The massive star is closer to center of mass, and moves more slowly than the planet, but it does move!

Worksheet #5: Sun and Jupiter orbit their common center of mass every 11.86 years.

Note $M_J / M_{Sun} = 1/1047$

What is the orbital speed of the Sun?

What is the astrometric displacement for a similar system at a distance of 10 pc?

Solution: First calculate the semi-major axis of the Jupiter's orbit

$P^2 = a^3$ so $a = 5.2$ AU $= 780 \times 10^9$ m

$M_s r_s = M_j r_j$ so $r_s = 740 \times 10^6$ m

Angular size $= 2 \times 740 \times 10^6$ m/10 x 3.09×10^{16} m

$= 4.8 \times 10^{-9}$ radians x 206265000 mas/rad

$= 0.99$ mas

Calculate orbital speed of Sun assuming Jupiter is only planet:

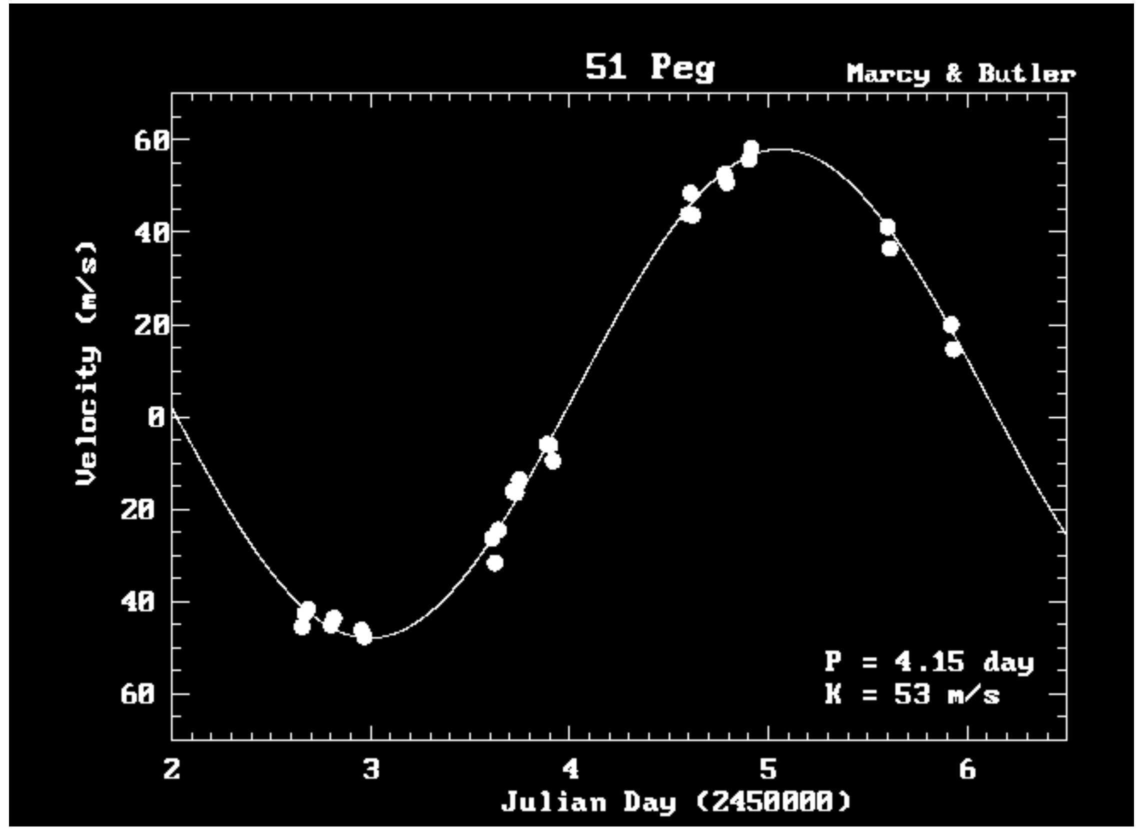
Moves in circular orbit of radius 742,000 km

How much Doppler shift? Consider H-alpha absorption line, at rest wavelength 656 nm:

$$V = \frac{2\pi r}{P} = \frac{2\pi(742{,}000\ \text{km})}{11.86\ \text{years}} = 12.5\ \text{m/s}$$

$$\Delta\lambda = \frac{V}{c}\lambda_0 = \frac{12.5\ \text{m/s}}{3\times10^{8}\ \text{m/s}} 656\ \text{nm} = 2.7\times10^{-5}\ \text{nm}$$

51 Pegasi



Michel Mayor & Didier Queloz at Geneva Observatory observed wobble in 1995

Sun-like star 15 pc distance

Wobble was 53 m/s, period 4.15 days

Implied a planet with 0.5 Jupiter mass orbiting at 0.05 AU!

First planet found around sun-like star

Michel Mayor & Didier Queloz
Nobel Prize 2019

Selection effects

Doppler wobble biased towards massive planets close to their star (leads to larger velocities and shorter periods). Now getting close to Earth-mass planets.

Limited by the orbital speed sensitivity (few m/s but always improving) and length of orbital period: for more than several year periods, hard to tell if motion is periodic

Inclination of binary orbit unknown (unless transits observed). More likely to be close to edge-on for detection. If not, wobble is larger than measured and so is planet.

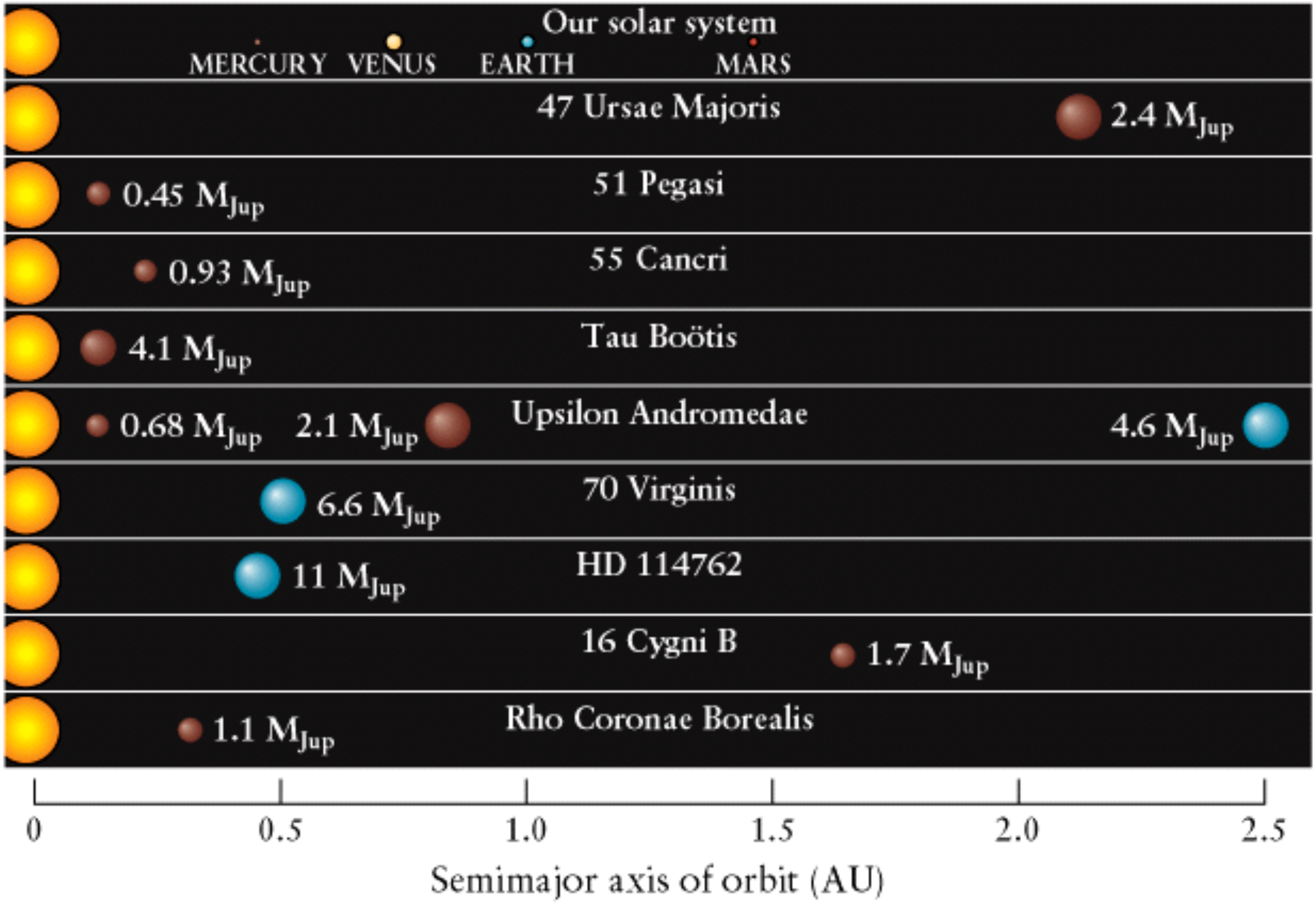
Characteristics of detections

Some hot Jupiters on small orbits (migration or formation?)

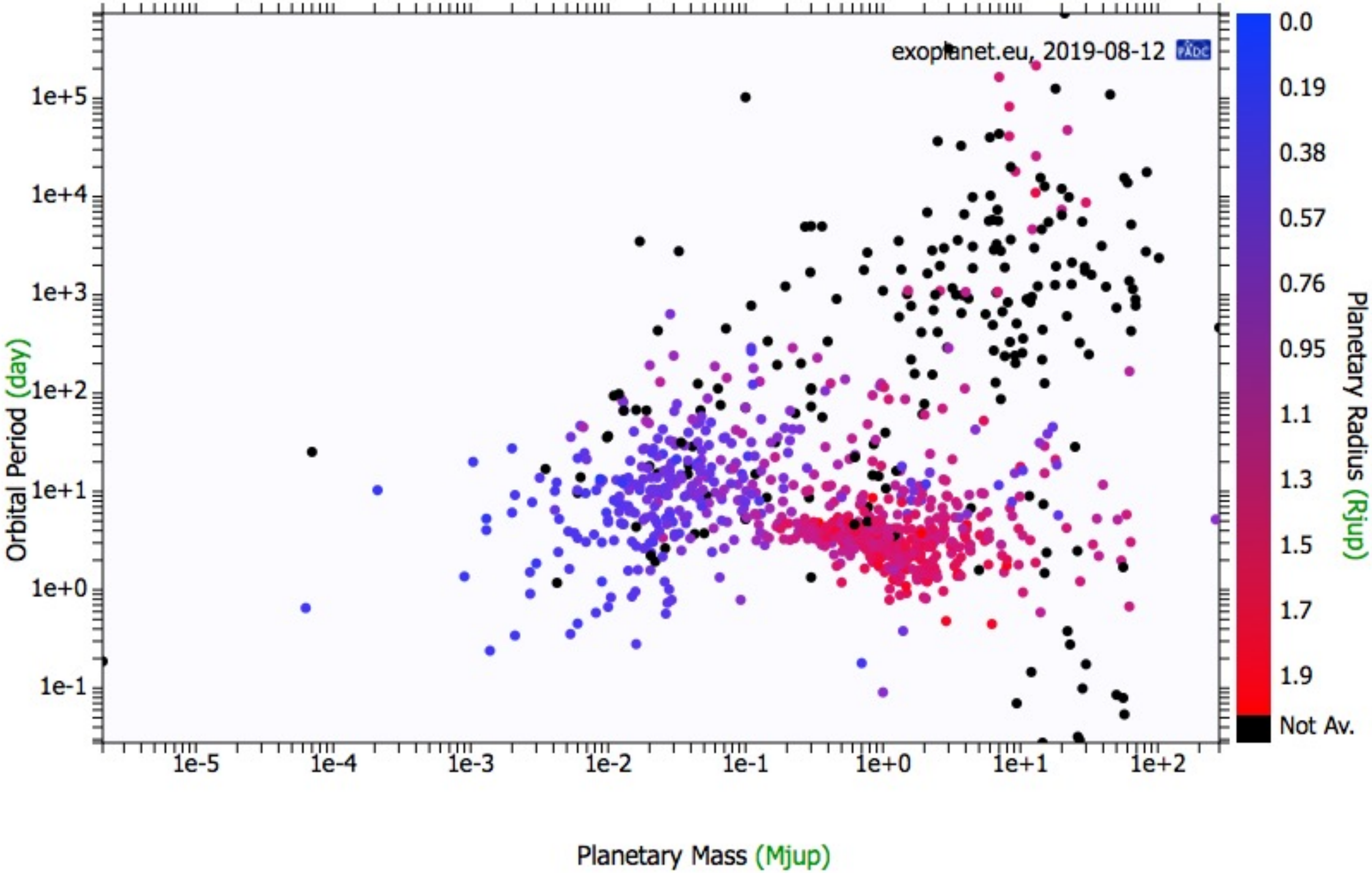
Some with very elliptical orbits (in the Solar System this is a sign of perturbation => supports migration idea?)

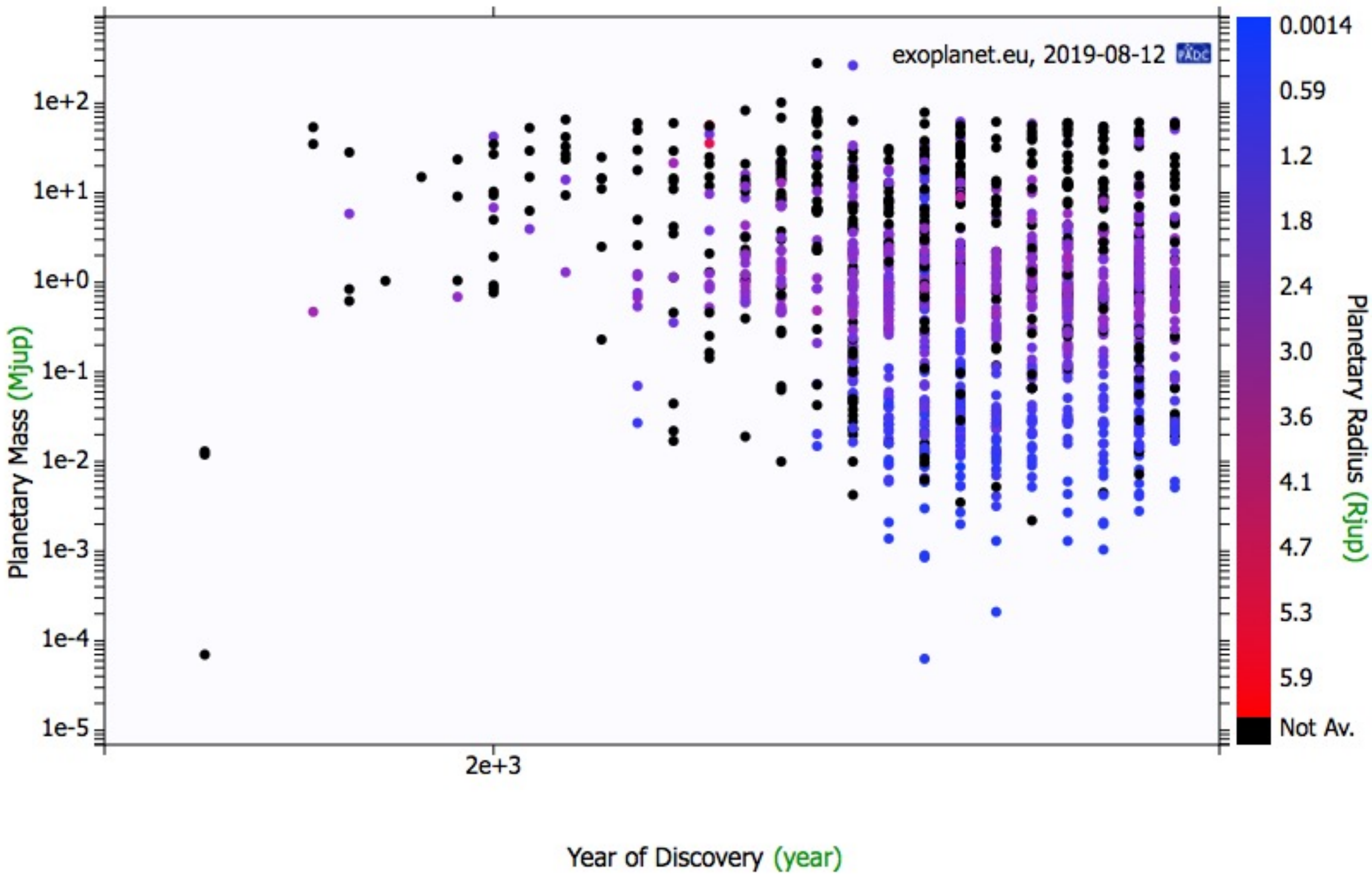
Rare around low-mass stars (smaller disks thus less material)

Rare around metal-poor stars

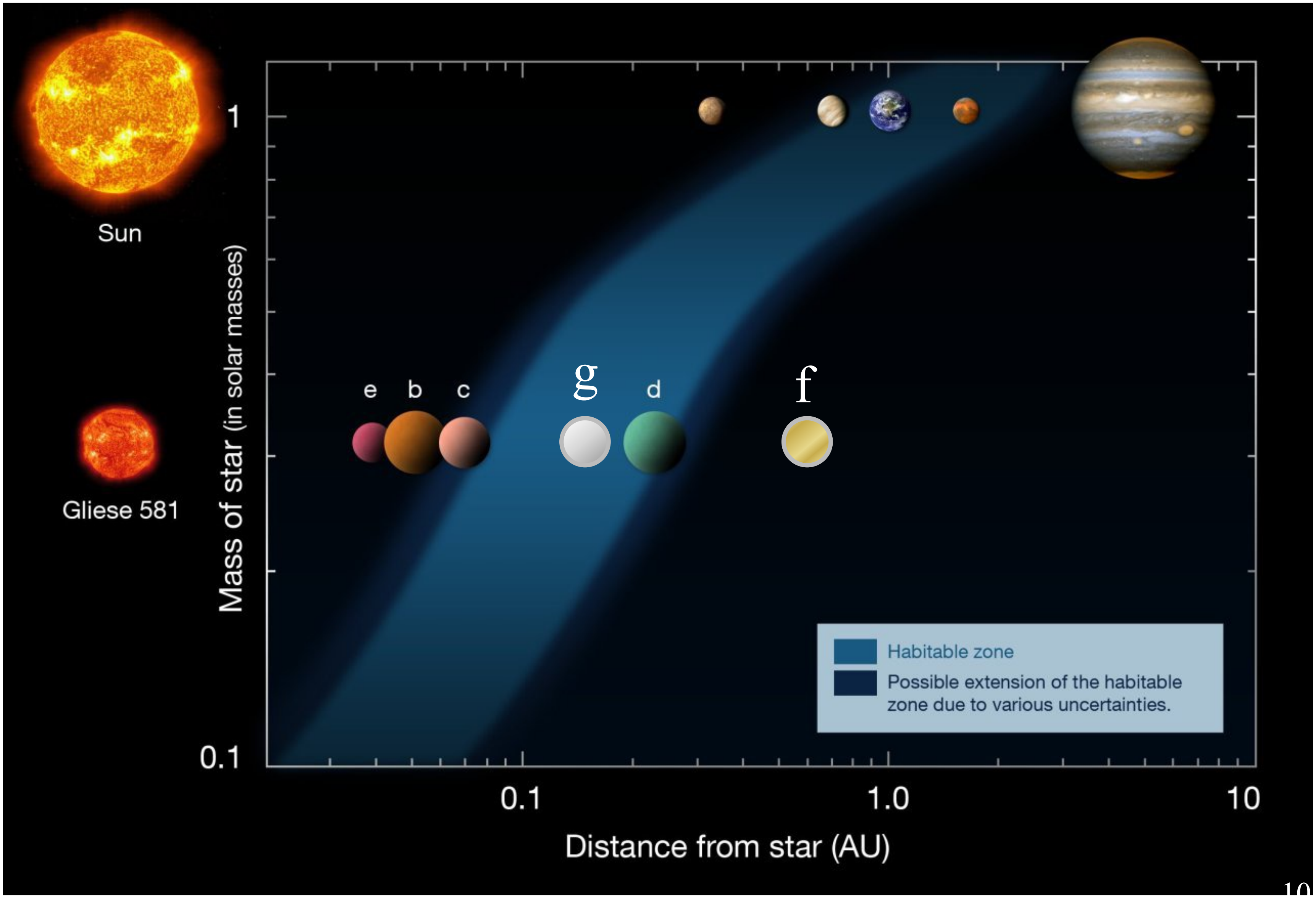


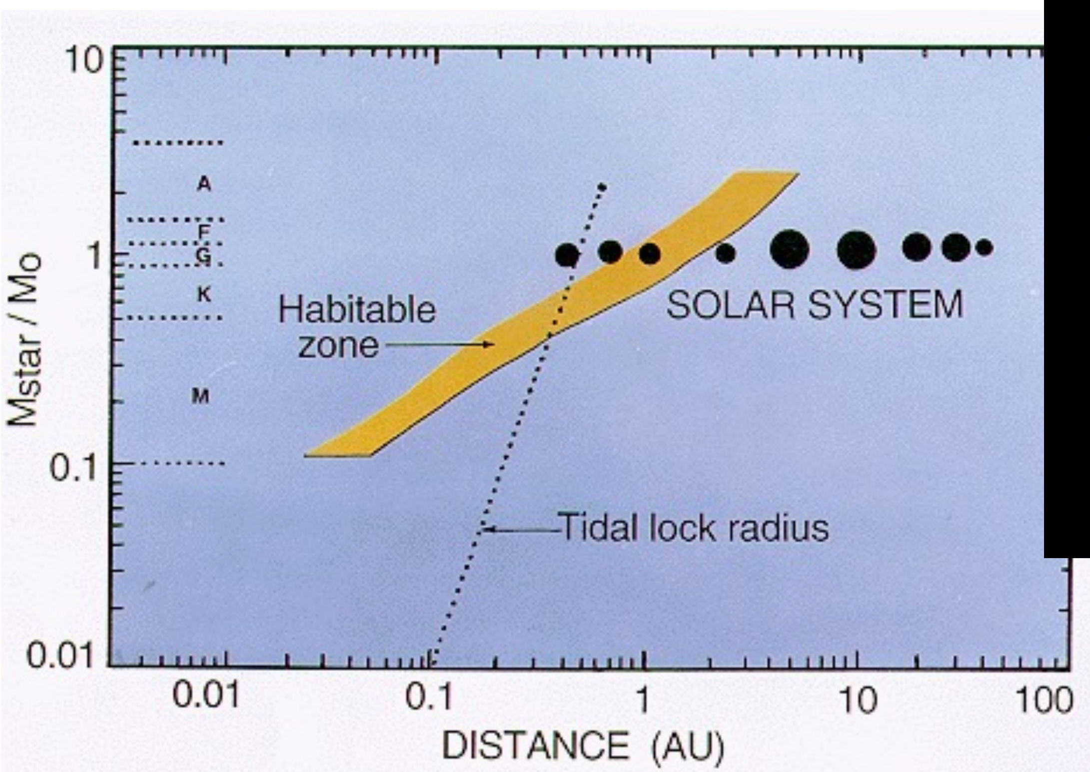
count: 4472 planets! What about these "hot Jupiters"?


exoplanet.eu, 2019-08-12
Orbital Period (day)
1e+5
1e+4
1e+3
1e+2
1e+1
1e+0
1e-1
1e-5
1e-4
1e-3
1e-2
1e-1
1e+0
1e+1
1e+2
Planetary Mass (Mjup)
0.0
0.19
0.38
0.57
0.76
0.95
1.1
1.3
1.5
1.7
1.9
Not Av.
Planetary Radius (Rjup)

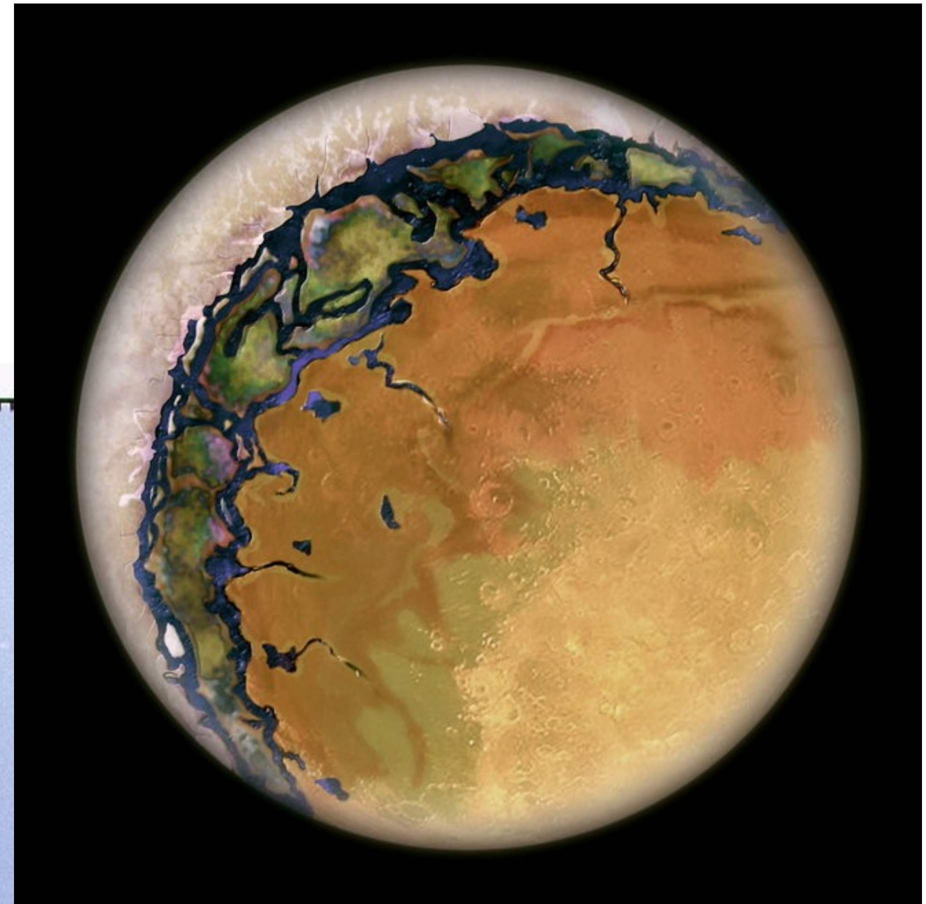


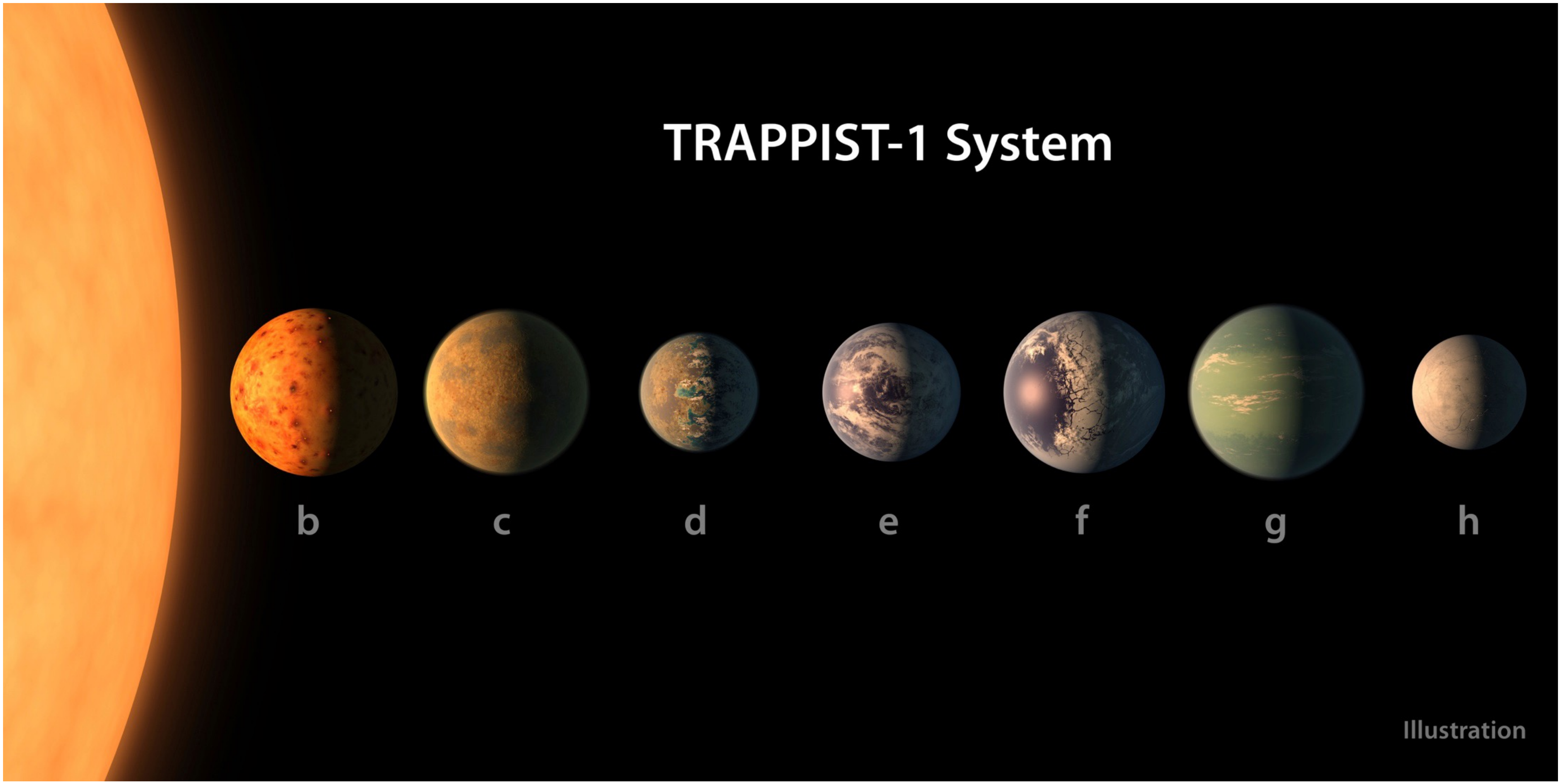
exoplanet.eu, 2019-08-12
1e+2
1e+1
1e+0
1e-1
1e-2
1e-3
1e-4
1e-5
Planetary Mass (Mjup)
2e+3
Year of Discovery (year)
0.0014
0.59
1.2
1.8
2.4
3.0
3.6
4.1
4.7
5.3
5.9
Not Av.
Planetary Radius (Rjup)

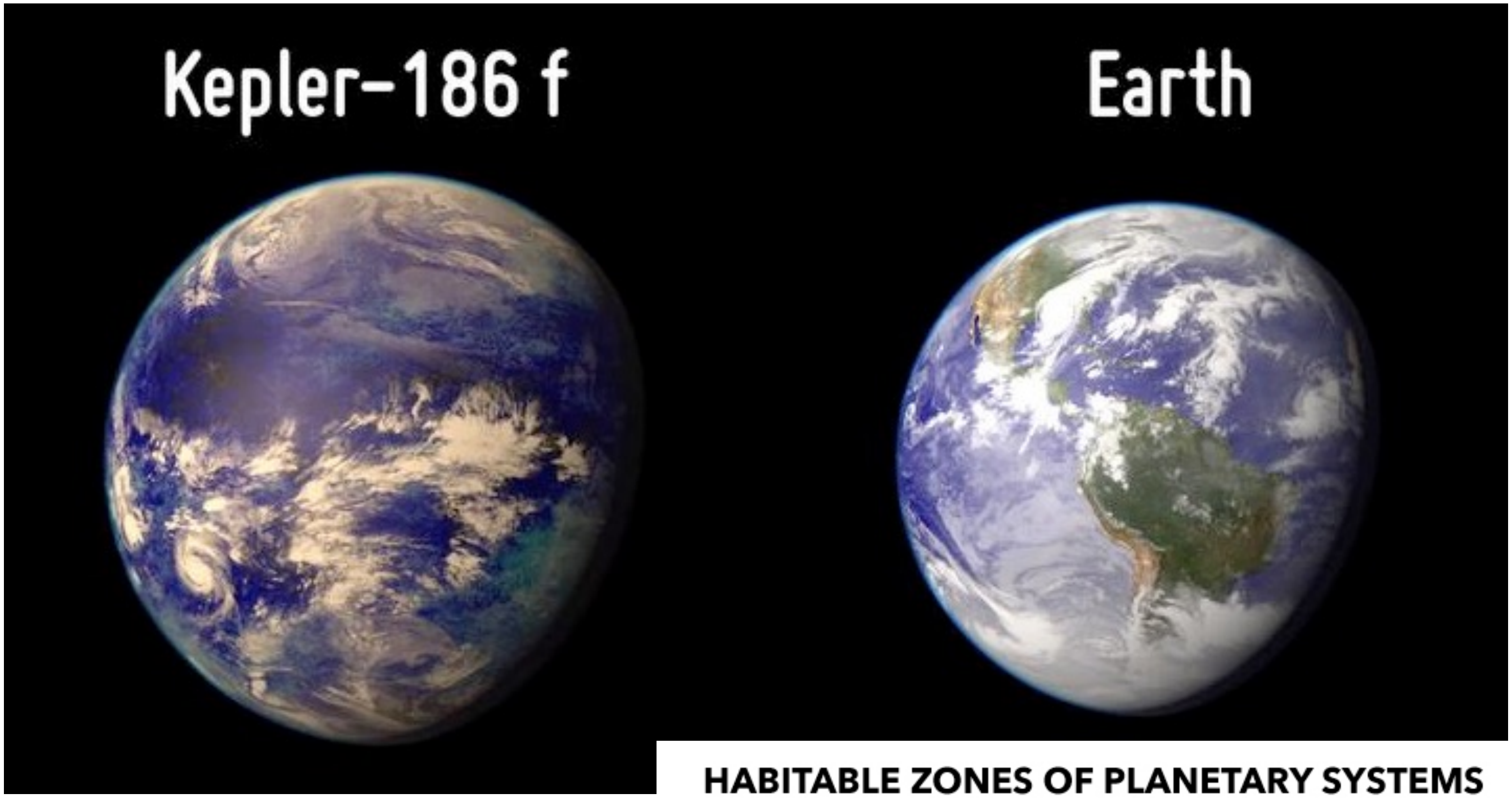

Sun
Gliese 581
Mass of star (in solar masses)
1
0.1
e
b
c
g
d
f
Habitable zone
Possible extension of the habitable zone due to various uncertainties.
0.1
1.0
10
Distance from star (AU)

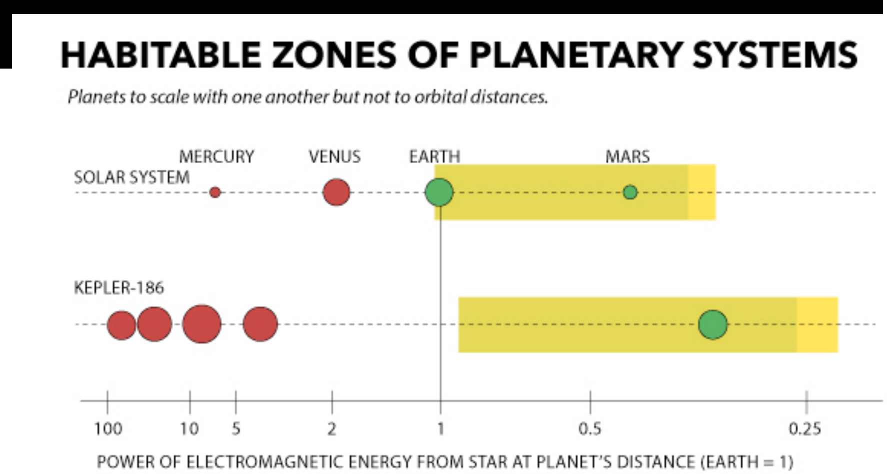


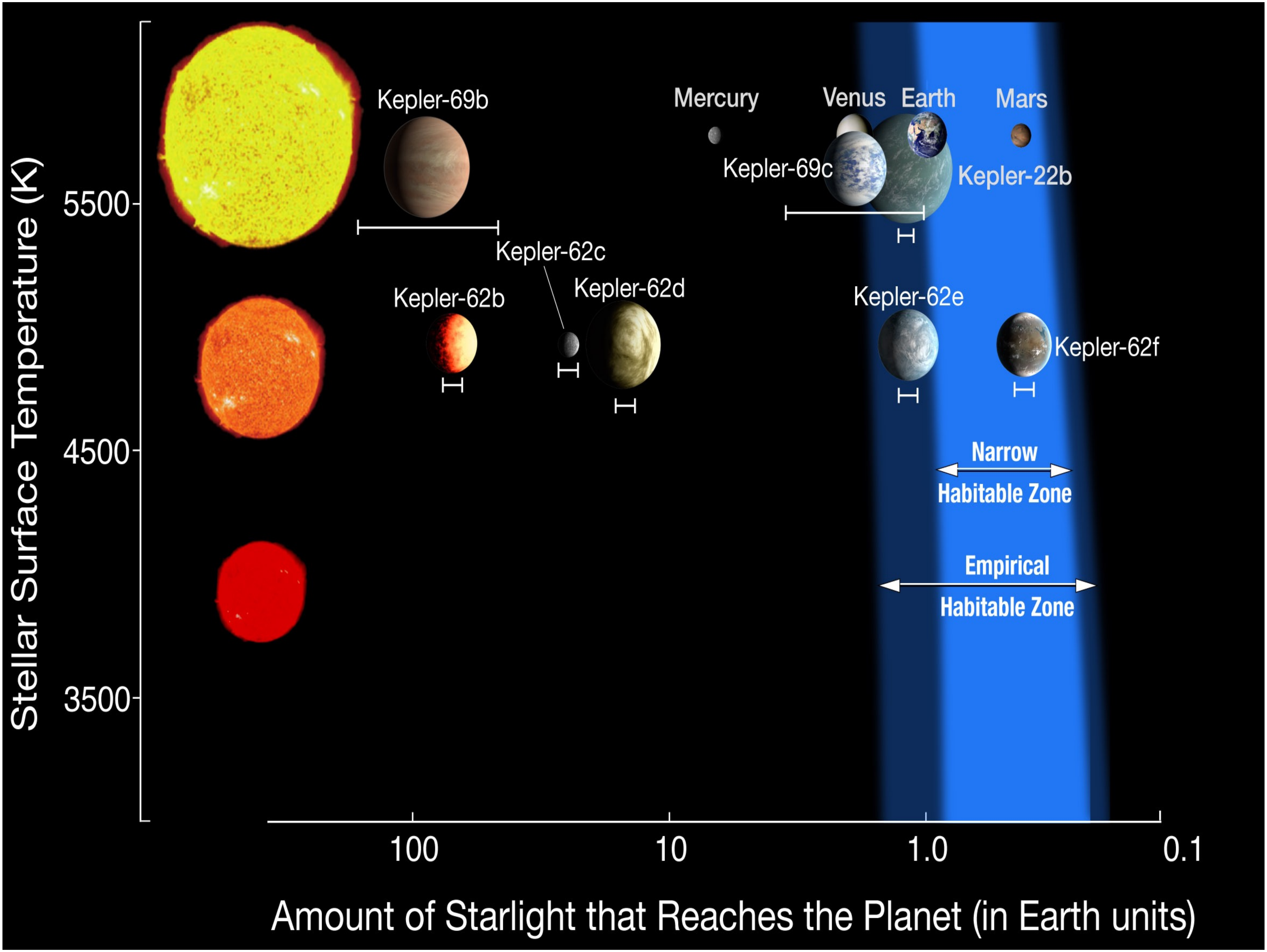
10
1
0.1
0.01
Mstar / Mo
A
F
G
K
M
Habitable
zone
SOLAR SYSTEM
Tidal lock radius
0.01
0.1
1
10
100
DISTANCE (AU)

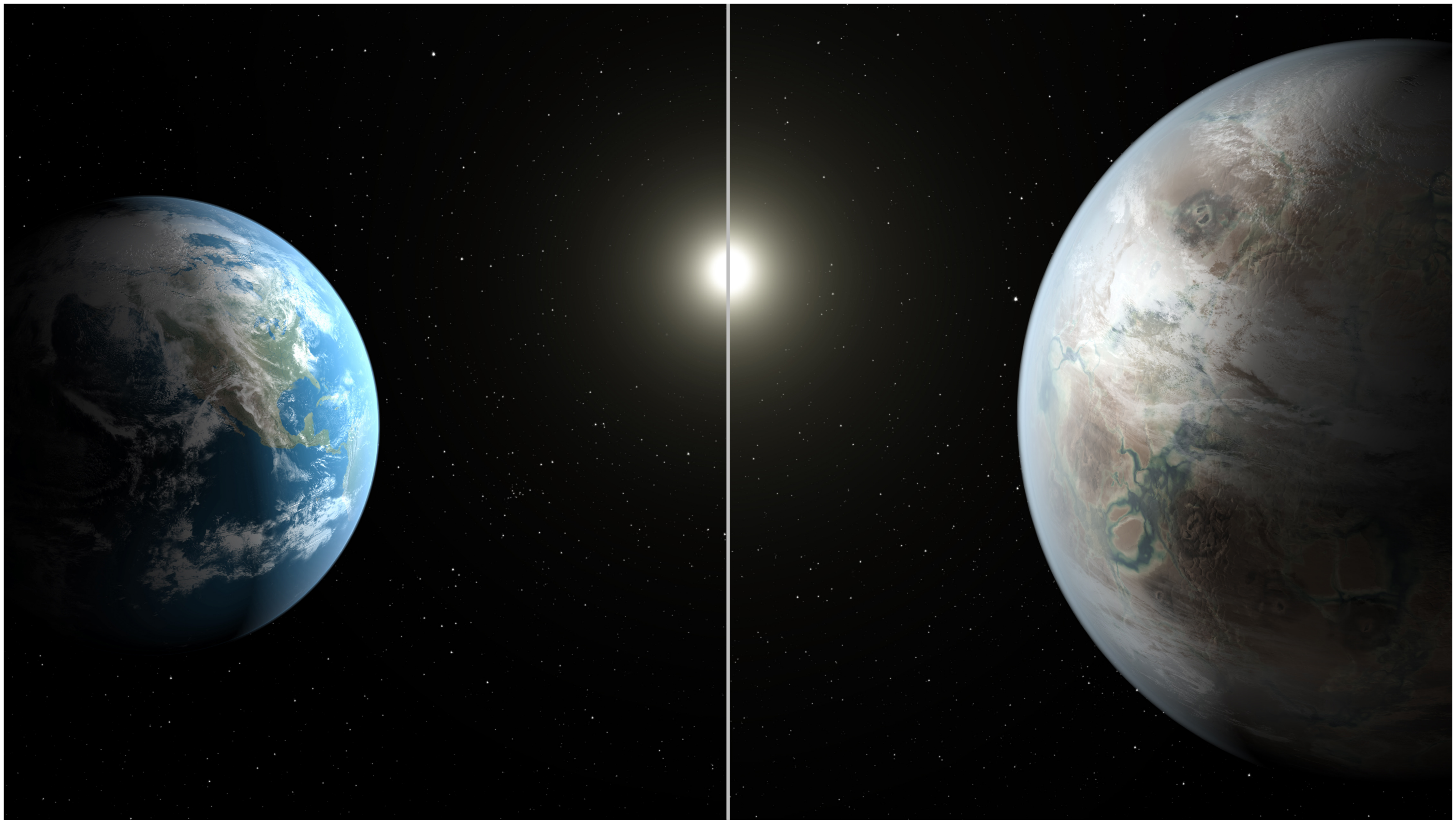



TRAPPIST-1 System
b
c
d
e
f
g
h
Illustration


Kepler-186 f
Earth


HABITABLE ZONES OF PLANETARY SYSTEMS
Planets to scale with one another but not to orbital distances.
SOLAR SYSTEM
MERCURY
VENUS
EARTH
MARS
KEPLER-186
100
10
5
2
1
0.5
0.25
POWER OF ELECTROMAGNETIC ENERGY FROM STAR AT PLANET'S DISTANCE (EARTH = 1)


Stellar Surface Temperature (K)
5500
4500
3500
Kepler-69b
Mercury
Venus
Earth
Mars
Kepler-69c
Kepler-22b
Kepler-62c
Kepler-62d
Kepler-62b
Kepler-62e
Kepler-62f
Narrow
Habitable Zone
Empirical
Habitable Zone
100
10
1.0
0.1
Amount of Starlight that Reaches the Planet (in Earth units)



Detecting Radio Bursts from Exoplanets

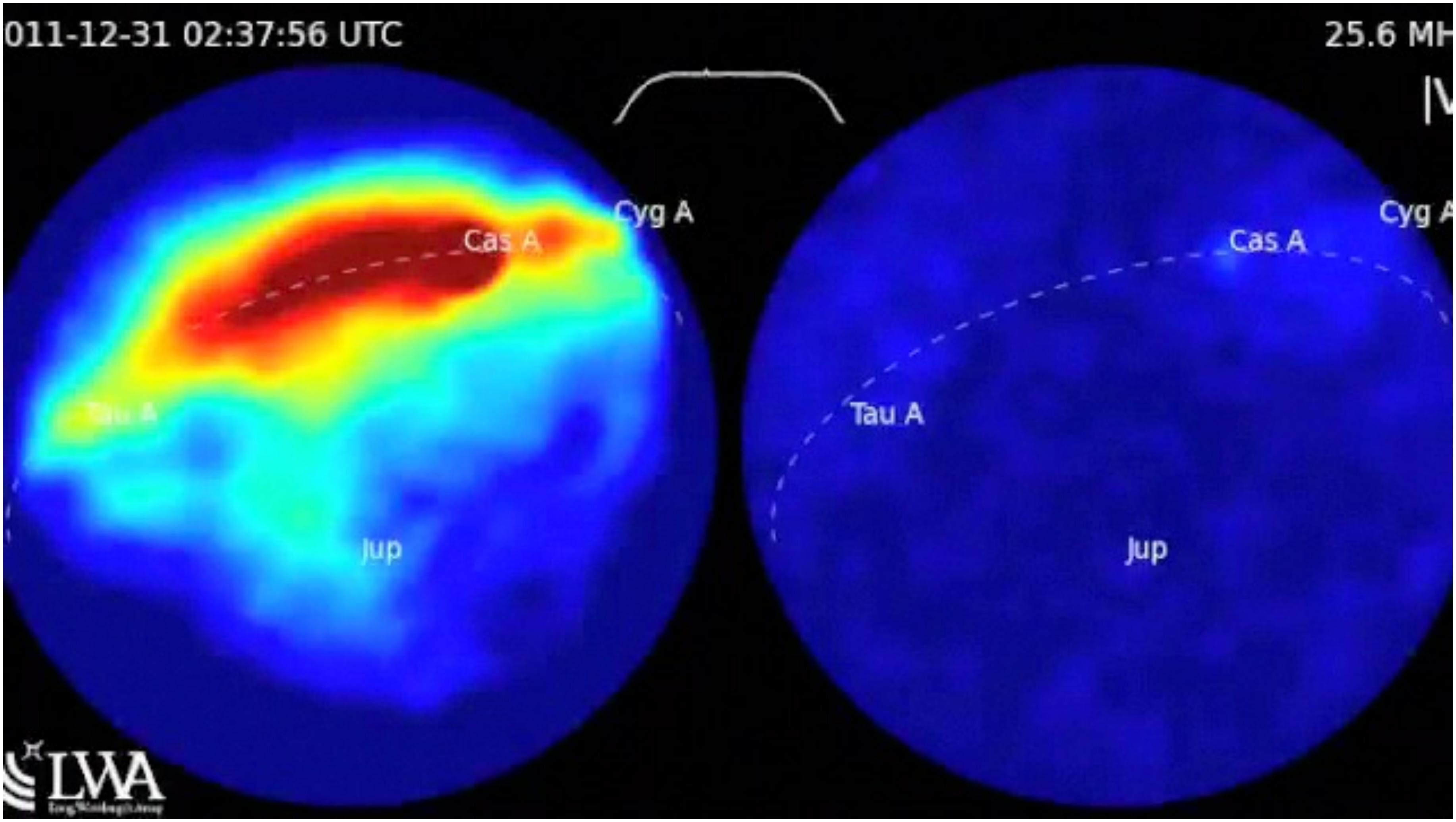
Suitability of the LWA1

Observations to date

Near future: Owens Valley

Farther future: the LWA swarm

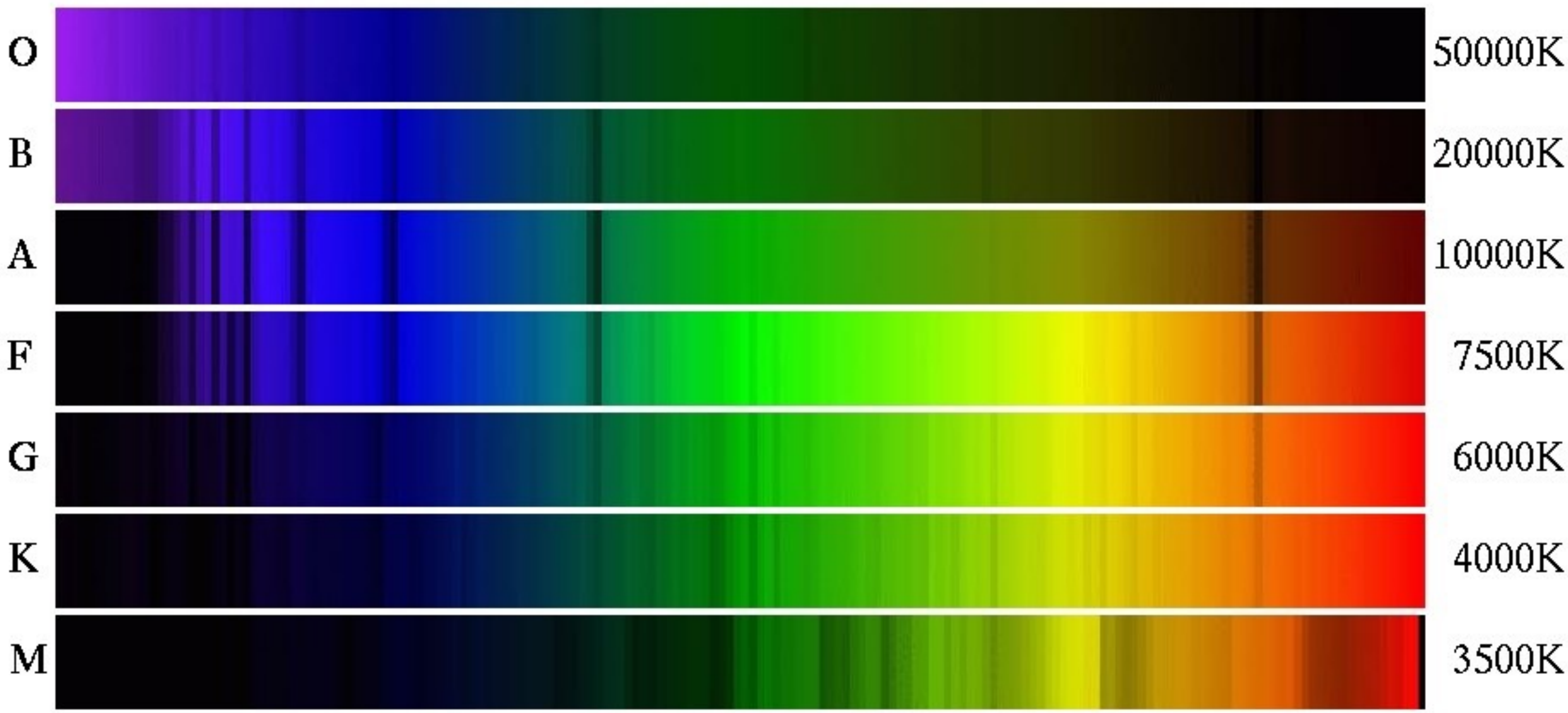
Emission from Jupiter



Also, sort the following table out on your own from the physics and geometry involved in each type of event:

Type of binary	Observations performed (or needed)	Parameters determined
Visual	a) Apparent magnitudes and π *b)* P, a, and π c) Motion relative to CM	Stellar luminosities Semi-major axis (a) Mass sum ($M+m$)
Spectroscopic	a) Single velocity curve b) Double velocity curve	Mass function f(M,m) Mass ratio (M/m)
Eclipsing	a) Shape of light curve eclipses b) Relative times between eclipses c) Light loss at eclipse minima	Orbital inclination (i) Relative stellar radii ($R_{l,s}/a$) Orbital eccentricity (e) Surface temperature ratio (T_l/T_s)
Eclipsing/spectroscopic	a) Light and velocity curves b) Spectroscopic parallax + apparent magnitude	Absolute dimensions (a, r_s, r_l) e and i Distance to binary Stellar luminosities Surface temperatures (T_l, T_s)

Astronomy 421



Lecture 8: Stellar Spectra

Key concepts:

Stellar Spectra

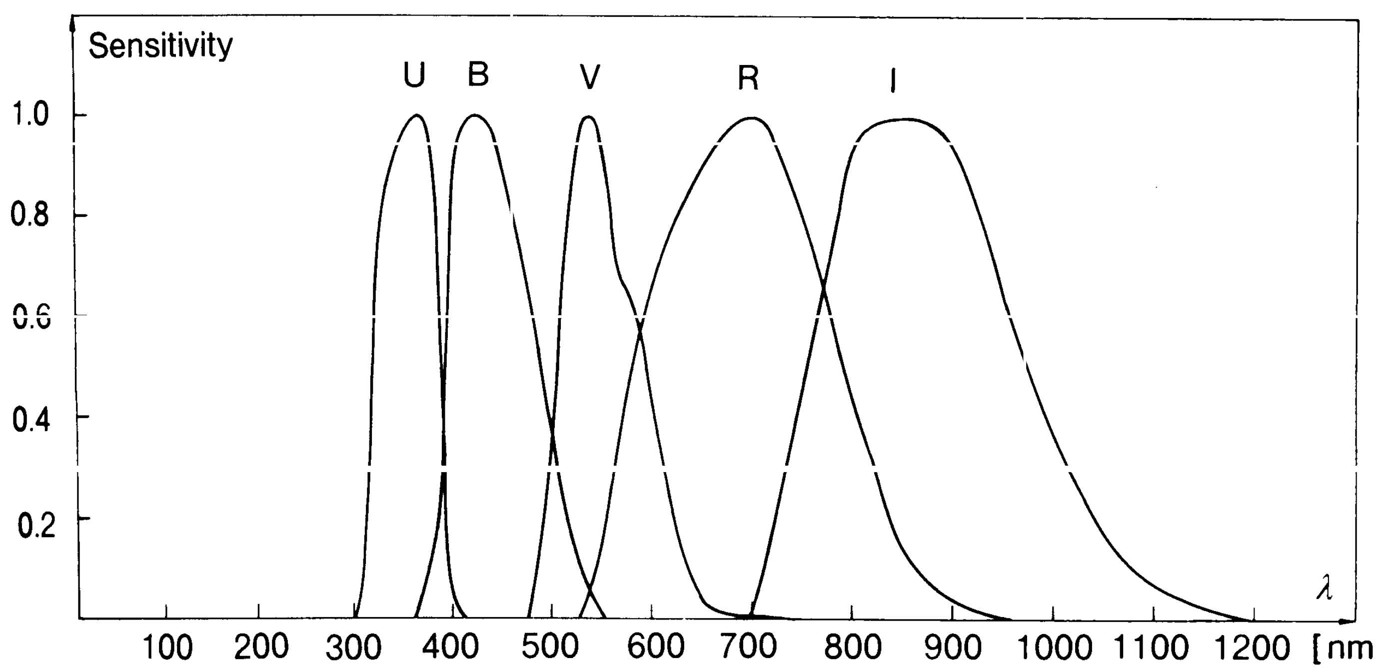
The Maxwell-Boltzmann Distribution

The Boltzmann Equation

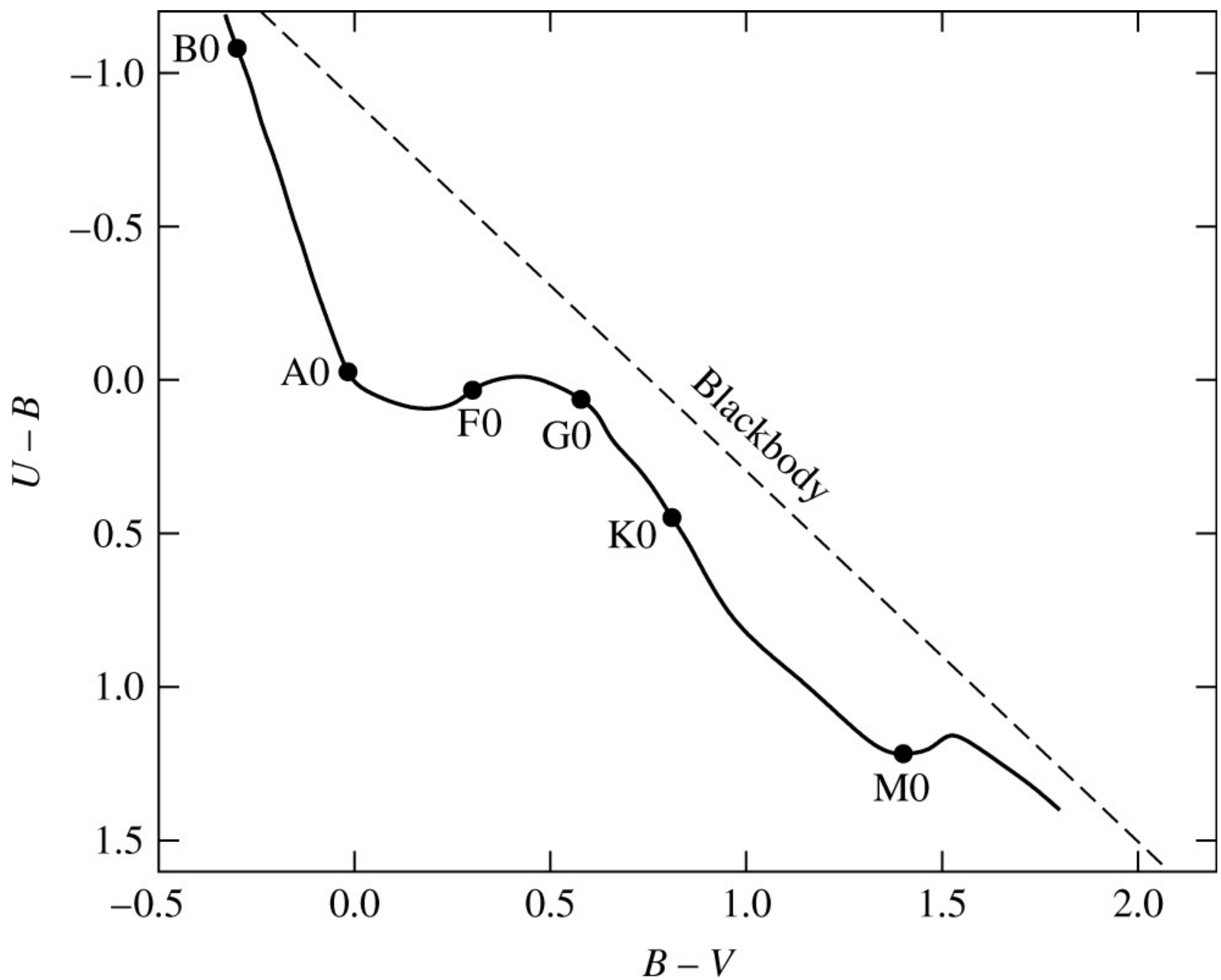
The Saha Equation

UVBRI system

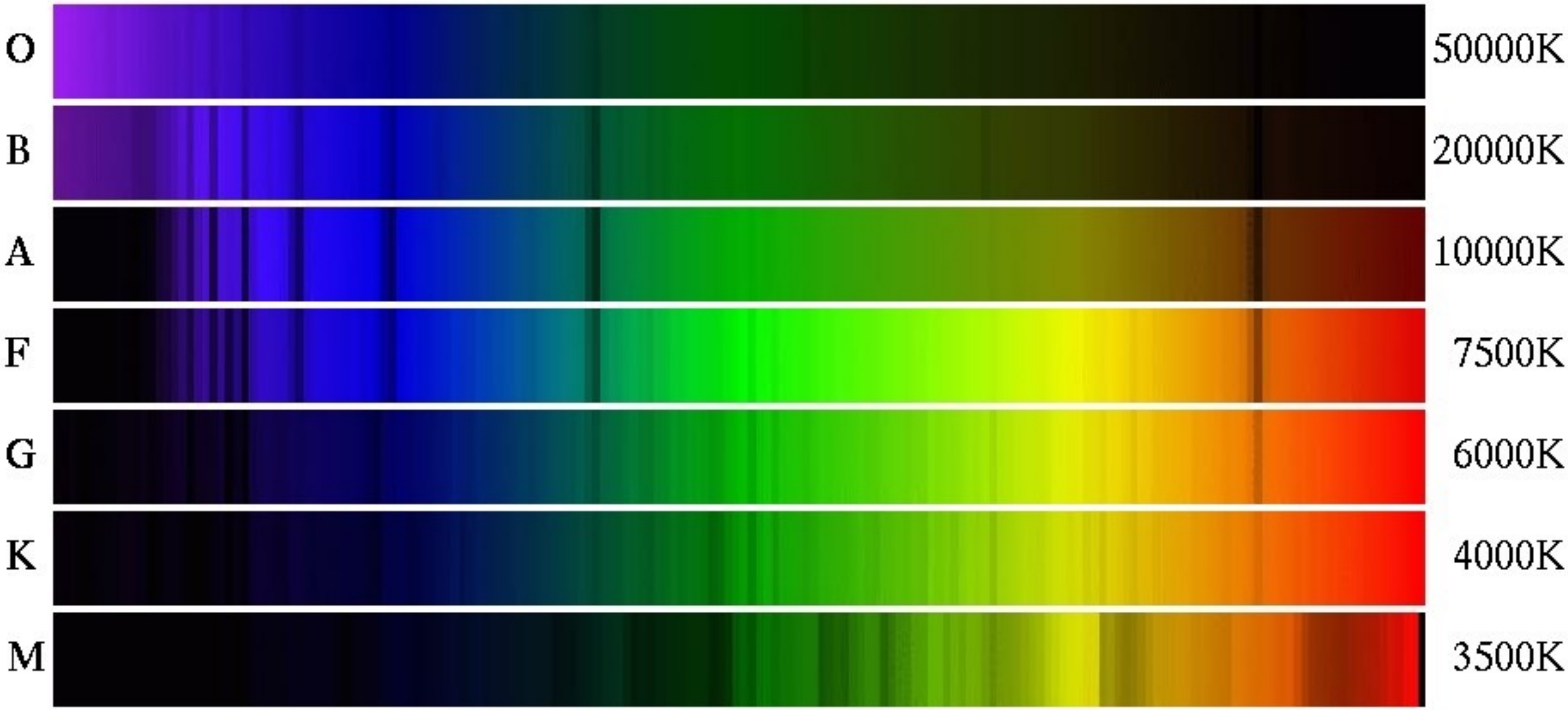
Filter name	Effective wavelength (nm)	0-magnitude flux (Jy)
U	360	1880
B	440	4400
V	550	3880
R	700	3010
I	880	2430



Color index useful since it defines a star's temperature



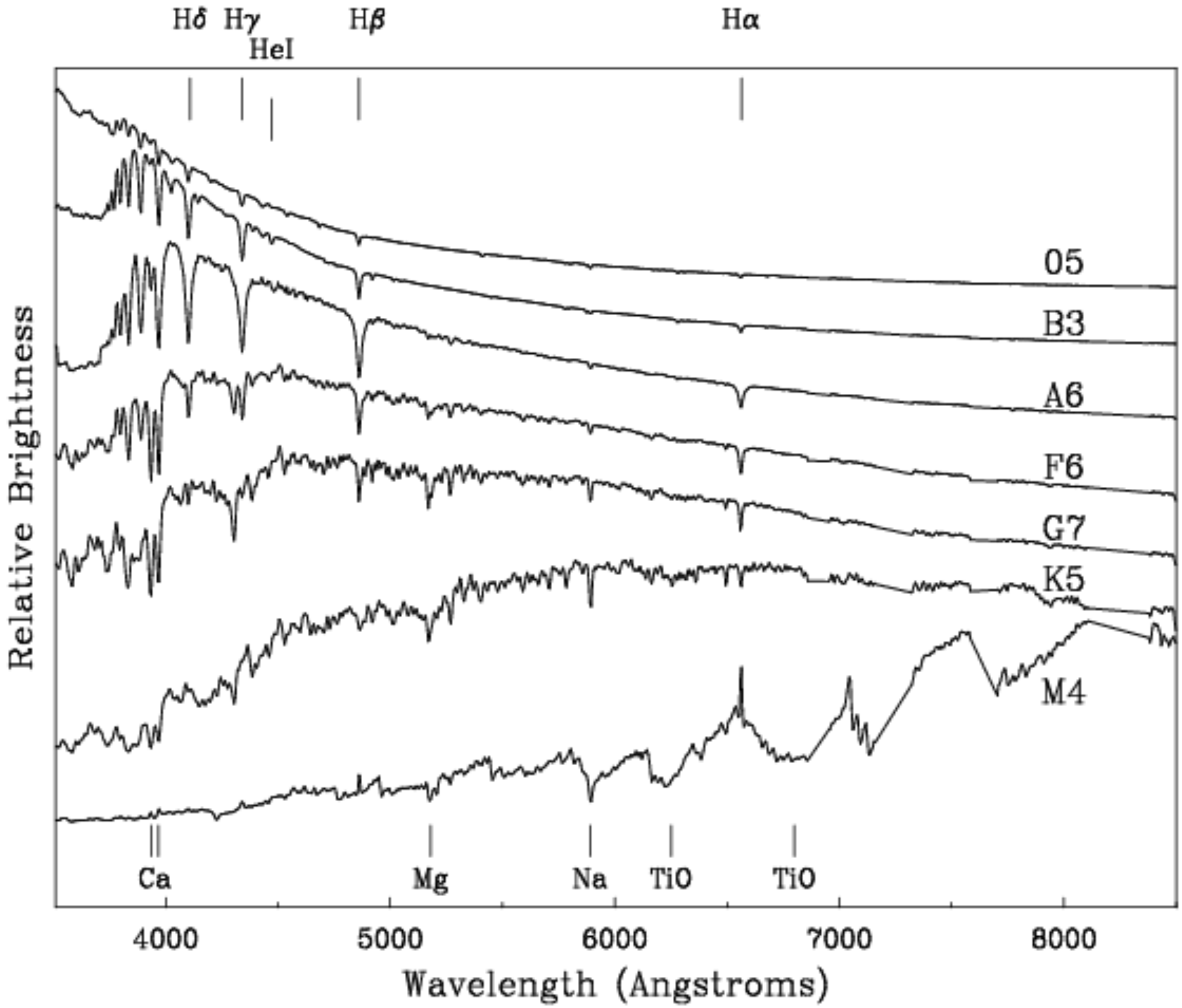
Stellar Spectra

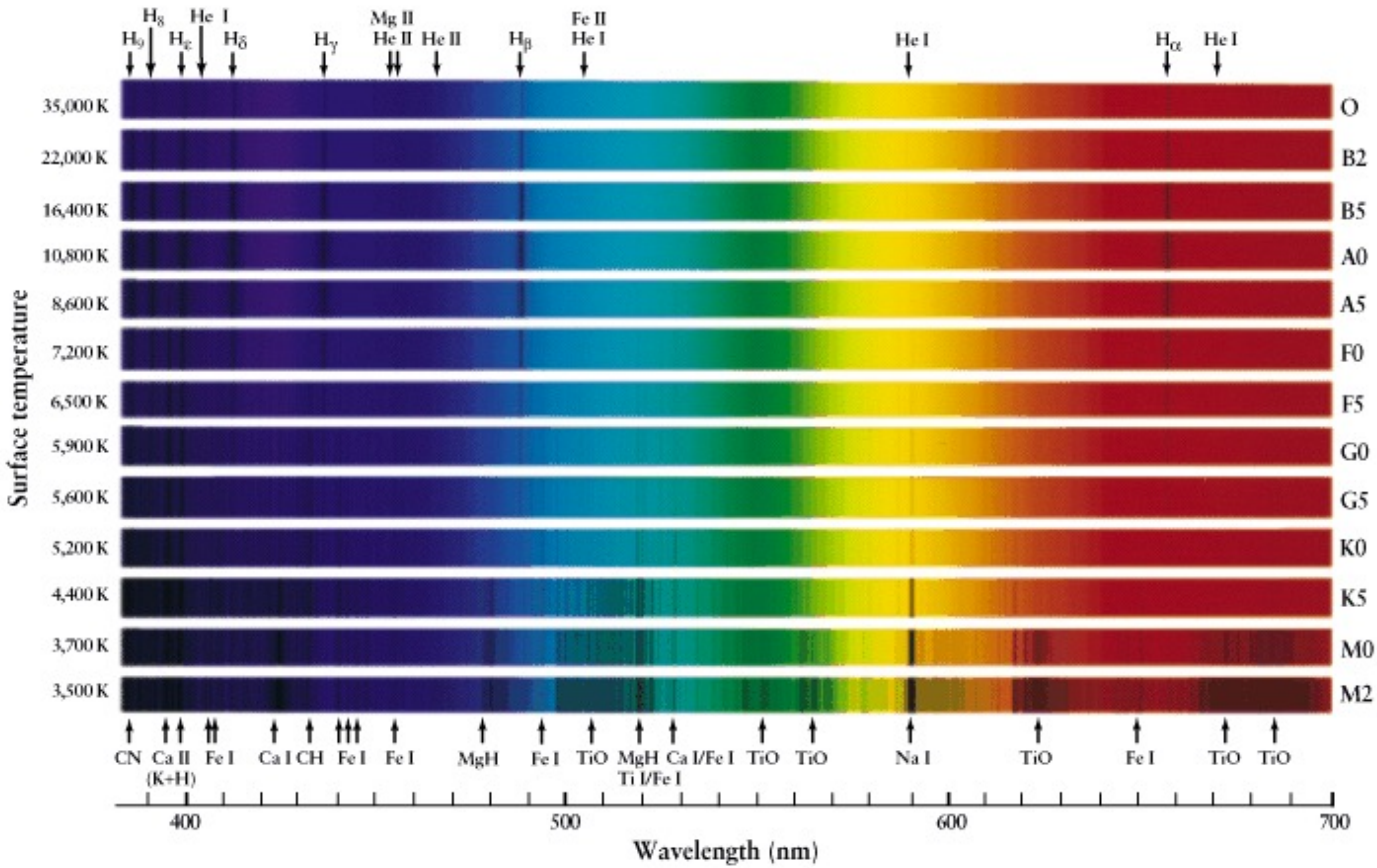


Stellar spectral types

Spectral type	Temperature of Atmosphere	Examples
O	30,000-50,000	
B	10,000-30,000	Rigel
A	8,000-10,000	Vega, Sirius
F	6,000-8,000	
G	5200-6000	Sun
K	4000-5200	
M	2000-4000	Betelgeuse
L	1300-2000	
T	<1300	

Further subdivision: e.g., B0-B9, G0-G9 etc. The Sun is a G2.

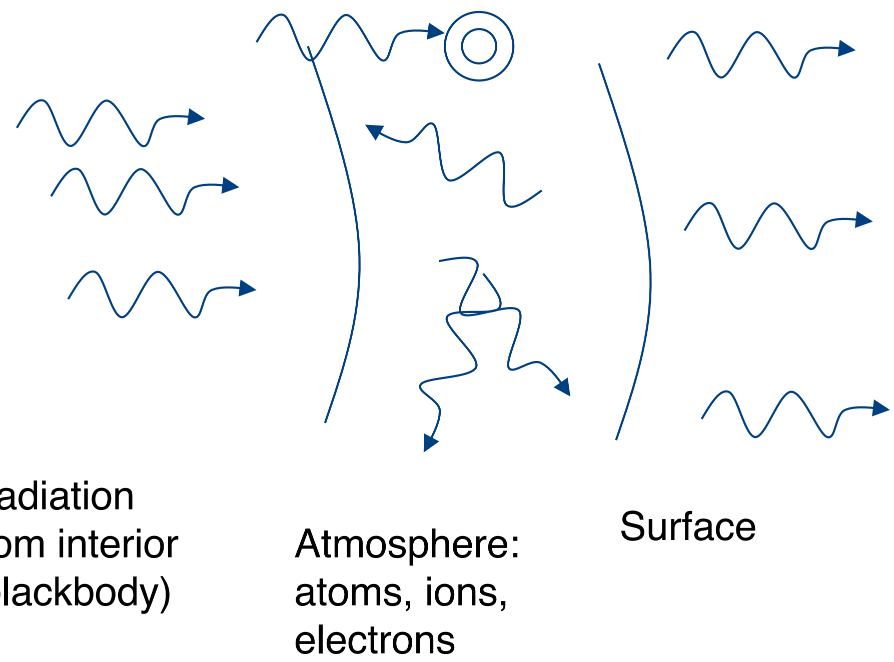

Hδ
Hγ
HeI
Hβ
Hα
05
B3
A6
F6
G7
K5
M4
Relative Brightness
Ca
Mg
Na
TiO
TiO
4000
5000
6000
7000
8000
Wavelength (Angstroms)

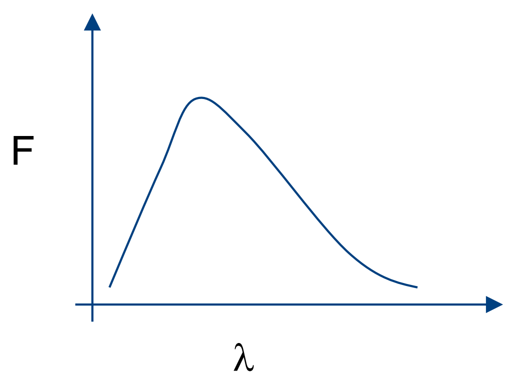


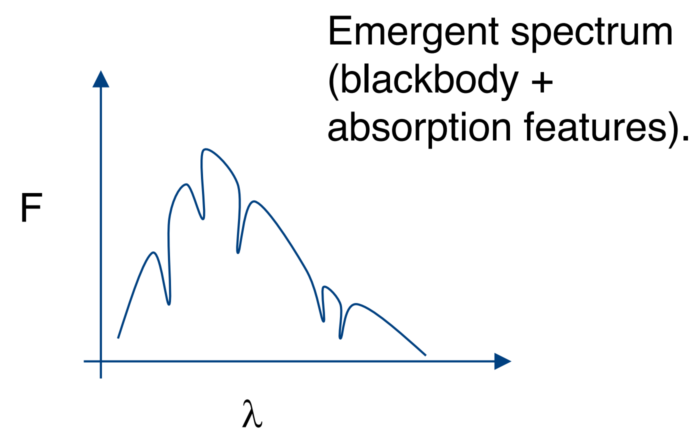
H9 H8 Hε He I Hδ Hγ Mg II He II He II Hβ Fe II He I He I Hα He I
35,000 K
22,000 K
16,400 K
10,800 K
8,600 K
7,200 K
6,500 K
5,900 K
5,600 K
5,200 K
4,400 K
3,700 K
3,500 K
Surface temperature
O
B2
B5
A0
A5
F0
F5
G0
G5
K0
K5
M0
M2
CN Ca II (K+H) Fe I Ca I CH Fe I Fe I MgH Fe I TiO MgH Ti I/Fe I Ca I/Fe I TiO TiO Na I TiO Fe I TiO TiO
400
500
600
700
Wavelength (nm)

Why is spectrum so sensitive to temperature (apart from blackbody)?

Schematic of stellar atmosphere







Maxwell-Boltzmann velocity distribution

Consider gas in a star with some density, temperature, chemical composition (may change with *R*).

Several different atomic processes occur:

- *Collisional excitation*
- *Collisional de-excitation*
- *Radiative excitation*
- *Radiative de-excitation*
- *Collisional ionization*
- *Radiative ionization*
- *Recombination*

Thermal Equilibrium: *T* of any parcel of gas equals that of surroundings. Not quite true in stars, but *T* varies slowly enough with *R* that it's a good approximation.

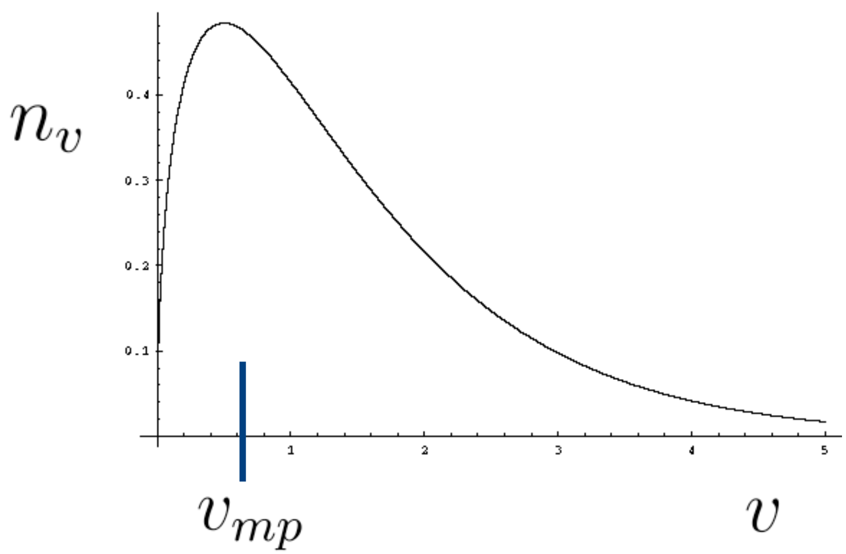
In TE, the number density of particles with speed between *v* and *v+dv:*

$$n_V dv = n \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} 4\pi v^2 dv$$

This is the Maxwell-Boltzmann velocity distribution function. *kT* is the *characteristic thermal energy* of a gas at temperature T.

Most particles have KE~kT, due to collisions. This process is called *thermalization.*

This is a result from "statistical mechanics".



Most probable speed $$v_{mp} = \sqrt{\frac{2kT}{m}}$$

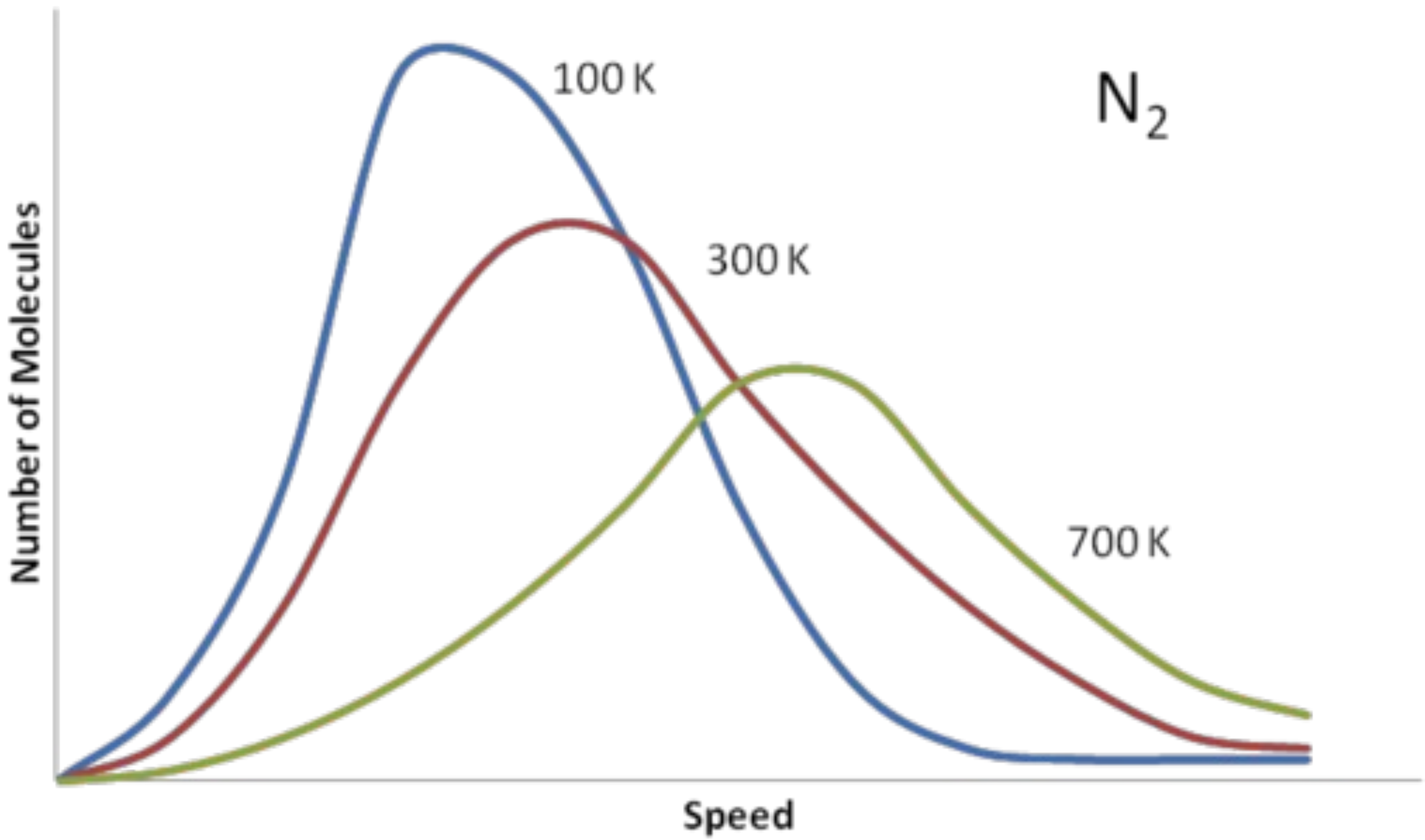
Root mean-square speed $$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Note: the speed depends on mass. At a given *T*, more massive particles slower on average, but each species has same average KE.

Point: Atoms of a gas gain and lose energy via collisions.

In typical stellar atmospheres, radiative transitions dominate.

Maxwell-Boltzmann Velocity Distribution for N_2



What dictates spectral line strengths?

We’ll see spectral lines are extremely useful - relative strengths give information on temperature (most clearly), density, and composition of stellar atmospheres.

Consider the case of thermal equilibrium (TE) => average number of atoms in a given energy (i.e. electron energies) state remains unchanged, e.g. each excitation balanced by a de-excitation ("steady-state").

The relative number of atoms or ions in each state is governed by the Boltzmann Equation:

$$\frac{N_b}{N_a} = \frac{g_b e^{-E_b/kT}}{g_a e^{-E_a/kT}} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

$$\frac{N_b}{N_a} = \frac{g_b e^{-E_b/kT}}{g_a e^{-E_a/kT}} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

N_b = #atoms per unit volume in state b

N_a = #atoms per unit volume in state a

E_b = energy of level b

E_a = energy of level a

k = Boltzmann's constant = 1.38×10^{-23} J K^{-1} = 8.6×10^{-5} eV K^{-1}

T = gas temperature

g_i = multiplicity of level i

= how many e^- you can put in level i before Pauli exclusion principle is violated

= "*statistical weight*" or "*degeneracy*" of energy level i (number of quantum states (different l, m_l, m_s) with same energy)

Statistical weight for H atom

n=1, l=0, m_l = 0, m_s = +/- ½ => g=2

n=2, l=0,1, m_l = 0, +/-1, m_s = +/- ½ => g=8

Can show $g_n = 2n^2$ for H.

Boltzmann Equation qualitatively:

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

Physical reasoning:

- $\frac{N_b}{N_a} \uparrow \quad \text{as} \quad T \uparrow$ more collisional, radiative excitations at higher *T*
- $\frac{N_b}{N_a} \rightarrow \frac{g_b}{g_a} \quad \text{as} \quad T \rightarrow \infty$ exponential decline becomes unimportant
- $\frac{N_b}{N_a}$ small if $E_b - E_a >> kT$

  Few excitations to level *b* if typical thermal energy too low.

Example

What is N_2/N_1 for hydrogen for the Sun? T~5780 K.

$$E_n = -\frac{13.6eV}{n^2}$$

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT}$$

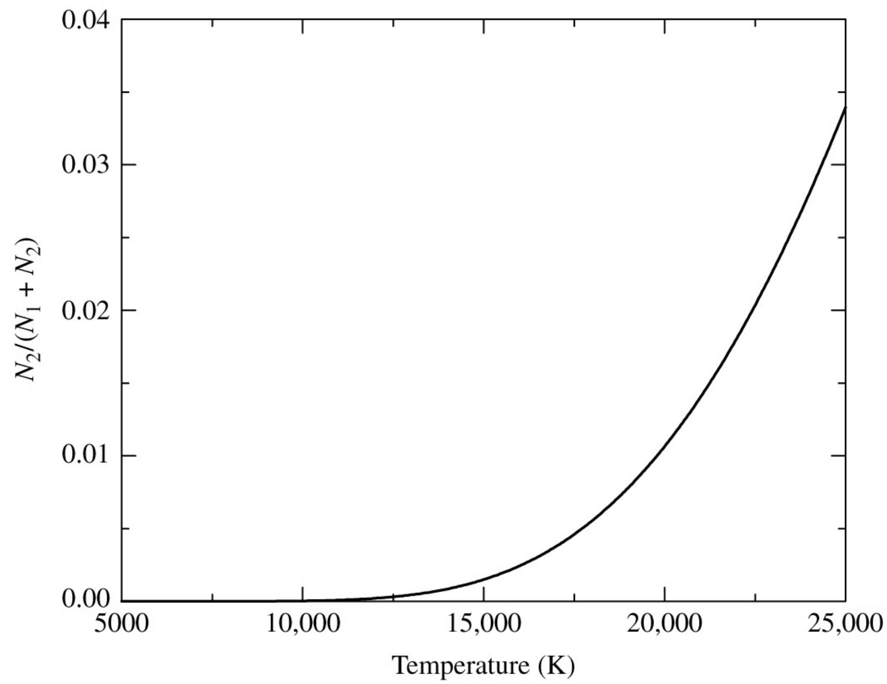
$$= \frac{2(2)^2}{2(1)^2} e^{-[(-13.6eV/2^2)-(-13.6eV/1^2)]/kT} = 4e^{-10.2eV/kT} \simeq 4 \times 10^{-9}$$

Balmer lines are not very intense in the Solar spectrum.

For stars T~10,000 K, $\frac{N_2}{N_1} \sim 3 \times 10^{-5}$

So stronger Balmer lines in Type A stars.

In fact, to reach $N_b/N_a = 1$, we need T=85,000 K.



Observationally though, Balmer lines reach maximum intensity at T=9250 K, and fall in intensity at higher temperatures. Why?

Ionization!

As T increases, there is more energy (both radiative and collisional) available to ionize the atoms.

In equilibrium, the ionization rate = recombination rate for every type of ion.

$$X \leftrightarrow X^{+} + e^{-}$$

The ratio of the number of atoms in ionization stage (i+1) to the number in stage i:

$$\boxed{\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}}$$

The Saha Equation

Where

χ_i = ionization energy needed to remove e^- from atom in ground state of state i to $i+1$

n_e= free electron number density

Z = *partition function,* weighted sum of ways the atom or ion can distribute its electrons among its energy levels

$$Z = \sum_{j=1}^{\infty} g_j e^{-(E_j - E_1)/kT}$$

weighted such that higher E is a less likely configuration.
Accounts for fact that not all atoms or ions will be in ground state.
Z must be calculated for each ionization state of the element.

For an ideal gas, can also express Saha Eqn in terms of pressure, using $P_e = n_e kT$

$$\frac{N_{i+1}}{N_i} = \frac{2kTZ_{i+1}}{P_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT}$$

Example: consider a pure Hydrogen atmosphere at constant pressure:

$$P_e = 20\,\mathrm{N\,m^{-2}}$$

We want to calculate the ionized fraction as a function of *T*, from 5000 K up to 25,000 K.

Then we need the partition functions Z_I (neutral) and Z_{II} (ionized).

Z_{II} = 1 - only one state available, just a proton.

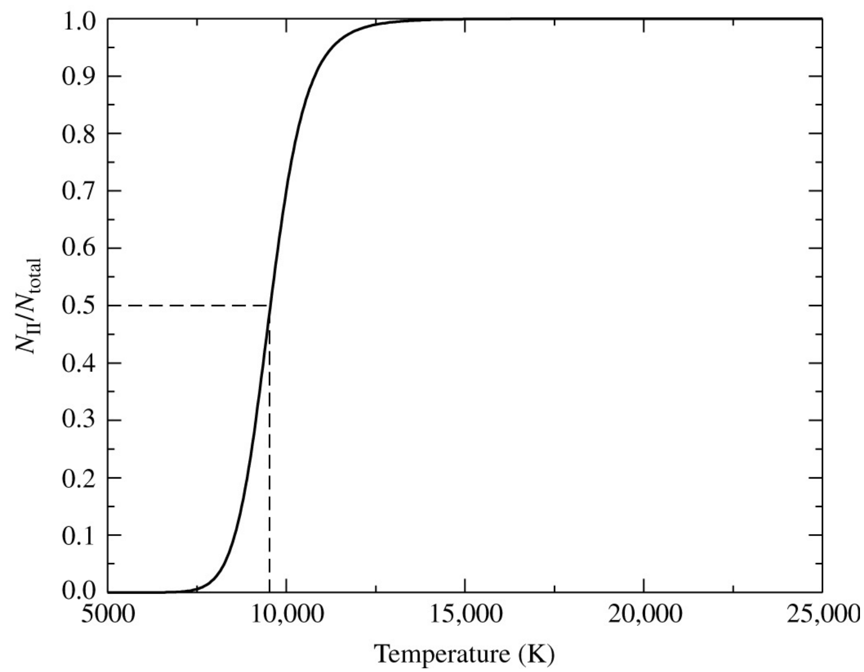
Z_I : at these *T*'s, E_2-E_1=10.2 eV >> kT. (check!). Thus, as before, most neutral H is in ground state, and only j=1 contributes significantly to Z_I:

$$Z_I \simeq g_1 = 2(1)^2 = 2$$

Plug this into the Saha Equation giving N_{II}/N_I
and then compute the fraction of ionized hydrogen which must be N_{II}/N_{tot}

$$\frac{N_{II}}{N_{tot}} = \frac{N_{II}}{N_I + N_{II}} = \frac{N_{II}/N_I}{1 + N_{II}/N_I}$$

Plot!



So, ionization happens in a very narrow range of *T*.
Almost completely ionized (95%) by *T* = 11,000 K.

Combine Boltzmann and Saha Equations to understand the Balmer lines:

Population of higher levels with higher *T* (Boltzmann) quenched by ionization (Saha)

Therefore, at some *T*, the population of e^- in higher levels will reach a maximum.

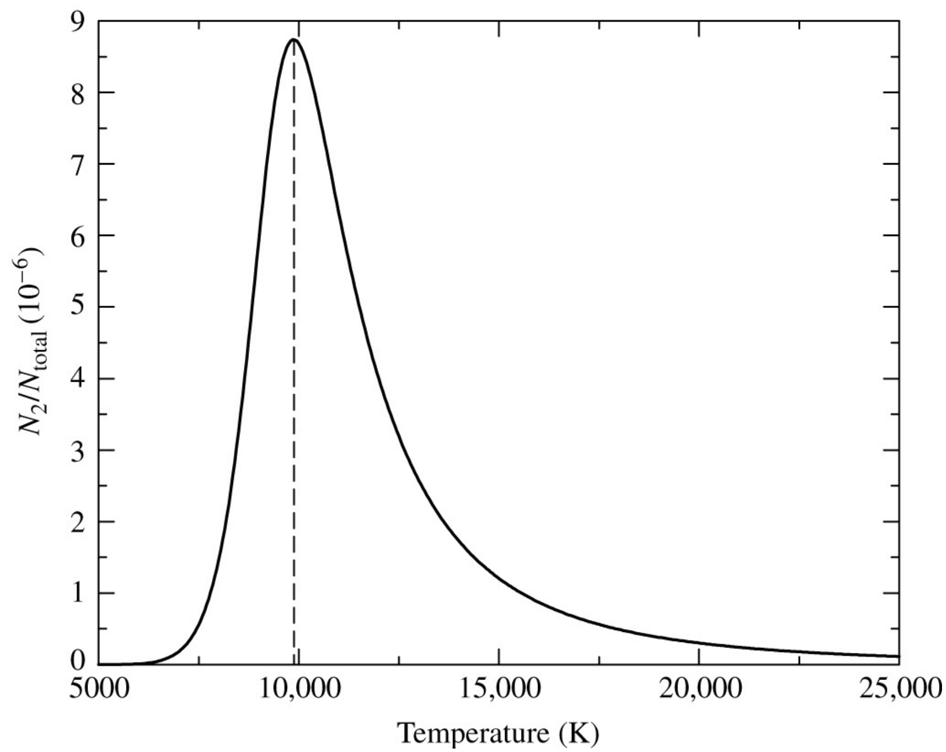
For our hydrogen example, at T < 25,000 K, almost all neutral atoms in *n*=1 or 2, so $N_I \approx N_1 + N_2$ and population of *n*=2 level is:

$$\frac{N_2}{N_{tot}} = \frac{N_2}{N_2 + N_1}\frac{N_I}{N_{tot}} = \frac{N_2/N_1}{1 + N_2/N_1}\frac{1}{N_{tot}/N_I}$$

since $N_{tot} = N_I + N_{II}$

$$\frac{N_2}{N_{tot}} = \frac{N_2/N_1}{1 + N_2/N_1}\frac{1}{(1 + N_{II}/N_I)}$$

Plot this



n=2 maximized at T~ 10,000K (A stars) => Balmer absorption lines (n_{lower}=2) strongest.

Why are lines of other elements often at least as strong as H?

For example, Ca in the Sun has $\frac{N_{Ca}}{N_H} = 2 \times 10^{-6}$!

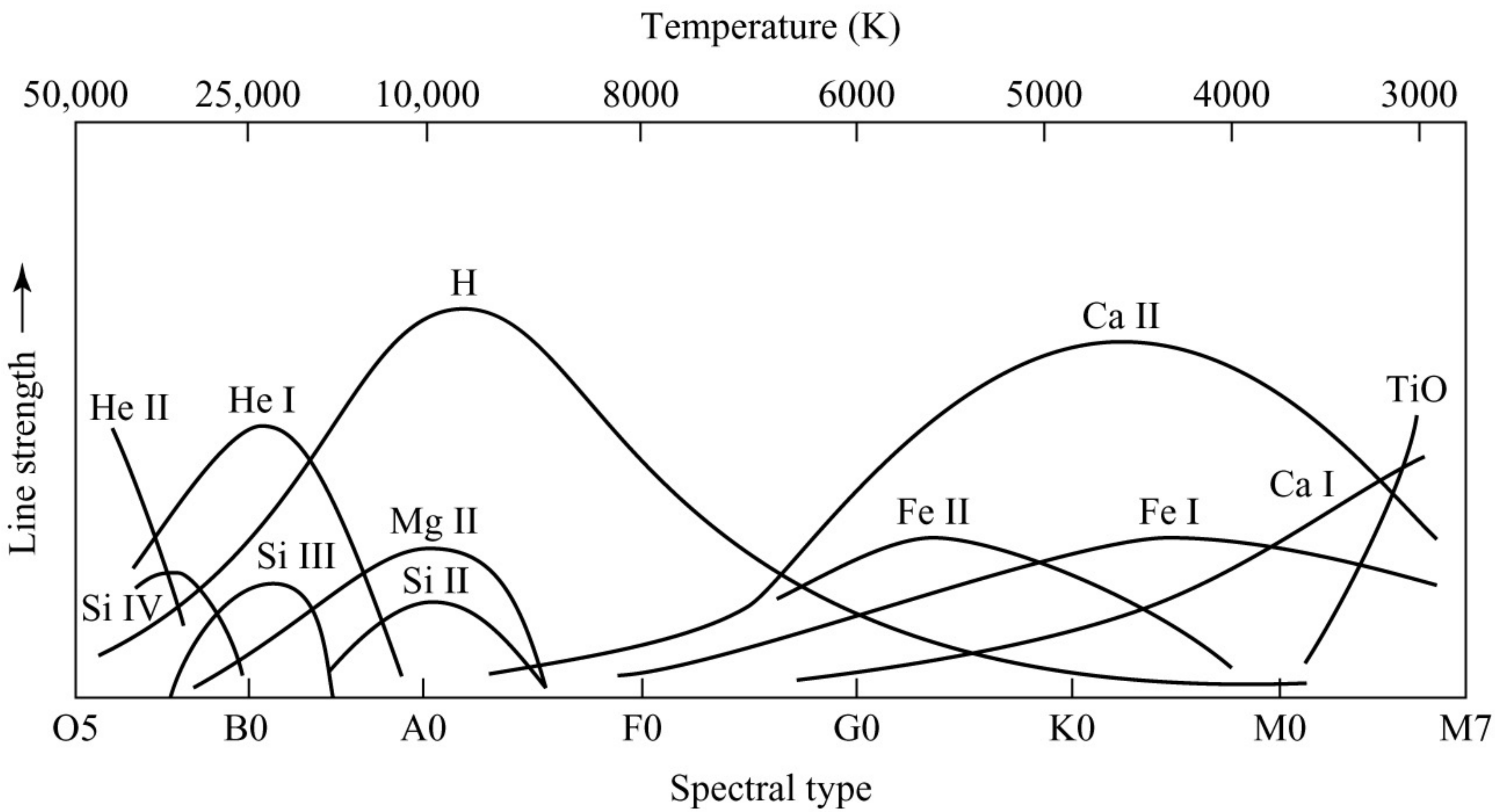
But Ca I => Ca II requires only 6.11 eV. So ~ all Ca is Ca II.

Ca II "H" and "K" lines at ~400 nm require only 3.12 eV photons. Absorption is from ground state.

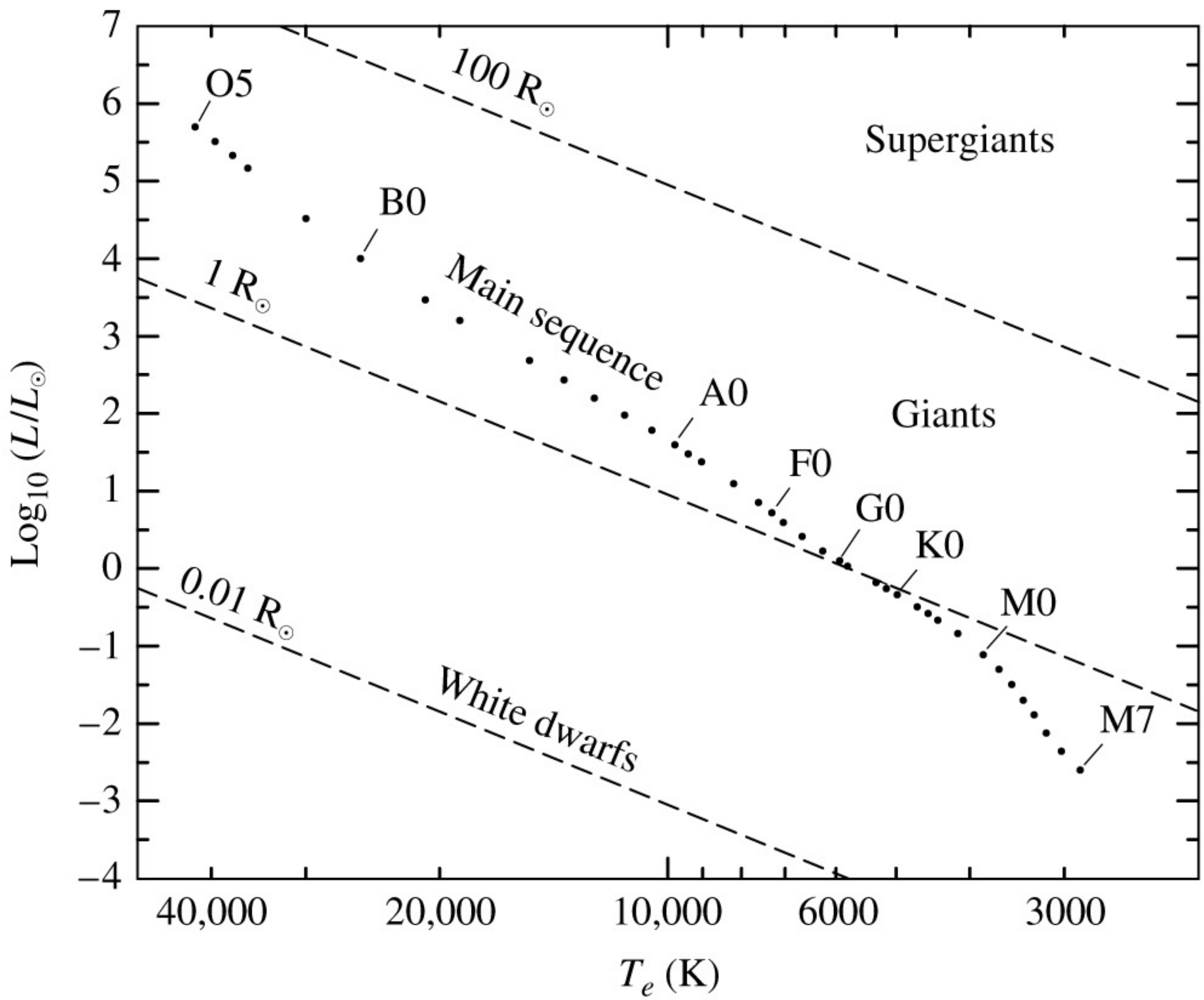
Balmer lines require electrons to be in n=2, 10.2eV above n=1.

Because of exponentials in Boltzmann and Saha equations, there are many more Ca II's in ground state than H's in n=2.

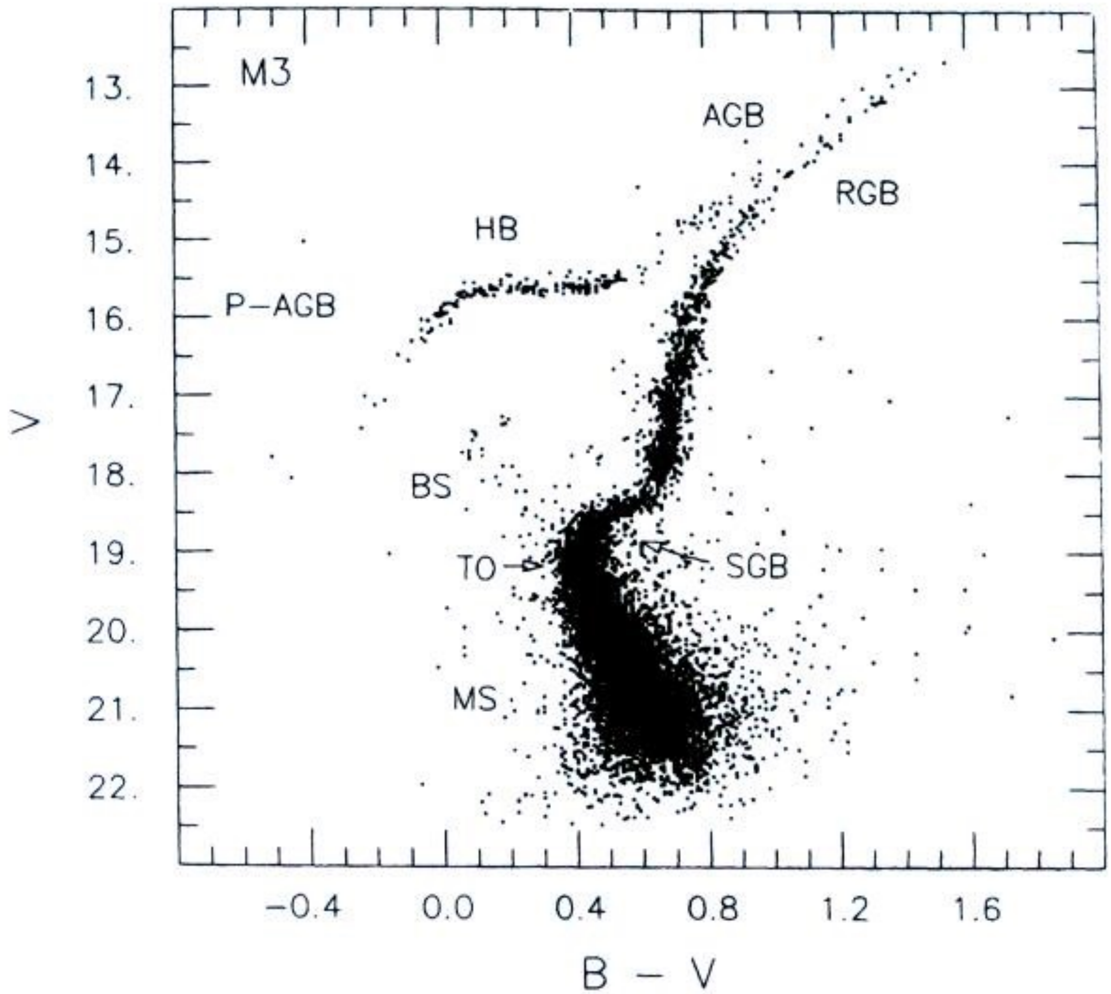
(see example 8.1.5 for numbers)


Temperature (K)
50,000
25,000
10,000
8000
6000
5000
4000
3000
Line strength
H
Ca II
He II
He I
TiO
Ca I
Fe II
Fe I
Mg II
Si III
Si II
Si IV
O5
B0
A0
F0
G0
K0
M0
M7
Spectral type

Hertzsprung Russell Diagram



More commonly made from observations: a color-magnitude diagram:



(globular cluster M13)

(Morgan-Keenan) Luminosity Classes

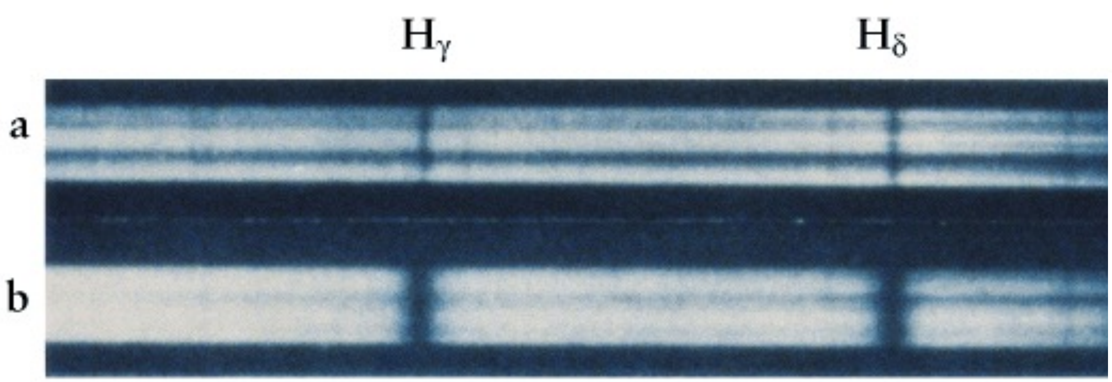
Atmospheric pressure affects width of absorption lines:

Lower pressure => decreased line width

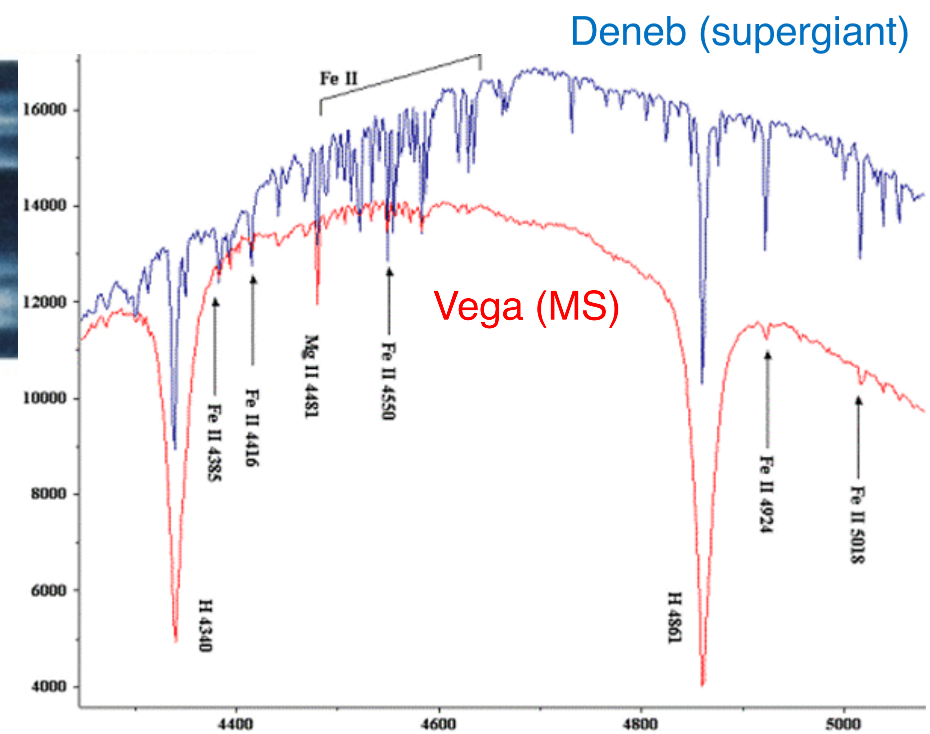
Higher pressure => increased line width

The atmospheric pressure is lower in the photosphere of an extremely large red giant than in a main sequence star of similar temperature

=> giant's spectral lines are *narrower*

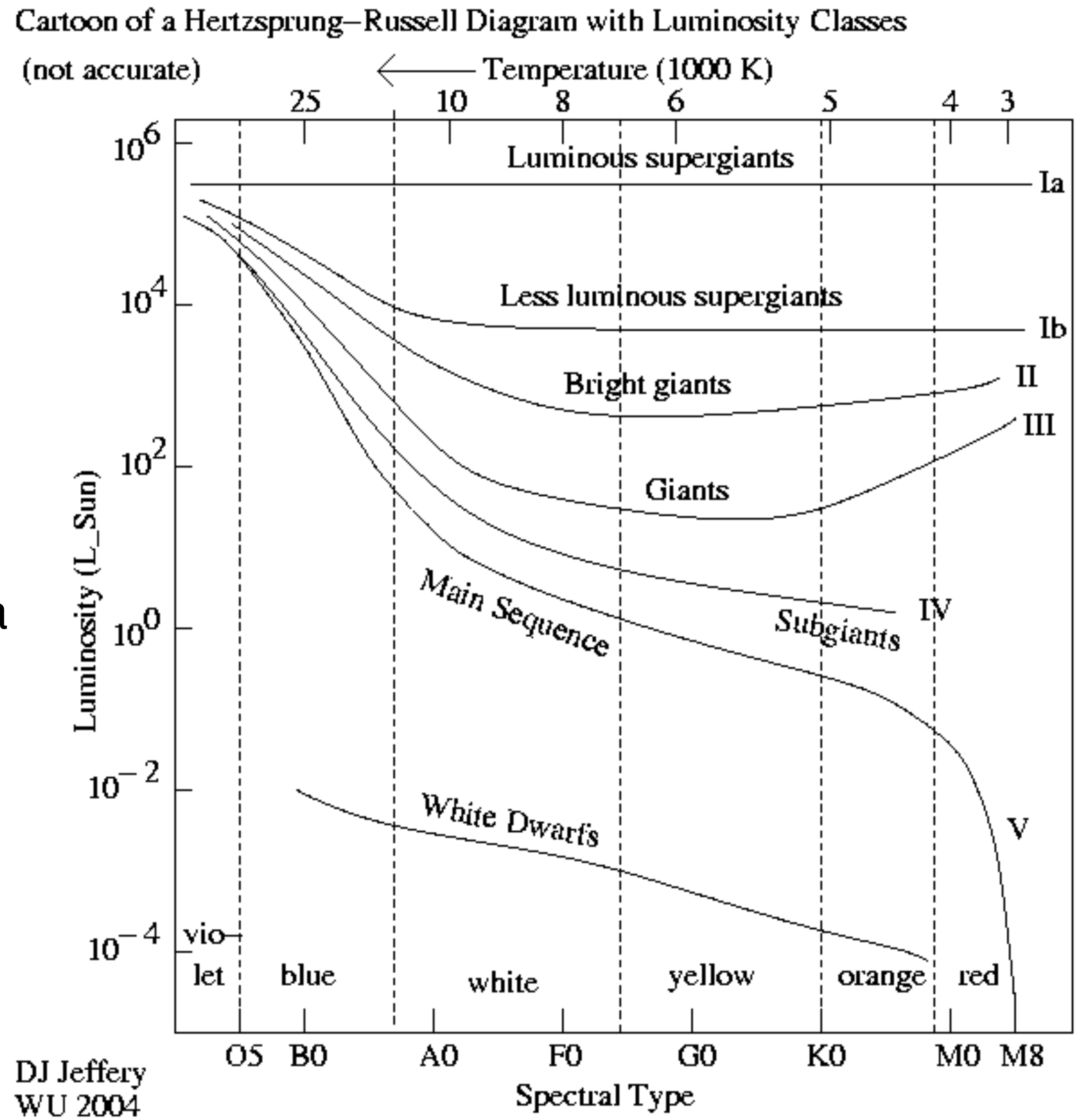


B8 supergiant (top), main sequence (bottom).



We will quantify this later

Sun G2V
Betelgeuse M2Ia



generally narrower lines

With spectral type and luminosity class, distance to star can be estimated: "spectroscopic parallax".