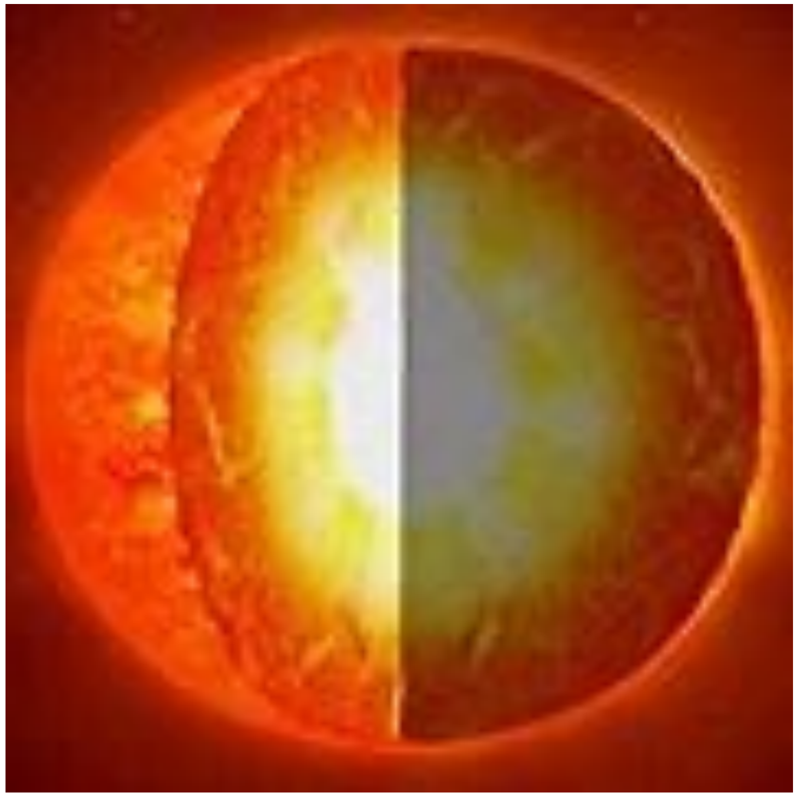
Astronomy 421



Lecture 16: Stellar Interiors II

Stellar Energy Sources

Historical possibility: gravitational potential energy gained in formation. How much available?

$$E \sim \frac{GM^2}{R} \sim 10^{41}\ J$$

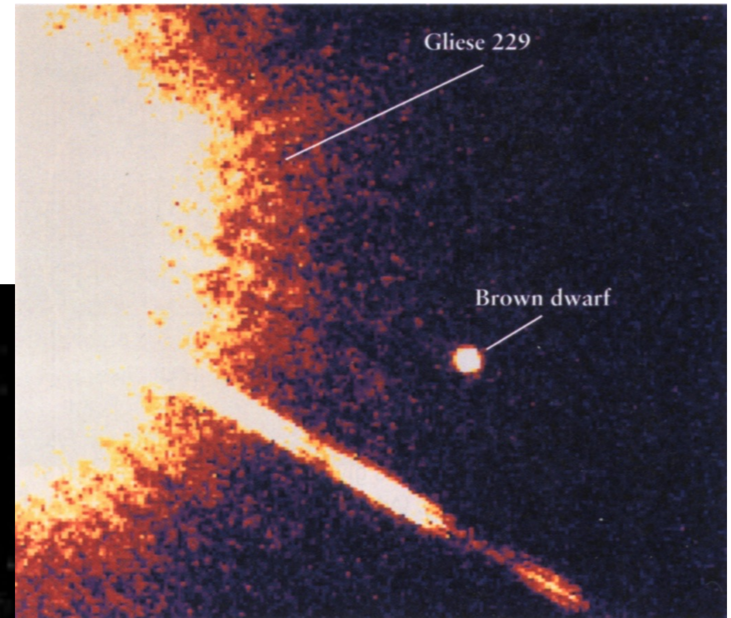
Sun could shine this way for

$$t = \frac{E}{L_\odot} \sim 10^7\ yrs$$

("Kelvin-Helmholtz timescale")

Need something else! (But, stars can convert gravitational energy to thermal energy, some of which may emerge as radiation, if in contracting phase, e.g. collapsing protostars.)

Brown Dwarfs



Gliese 229B discovered in 1995, just 19 light-years away

Nuclear energy

Essence of Main Sequence fusion reactions:

$$4\text{H} \rightarrow \text{He (+ low mass remnants)}$$

$$\text{mass: } 4m_p \rightarrow 3.97m_p \ \text{(0.7\% less)}$$

Deficit of $0.03m_p$ converted to energy:

$$E = mc^2 = 0.03m_pc^2 \quad \text{(for every four H's)}$$

Atomic nuclei: consist of *nucleons* (protons and neutrons):

$$A = Z + N$$

mass number = # of nucleons

# of neutrons

***atomic number* or # of protons**

Notation: ${}^{A}_{Z}X$ $\quad$ X is the chemical symbol of the element. Examples: ${}^{1}_{1}H,\ {}^{4}_{2}He$

Element identified by # protons. Isotope identified by # neutrons.

Examples: ${}^{1}_{1}H$, vs. ${}^{2}_{1}H$, ${}^{4}_{2}He$ vs. ${}^{3}_{2}He$

Nucleus held together by the attractive *strong nuclear force.*

In $4\,{}^{1}_{1}H \rightarrow {}^{4}_{2}He$, $m_{He} < 4m_H$ so energy is released.

$$\Delta mc^2 = (4m_H - m_{He})c^2$$

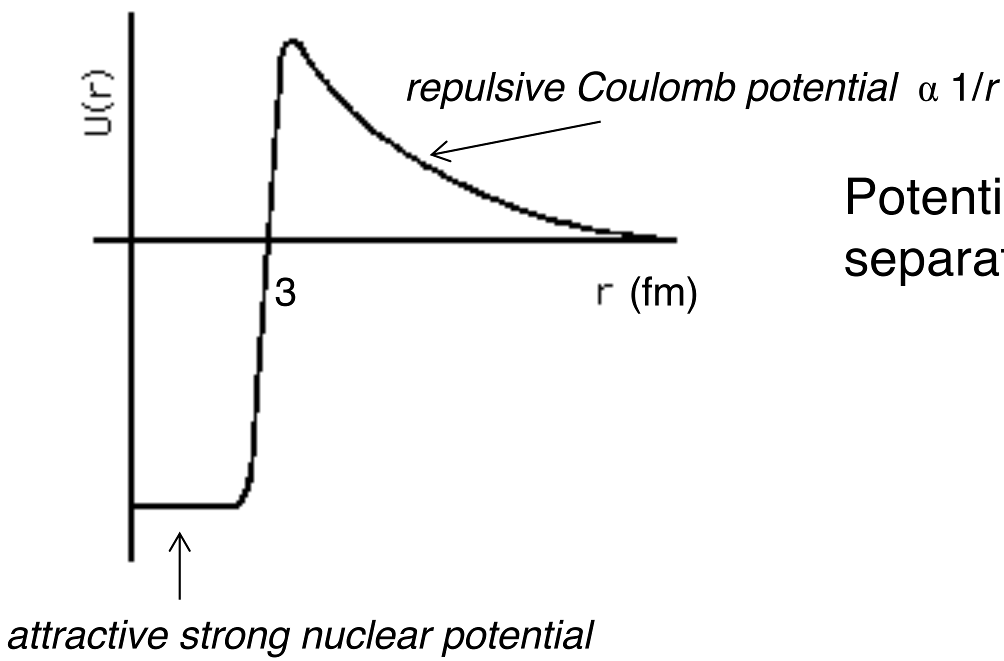
This is also called *binding energy* of the nucleus. Must put in this energy to break up He into protons.

Fusion reactions

Under what conditions can fusion occur?

1) Nuclei can interact via the four fundamental forces, but only EM and strong nuclear force important here.

2) To fuse, two positively charged nuclei must overcome the Coulomb barrier (the long range force $\propto 1/r^2$) to reach separation distances where the strong force dominates (10^{-15} m, typical nuclear size)



Potential energy versus nuclei separation.

The height of the Coulomb barrier is given by:

$$\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r}$$

e = charge of electron = 1.6×10^{-19} C,

ε_0 = permittivity of free space = 8.85×10^{-12} $C^2N^{-1}m^{-2}$

Calculate potential energy required for fusion of two H nuclei for r = 1 fm.
Compare to the average kinetic energy of a particle ($3kT/2$) to find $T \sim 10^{10}$ K!

But T at center of Sun only 1.6×10^7 K.

Quantum tunneling

According to Quantum Mechanics, there is a finite probability that a particle will penetrate the Coulomb barrier, due to the Heisenberg uncertainty in its position, even if it does not come close enough classically.

The probability for this tunneling for two like charges colliding at speed *v* depends on (Gamow 1928):

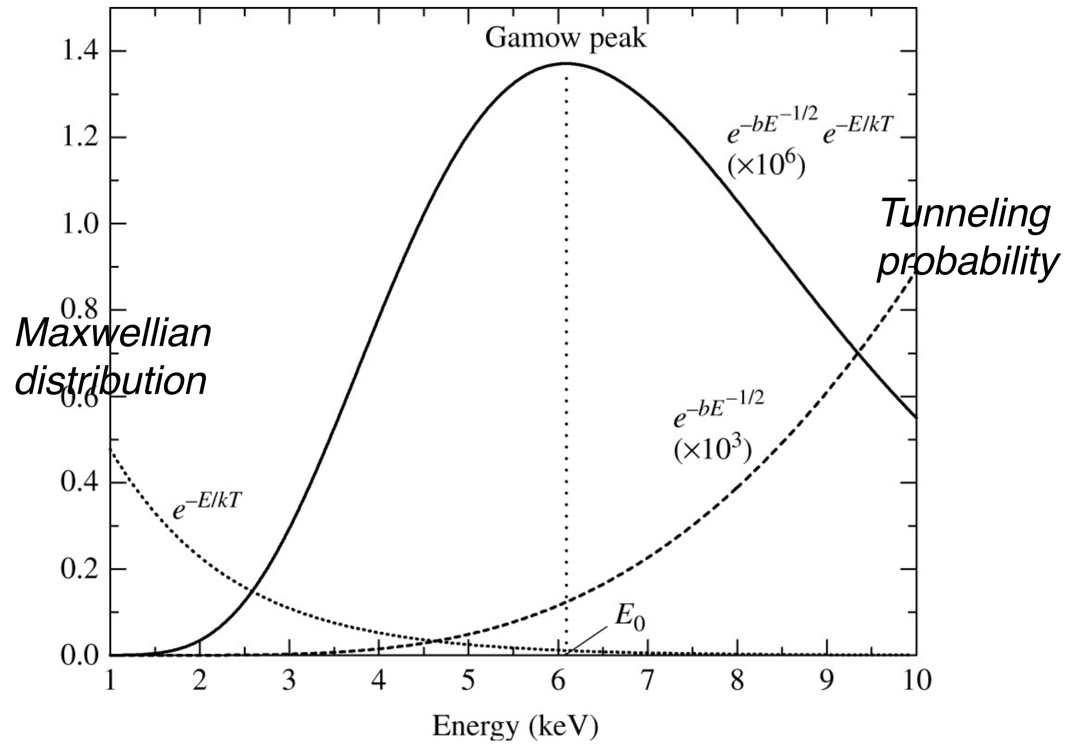
$$e^{-\frac{\pi Z_1 Z_2 e^2}{\epsilon_0 h v}}$$

Hence, this decreases with higher charge and increases with particle velocity *v* (thus energy of collision). But we also know that the velocity follows the Maxwell-Boltzmann distribution for an ideal gas. The fusion probability is therefore proportional to the product

$$e^{-\frac{\pi Z_1 Z_2 e^2}{\epsilon_0 h v}} (kT)^{-\frac{3}{2}} e^{-\frac{m v^2}{2kT}}$$

The Gamow peak

Fusion is most likely to occur in the energy window defined as the Gamow peak, which reflects the product of the Maxwell-Boltzmann distribution and tunneling probability. Area under Gamow peak determines reaction rate!



A higher electric charge means a greater repulsive force => higher E_{kin} and T required before reactions occur. For two protons, Gamow peak is at 10^6 keV which is, using $E=3kT/2$, about $T \sim 10^7$ K.

Simplified treatment – see C+O for complications.

Nuclei that are highly charged are also the more massive ones, so reactions between light elements occur at lower T's than reactions between heavy elements.

Nuclear reaction rates:

Can parameterize the number of reactions per unit volume and time:

$$r_{1,2} \approx r_0 X_1 X_2 \rho^{\alpha\prime} T^{\beta}$$

r_0 is a constant

X_i is mass fraction of nucleus *i*

$\alpha\prime,\ \beta$ are constants, $\alpha\prime = 2$ for two-body collisions

Most importantly: the rate depends on *T.*

If ϵ_0 = energy released per reaction (Δmc^2) , then the energy liberated per second per kg of stellar material, $\epsilon_{1,2}$, is then:

$$\epsilon_{1,2} = \frac{\epsilon_0}{\rho} r_{1,2}$$

$$\boxed{\epsilon_{1,2} = \epsilon_0' X_1 X_2 \rho^{\alpha} T^{\beta}}$$

$$\alpha = \alpha' - 1$$
$$\epsilon_0' = \text{constant}$$

If ϵ is total energy released per kg per sec by <u>all</u> reactions, then the luminosity from a mass *dm* is:

$$dL_r = \epsilon dm = \epsilon \rho dV = 4\pi r^2 \rho \epsilon dr \quad \text{(spherical shell)}$$

$$\text{Thus } \boxed{\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon}$$

(fourth fundamental differential equation of stellar structure)

Note, as with the mass conservation equation, $L_r = \int_0^r 4\pi r^2 \rho \epsilon dr$ is luminosity generated *within r*. The difficult part here is to know ϵ .

Hydrogen fusion reactions

Conservation laws for nuclear reactions:

1) Mass-energy
2) Momentum
3) Charge
4) Nucleon number
5) Lepton number

Leptons: *light particles,* for example e^-, e^+, ν_e, $\overline{\nu_e}$
and anti-particles have opposite baryon/lepton number to their particles

$$e^-, \nu_e \text{ have lepton number } +1$$
$$e^+, \overline{\nu_e} \text{ have lepton number } -1$$

<u>The proton-proton chain</u>

$$I \quad {}_1^1H + {}_1^1H \;\longrightarrow\; {}_1^2H + e^+ + \nu_e$$

$$II \quad {}_1^2H + {}_1^1H \;\longrightarrow\; {}_2^3He + \gamma$$

$$III \quad {}_2^3He + {}_2^3He \;\longrightarrow\; {}_2^4He + {}_1^1H + {}_1^1H$$

III) happens 69% of the time in the Sun. The other 31%:

$${}_2^3He + {}_2^4He \;\longrightarrow\; {}_4^7Be + \gamma$$

$${}_4^7Be + e^- \;\longrightarrow\; {}_3^7Li + \nu_e$$

$${}_3^7Li + {}_1^1H \;\longrightarrow\; 2\,{}_2^4He$$

Second chain also more common when more ${}_2^4He$ has been made.
In very cool stars, the chain stops at ${}_2^3He$.

For only 0.3% of the time, the ^{7}Be nucleus can react with a proton rather than an electron:

$$^7Be + ^1H \rightarrow {}^8B + \gamma$$
$$^8B \rightarrow {}^8Be + e^+ + \nu_e$$
$$^8Be \rightarrow 2\,^4He$$

Energy generation rate of p-p chain

$$\epsilon_{pp} \approx 10^{-12} \rho X^2 T_6^4 \;\; \mathrm{W\,kg^{-1}}$$

(X = H mass fraction). For T's near 1.5 x 10^7 K. $T_6 = T/10^6$ K

CNO cycle:

4H $\rightarrow$ He but with *catalysts.*

$$I \quad {}^{12}_{6}C + {}^{1}_{1}H \longrightarrow {}^{13}_{7}N + \gamma$$

$$II \quad {}^{13}_{7}N \longrightarrow {}^{13}_{6}C + e^{+} + \nu$$ (${}^{13}_{7}N$ unstable, $\tau \sim$ 7 min)

$$III \quad {}^{13}_{6}C + {}^{1}_{1}H \longrightarrow {}^{14}_{7}N + \gamma$$

$$IV \quad {}^{14}_{7}N + {}^{1}_{1}H \longrightarrow {}^{15}_{8}O + \gamma$$ (rate determining step)

$$V \quad {}^{15}_{8}O \longrightarrow {}^{15}_{7}N + \nu + e^{+}$$ (${}^{15}_{8}O$ unstable, $\tau \sim$ 82 sec)

$$VI \quad {}^{15}_{7}N + {}^{1}_{1}H \longrightarrow {}^{12}_{6}C + {}^{4}_{2}He$$

Requires higher *T*'s than the p-p chain (why?) but is much more efficient => short lives of high mass stars.

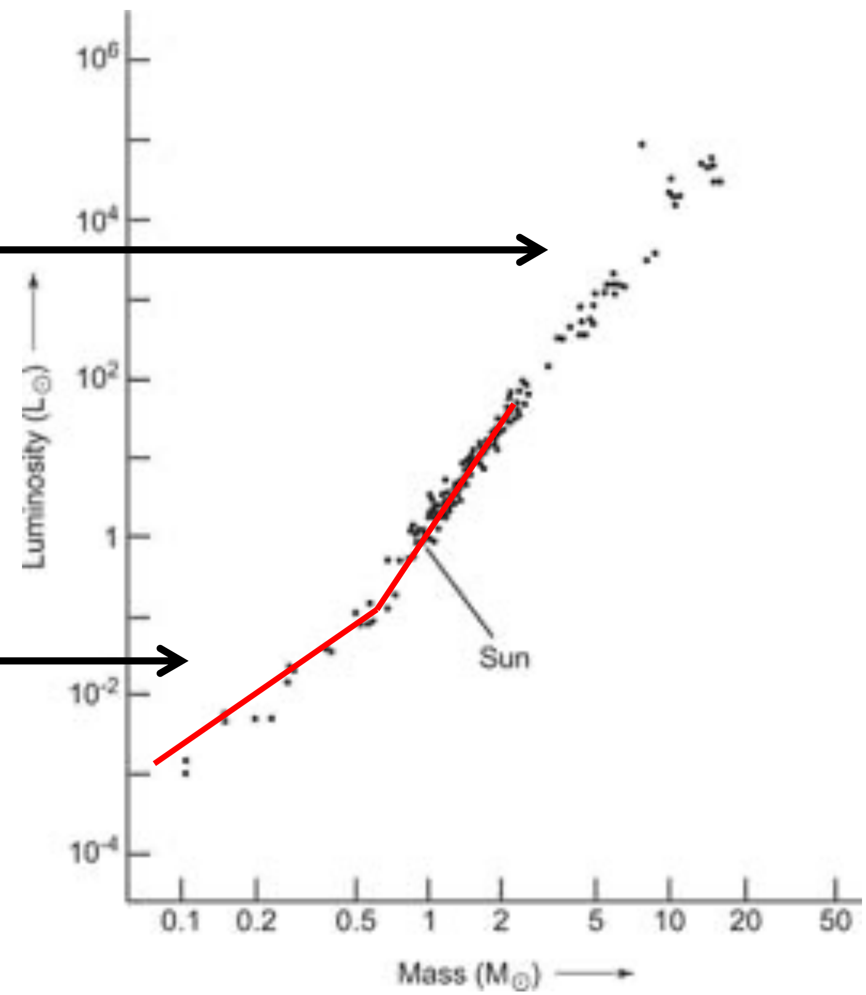
$$\epsilon_{CNO} \approx 8.24 \times 10^{-31} \rho X X_{CNO} T_6^{19.9} \ \mathrm{W\,kg^{-1}}$$

X_{CNO} is the total mass fraction of C,N,O.

Explains the break in the mass-luminosity relation:

$$\left(\frac{L}{L_\odot}\right) = \left(\frac{M}{M_\odot}\right)^4$$

$$\left(\frac{L}{L_\odot}\right) = \left(\frac{M}{M_\odot}\right)^2$$



Worksheet 10. Calculate the lifetime in years on the main sequence for a 1 solar mass star, and for a 10 solar mass star.

Assume core was initially all H, and *L* has been constant

$$\text{Lifetime} = \frac{0.1 \times 0.007\, M_{\odot} c^2}{L_{\odot}}$$

$$t = \frac{8.78 \times 10^{43}\ \text{J}}{3.96 \times 10^{26}\ \text{J/sec}}$$

$$t = 7.1 \times 10^{9}\ \text{years}$$

What about a 10 solar mass star?

$t \sim E/L \sim M/M^4 \sim 1/M^3$

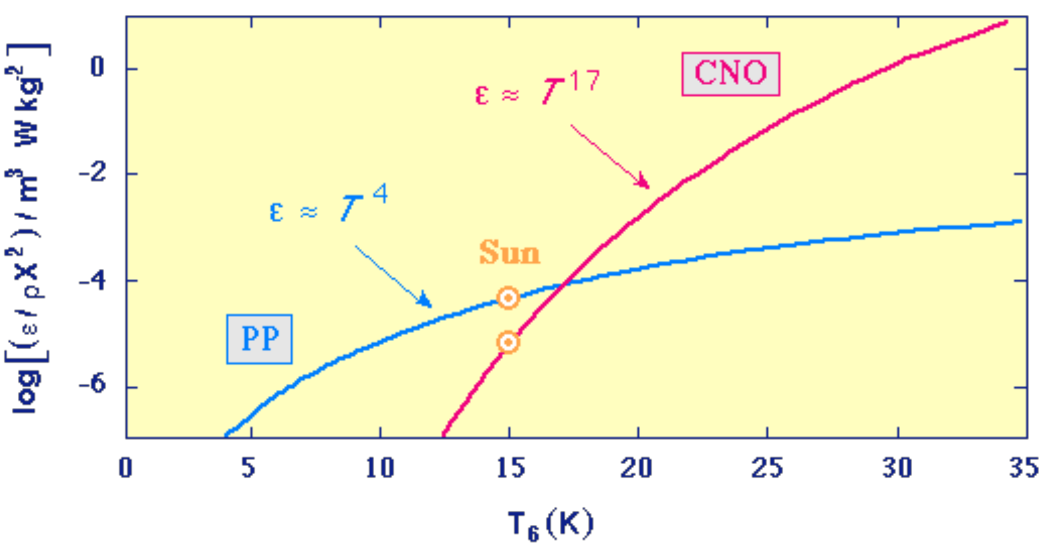
1000 times shorter so lifetime is 7.1×10^6 years.

If we equate the rate of energy production in the PP chain and the CNO cycle, we can find a *T* at which they produce the same rate of energy production.

This occurs around $T \sim 1.7 \times 10^7$ K.

Below this temperature the PP chain dominates, and above it the CNO cycle dominates.

This temperature limit occurs in stars slightly more massive than the Sun, around 1.2-1.5 solar masses.



Triple alpha process:

He $\rightarrow$ C in post-MS stars. Simplest reaction should be fusion of two He nuclei. But, there is no stable configuration with A=8. For example, ^{8}Be has a lifetime of about 10^{-16} s!

However, a third He nucleus can be added before ^{8}Be decay, forming ^{12}C by the triple alpha process. This makes it essentially a three-body interaction.

$$I \;\; {}_2^4He + {}_2^4He \;\rightarrow\; {}_4^8Be + \gamma$$

$$II \;\; {}_4^8Be + {}_2^4He \;\rightarrow\; {}_6^{12}C + \gamma$$

$$\epsilon \propto Y^3 \rho^2 T^{41}$$ (*Y* is mass fraction of He)

This requires T > 10^8 K. Occurs in cores after H exhausted, and they have compressed and heated up.

Other reactions at higher T's produce O, Ne, Na, Mg, Si, P and S.

Common reactions include:

$$X +^4_2 He \;\rightarrow\; X'$$

$$^{12}_{6}C +^4_2 He \;\rightarrow\; ^{16}_{8}O + \gamma$$

$$^{16}_{8}O +^4 He \;\rightarrow\; ^{20}_{10}Ne + \gamma$$

$$^{12}_{6}C + ^{12}_{6}C \;\rightarrow\; ^{24}_{12}Mg + \gamma$$

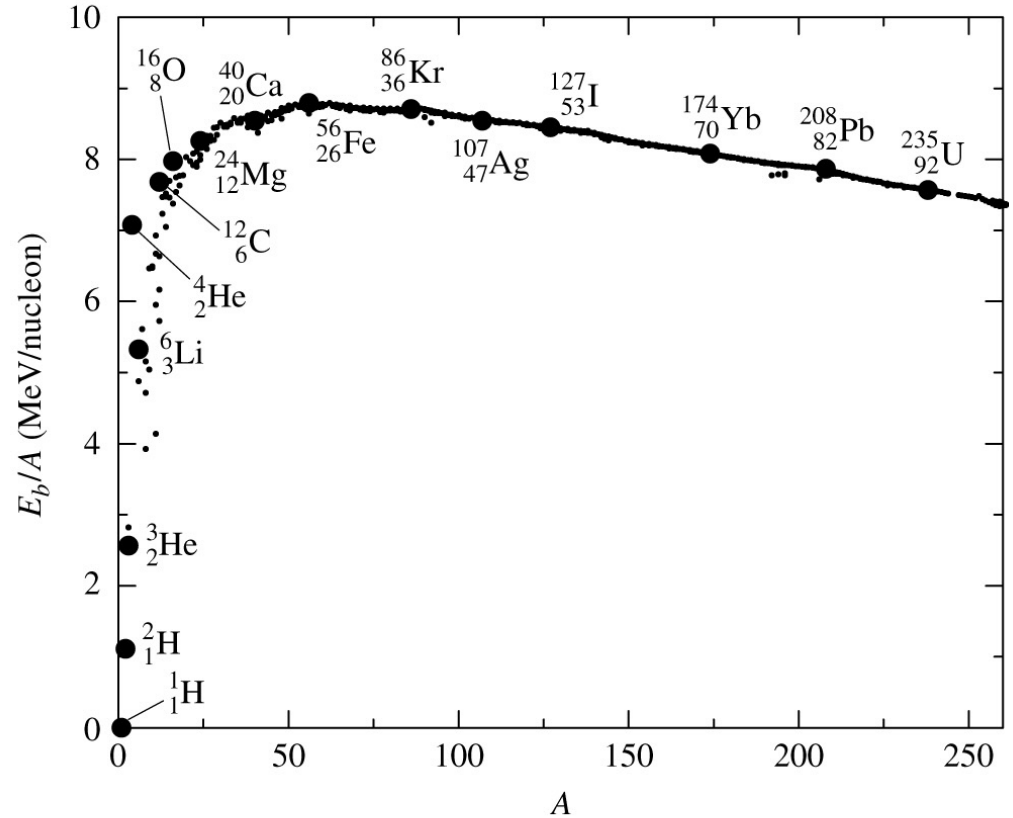
$$^{12}_{6}C + ^{12}_{6}C \;\rightarrow\; ^{23}_{12}Mg + n$$

note: some reactions make free neutrons, important later

$$^{16}_{8}O + ^{16}_{8}O \;\rightarrow\; ^{32}_{16}S + \gamma$$

(=> elements with A/4 = integer are abundant)

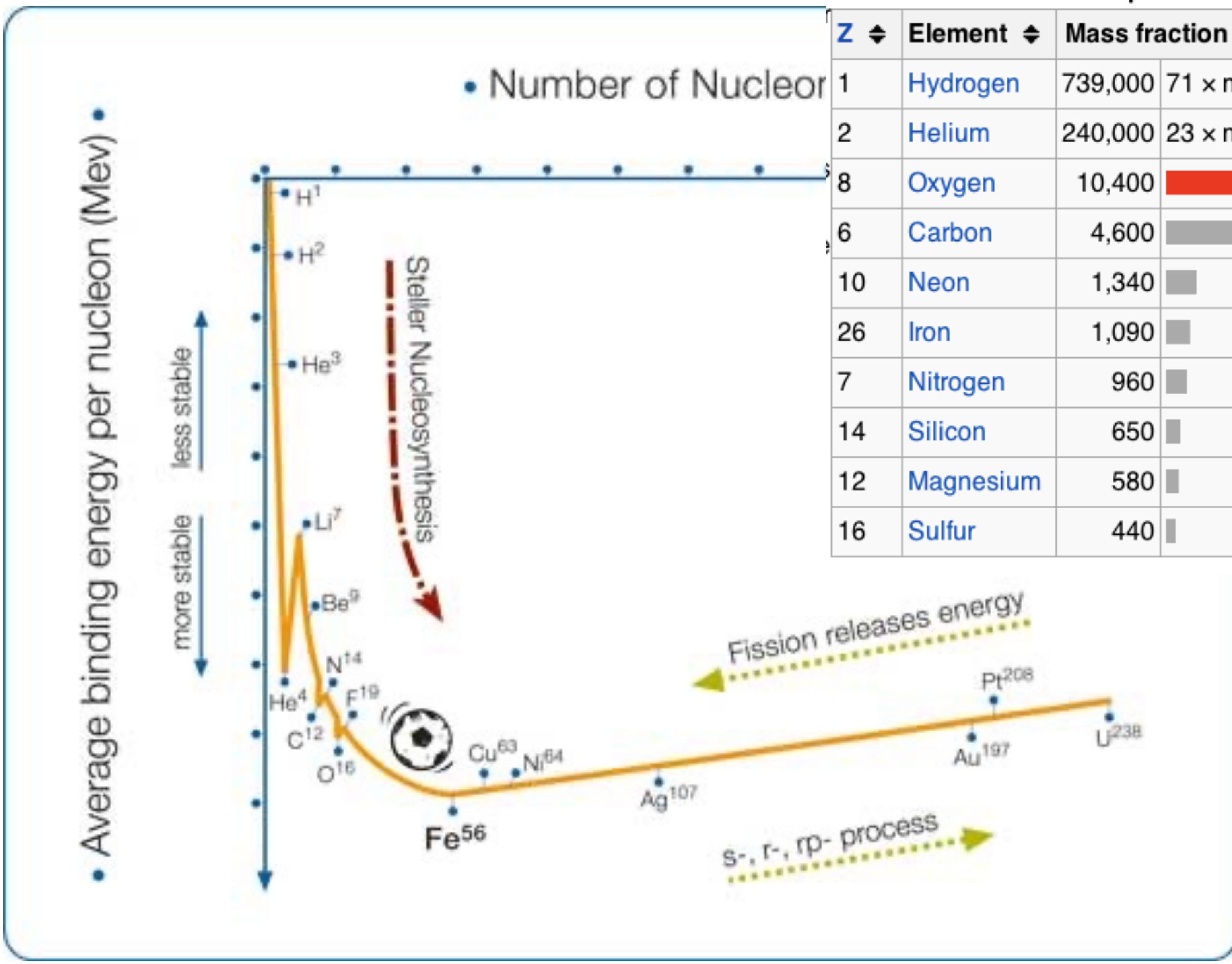
The general trend is an increase of binding energy per nucleon with atomic mass up to A=56 (Fe), then a slow decline.



Fe most strongly bound nucleus, adding more protons will cause the additional Coulomb force to become more important than the strong nuclear force.

Must put energy in to make nuclei larger than Fe, thus no energy generation from fusion beyond Fe. Fission of heavy nuclei into lighter ones can release energy (down to Fe).

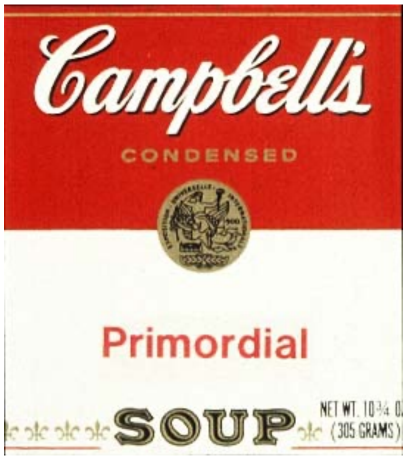
Binding Energy per nucleon



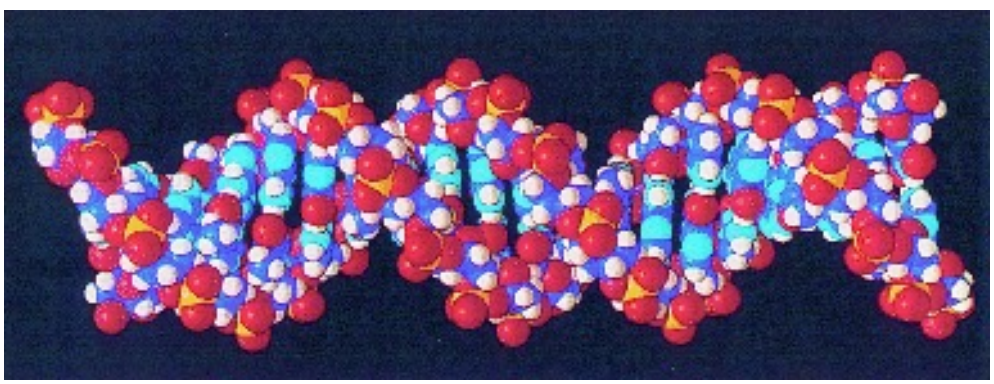
Ten most common elements in the Milky Way Galaxy estimated spectroscopically[1]

Z	Element	Mass fraction in parts per million	
1	Hydrogen	739,000	71 × mass of Oxygen (red bar)
2	Helium	240,000	23 × mass of Oxygen (red bar)
8	Oxygen	10,400	
6	Carbon	4,600	
10	Neon	1,340	
26	Iron	1,090	
7	Nitrogen	960	
14	Silicon	650	
12	Magnesium	580	
16	Sulfur	440	

Cosmic Abundances



- All life (as we know it) is made of carbon based molecular chains
- Only 30 complex molecules comprised of only five (5) basic elements
- Urey-Miller experiment in 1953 showed that we could build amino acids



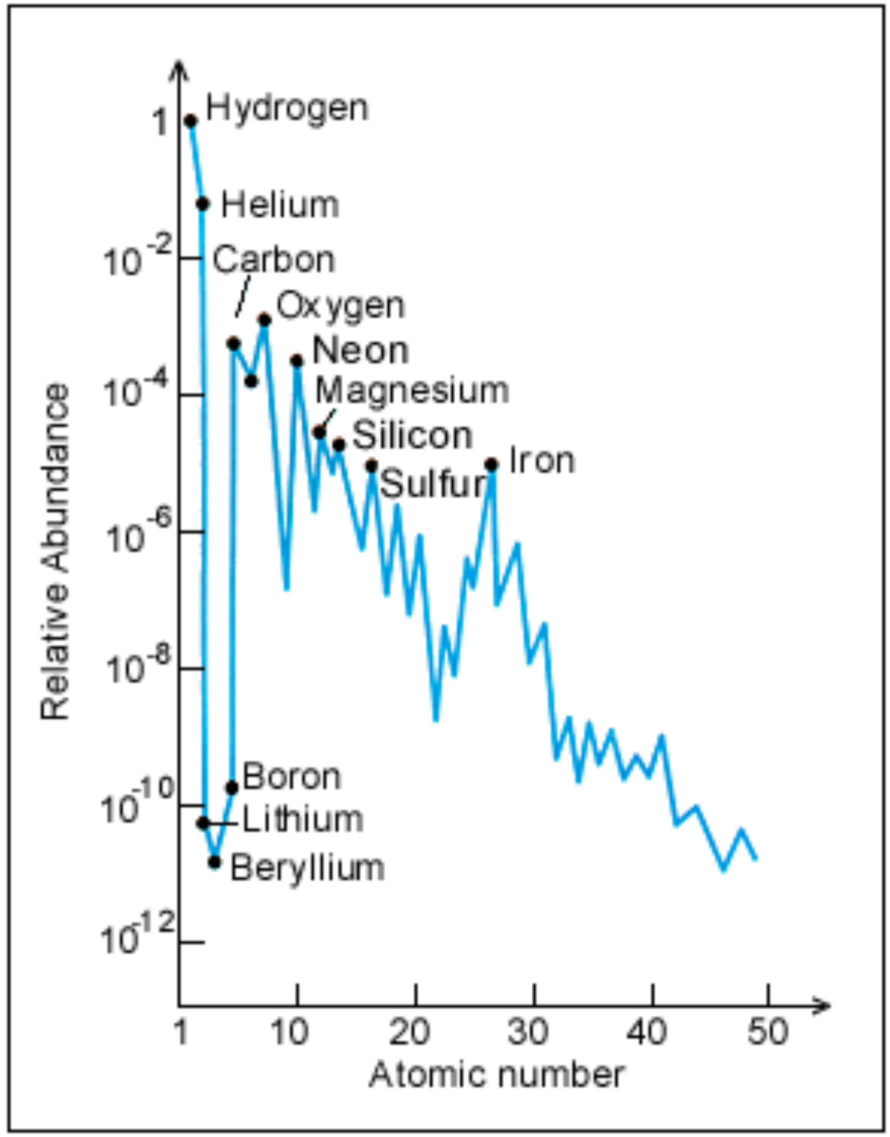
DNA molecule

C = carbon
H = hydrogen
N = nitrogen
O = oxygen
P = phosphorous

Sun's photospheric abundances reflect abundances of pre-solar nebula, enriched by stellar winds and supernovae of previous generations.

Note peaks where A/4 = integer (or Z/2 = integer). Note drop after Fe, where steady fusion no longer creates energy.

Li abundance low because it is destroyed in stars. 33rd most abundant element.



Summary: stellar model building

$$\frac{dP}{dr} = -\rho g$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{dT_0}{dr} < -\left(1 - \frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2} = \frac{dT}{dr}\Big|_{ad}$$

Also have ideal gas law and equations for specify fusion energy generation rate and tables for opacity. Solve these DE numerically in narrow spherical shells, subject to boundary conditions:

$$M_r \rightarrow 0,\; L_r \rightarrow 0 \;\; \text{as} \;\; r \rightarrow 0$$

$$T, \rho, P \rightarrow 0 \;\; \text{as} \;\; r \rightarrow r_{star}$$

Next:

Stellar evolution: read chapter 12 (skip chapter 11 on Sun)

EXAM 2 on Thursday Oct 27.
Covers chapters 6, 9 and 10.

Calculator allowed, equations provided.