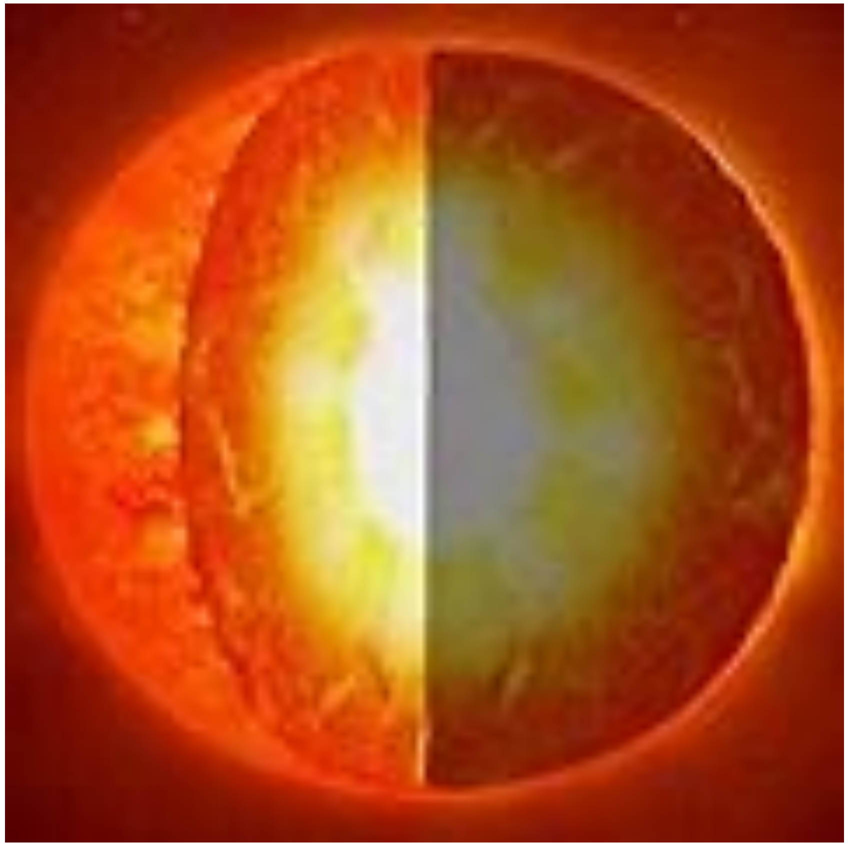
Astronomy 421



Lecture 15: Stellar Interiors I

(skip “Polytropic models” in 10.5)

Key concepts:

Hydrostatic Equilibrium

Mass Conservation

Pressure Equation of State

Energy Transport and Temperature Gradient

Stellar Energy Sources and Fusion Reactions

By observing stars, we can directly infer *T* of photosphere, composition and physical conditions of stellar *atmosphere*. There is no way to observe directly the *interior* (except via neutrinos)

However, physical conditions in the upper, observable layers are determined by interior conditions, where certain physical laws apply:

-> flow of radiation

-> balance of pressure and gravity

-> nuclear energy generation rates

-> equation of state (e.g. ideal gas law)

Deducing stellar structure

Given: $M, R, L, T,$ composition

Find: physical conditions at each radius within star (e.g. T, P, ρ, κ)

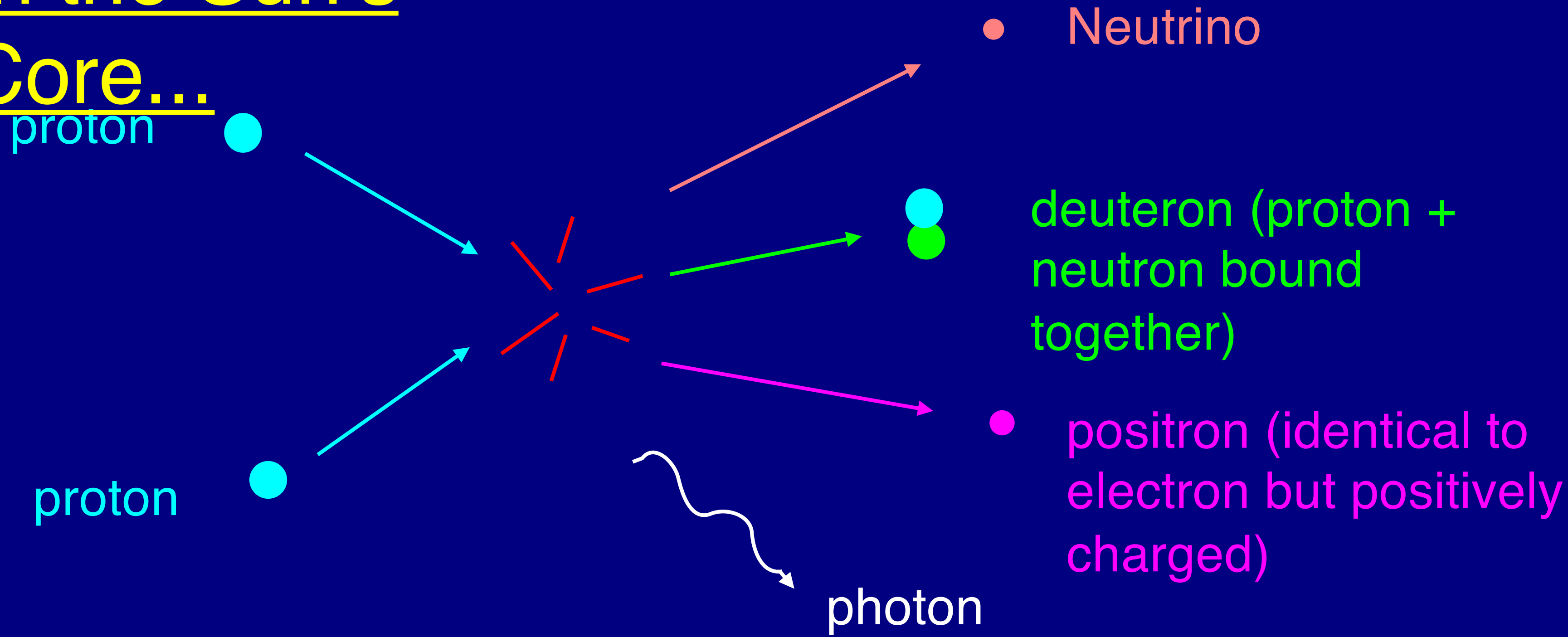
Assume: 1) Star is stable

2) Energy generation rate = energy radiation rate

3) Star is a sphere

4) Material is an ideal gas

In the Sun's Core...



1) proton + proton → proton+neutron + neutrino + positron) + energy (photon)

(deuteron) (heavy hydrogen) + energy (photon)

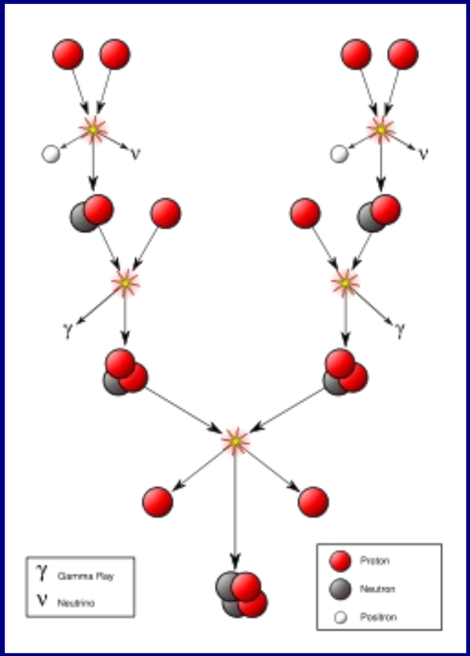
2) deuteron + proton → ^{3}He + energy

He nucleus, only 1 neutron

3) ^{3}He + ^{3}He → ^{4}He + proton + proton + energy

Net result:

4 protons → ^{4}He + 2 neutrinos + energy



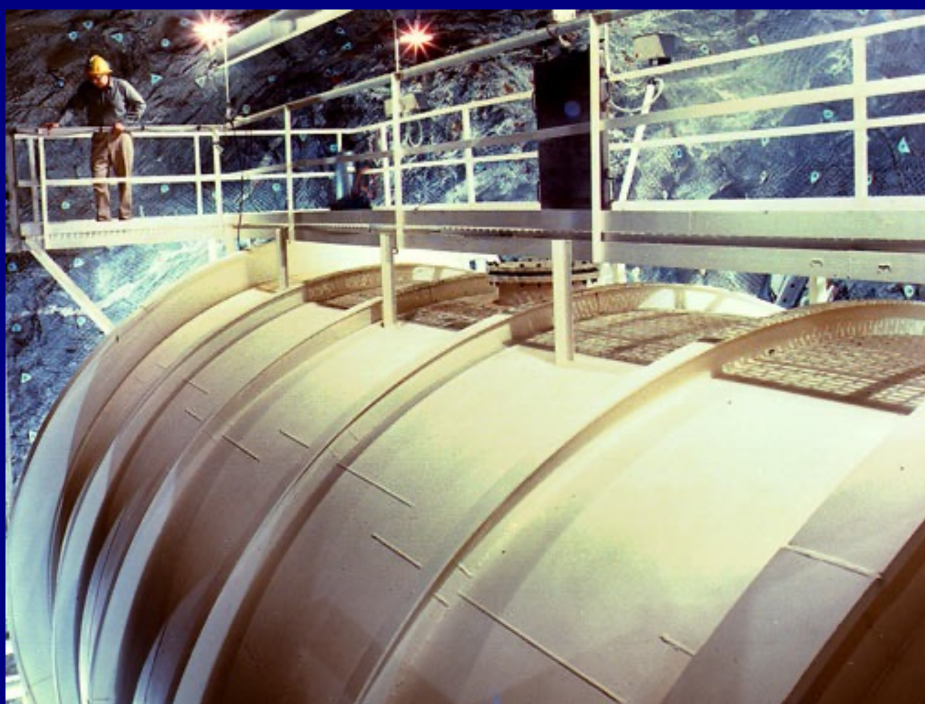
Mass of end products is less than mass of 4 protons by 0.7%.
Mass converted to energy.

600 millions of tons per second fused. Takes billions of years to convert p's to ^{4}He in Sun's core. Process sets lifetime of stars.

Solar neutrino problem

In 1960s Ray Davis and John Bahcall measured the neutrino flux from the Sun and found it to be lower than expected (by 30-50%)

Confirmed in subsequent experiments
Theory of p-p fusion well understood
Solar interior well understood

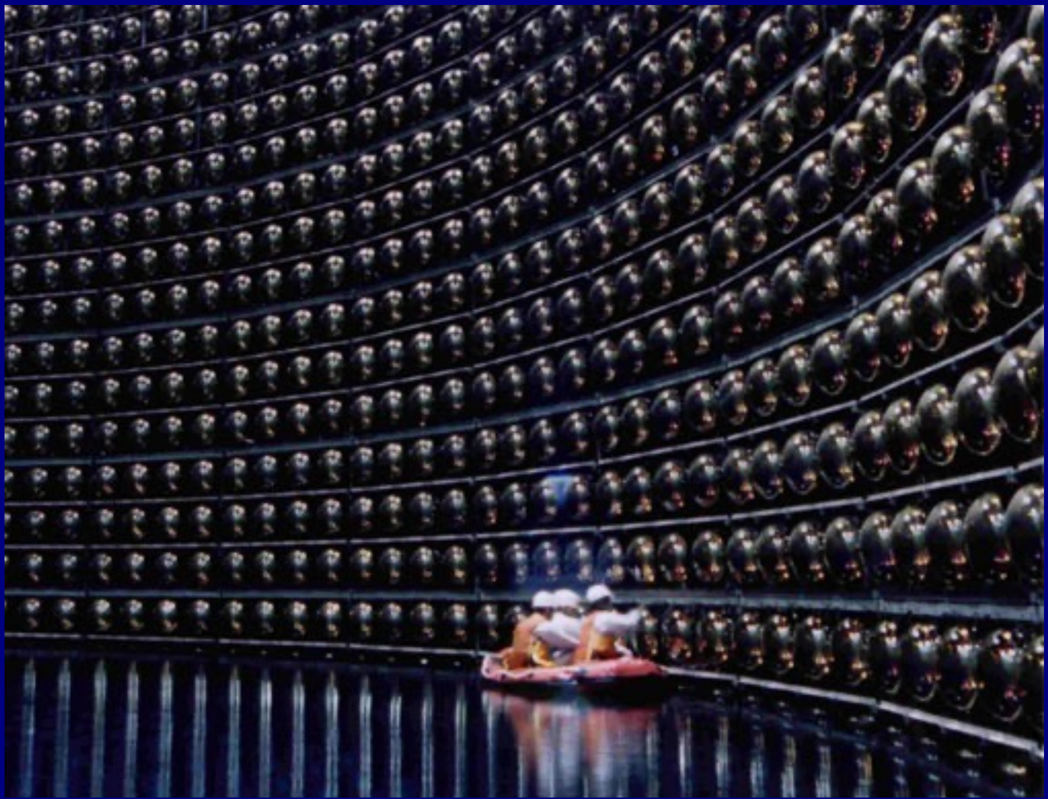


Answer to the Solar neutrino problem

Theoriticians like Bruno Pontecorvo realized

- There was more than one type of neutrino
- Neutrinos could change from one type to another

Confirmed by Super-Kamiokande experiment in Japan in 1998



50,000 gallon tank

Total number of neutrinos agrees with predictions

We will deduce four differential equations of stellar structure.
Combined with:

(i) an equation of state (e.g. the ideal gas law)

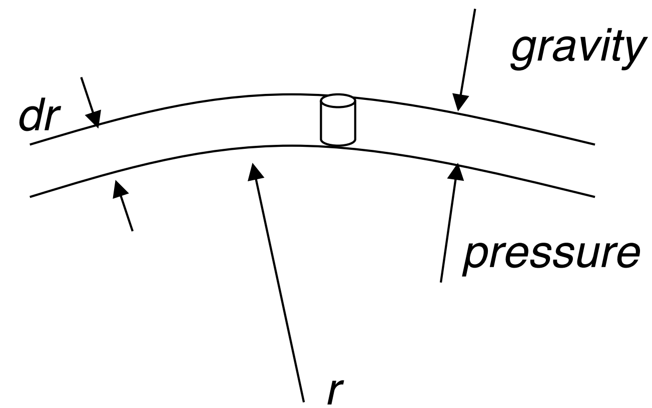
(ii) a condition for convection

(iii) tables of opacity for different temperature, density, composition

(iv) and equations relating energy generation by fusion to density, temperature and composition

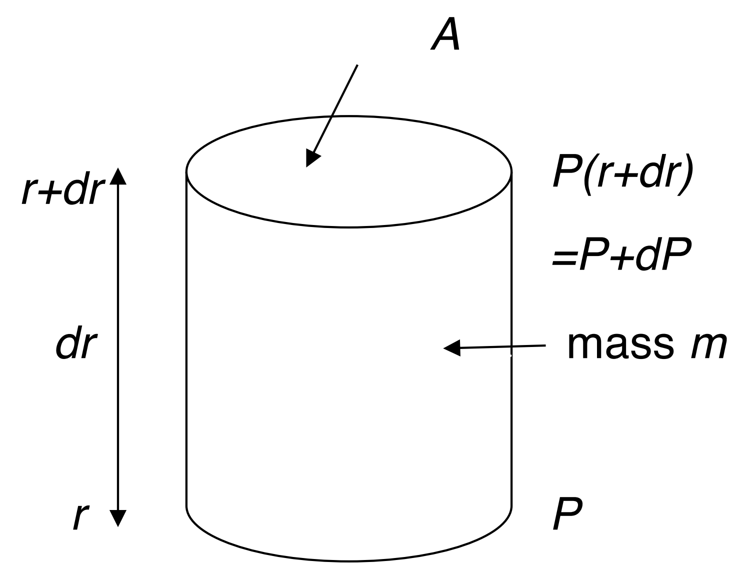
we can completely describe the interior structure.

Hydrostatic Equilibrium



Stability of star requires cylinder of gas is static -> no net forces.

At every *r*, gravity balanced by pressure.



Static => weight must be balanced by pressure difference over *dr*.

Or $AdP = -mg$.

Since $m = \rho V = \rho Adr$, then $AdP = -\rho gAdr$,

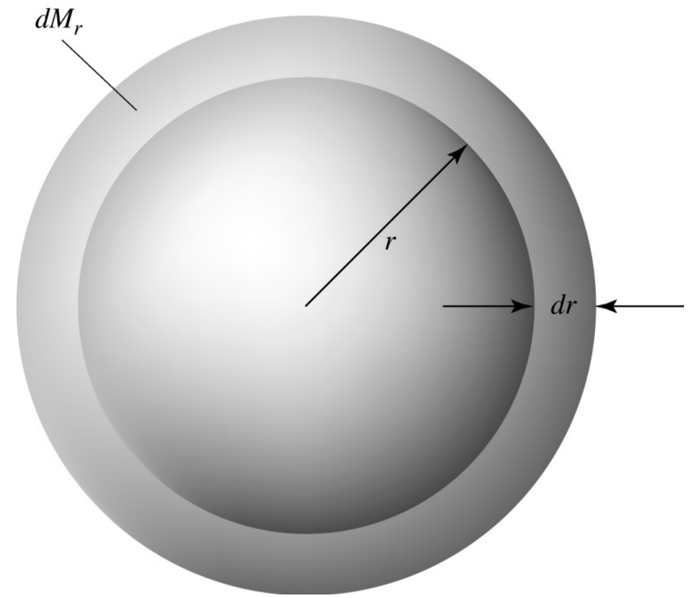
So
$$\frac{dP}{dr} = -\rho g$$

Equation of Hydrostatic Equilibrium (first of four fundamental differential equations of stellar structure)

g, the acceleration due to gravity, depends only on mass interior to *r*:

$$g(r) = \frac{GM_r}{r^2}$$

Mass Conservation Equation



For a thin spherical shell (*dr<<r),* with local density $\rho=\rho(r)$

$$dM_r = \rho(4\pi r^2)dr$$

$$\boxed{\frac{dM_r}{dr} = 4\pi r^2 \rho}$$

(second fundamental differential equation of stellar structure)

$$M_{r_0} = \int_0^{r_0} 4\pi r^2 \rho(r) dr$$

The integral gives mass *enclosed* in r_0.

Pressure Equation of State

What provides the *P* in $dP/dr = -\rho g$?

Under normal stellar conditions, material obeys the *ideal gas law.*

$$\boxed{P = nkT}$$

P = pressure, *n* = number density, *k* = Boltzmann's constant, *T* = temperature

Also useful to express in terms of ρ, since it appears in our differential equations and directly relates to gravity. In general, will have a variety of particles of different masses (p, e, He nuclei, Fe ions, etc).

$$n = \frac{\rho}{\bar{m}} \qquad P = \frac{\rho k T}{\bar{m}}$$

where *m* is average mass of a particle

Define *mean molecular weight* $\mu \equiv \frac{\bar{m}}{m_H}$ where m_H is mass of H atom

$$\boxed{P = \frac{\rho k T}{\mu m_H}}$$

μ depends on composition and degree of ionization

What is μ for a star of a) neutral H?

b) ionized H (approximately)?

What is μ in general?

Define:

$$X = \frac{\text{total mass of H}}{\text{total mass of gas}}$$

$$Y = \frac{\text{total mass of He}}{\text{total mass of gas}}$$

$$Z = \frac{\text{total mass of metals}}{\text{total mass of gas}}$$

$$X + Y + Z = 1$$

For a neutral gas, can show $$\frac{1}{\mu_n} \approx X + \frac{1}{4}Y + < \frac{1}{A} >_n Z$$

For a given metal, $A = (m_{metal\ atom})/m_H$

For solar abundances, $<1/A_n> \sim 1/15.5$

For ionized gas, can show: $$\frac{1}{\mu_i} \approx 2X + \frac{3}{4}Y + < \frac{1+z}{A} >_i Z$$

For a given metal, z is the number of protons.

For X=0.70, Y=0.28, Z=0.02 $$\mu_n = 1.30 \text{ and } \mu_i = 0.62$$

(note Z is minor contributor to μ)

Worksheet 9. Calculate the Temperature at the Center of the Sun

$M_{sun} = 1.99 \times 10^{30}$ kg

$R_{sun} = 6.95 \times 10^{8}$ m

Finally, pressure due to radiation can be important, so

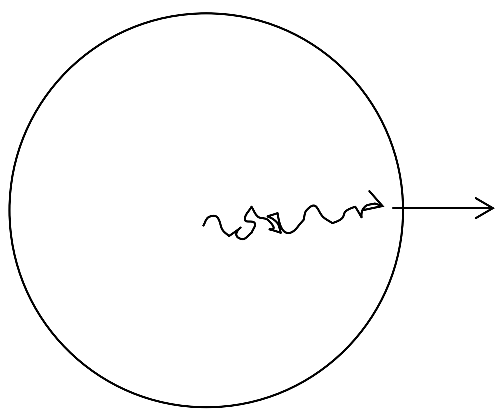
$$P_{total} = \frac{\rho k T}{\mu m_H} + \frac{1}{3} a T^4$$

Energy transport

Mechanisms:

1) Radiation
2) Convection

1) Radiation - net outward flux of energy carried by photons. Photons created in interior interact frequently with matter (mean free path for the Sun ~ 0.5cm). Energy degrades from X-ray to optical wavelengths. Eventually reaches τ~1 at “surface” and escapes.



e^- scattering is main opacity in interior.

Radiation pressure gradient:

$$\frac{dP_{rad}}{dr} = -\frac{\bar{\kappa}\rho}{c} F_{rad}$$

where F_{rad} is the *flux* passing through radius r.

(the pressure gradient keeps the radiation moving towards the photosphere. Derived in 9.4, not responsible for derivation).

$$P_{rad} = \frac{1}{3} a T^4 \Rightarrow \frac{dP_{rad}}{dr} = \frac{4}{3} a T^3 \frac{dT}{dr}$$

Thus $$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\overline{\kappa}\rho}{T^3} F_{rad}$$

Also $$F_{rad} = \frac{L_r}{4\pi r^2}$$

where L_r is the luminosity *generated within* radius r

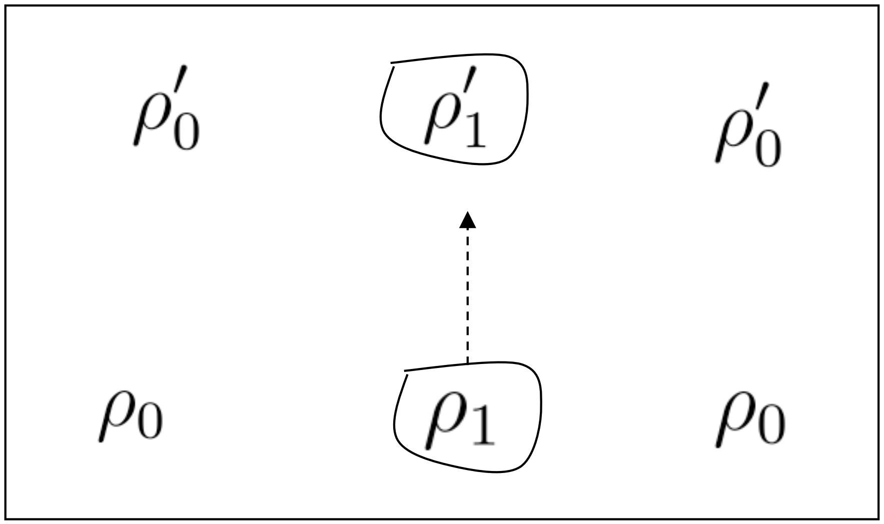
$$\Rightarrow \boxed{\frac{dT}{dr} = -\frac{3}{4ac} \frac{\overline{\kappa}\rho}{T^3} \frac{L_r}{4\pi r^2}}$$

Radiative temperature gradient

(third fundamental differential equation of stellar structure)

Meaning: As either flux or opacity increases, temperature (or pressure) gradient must become steeper if radiation is to get rid of the luminosity.

2) Convection - if *dT/dr* too steep, underdense pockets of gas start to rise (also applies to Earth's atmosphere, interior, Jupiter, etc). To see why:



Assume ρ_1 is just slightly less than ρ_0

Gas element rises by buoyancy

-> new density is ρ'_1

If $\rho'_1 < \rho'_0$, it will keep rising, thus being unstable. Now, let's assume <u>pressure equilibrium</u>, so that $P'_1 = P'_0$ $T'_1 > T'_0$,
so since

the underdense pocket rises, heat is transported upwards, as long as it doesn't start transferring heat to its surroundings.

So condition for gas to keep rising is:

$$\frac{d\rho_1}{dr} < \frac{d\rho_0}{dr} \quad (\text{both} \quad < 0)$$

If adiabatic (rises without transferring heat to surroundings), the adiabatic gas law for an ideal gas is obeyed:

$$P_1 V_1^{\gamma} = Const$$

(what should γ be in a stellar interior?)

or

$$P_1 = k\rho_1^{\gamma}$$

$$\text{Then } \frac{dP_1}{dr} = k\gamma\rho_1^{\gamma-1}\frac{d\rho_1}{dr} = k\gamma\frac{\rho_1^{\gamma}}{\rho_1}\frac{d\rho_1}{dr} = \gamma\frac{P_1}{\rho_1}\frac{d\rho_1}{dr}$$

$$\text{So } \frac{d\rho_1}{dr} = \frac{1}{\gamma}\frac{\rho_1}{P_1}\frac{dP_1}{dr} < \frac{d\rho_0}{dr} \qquad \textbf{(1)}$$

We still have the ideal gas law: $P = \frac{\rho k T}{\mu m_H}$

So we'll write: $\frac{dP_0}{dr} = \frac{P_0}{\rho_0}\frac{d\rho_0}{dr} + \frac{P_0}{T_0}\frac{dT_0}{dr}$

Solve for $d\rho_0/dr$, substitute in (1). Assuming pressure equilibrium,

$$P_1 = P_0, \frac{dP_1}{dr} = \frac{dP_0}{dr}$$

and now assume that the initial densities were nearly equal,
so approximately $\rho_1 = \rho_0$.

Then

$$\frac{dT_0}{dr} < \left(1 - \frac{1}{\gamma}\right)\frac{T_0}{P_0}\frac{dP_0}{dr}$$

But $\frac{dP_0}{dr} = -\rho_0 g = -\rho_0 \frac{GM_r}{r^2}$ and $\frac{T_0}{P_0} = \frac{\mu m_H}{\rho_0 k}$

$$\frac{dT_0}{dr} < -\left(1 - \frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2} = \frac{dT}{dr}\Big|_{ad}$$

condition for convection

(Remember, $dT/dr < 0$)

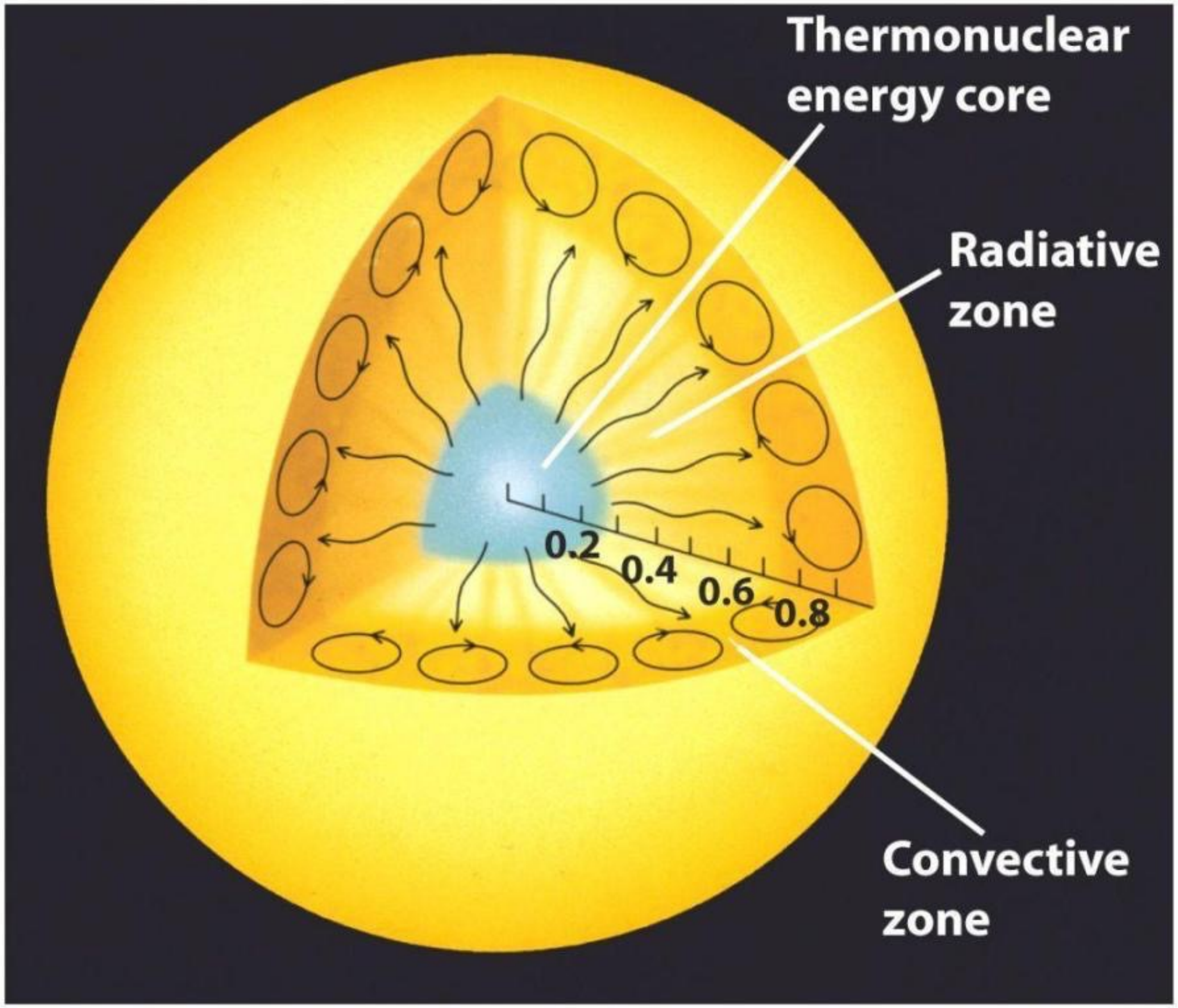
Since the temperature gradients are negative, then if $\left|\frac{dT}{dr}\right| > \left|\frac{dT}{dr}\right|_{ad}$, bubbles will rise, and heat will be carried by convection.

Stops when bubble can finally cool (i.e. is no longer adiabatic), e.g. near the top of the Sun's atmosphere.

With the convection condition in this form, we can see when convection is likely, namely when

1. $\overline{\kappa}$ is large (so $\left|\frac{dT}{dr}\right|$ is large)
2. $\frac{L_r}{4\pi r^2}$ is high, deep in cores of massive stars (so $\left|\frac{dT}{dr}\right|$ is large)
3. $g = \frac{GM_r}{r^2}$ is low (so $\left|\frac{dT}{dr}\right|_{ad}$ is low)

(skip "Mixing-Length Theory" part of 10.4)


Thermonuclear
energy core
Radiative
zone
0.2
0.4
0.6
0.8
Convective
zone