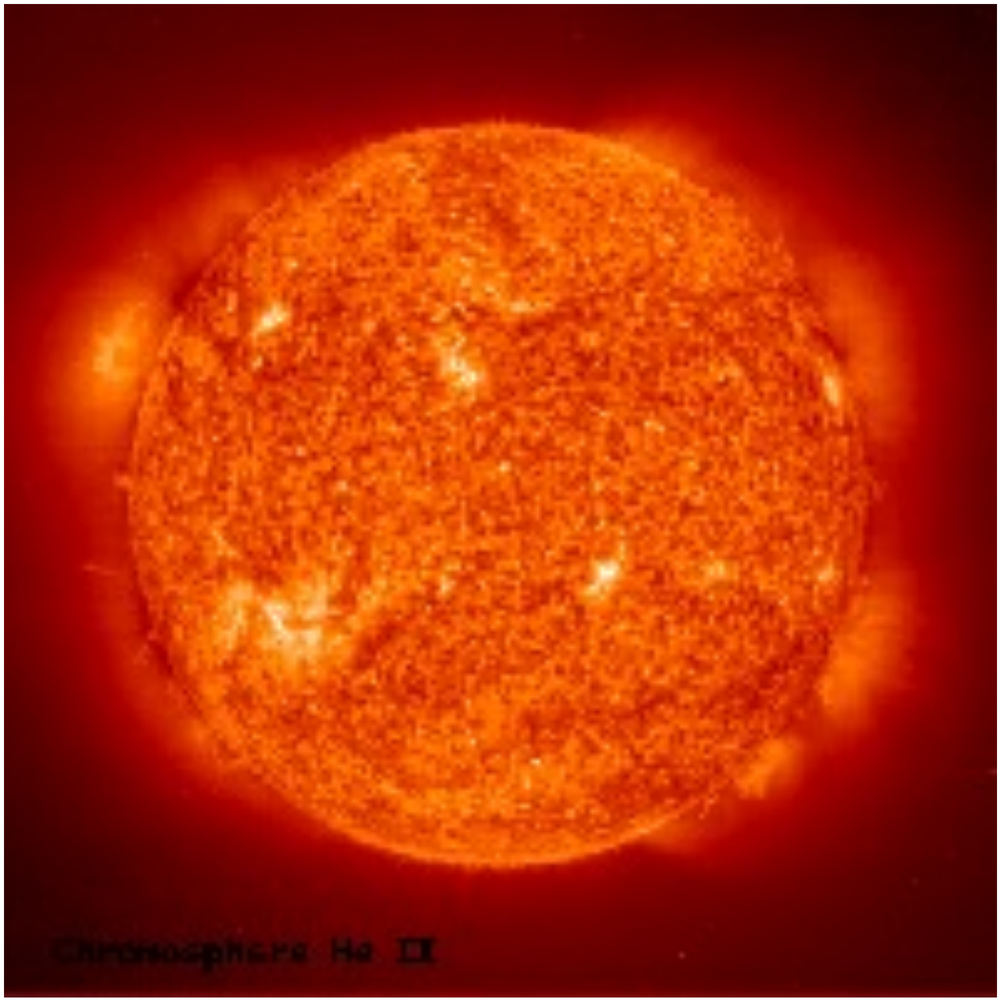
Astronomy 421



Lecture 12: Stellar Atmospheres I

Key concepts:

Radiation and how it propagates (aka Radiative Transfer)

Opacity and optical depth

Sources of opacity in stars

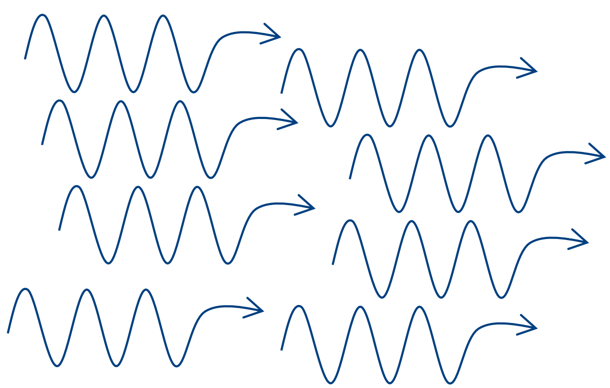
Diffusion, random walks and optical depth

Radiation field definitions

Aim: to understand how light travels through gas.

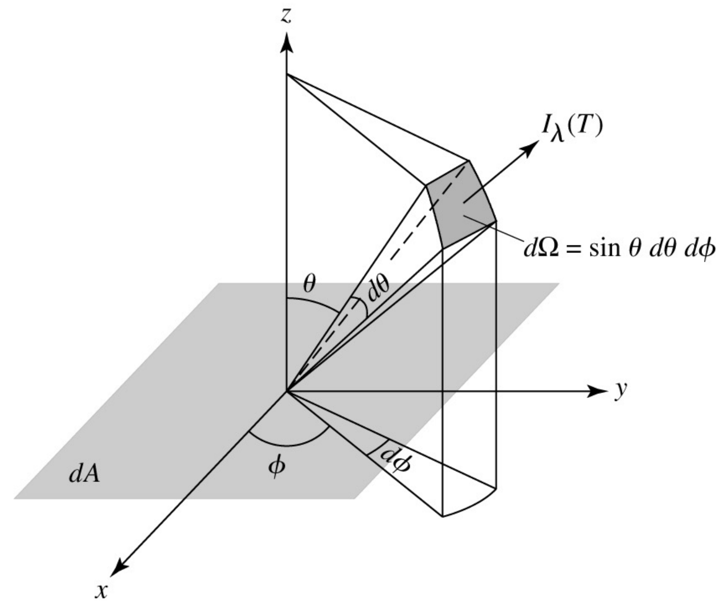
We need a language to describe how radiation propagates in presence of matter (works both for stellar atmospheres, interiors, and in the ISM), so we can quantify flux, radiation pressure, radiation energy density, opacity, etc.

Consider a light beam traveling through space:



(This treatment builds on what we did in Ch 3)

A general quantity is <u>(specific) intensity</u>.
For radiation in the range λ to $\lambda+d\lambda$ passing through an area dA at angle θ into solid angle $d\Omega$:



$$I_\lambda d\lambda = \frac{E_\lambda d\lambda}{dt\,dA\cos\theta\,d\Omega}$$

where $E_\lambda d\lambda$ is the radiation energy between λ and $\lambda+d\lambda$ ($dA\cos\theta$ is the projected area through which the radiation is traveling)

Units of I_λ are $\mathrm{W\,m^{-2}\,m^{-1}\,sr^{-1}}$

energy

per unit time

per unit area

per unit wavelength

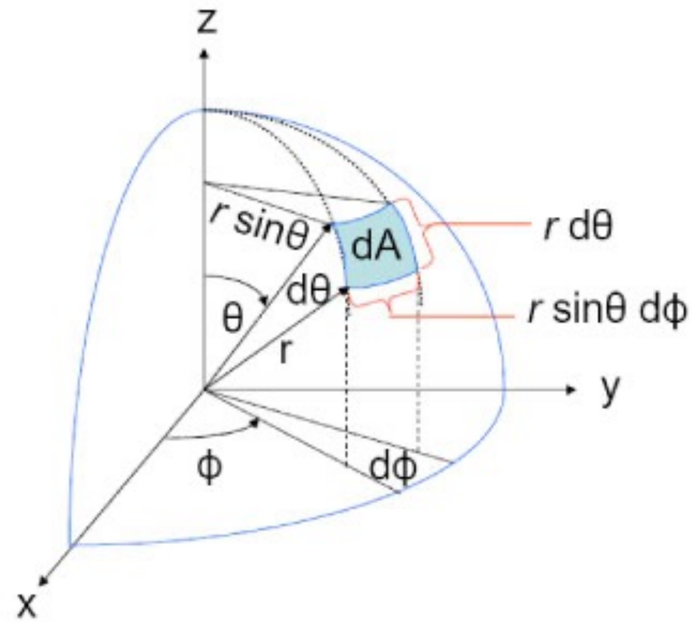
per unit solid angle

In spherical coordinates:

$$d\Omega = \sin\theta\, d\theta d\phi$$

Solid angle

- 2-D analog of an angle: the apex of a cone.



- 1-D angle gives you arc length $rd\theta = s$

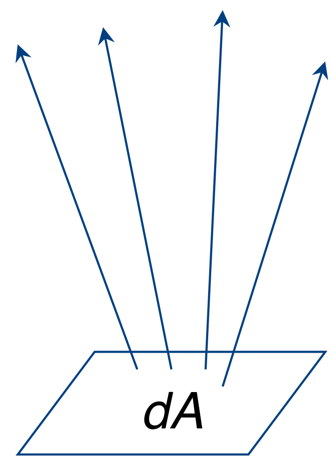
- Solid angle gives you surface area $r^2 d\Omega = dA$

- Unit is steradian (sr), and there are 4π sr in a spherical surface.

- A small element of area *dA* in spherical coordinates:
  - Side 1 has length $r d\theta$
  - Side 2 has length $r \sin\theta d\varphi$
  - Area $dA = r^2 d\theta \sin\theta d\varphi$ so $d\Omega = \sin\theta d\theta d\phi$

$$\int_\Omega d\Omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta d\theta d\phi = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi = 2 \times 2\pi$$

First, let's specify an area the beam passes through, *dA*, per unit time, *dt*.



Then, the amount of energy passing through *dA* during *dt* is called (already familiar to us) the *flux* (of energy).

$$F = \frac{E}{dA\,dt}$$

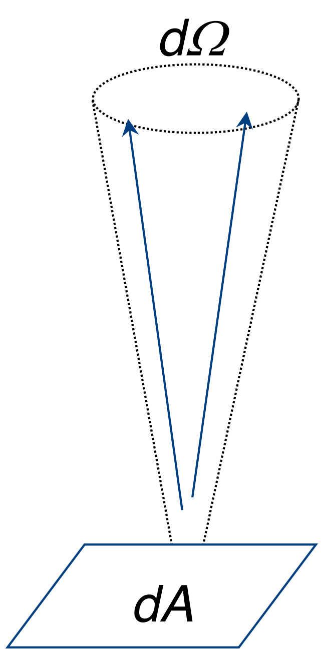
Units?

Now, we will restrict our calculations to a limited range of wavelengths, $d\lambda$, so that the energy contained in this range is $E_\lambda d\lambda$.

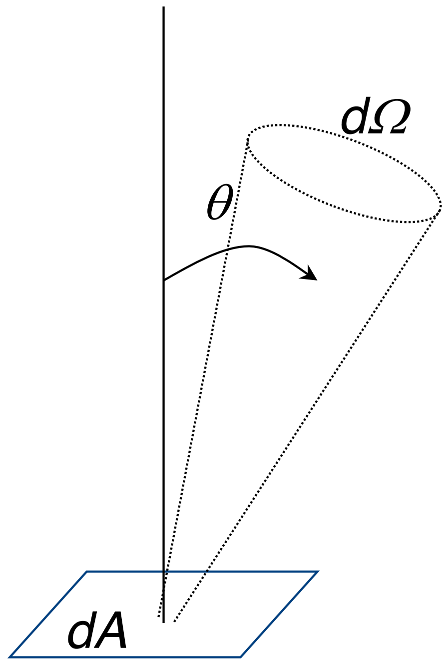
Then,the corresponding flux per unit wavelength (net energy passing through unit area in unit time):

$$F_\lambda d\lambda = \frac{E_\lambda d\lambda}{dAdtd\lambda}$$

Be more specific by choosing only light emerging in a specific solid angle, $d\Omega$, normal to the unit area:



$$I_\lambda = \frac{E_\lambda d\lambda}{dAdtd\lambda d\Omega}$$



If the solid angle is not perpendicular to the surface (corresponding to light passing straight through), then we have to take into account the angle, θ.

Specific intensity

With this angle, the area of *dA* perpendicular to the beam is smaller, *dA cos(θ).* Now we have the *specific intensity* (or just *intensity*)*:*

$$I_\lambda = \frac{E_\lambda d\lambda}{dA\,dt\,d\lambda\ \cos\theta d\Omega}$$

I_λ can vary with direction. So define

$$\text{mean intensity} = \frac{\int I_\lambda d\Omega}{4\pi} = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} I_\lambda \sin\theta d\theta d\phi = < I_\lambda >$$

Energy density

Next, consider light traveling ⊥ to *dA*. Travels a distance *dL/c* in time *dt*. So energy per unit volume in the range λ to λ+dλ is $E_\lambda d\lambda/dAdL$ = $E_\lambda d\lambda/cdAdt = I_\lambda d\lambda d\Omega/c$ for θ=0º. C+O (9.1) generalize this to all directions θ to show that the energy per unit volume in range λ to λ+*d*λ is the energy density:

$$u_\lambda d\lambda = \frac{1}{c}\int_\Omega I_\lambda d\lambda d\Omega = \frac{4\pi}{c} < I_\lambda > d\lambda$$

Use this idea to go from intensity or flux to energy density by using dt = dL/c in Prob. 9.1 of C&O

Integrated (or Total) energy density

$$u = \int_0^\infty u_\lambda d\lambda$$

In the specific case of an isotropic blackbody radiation field:

$$< I_\lambda >= I_\lambda = B_\lambda$$

Which then gives

$$u_\lambda d\lambda = \frac{4\pi}{c} B_\lambda d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/kT} - 1} d\lambda$$

$$u = \frac{4\pi}{c} \int_0^\infty B_\lambda d\lambda = \frac{4\pi}{c} \frac{\sigma T^4}{\pi} = \frac{4\sigma}{c} T^4 = aT^4$$

$a = \dfrac{4\sigma}{c}$ is called the "radiation constant"

Emergent flux reminder

This is the intensity of radiation passing through *dA*, integrated over all angles:

$$F_\lambda d\lambda = \int_\Omega I_\lambda d\lambda \cos\theta d\Omega$$

$$\int_\lambda F_\lambda d\lambda = \sigma T^4 \text{ for blackbody}$$

Luminosity L = Power = $A \int_\lambda F_\lambda d\lambda$

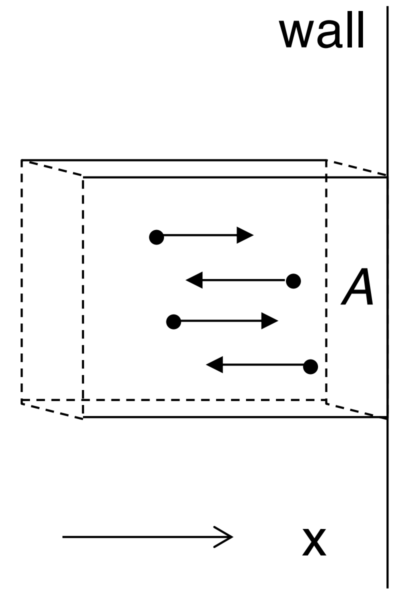
Define 1 Jy = 10^{-26} W m^{-2} Hz^{-1} small because astronomical sources are far away

Worksheet: The cosmic microwave background has a uniform brightness of 10,000 Jy. How much power would a 100 meter antenna collect with a bandwidth of 1 GHz?

Radiation pressure

Understand gas pressure first.

Assume all particles have speed v_x, and momentum p_x. Number density is n.



$$\text{Pressure} = \frac{\text{force}}{\text{area}} = \frac{\Delta p_x}{\Delta t}\frac{1}{A}$$

What is momentum transferred across area A in time Δt ?

If N = number of particles in volume V, then $N/2$ are moving to the right

and $V = A v_x \Delta t$

$$\Delta p_x = \frac{N}{2} 2 p_x = n V p_x$$

$$\text{so pressure} = \frac{\Delta p_x}{\Delta t A} = n v_x p_x$$

For some distribution of *x* velocities and momenta,

$$\text{Pressure} = n \langle v_x p_x \rangle$$

For an isotropic distribution:

$$P = \frac{1}{3} n \langle vp \rangle \quad \text{since} \quad vp = \bar{v} \cdot \bar{p} = v_x p_x + v_y p_y + v_z p_z$$

Photons can also exert a pressure by transferring momentum to matter. For photons, $v=c$, and $p=E/c$.

$$P_{rad} = \frac{1}{3} n_{rad} < c\frac{E}{c} > = \frac{1}{3} n_{rad} < E >$$

But $n_{rad} \langle E \rangle$ = energy density, u, so for BB radiation

$$P_{rad} = \frac{1}{3} a T^4$$

Radiation pressure