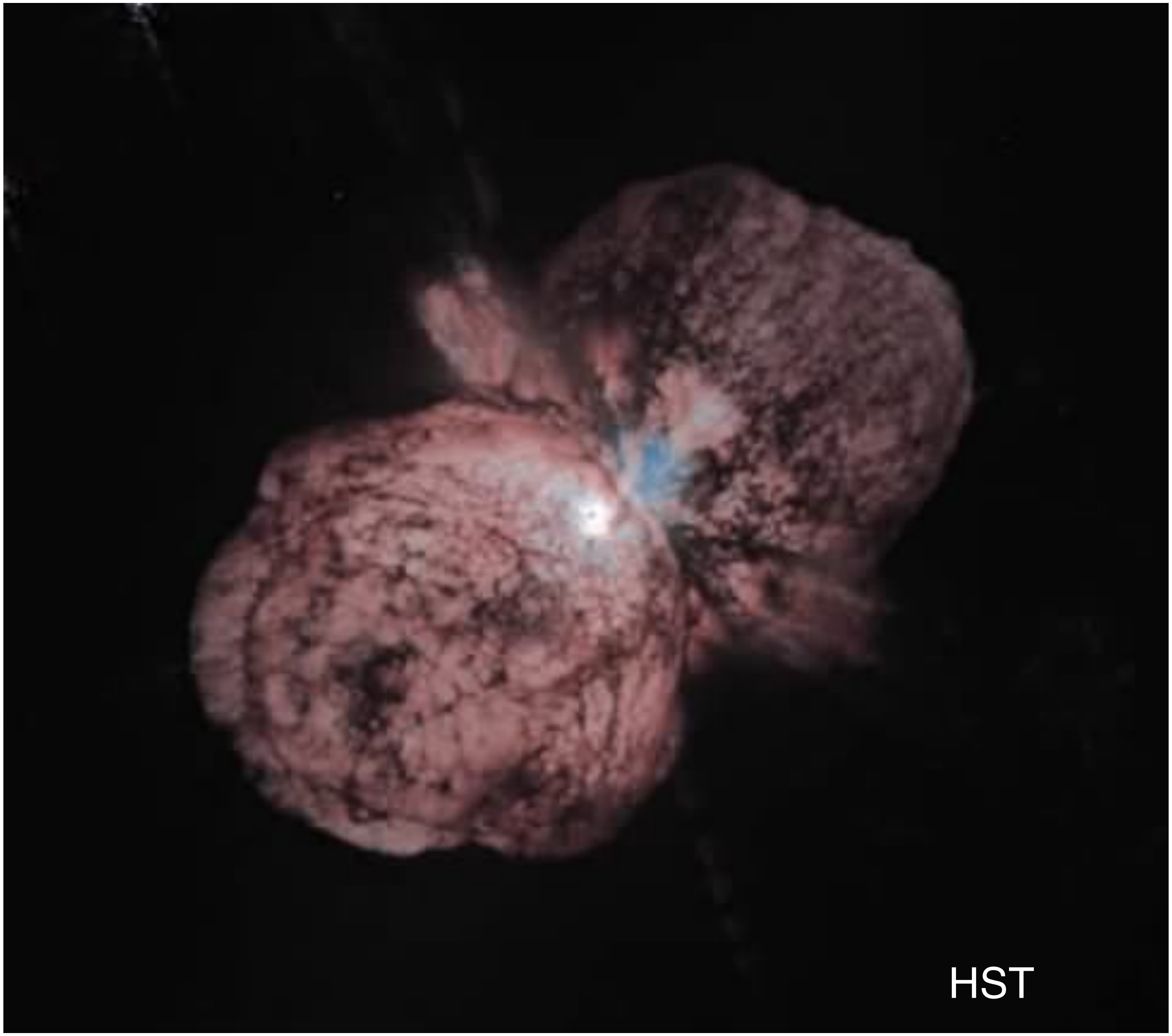
$$L_{edd} \simeq 1.5 \times 10^{31} \frac{M}{M_\odot}\ \mathrm{W}$$

$$\frac{L_{edd}}{L_\odot} \simeq 38{,}000 \frac{M}{M_\odot}$$

From massive star models + observations, we know $L \propto M^3$

$$L \sim L_{edd} \text{ for } M \sim 100 M_\odot$$

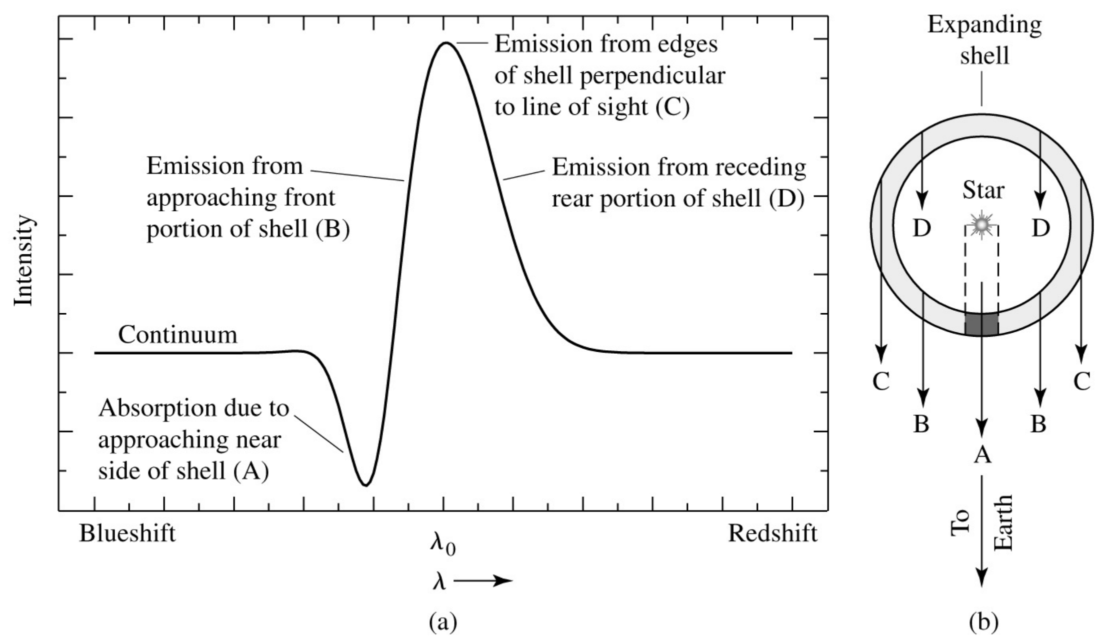
Thus, for $M > 100\ M_\odot$, there is huge mass loss, unstable.
Example is Eta Carina, M ~ 120 $M_\odot$.


HST

So are there observational evidence of this theory of star formation? How do we know this is happening when it is deep inside dark, opaque clouds?

1) IR sources embedded in molecular clouds (evidence of energy from collapsing clouds)

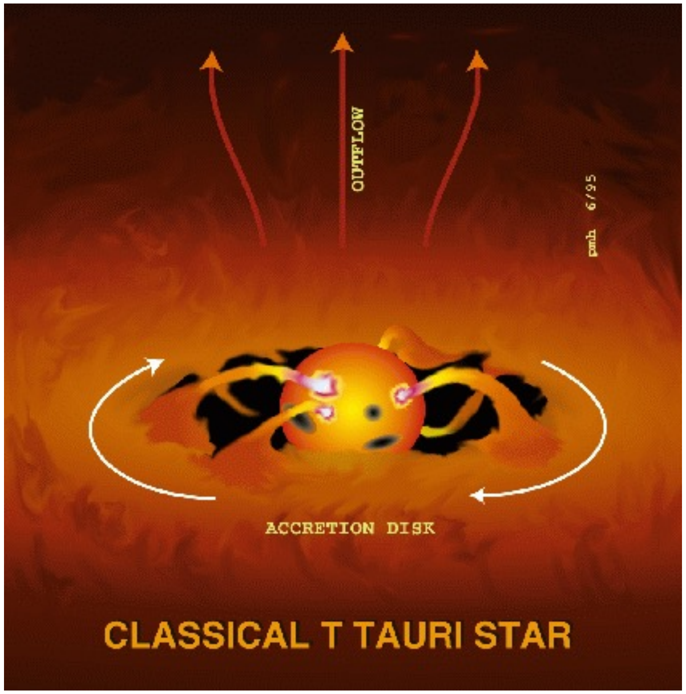
2) T Tauri stars - low mass pre-MS stars often highly variable with strong emission lines. Exhibits *P Cygni* line profiles, interpreted as mass loss.

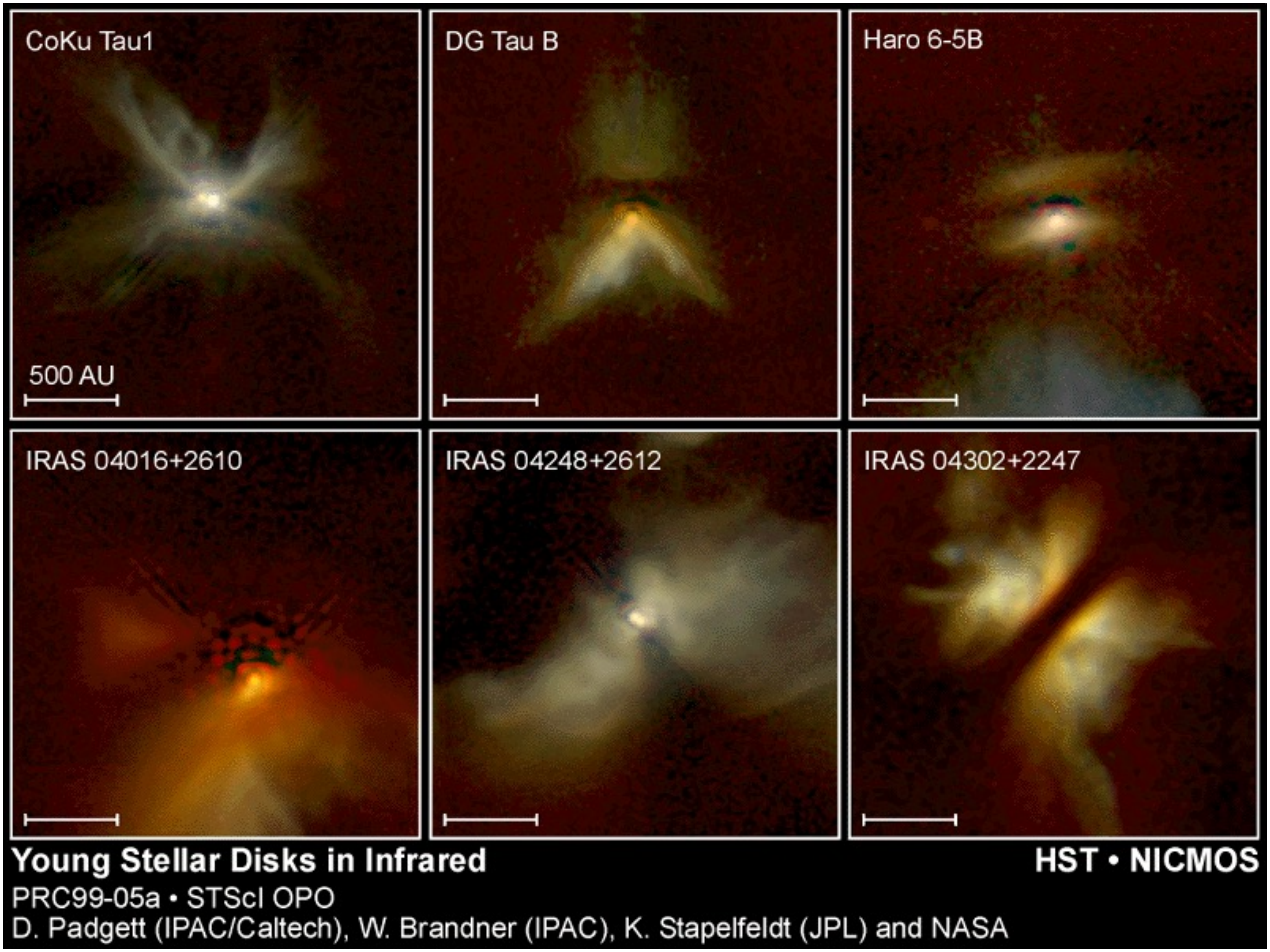


Classical T Tauri stars surrounded by discs of gas and dust, due to initial rotation of cloud.

Weak-lined T Tauri stars have lost their disks and are more evolved.

Found embedded in cloud of gas in which they were born (Trapezium Cluster in Orion Nebula is an example).




CoKu Tau1
DG Tau B
Haro 6-5B
500 AU
IRAS 04016+2610
IRAS 04248+2612
IRAS 04302+2247
Young Stellar Disks in Infrared
HST • NICMOS
PRC99-05a • STScI OPO
D. Padgett (IPAC/Caltech), W. Brandner (IPAC), K. Stapelfeldt (JPL) and NASA

3) Herbig-Haro objects - mass loss in jets from protostellar objects. Somehow a fraction of the material accreted onto the star is ejected perpendicular to the disk plane in a highly collimated stellar jet. When jet collides with surrounding gas it forms shock waves observed as HH objects.