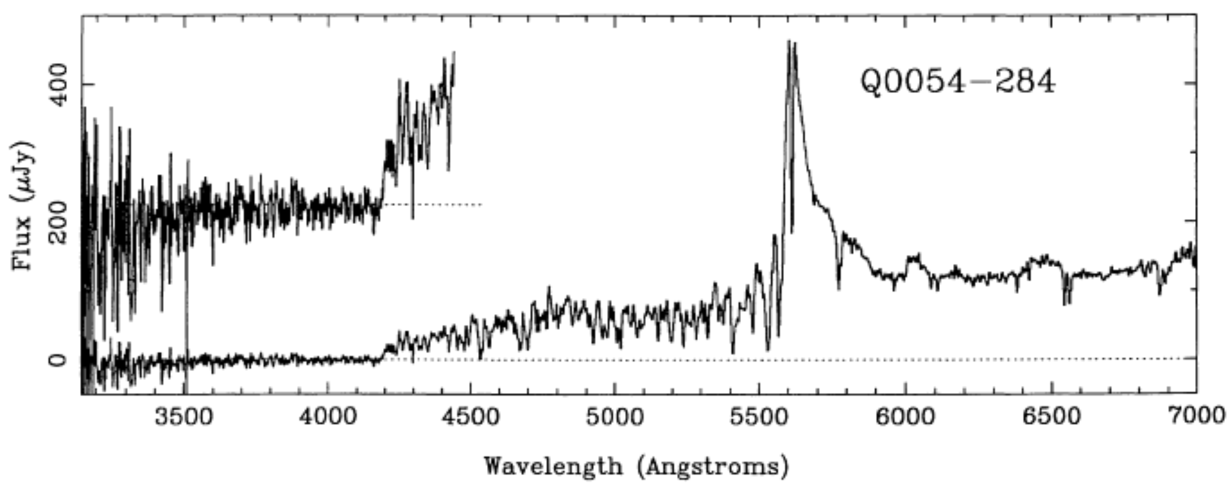
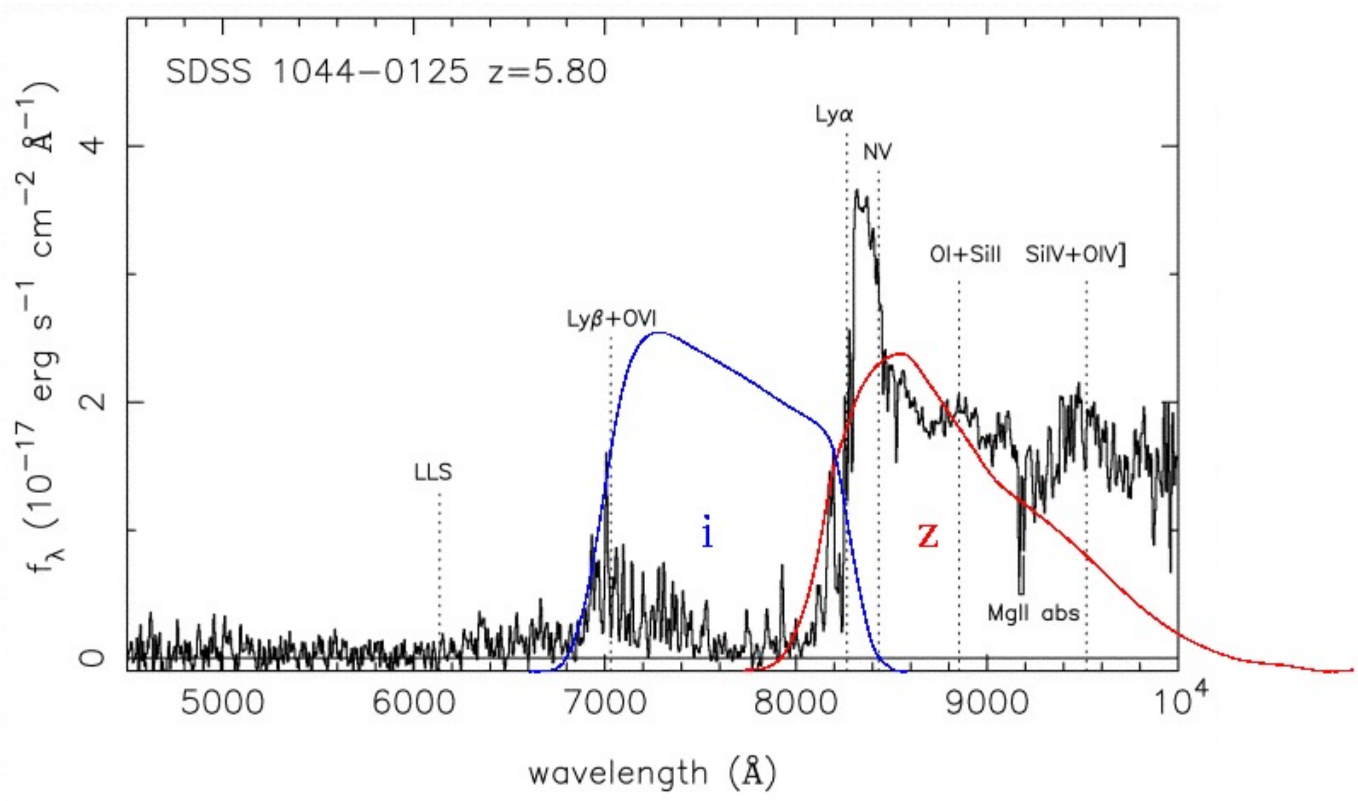
There exists a stronger jump, the *Lyman limit,* occurring at the wavelength corresponding to the energy required to ionize an H atom from the ground state (91.2 nm).

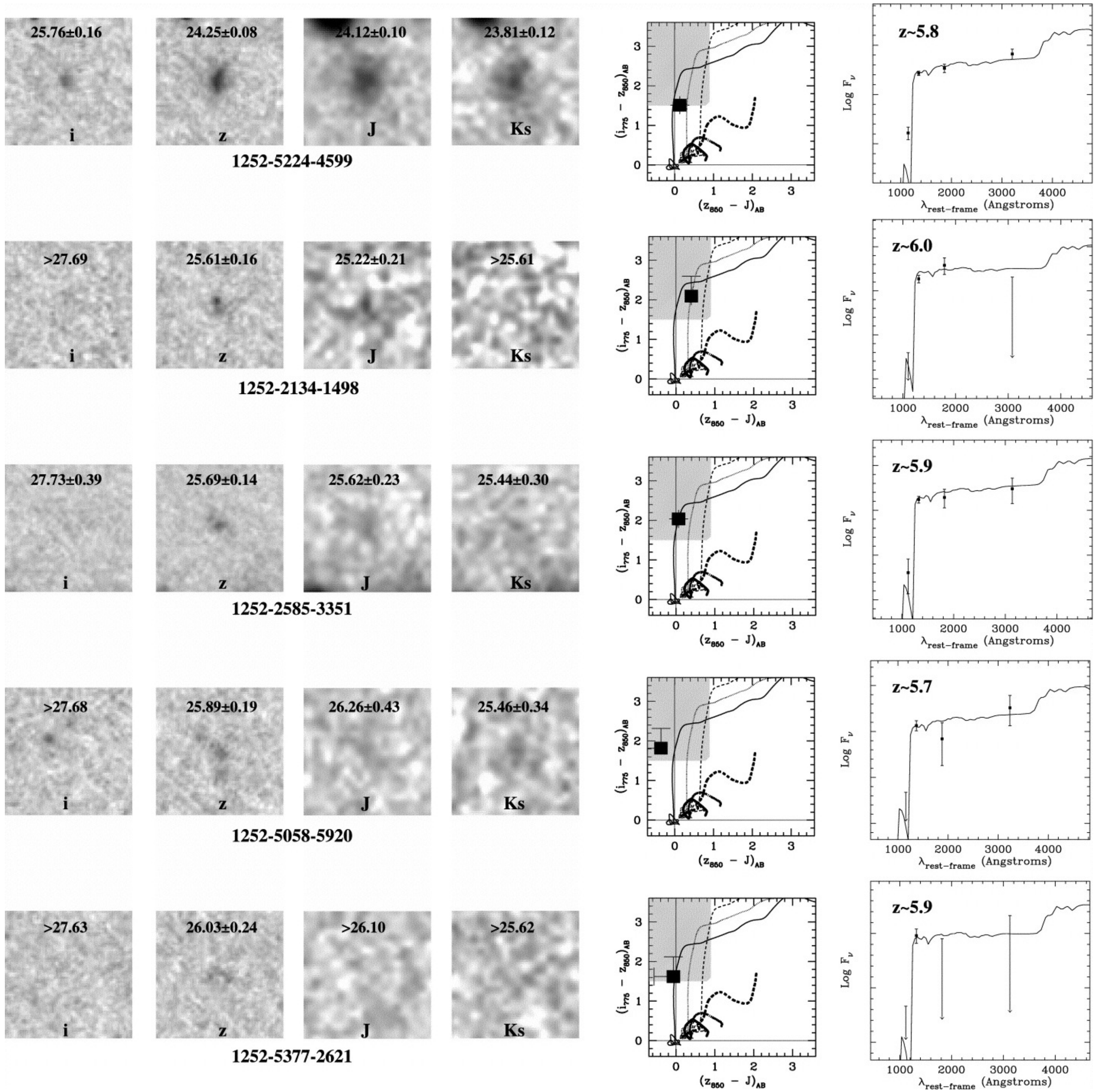
This region is not in the visible, can't be seen from the ground for nearby stars. However, it can be detected in some quasars (why?)





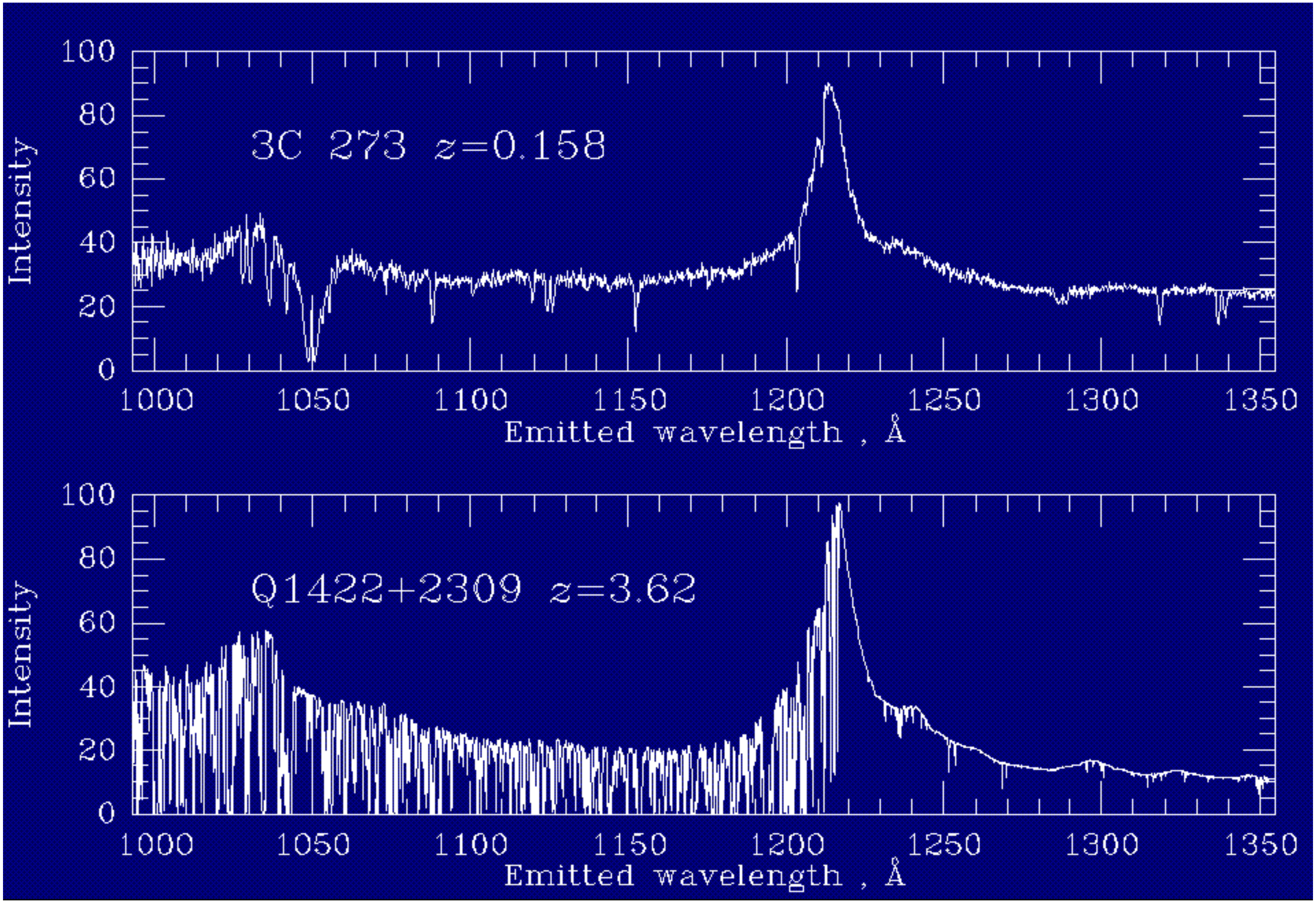

Can be used to search for high-redshift objects, called the 'drop-out' technique.

Look for objects that are faint in the bluest filters.

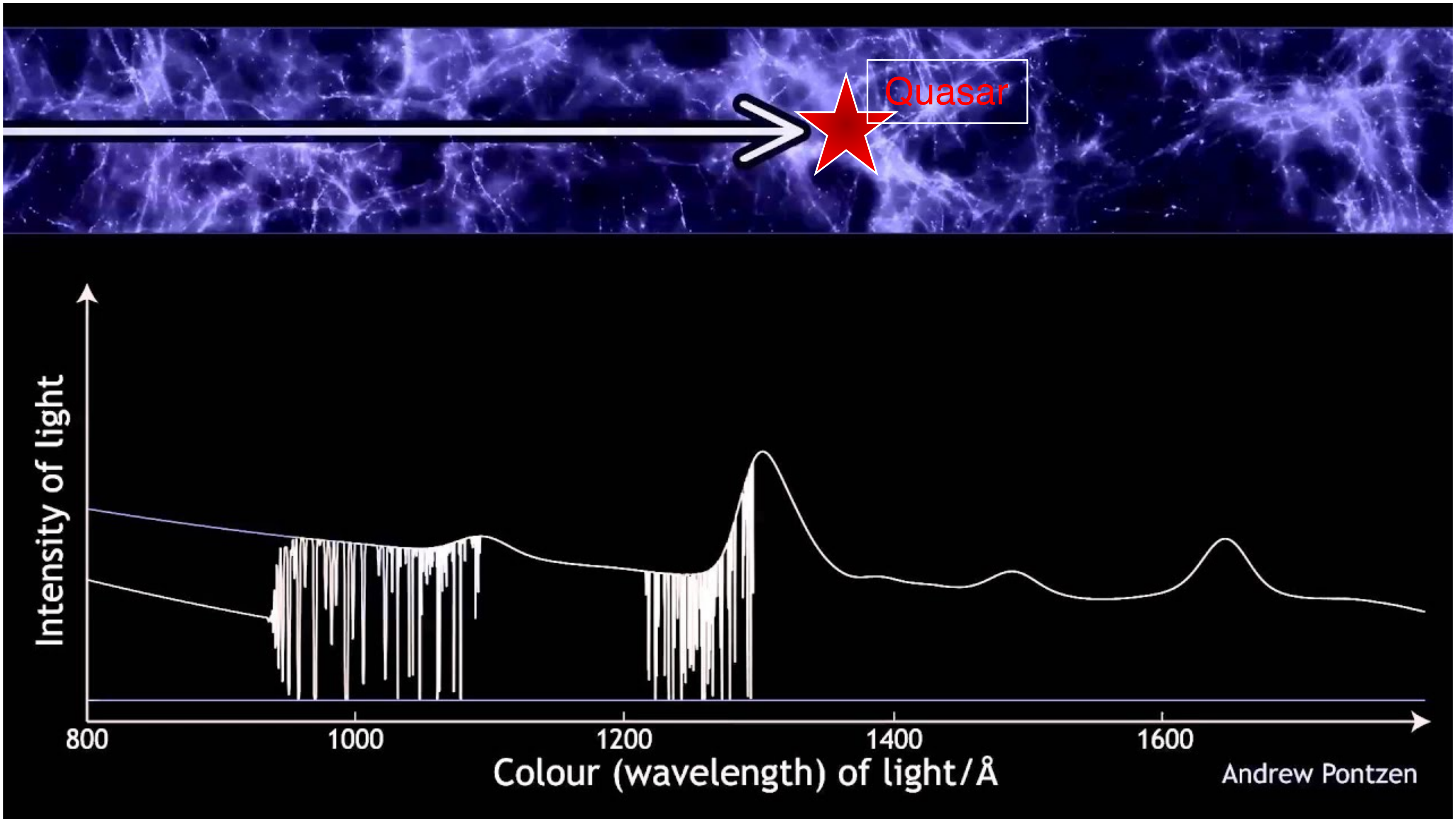


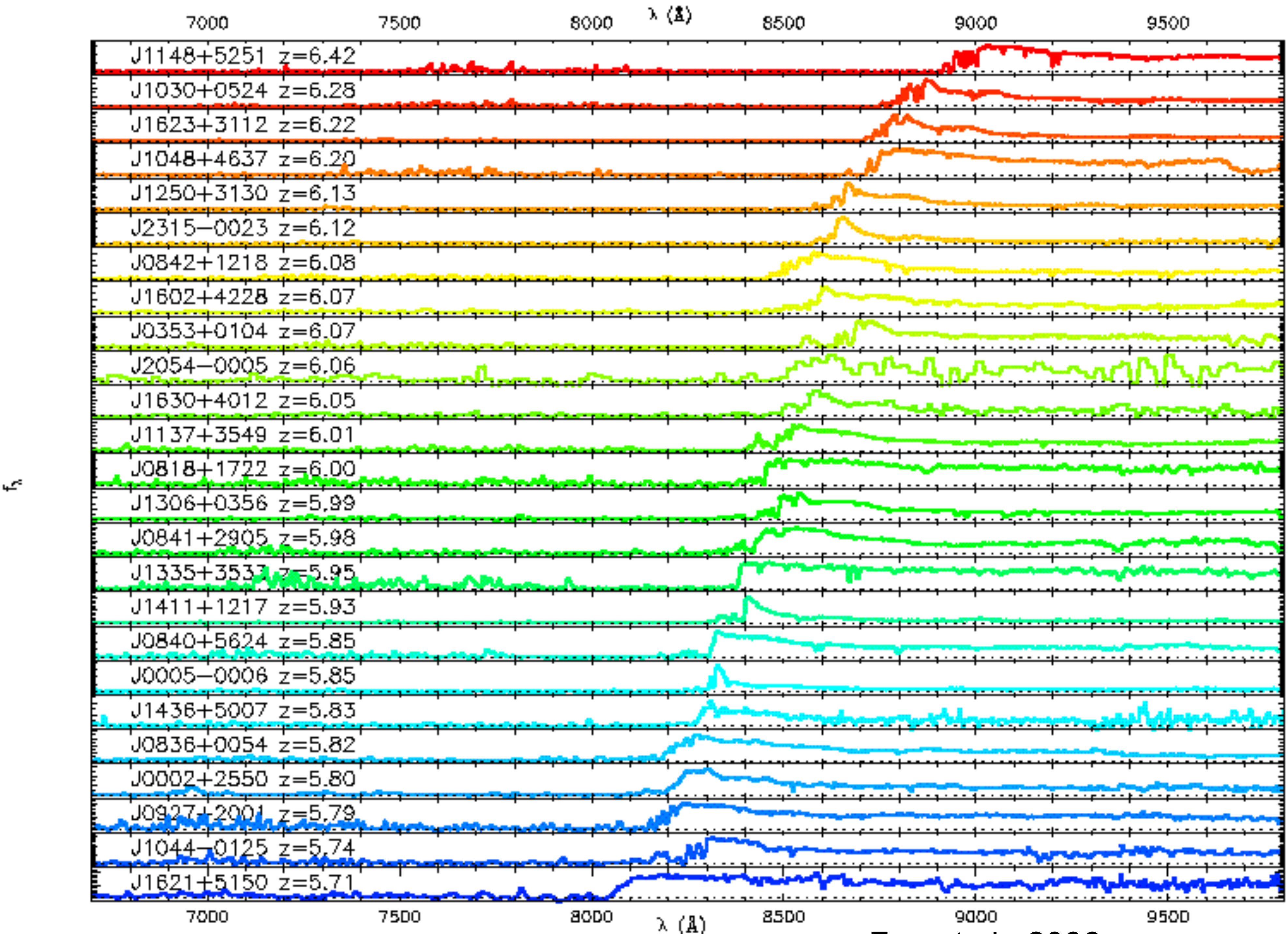
25.76±0.16
24.25±0.08
24.12±0.10
23.81±0.12
i
z
J
Ks
1252-5224-4599
z~5.8
>27.69
25.61±0.16
25.22±0.21
>25.61
i
z
J
Ks
1252-2134-1498
z~6.0
27.73±0.39
25.69±0.14
25.62±0.23
25.44±0.30
i
z
J
Ks
1252-2585-3351
z~5.9
>27.68
25.89±0.19
26.26±0.43
25.46±0.34
i
z
J
Ks
1252-5058-5920
z~5.7
>27.63
26.03±0.24
>26.10
>25.62
i
z
J
Ks
1252-5377-2621
z~5.9
$(i_{775} - z_{850})_{AB}$
$(z_{850} - J)_{AB}$
Log $F_\nu$
$\lambda_{rest-frame}$ (Angstroms)

# Lyman alpha forest

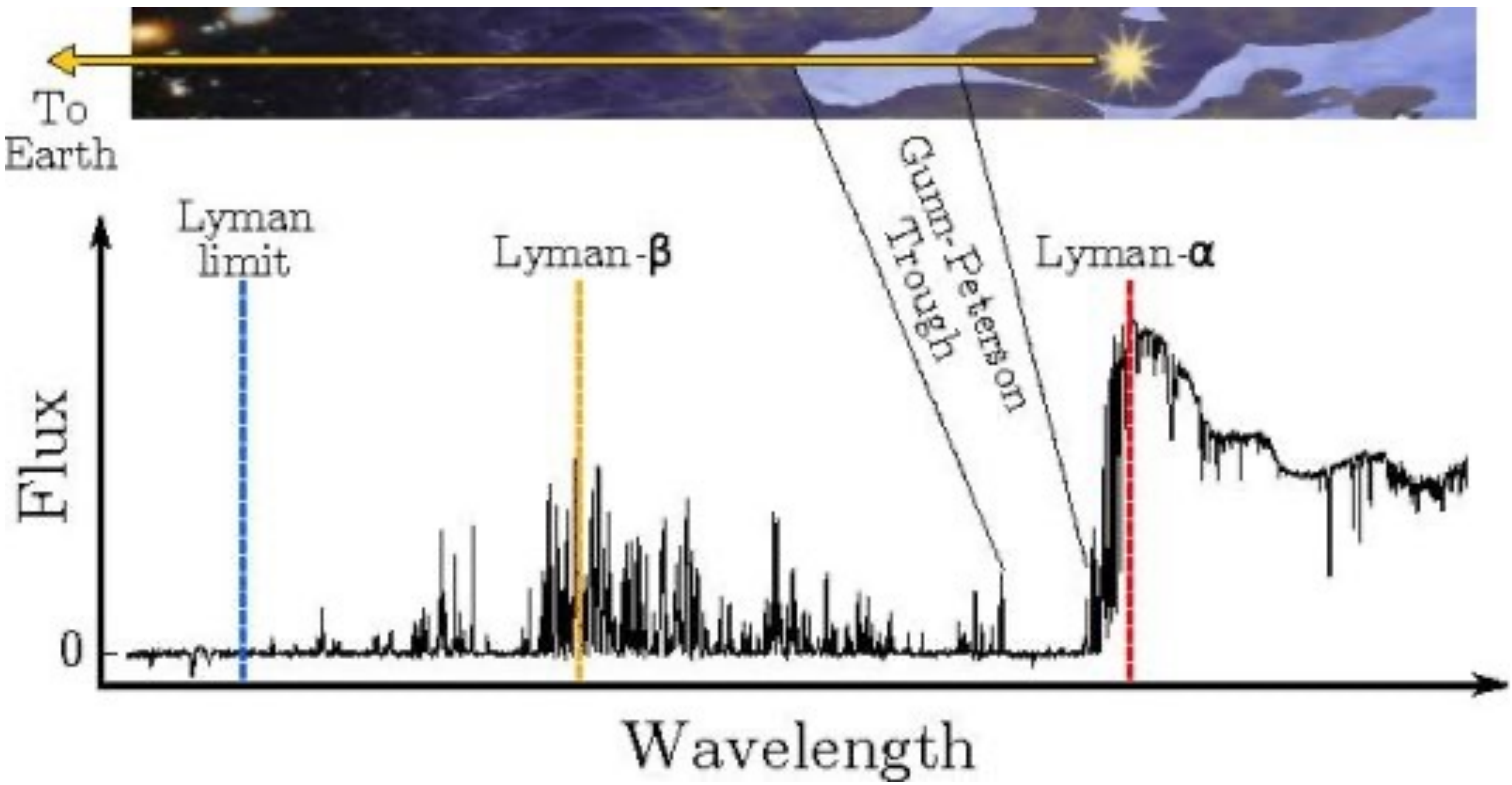


# Lyman alpha forest



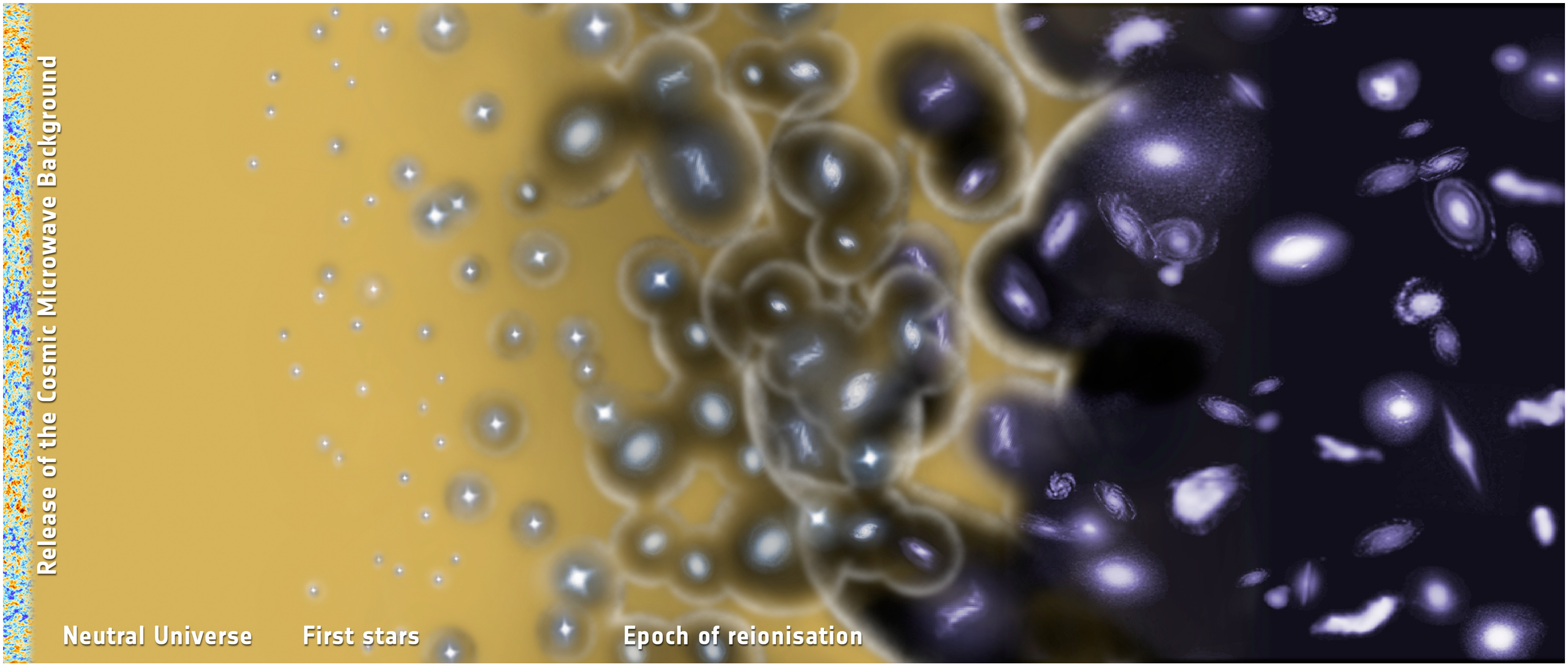



Fan et al. 2006




Credit: John Wise

# Epoch of Reionization




z =30-14

45-100 MHz

z = 14-6

100-200 MHz

Main source of continuum opacity in stellar atmospheres of type:

F and cooler: Photoionization of $\mathrm{H}^-$ ions.

Any photon with

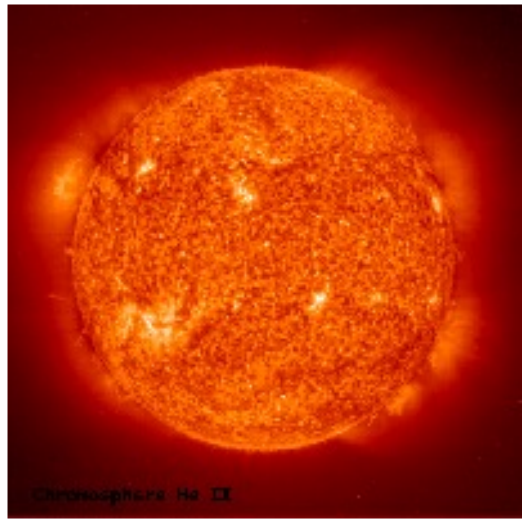
$$\lambda \leq \frac{hc}{\chi} = \frac{hc}{0.754\,\mathrm{eV}} = 1640\,\mathrm{nm} \qquad \text{(IR)}$$

B, A: Bound-free of H and free-free processes

O stars: Electron scattering and bound-free processes of He

Interiors of stars: Electron scattering

# Astronomy 421



Lecture 14: Stellar Atmospheres III

**Lecture 14 - Key concepts:**

Spectral line widths and shapes

Curve of growth

**RECAP: Sources of stellar opacity and emissivity:**

How photons interact with particles.

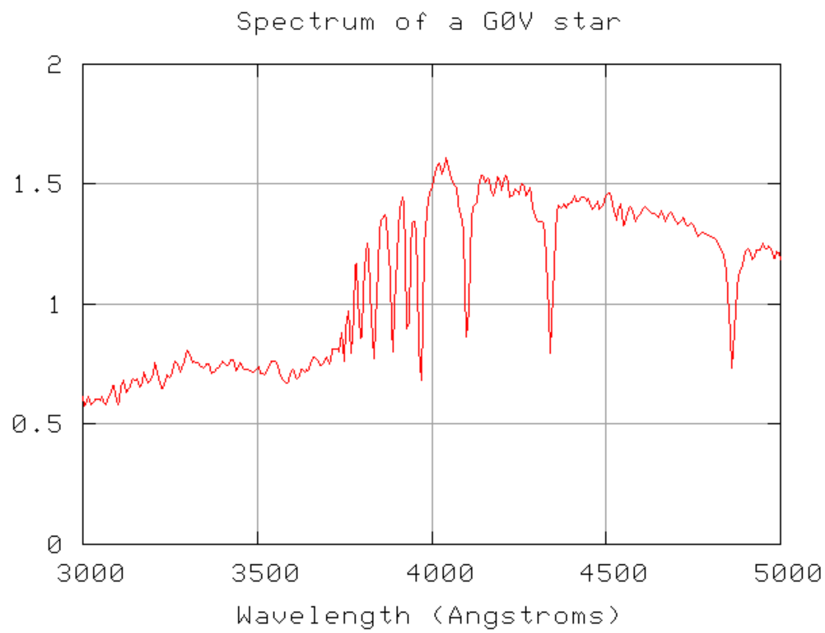
1) Bound-bound transitions.

- $\kappa_{\lambda,bb}$ small except at wavelengths capable of producing upward atomic transitions => absorption lines in stellar spectra.

- When $e^-$ makes downward transition:
  - could go directly to initial orbit (scattering process)
  - could drop to different orbit (true absorption process)
  - if one photon absorbed, but more than one emitted in downward cascade => energy degradation of photons in radiation field.

2) Bound-free absorption = photoionization

- $\kappa_{\lambda,\text{bf}}$ is a source of continuum opacity. Any photons with $\lambda \leq \dfrac{hc}{\chi_n}$ (where $\chi_n$ is the ionization potential of $n^{th}$ orbital) will do. Subsequent recombination also degrades photon energies.




Bound-free transitions are responsible for the presence of 'edges' in stellar spectra.

3) Free-free absorption

- $\kappa_{\lambda,ff}$ another source of continuum opacity. Free *e*$^-$ near ion absorbs photon and increases velocity.

- (converse: free-free emission, or *brehmsstrahlung,* e$^-$ loses energy passing by an ion, emits a photon)

4) Electron-scattering (Thomson scattering)

- $\kappa_{es}$ Photon scatters off free e$^-$. Depends on the *Thomson cross section* of the *e*$^-$ (relatively small, sometimes used as the 'radius' of an electron).

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-25}\ \mathrm{cm}^2$$

- Dominates in high-temperature situations.

*Why won't isolated e$^-$ absorb photons?*

**Rosseland Mean Opacity**

We'd like to define the temperature, density and composition and then estimate the total opacity from all the contributors:

The Rosseland Mean Opacity is an average $\kappa$ over all $\lambda$. A function of $T$, $\rho$, and composition only.

$$\overline{\kappa} = \overline{\kappa_{bb} + \kappa_{bf} + \kappa_{ff} + \kappa_{es}}$$

C+O give approximation expressions for $\overline{\kappa_{bf}}$, $\overline{\kappa_{ff}}$, but no simple equation for $\overline{\kappa_{bb}}$ !

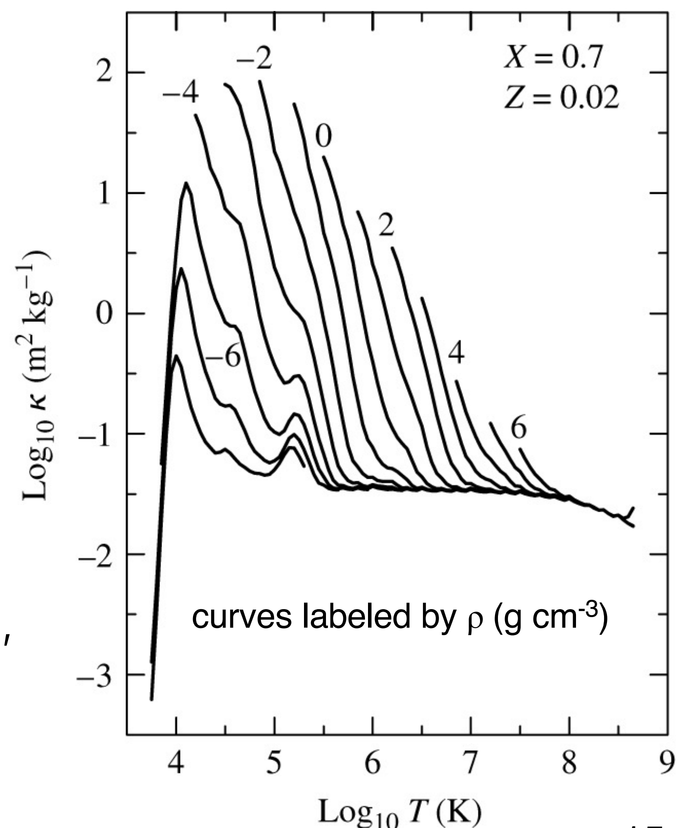
These opacity sources cause the deviations from the pure BB spectrum.

*Example: Rosseland mean opacity*

*X= abundance of mass for H*

*Z= abundance by mass for 'metals'*

*Rest: He*



The Rosseland mean opacity is one example of opacity averaged over wavelength.

The opacity depends on the temperature and the density of the medium in a complicated way but when bound-free and free-free opacity dominate, can be approximated as:

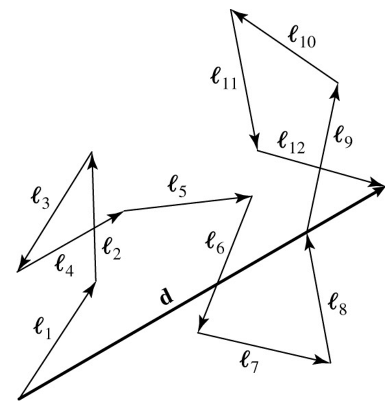
$$\bar{\kappa} = C\rho T^{-3.5}$$

This is called the *Kramers opacity.* Approximately describes falling part of opacity curves.

## Diffusion, Random Walks and Optical Depth

How do photons get out of a star?

Absorptions + emissions change path almost randomly => *random walk.*




Take 1-dim case with equal step size *l* (thus *l* represents the typical distance between interactions, i.e. the mean free path). After 1 step, the displacement is

$$x_1 = \pm l$$

After 2 steps: $$x_2 = x_1 \pm l$$

Squaring both sides:

$$x_2^2 = x_1^2 \pm 2lx_1 + l^2 = 2l^2 \pm 2lx_1$$

On average, $x_1$ =0, so expected average of $x^2_2$ is

$$< x_2^2 >= 2l^2$$

Likewise, after $N$ steps

$$< x_N^2 >= Nl^2$$

The rms displacement is then

$$x_{rms} = \sqrt{N}l$$

So, e.g., it takes a million steps to random walk 1000 mean free paths

Remember: $$\tau_\lambda = \kappa_\lambda \rho s = n\sigma_\lambda s$$

$$\tau_\lambda = \frac{s}{l}$$

where *l* is the mean free path = $1/n\sigma_\lambda$

So to random walk a distance $x_{rms}=s$

$$\tau_\lambda = \sqrt{N}$$

Physical interpretation:

$\tau_\lambda \leq 1$ easy to travel distance *s*

$\tau_\lambda \gg 1$ many scatterings before reaching *s*

(In practice, there must be a net preference to move towards surface instead of center. Will return to this in Ch 10.)

So when $\tau_\lambda = \frac{s}{l} \leq 1$, photons can easily escape the star from depth *s*.

More accurately (see C&O): $\tau_\lambda = \frac{2}{3}$ is the average point of origin of escaping photons.
$\Rightarrow$ we see into a star to a depth corresponding to $\tau_\lambda = \frac{2}{3}$

<u>Consequences:</u>

1) Absorption lines: $\tau_\lambda = \frac{2}{3} = \int_0^s \kappa_\lambda \rho ds$

At the line center, $\kappa_\lambda$ is highest => we don't see as deeply into atmosphere relative to neighboring $\lambda$'s in line.

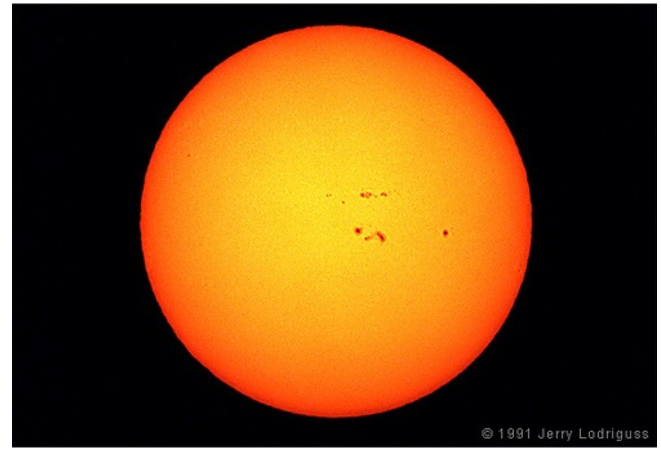
For $\lambda$'s with no line, we see even deeper.

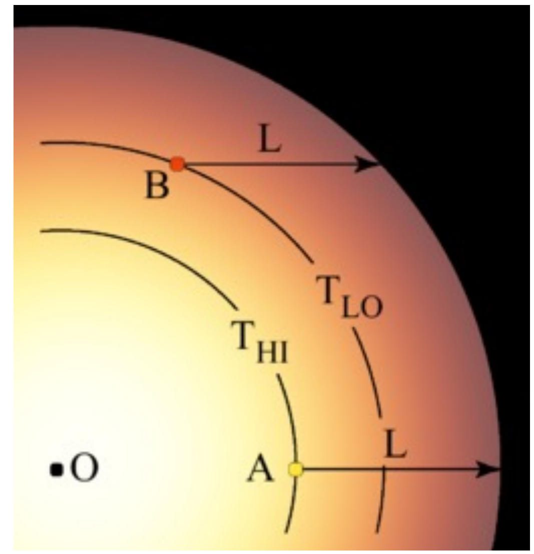
2. Limb darkening.

We see down to some $L$ $(\leftrightarrow \tau_\lambda = 2/3)$ across the disk of the Sun ($L$~ a few 100 km).
A depth $L$ does not penetrate as deeply into the atmosphere at limb, as it does at the center.

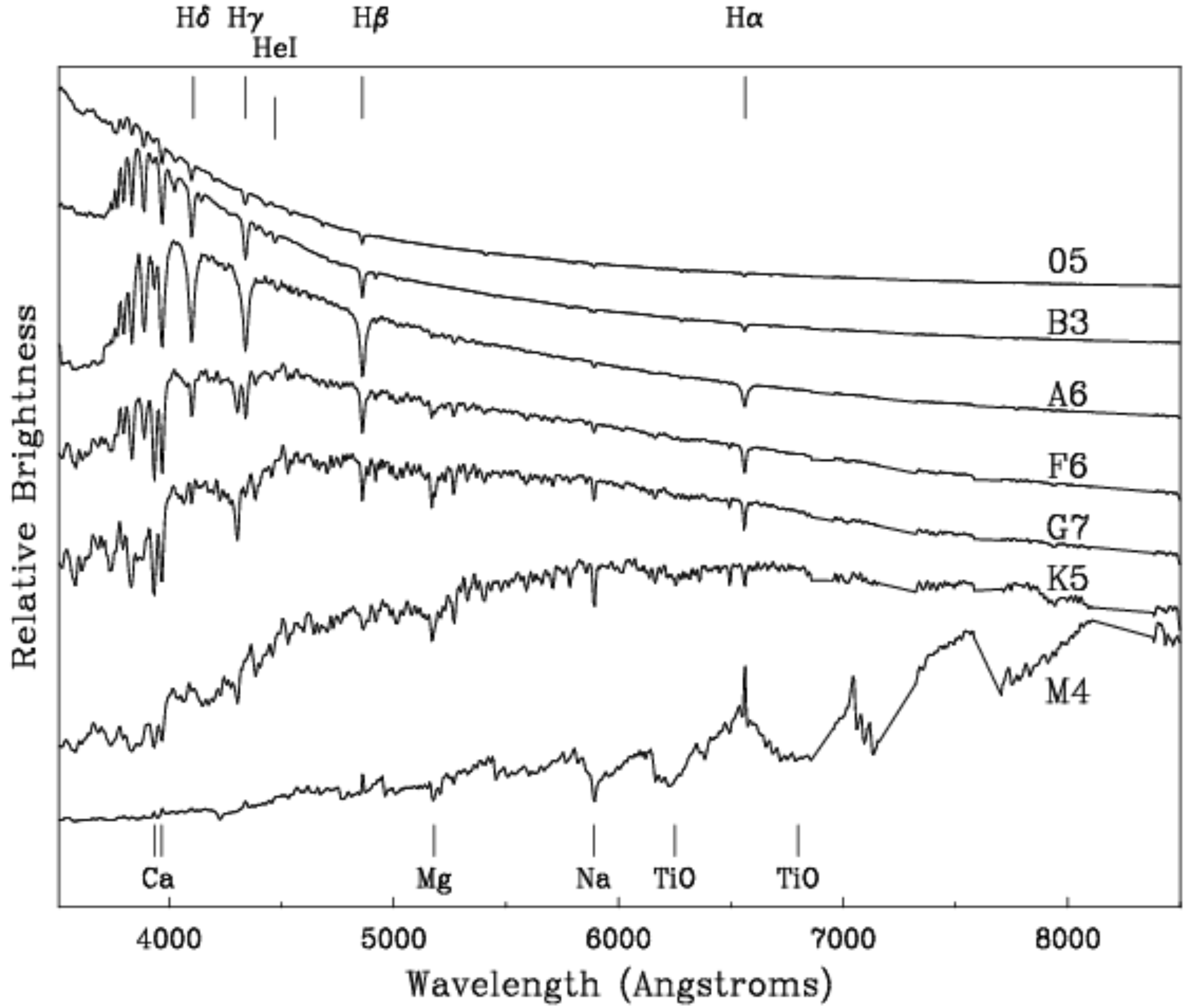
If $T$ drops with height $R$ (=distance from center of Sun), blackbody radiation less intense at limb => darker.

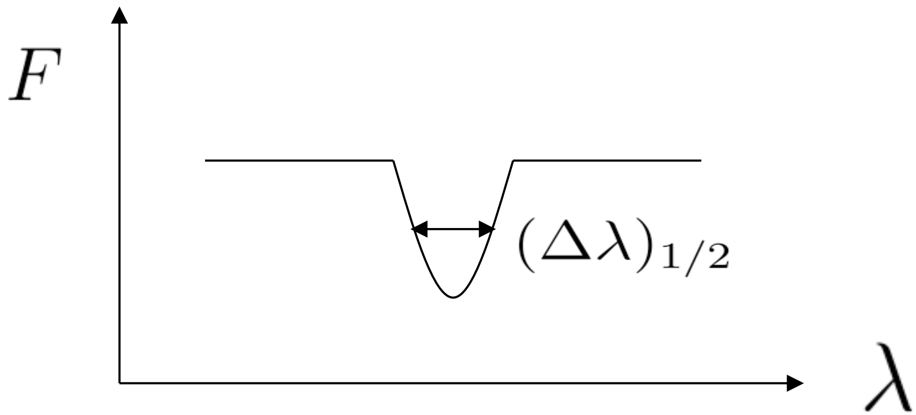
What would we see if $T$↑ height?



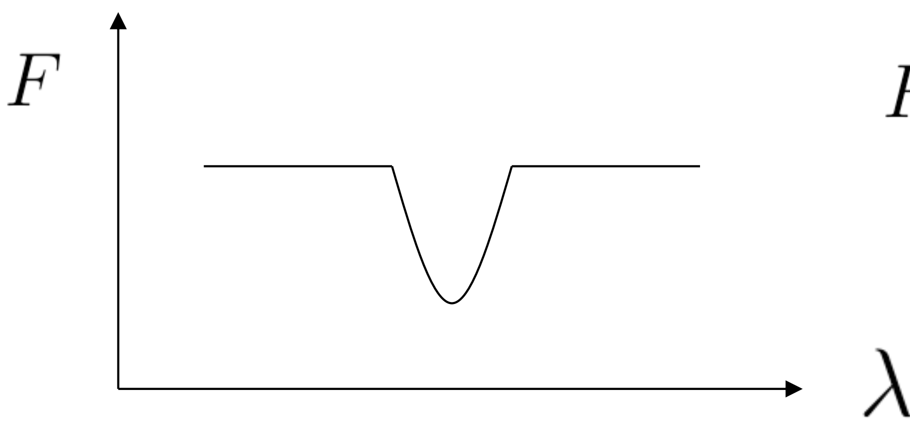



## Spectral line widths and shapes



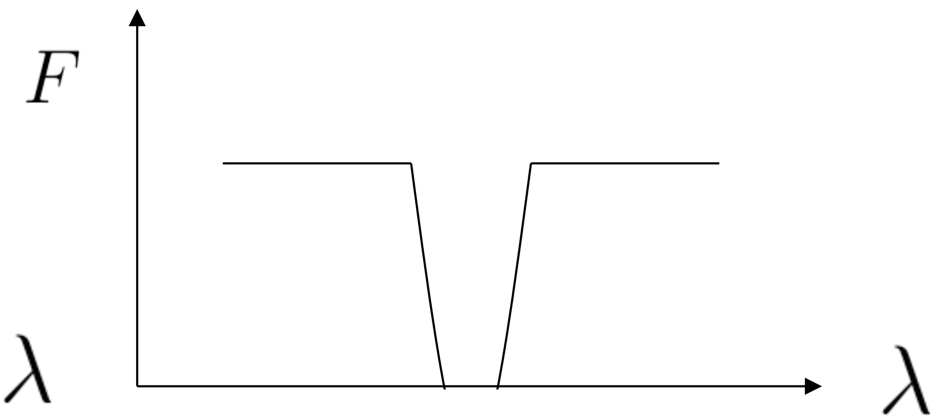



$(\Delta\lambda)_{1/2}$ = "full width at half maximum" = FWHM




Optically thin spectral line.

$(\tau_\lambda \sim 1)$




Optically thick spectral line.

Radiation almost completely absorbed at some $\lambda$'s. $(\tau_\lambda \gg 1)$

What governs width of spectral lines, and what can we learn?

## Three main broadening mechanisms

1. Natural broadening

   Even low density, motionless gas can't produce infinitely sharp lines. Because of small lifetime of excited state $\Delta t$, orbital will uncertainty in energy

$$\Delta E \sim \frac{\hbar}{\Delta t}$$

=> there is an uncertainty in $\lambda$ of emitted photon:

$$E = \frac{hc}{\lambda}$$

$$\Delta E = -\frac{hc}{\lambda^2} \Delta\lambda$$

$$\Rightarrow \ \Delta\lambda = \frac{\lambda^2}{2\pi c}\left(\frac{1}{\Delta t_i} + \frac{1}{\Delta t_f}\right)$$

e.g. H lines, $\Delta t \sim 10^{-8}$ sec, $\Delta\lambda \sim 10^{-5}$ nm – not important usually

2. Doppler broadening

Absorptions and emissions do not occur at rest because of particle motions. Thermal motion, with Maxwell-Boltzmann distribution of velocities, can estimate effect for:

$$v_{\mathrm{mp}} = \sqrt{\frac{2kT}{m}} \quad \text{from Doppler equation} \quad \frac{\Delta\lambda}{\lambda} = \pm\frac{v_r}{c}$$

$$\Delta\lambda = \frac{2\lambda}{c}\sqrt{\frac{2kT}{m}}$$

For hydrogen in Sun's photosphere, $\Delta\lambda \sim 0.04\,\mathrm{nm}$

A more accurate calculation, taking 3-dim motion into account shows:

$$\Delta\lambda_{1/2} = \frac{2\lambda}{c}\sqrt{\frac{2kT\ln 2}{m}}$$

Other kinds of Doppler broadening:

A. due to large-scale turbulent motion
B. outflows
C. rotation

e.g. if turbulent motion with typical speed $v_{turb}$,

$$\left(\Delta\lambda_{1/2}\right) = \frac{2\lambda}{c}\sqrt{\left(\frac{2kT}{m} + v_{turb}^2\right) ln2}$$

3. Pressure or collisional broadening

   Collisions can cause de-excitation, further limiting the lifetime of the excited state, and thus the energy uncertainty of the state.

   If we equate the lifetime to the time between collisions, we can treat it the same way as natural broadening:

$$\Delta t = \frac{mfp}{v_{mp}} = \frac{1}{n\sigma\sqrt{2kT/m}}$$

So

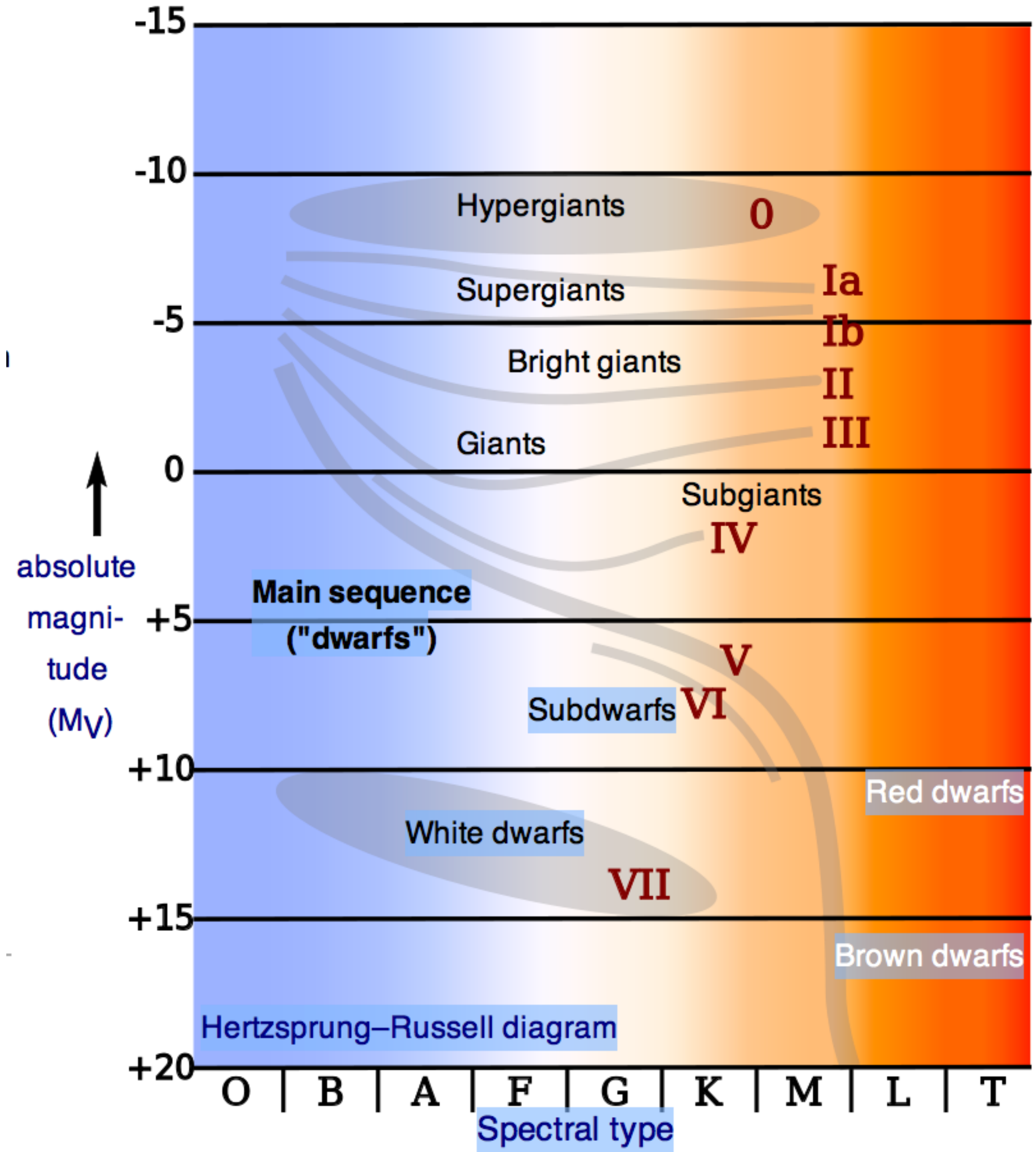
$$\Delta\lambda \sim \frac{\lambda^2}{\pi c} n\sigma \sqrt{\frac{2kT}{m}}$$

Another, more subtle effect has to do with the perturbing of energy levels by passing ions. But again, the key factor is the time between encounters.

Higher density -> more collisional broadening (weaker dependence on T, but n varies more than T among stars (e.g. MS stars, how does density depend on mass?).

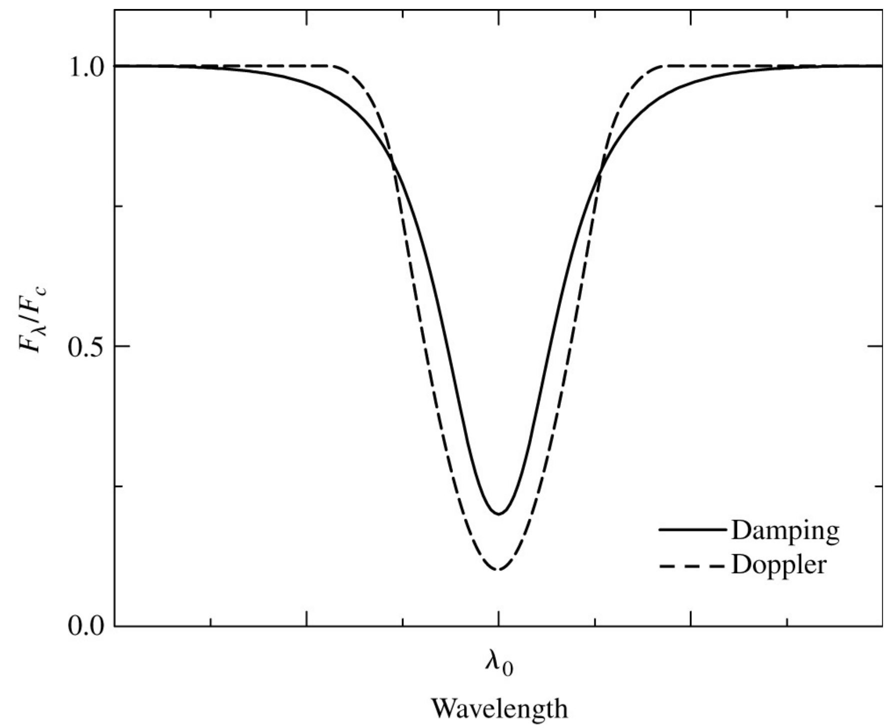
=> physical basis for Morgan-Keenan luminosity classes.

# Morgan Keenan (MK) Luminosity Classes



Doppler versus pressure broadening line shapes:

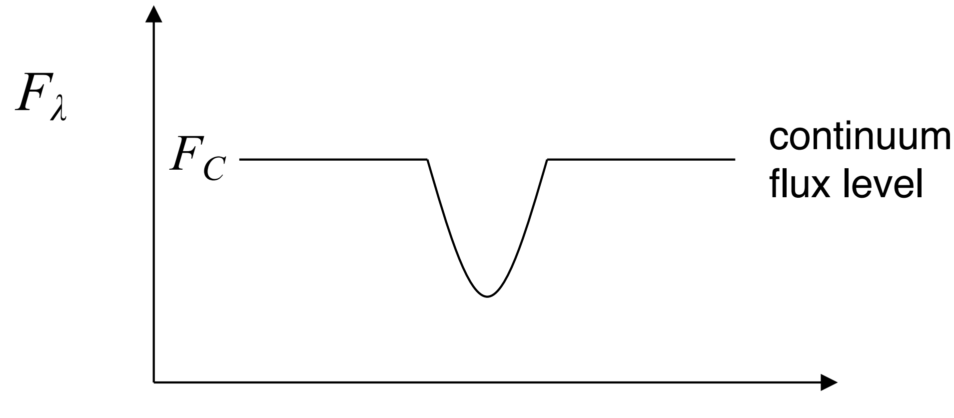
Doppler has larger FWHM in stars, but pressure broadening noticeable in "wings" of the line, leading to observable width variation among MK luminosity classes.



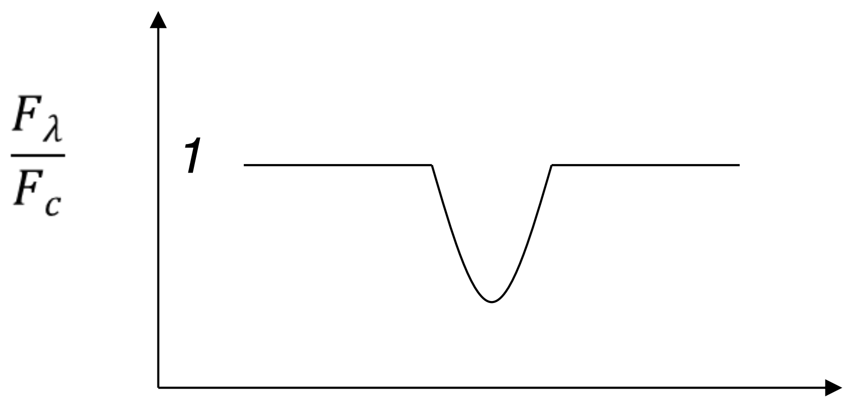

The total line shape is the sum of these three effects => called a "Voigt profile".

So how do we relate absorption line strength to abundances, i.e. how many atoms or ions are in the initial energy state?

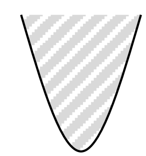
## Equivalent width and curve of growth




Plot flux relative to $Fc$: $F_\lambda/F_c$



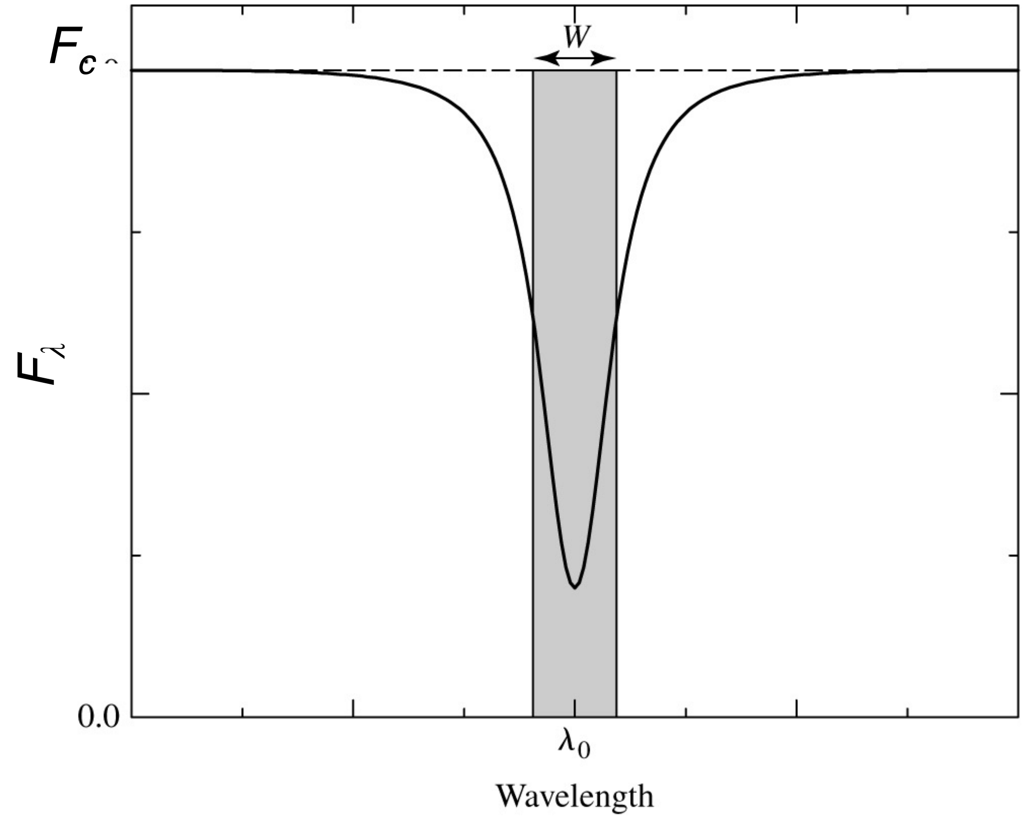

Equivalent width *W* (or *EW*):



i.e. in top plot, this area divided by $F_c$

$$W = \int_{abs\ line} \frac{F_c - F_\lambda}{F_c} d\lambda = \int_{abs\ line} \left(1 - \frac{F_\lambda}{F_c}\right) d\lambda$$

Illustration:




*EW* is width of box with height $F_c$ and same area as spectral line.

Note: *EW* is independent of line shape.

*EW* is proportional to the fraction of light removed by the absorption line integrated over all wavelengths, and thus to the optical depth (over all wavelengths), which depends on the *column density* of absorbing atom or ion along line of sight (number per $m^2$), and its ability to absorb radiation.

Consider "extra" absorption by line compared to adjacent continuum, amount $\tau_\lambda$ .

$$F_\lambda = F_{0,\lambda} e^{-\tau_\lambda} = F_{0,\lambda}$$ in continuum

In line, if $\tau_\lambda \ll 1$

$$F_\lambda \simeq F_{0,\lambda}(1 - \tau_\lambda)$$ or $$\tau_\lambda = 1 - \frac{F_\lambda}{F_{0,\lambda}} = 1 - \frac{F_\lambda}{F_c}$$

$$W \simeq \int_{line} \tau_\lambda d\lambda = \int_{line} \kappa_\lambda \rho s d\lambda = \int_{line} \sigma_\lambda n s d\lambda =$$

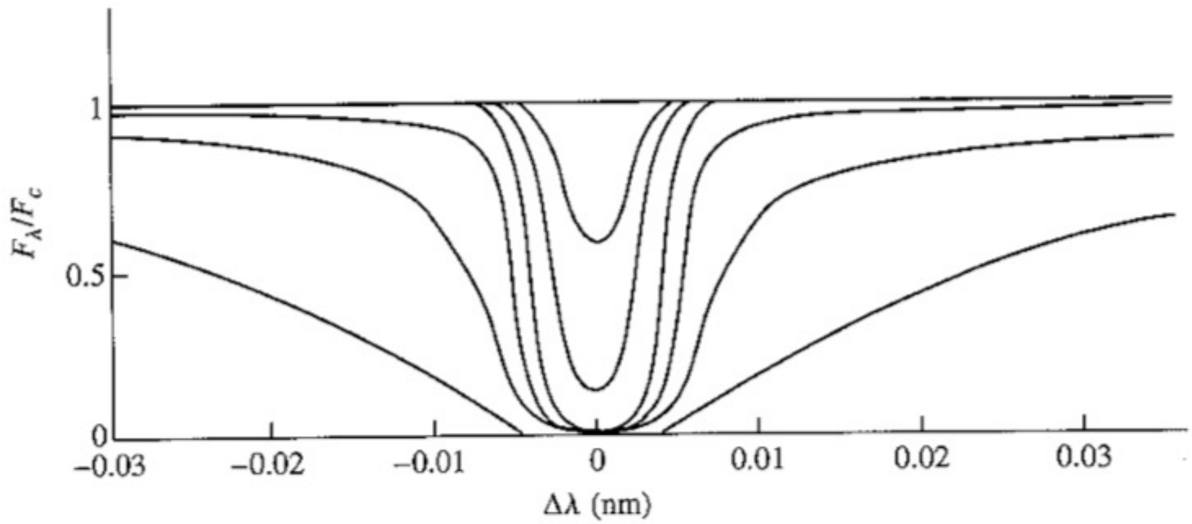
$$= \int_{line} \sigma_\lambda N d\lambda = N \int_{line} \sigma_\lambda d\lambda$$

*N* is the <u>column density</u> of atoms or ions in the lower energy state

$$f \propto \int_{line} \sigma_\lambda d\lambda$$ is "f-value" or "oscillator strength".
e.g. f=0.637 for Hα, f=0.119 for Hβ).

So $W \propto Nf$ for $\tau_\lambda$

Voigt profiles
Ca II K line.




Doppler contributes mainly to the core, and pressure effects to the line wings.

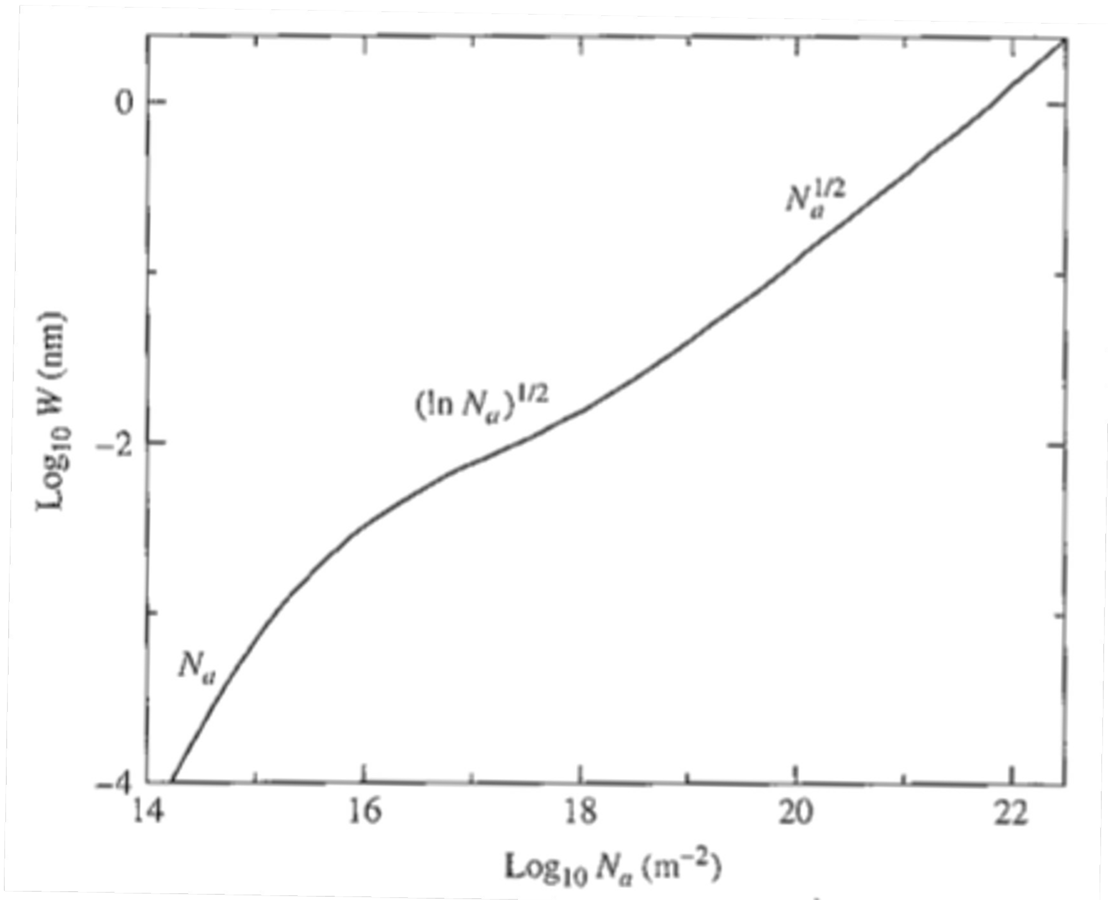
- At low abundances, $W \propto Nf$, #atoms/ions in path times oscillator strength.

- As abundance increases, center of line becomes optically thick (line saturates), only wings can absorb more photons. Core gets flatter, *W* grows approximately as $(\ln Nf)^{1/2}$

- Increasing density further increases *W* through pressure broadening, increasing rate of growth, now W grows approximately as $(Nf)^{1/2}$

**Curve of growth**

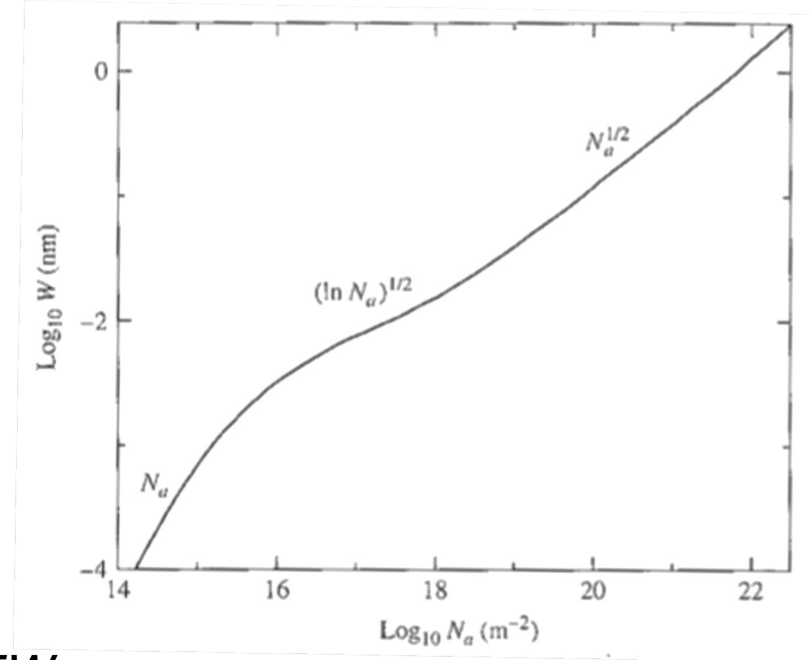
A plot of the line width as a function of the column density for a given transition with a certain *f*.




Boltzmann and Saha Eqn can then be used to find the total number of atoms of a given element.

**Using the curve of growth**

This graph can be applied to any line originating from the same energy state, not just one specific transition.




1. Observe absorption line and measure the *EW*.
2. Locate position on the curve of growth for the transition, read *N* value.
3. Use Boltzmann and Saha Equations to find the fraction of all atoms and ions in this state.
4. Calculate the total column density of the element.

See Ex. 9.5.5

Problem 9.13:

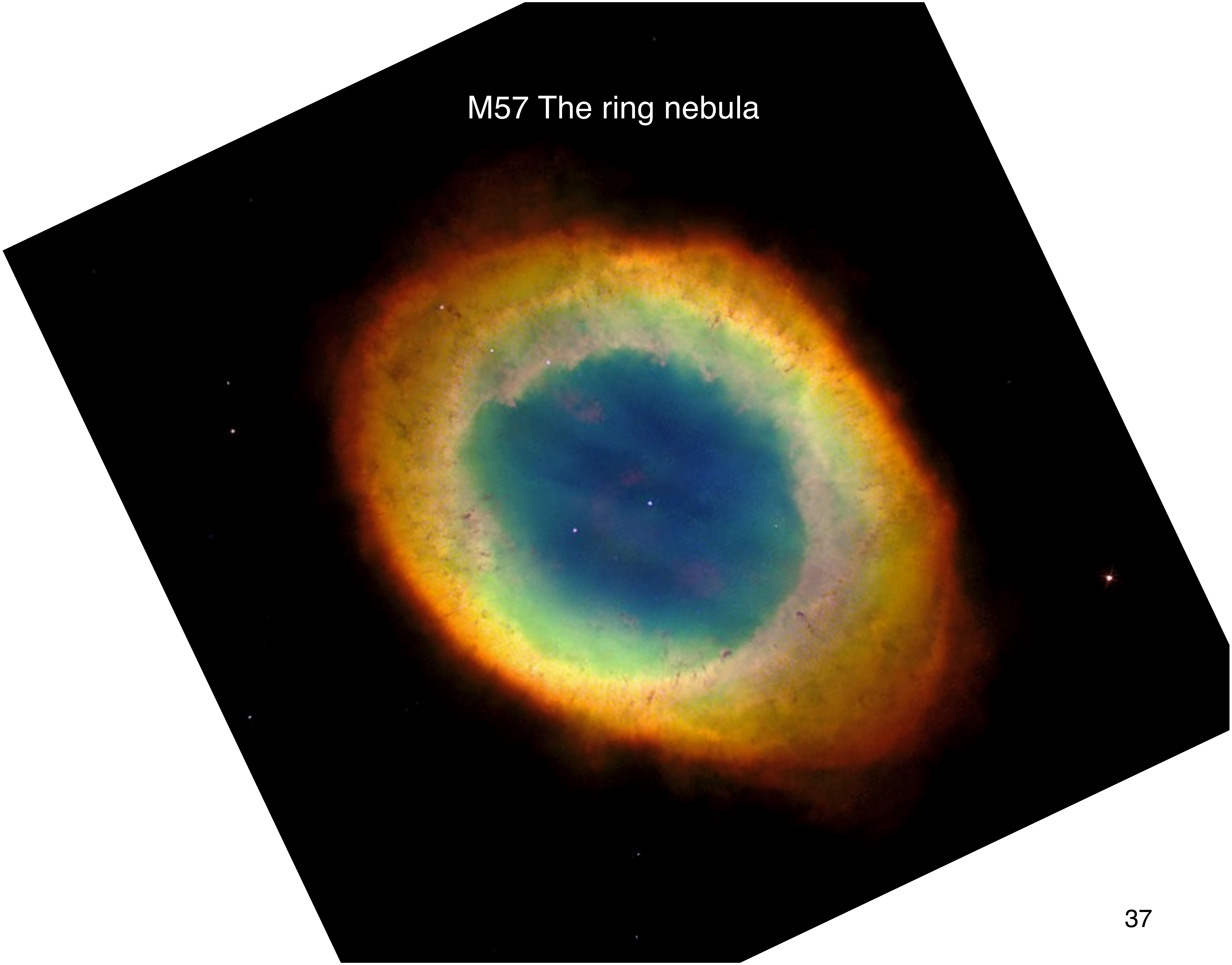
Consider a large, hollow spherical shell of hot gas surrounding a star. Under what circumstances would you see the shell as a glowing ring around the star?

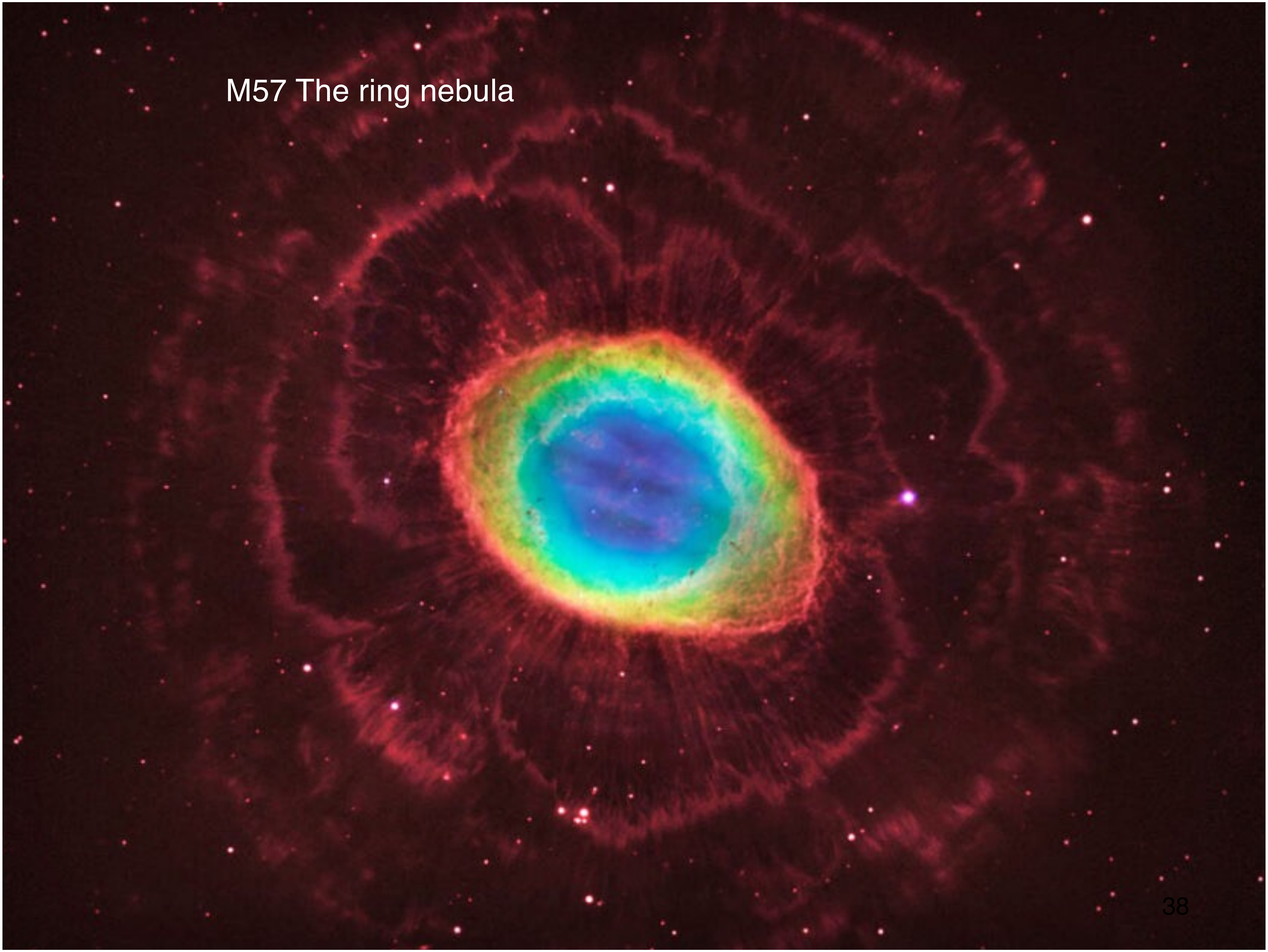
What can you say about the optical thickness of the shell?

If we have a hollow shell, then it will look like a ring if we can see through the middle regions. That means the shell must be *optically thin.*

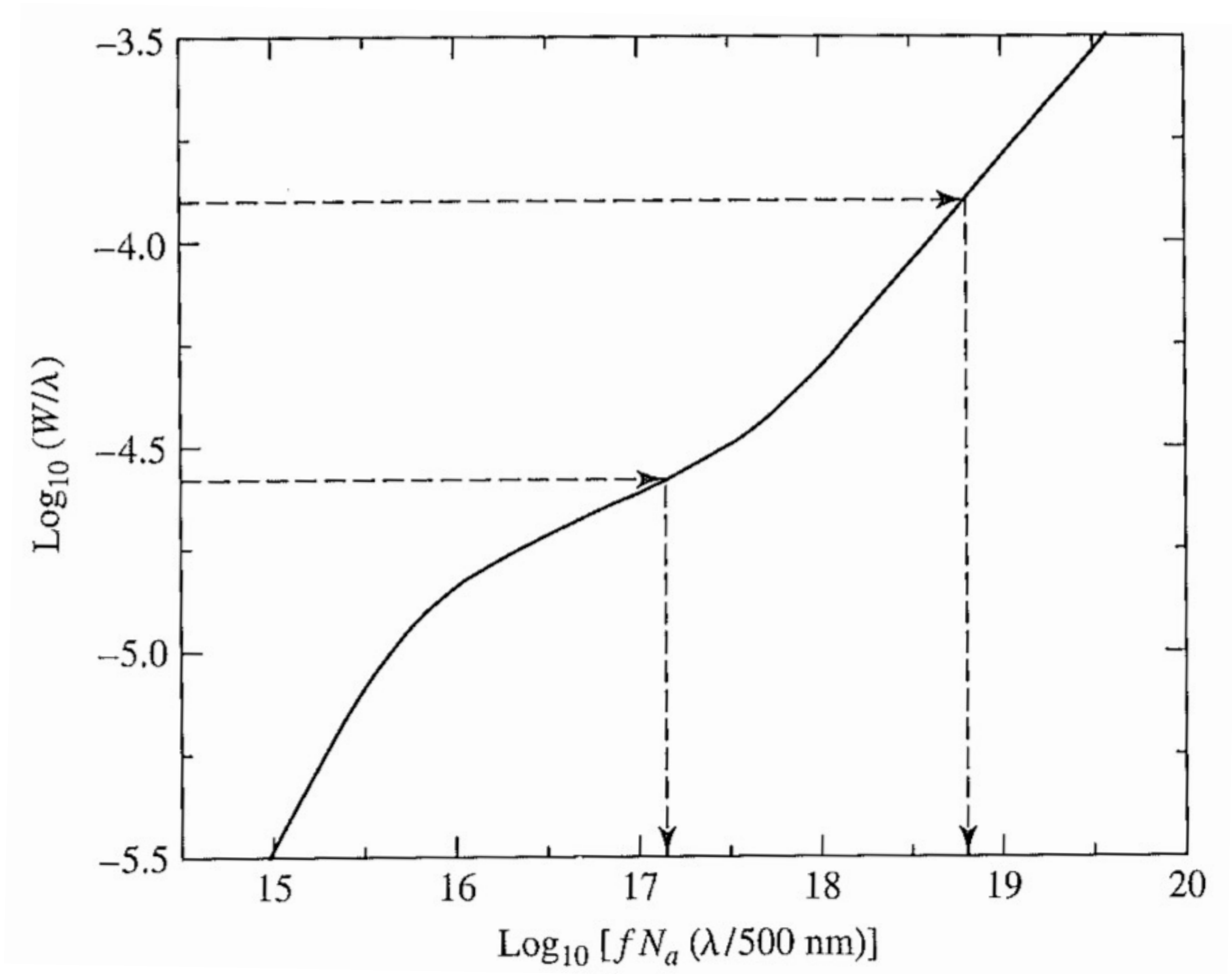
Optically thin, hot gas will produce emission lines (Kirchoff's!). Near the edge of the shell, where LOS passes through more gas, the shell will look brighter.

# M57 The ring nebula

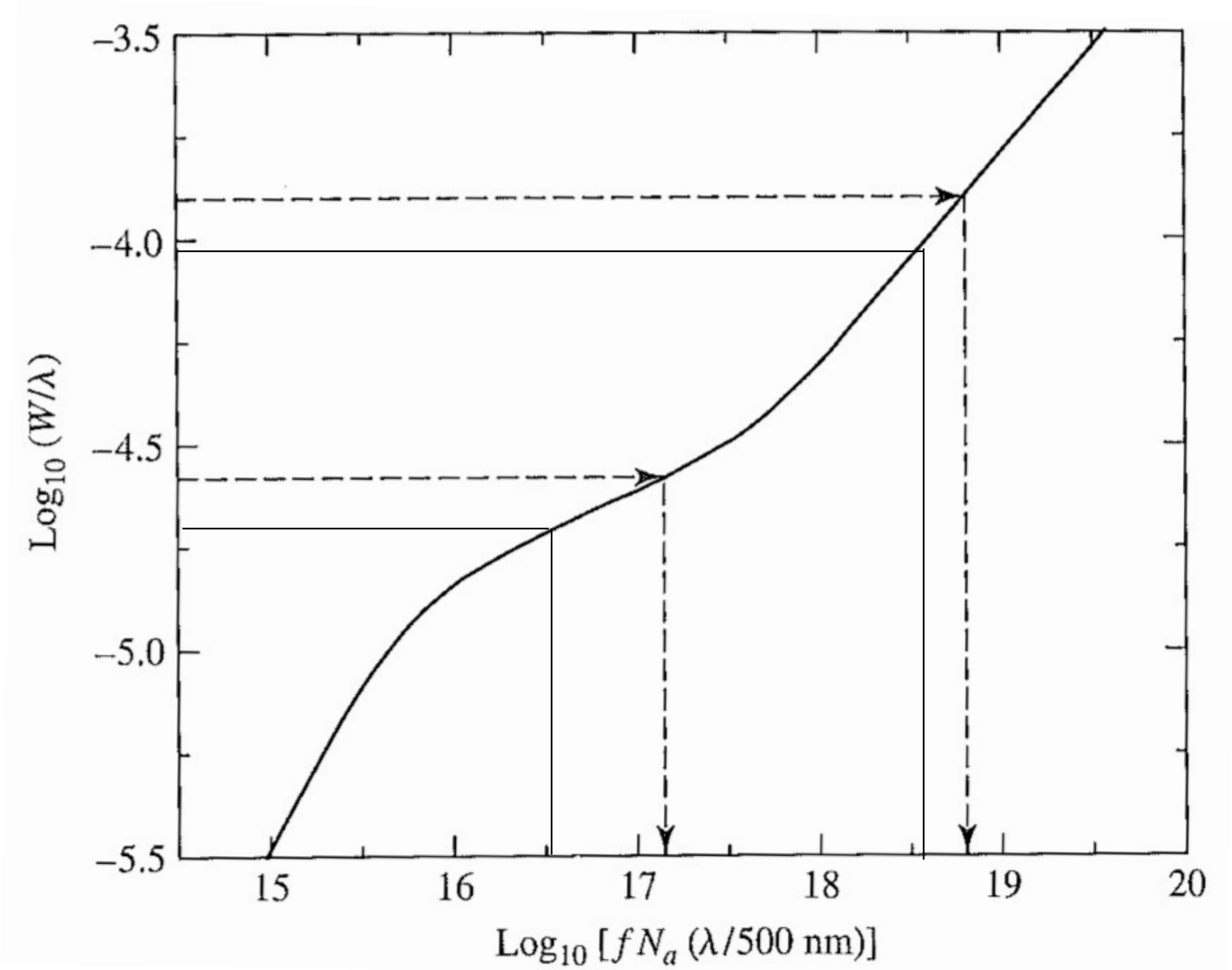



M57 The ring nebula

Problem 9.26



Problem 9.26



−3.5
−4.0
−4.5
−5.0
−5.5
15
16
17
18
19
20
$\mathrm{Log}_{10}\,(W/\lambda)$
$\mathrm{Log}_{10}\,[f N_a\,(\lambda/500\ \mathrm{nm})]$