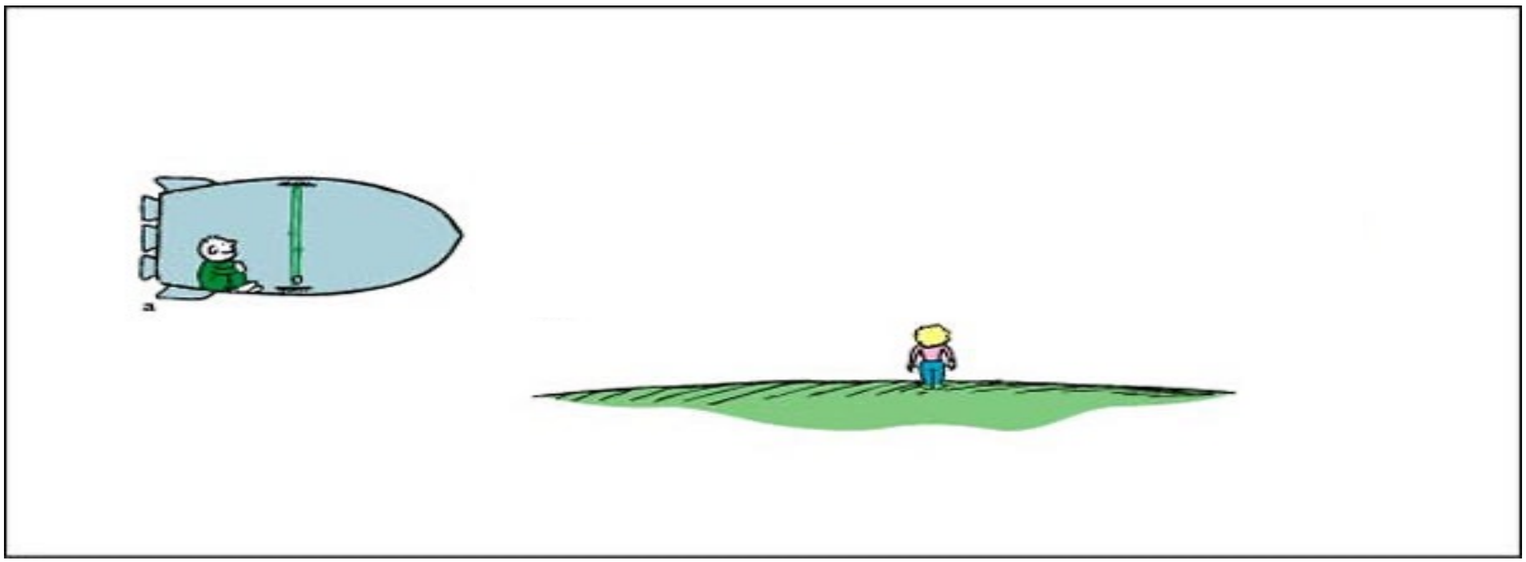
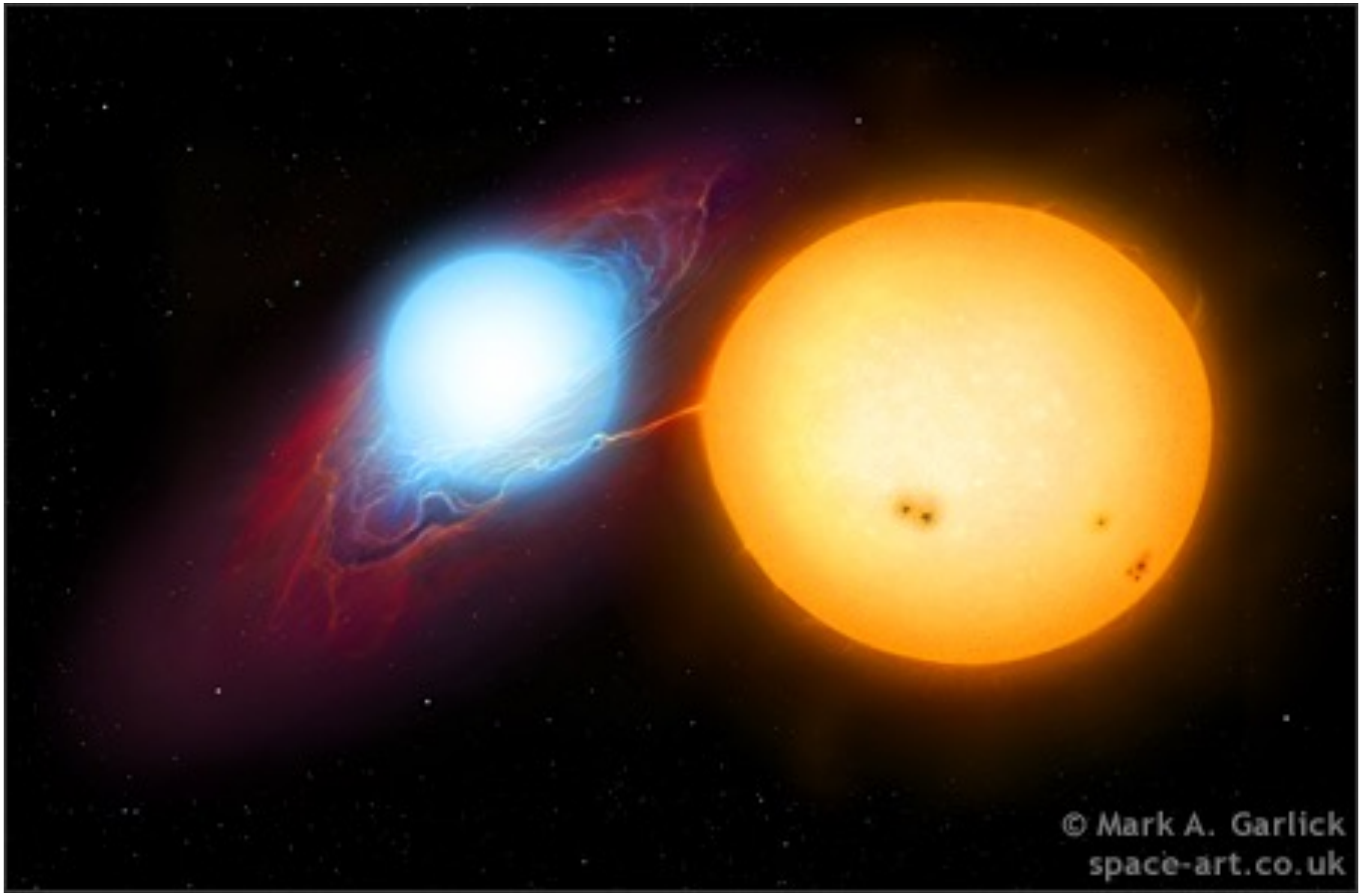
Twin Paradox

Suppose there are two twins, Al and Bill age 10. Al goes to summer camp 25 light-years away. If he travels at 0.9999c then it takes 25 years each way and Bill is age 60 when Al gets back. But Al is only 10 and a half because time for him was moving slower. But from Al's point-of-view Bill was the one moving so how did Bill get so old?



Astronomy 421



Lecture 8: Binary stars

Four Corners Meeting

How to Register for the Meeting

1) Download and fill out "Purchasing Goods and Services" form from

http://physics.unm.edu/pandaweb/mainoffice/forms/Interna1.pdf

See example next page

2) Go to the web site for the meeting and click on " Four Corners Section Meeting Registration Form"
3) Login with your APS account, or create a free one
4) Fill in the registration form until you get to the credit card request
5) Take your Purchasing form and your laptop computer down to Chris Moroney in PAIS 1211 and get him to put in the info for his UNM P-card to pay for the conference.
6) Done!

P&A Internal Order Form for Goods & Services

Date: 9/14/2022
*Version: May 2021

Name: Ima Student
Email: istudent@unm.edu
Phone: (505)123-1234

Check one: Equipment ☐ Consumable ☐
Replacement Part: Yes ☐ No ☐
UNM Tag / Assembly No. ______________
Location (Required) ______________

Shipping Preference: ☐ FedEx ☐ UPS ☐ Pick-Up ☐ Overnight ☐ 2nd Day ☐ Ground

Vendor: APS
Phone/Fax: ______________
Prices confirmed with/quote # : ______________
Website/E-mail: ______________

Additional Requirements for Purchase Type:

Non-dell computer: Attach UNM non-standard computer form

Computer parts: Include model, serial, and UNM tag number for computer parts will be used for ______________

Purchase requiring login to personal account(such as conference registration): Include username/password, or indicate time to meet in person to make purchase ______________

If any requested item(s) is back-ordered: (check one)

☐ Order item anyway and notify me afterwards
☐ Notify me first before purchasing item(s).

Qty	Part #	Description	Item cost	Total
1		Registration for APS 4 Corners Meeting	50.00	$50.00
				$0.00
				$0.00
				$0.00
				$0.00
				$0.00
				$0.00
				$0.00
				$0.00
				$0.00
				$0.00
				$0.00
				$0.00
Sub Total				$50.00
Shipping Costs				

Total w/shipping ______________

Index	Acct. Code	Amount

←If splitting indexes, indicate index to use for additional shipping charges

(for office use only)

___ Odrive
___ Index
___ Scan
___ Email Ntfctn
___ Invoice Rcvd
___ Odrive Updated
___ Index Updated
___ Packing Slip Rcvd
___ Reallocated -
___ Reconciled

Purchase Orders:

(Federal grants: always attach 3 quotes)
Services >$5,000 (SPQ required)
Goods $10,000-$20,000 (attach a quote)
Goods $20,000-$60,000 (attach 3 quotes)
Goods >$60,000 (must be competitively solicited for 30 days, include Sole Source if applicable)

Pcard orders:

Services < $5,000
Goods < $10,000 (attach a quote)

Authorized Signature: ______________

Description of items: Registration fee

Business Purpose*: Attend APS 4 Corners meeting

Key concepts:

Binary types

How to use binaries to determine stellar parameters

The mass-luminosity relation

Binary stars

So far, we've looked at the basic physics we need for understanding stars:

Newton's laws and binary orbits

BB radiation: luminosity, effective temperature, radius

Spectra: composition, radial velocity (and much more to come!)

Now we start to apply these to learn about stars. The most fundamental property that dictates a star's structure and evolution is its *mass*.

The direct way to measure mass is by observing binary systems (will also give info on radii, temperature, luminosity, density).

Types of binary stars

About half of the stars are in binary or multiple systems. We divide them into groups from an observational standpoint.

- Visual binaries: nearby binaries where both stars can be imaged and motions across sky measured. Masses of each star can be found.

- Astrometric binaries: Only one star visible (other too dim). Infer existence of unseen companion causing oscillatory motion of the visible star.

Types of binary stars cont.

- Eclipsing binaries: orbital plane has orientation relative to our LOS that the stars periodically eclipse each other - studies of light curves yields much information.

- Spectrum binaries: can't distinguish two stars on the sky, but spectrum is two superimposed spectra with different Doppler shifts, but period too long to measure.

- Spectroscopic binaries: period is sufficiently short to cause observable periodic shift of spectral lines (if $v_r \neq 0$). Very useful when they also eclipse.
  - double-lined binaries (spectral lines from both stars visible)
  - single-lined binaries (spectral line from only one star visible)

Optical doubles: a chance line-of-sight (LOS) alignment. Not gravitationally bound, no physical connection so not useful.

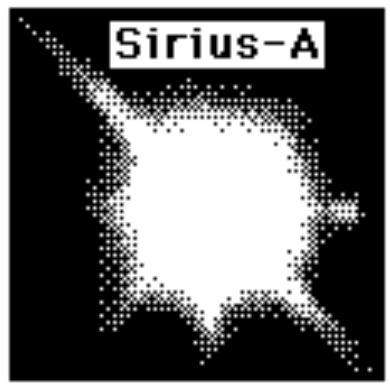
Visual binaries

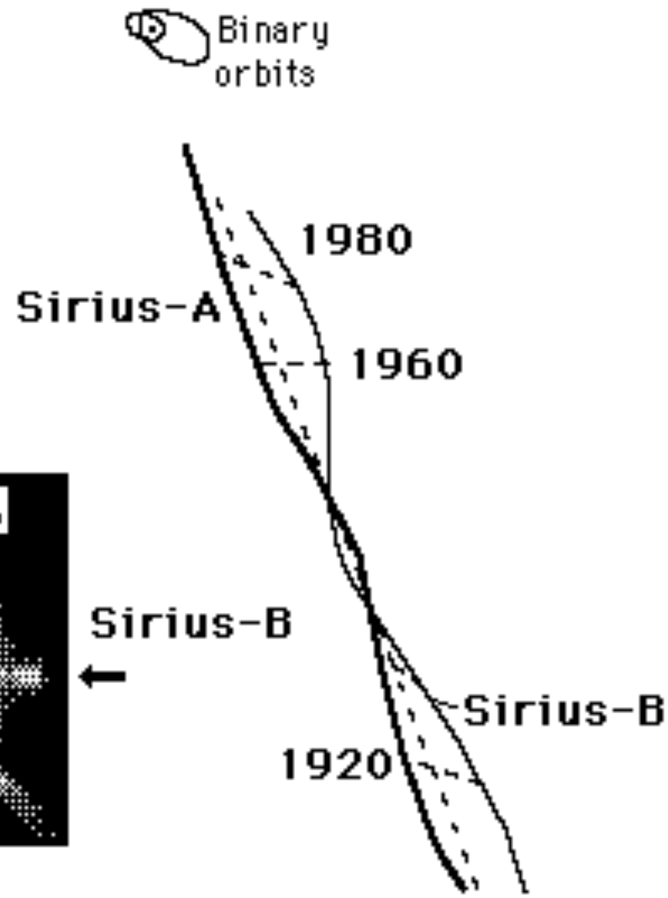
Wobble of Sirius A and B. Straight line describes motion of CM through space.

Which one is more massive?

Terminology: *brighter* = primary

fainter = secondary





We observe:

- Angular separation of each star from CM as a function of time
- Period

To get masses, we also need to know

- Distance
- Inclination (i)

Mass determination from visual binaries

Assume we have two stars with masses m_1 and m_2 , that orbit around the center of mass with semi-major axes a_1 and a_2. Definition of the center of mass gives

$$\frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{a_2}{a_1}$$

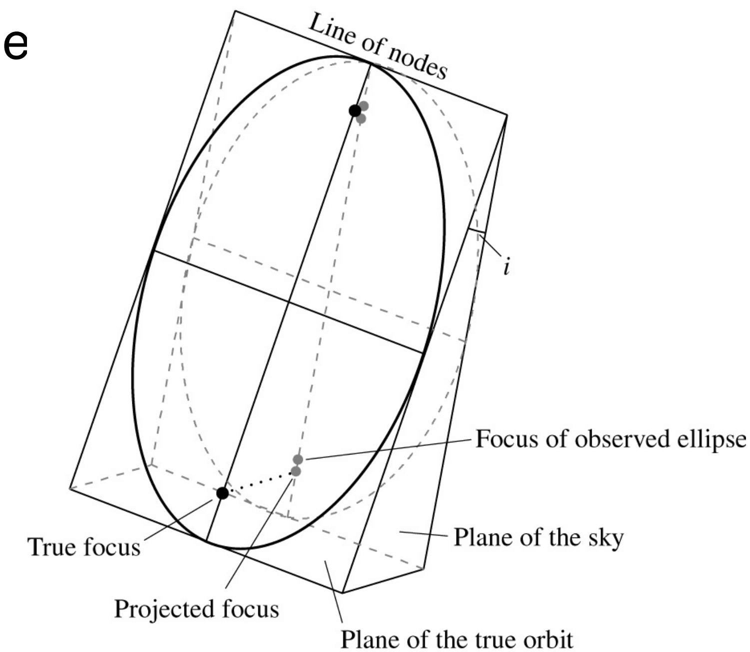
and Kepler's third law gives

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

where $a = a_1 + a_2$ = semi-major axis of orbit of reduced mass. So we have two equations which can give us both masses. But we need to measure P, a_1 and a_2.

In practice, the binary is located at a distance *d* and its orbital plane will form an angle *i* with the plane of the sky. Assume as a special case that the orbit is tilted around the minor axis.

Then we observe the semi-major axes as angle projected onto the sky:



$$\alpha_1 = a_1 \frac{\cos i}{d} \qquad \alpha_2 = a_2 \frac{\cos i}{d}$$

Then we write the mass ratio using these observable quantities as

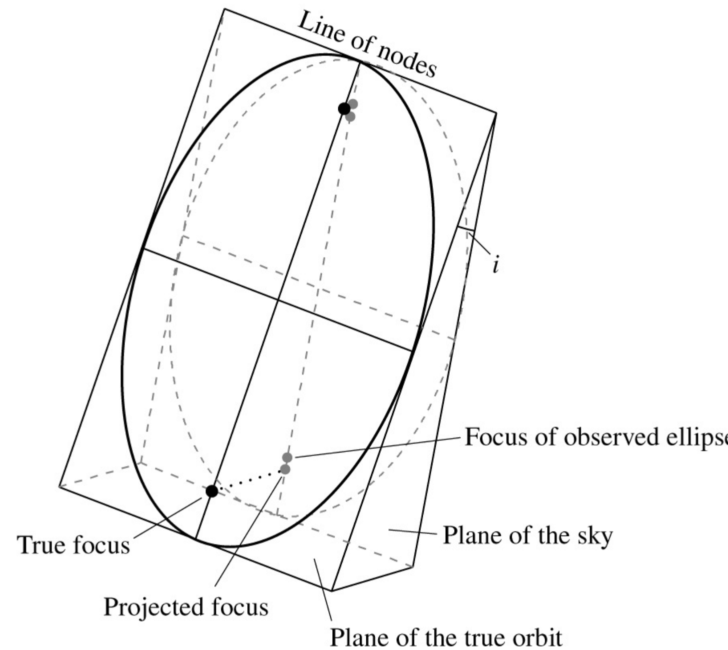
$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{\alpha_2 d \cos i}{\alpha_1 d \cos i} = \frac{\alpha_2}{\alpha_1}$$

(convince yourself that this is true regardless of axis around which orbit is tilted).

To find $m_1 + m_2$, we need to measure *P* and determine *a* (= a_1+a_2). If *d* and *i* are known, then we know a_1 and a_2, and thus *a*.

Now we can solve for m_1/m_2 and $m_1+m_2 \Rightarrow m_1, m_2$.

The orbital plane will generally not be in the plane of the sky: the *apparent orbit* will still be an ellipse (of different eccentricity and focus location).



How can we find *i* ?

The CM is found by defining a straight line in space for which the ratio of the angular distances of each star to it is the same at all times. But if orbit inclined, this will not be at apparent focus of ellipse. Must find ellipse that, when projected onto sky, has focus at CM. Harder if ellipse not tilted around minor axis, as in figure.

How can we find *d*?

Possibly Earth-orbit parallax (e.g.with Hipparcos satellite), or other methods we'll see later.

What about astrometric binaries?

Assume distance is known and orbit in plane of sky again.
Can only measure P, a_1 (star oscillates around a point that exhibits straight line, unaccelerated motion).

Then use

$$a_2 = \frac{m_1}{m_2} a_1$$

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} (a_1 + a_2)^3$$

To find

$$P^2 = \frac{4\pi^2 a_1^3}{G m_2^3} (m_1 + m_2)^2$$

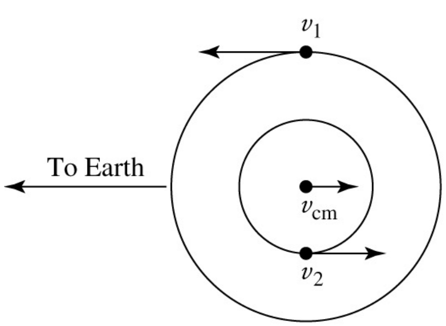
If m_1 known some other way, can get m_2. Applications?

Spectroscopic binaries

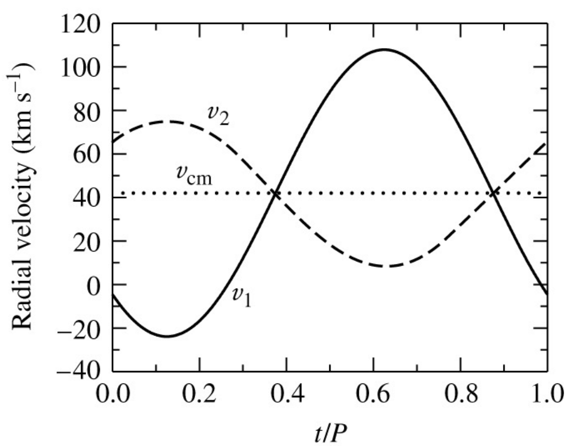
Measure the Doppler shift of spectral lines to get the radial velocity.

Consider case where both sets of lines are visible (doesn't have to be true!) and circular orbits.

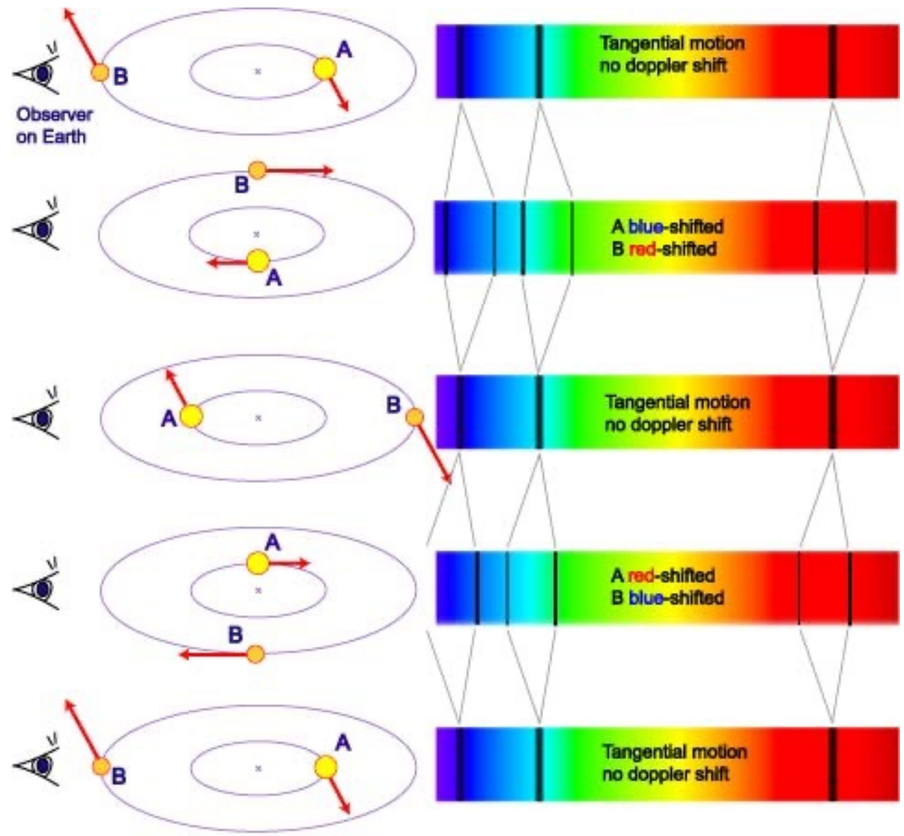
Then radial velocity curves will be sinusoidal.



(a)



(b)

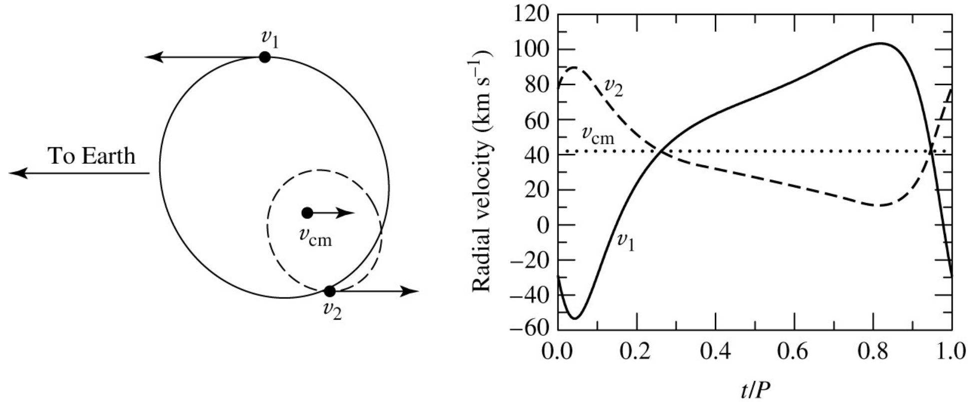


A Spectroscopic Binary System

High-mass star A and lower-mass B orbit around a common centre of mass. The observed combined spectrum shows periodic splitting and shifting of spectral lines. The amount of shift is a function of the alignment of the system relative to us and the orbital speed of the stars.

Changing *i* just changes amplitude, not the shape.

Note: when orbits are not circular, the velocity curves will be skewed:



(a) (b)

Fortunately, spectroscopic binaries tend to be close (why?) and their orbits circularize over relatively short timescales.

For circular orbits at some inclination *i* ,

$$v_{1r}^{max} = v_1 \sin i$$

$$v_{2r}^{max} = v_2 \sin i$$

For circular orbits: $v_1 P = 2\pi a_1, \quad v_2 P = 2\pi a_2$

$$\text{since} \quad \frac{m_1}{m_2} = \frac{a_2}{a_1} \quad \Rightarrow \frac{m_1}{m_2} = \frac{v_2}{v_1}$$

Or, using the observed radial velocities:

$$\frac{m_1}{m_2} = \frac{v_{2r}^{max}}{v_{1r}^{max}}$$

inclination drops out again!

To get the sum of the masses, use:

$$a_1 = \frac{v_1 P}{2\pi}, \quad a_2 = \frac{v_2 P}{2\pi}, \quad a = a_1 + a_2$$

Kepler's third law then becomes: $$m_1 + m_2 = \frac{P}{2\pi G}(v_1 + v_2)^3$$

Or, in terms of observed velocity:

$$m_1 + m_2 = \frac{P}{2\pi G}\frac{(v_{1r}^{max} + v_{2r}^{max})^3}{sin^3 i}$$

Problem: we cannot measure *i* as before with visual binaries.
Must make statistical arguments.

Is an *i* close to 0 degrees likely?

Would you be happier if this were an eclipsing spectroscopic binary?

Another problem:

sometimes only one set of lines is visible. If only star 1 is observable:

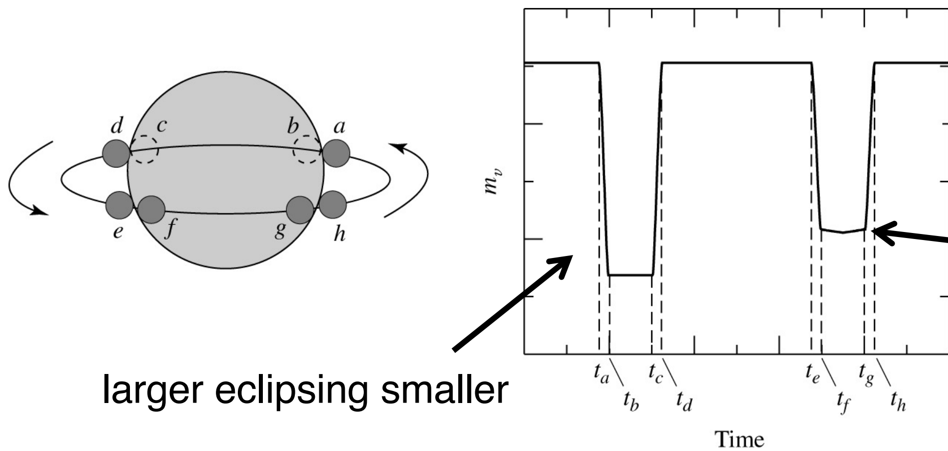
$$\frac{m_1}{m_2} = \frac{v_{2r}^{max}}{v_{1r}^{max}} \qquad \Rightarrow \frac{m_2^3}{(m_1 + m_2)^2} \sin^3 i = \underbrace{\frac{P}{2\pi G} (v_{1r}^{max})^3}_{\text{The 'mass function'}}$$

The RHS is called the mass function and contains observable parameters (also looks familiar from our eqn for astrometric binaries)

This sets a lower limit on m_2 (e.g. still useful for investigating neutron star and black hole binaries).

Eclipsing binaries:

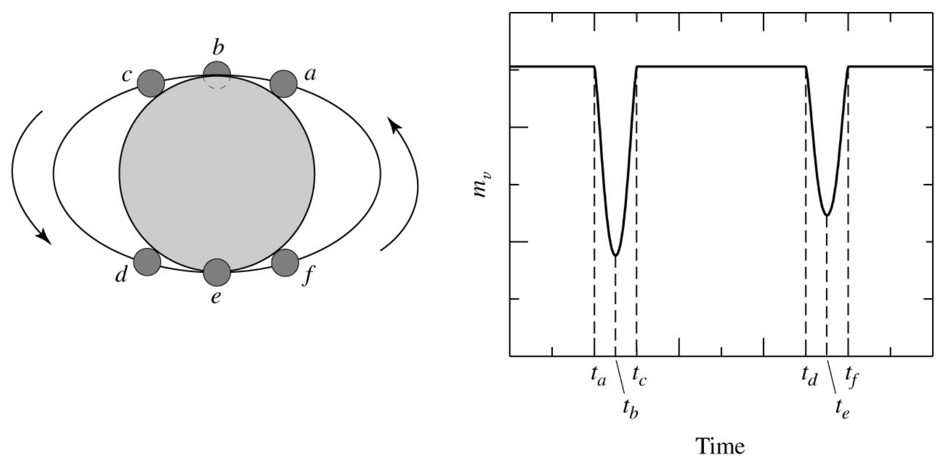
Can't resolve the stars, but the shape of the "light curve" gives information on the inclination and the radii of the stars.



Which star is hotter?

smaller eclipsing larger

larger eclipsing smaller



Can compare depths of minima to find ratio of temperatures, since $F_e \propto T^4$

and same cross-sectional areas are obscured during eclipses.

Assume the smaller star is hotter. Then the incident fluxes at key times are:

$$d \longrightarrow e \qquad F_o \propto r_l^2 T_l^4 + r_s^2 T_s^4$$

$$b \longrightarrow c \qquad F_{prim} \propto r_l^2 T_l^4$$

"primary", or deeper minimum

$$f \longrightarrow g \qquad F_{sec} \propto (r_l^2 - r_s^2) T_l^4 + r_s^2 T_s^4$$

"secondary" minimum

$$\Rightarrow \left(\frac{T_s}{T_l}\right)^4 = \frac{F_0 - F_{prim}}{F_0 - F_{sec}}$$

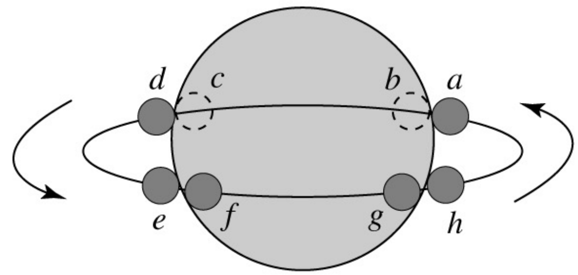
Note: if we assume the larger star is hotter, it can be shown that the same equation holds, but with the LHS becoming $\left(\frac{T_l}{T_s}\right)^4$

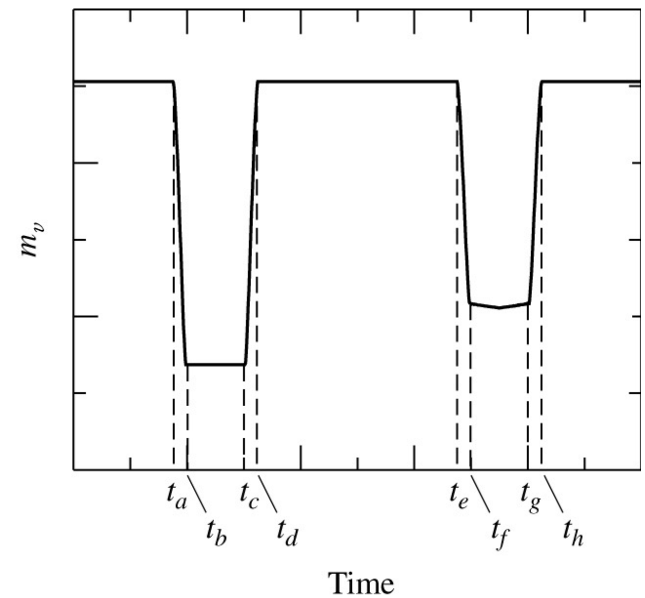
Thus, we can get T ratios, but can't tell which one is hotter.

Finally, if eclipsing and double lined spectroscopic binary and orbits are circular, then we can get the stellar radii. We know $v \sin i \approx v$. Then

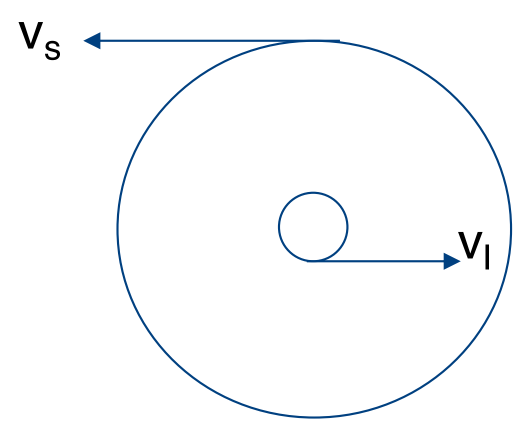
$$r_s = \frac{v}{2}(t_b - t_a)$$

$$r_l = \frac{v}{2}(t_c - t_a)$$





Where $v = v_s + v_l$ which we determined from spectra



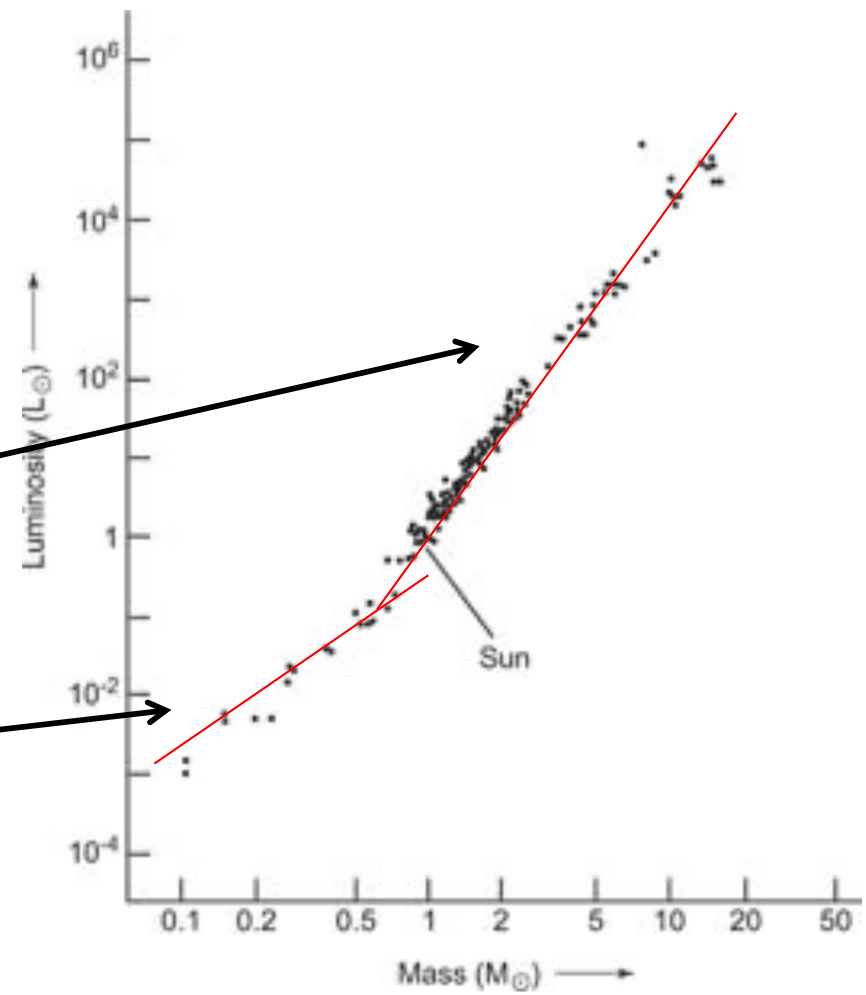
Application of mass determinations:

Study of binary stars is the source of the calibrated *mass-luminosity relation.*

Will look at physics of stars to understand where the relation comes from, and why there is a break in the slope.

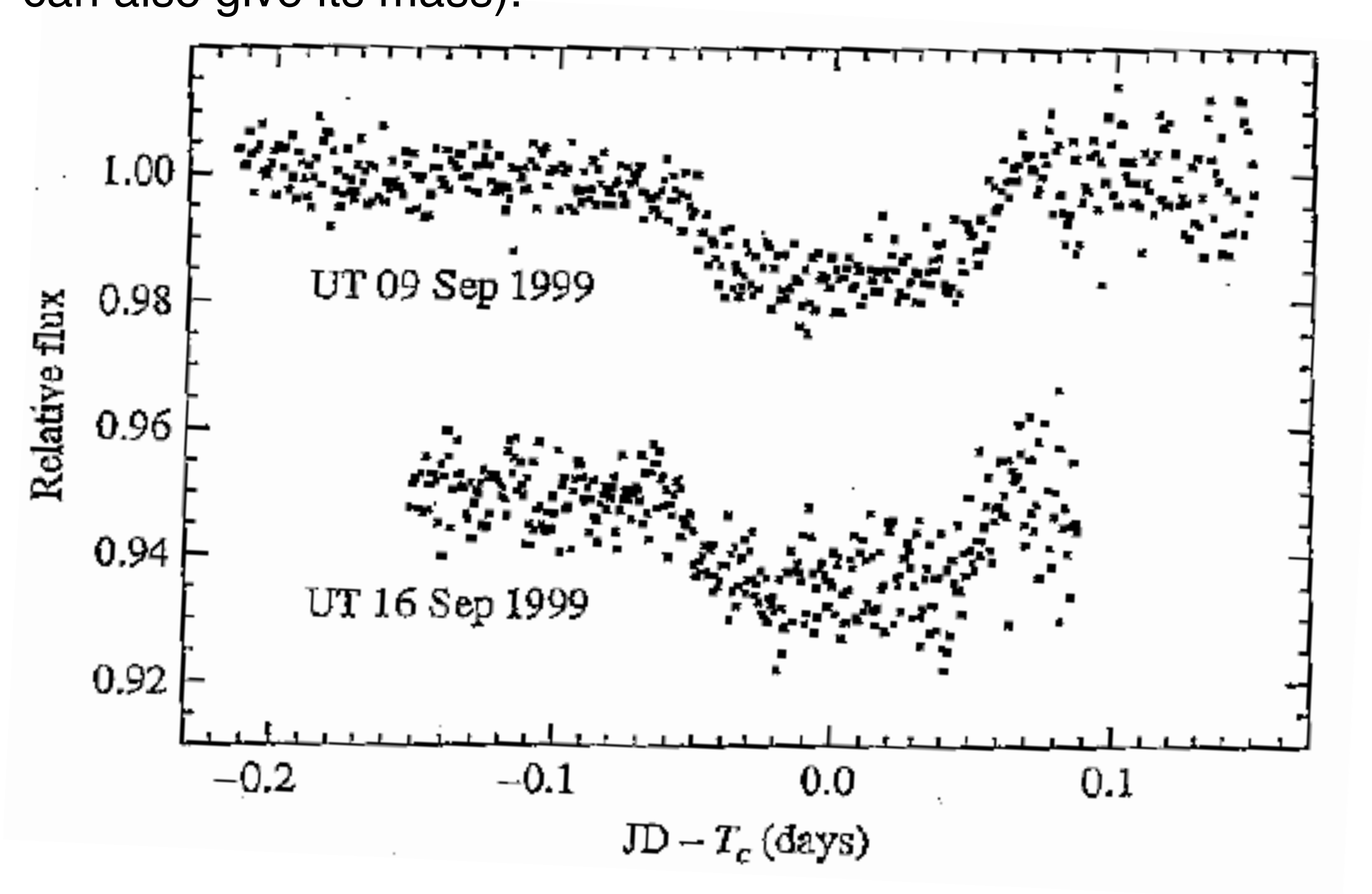
$$\left(\frac{L}{L_\odot}\right) = \left(\frac{M}{M_\odot}\right)^4$$

$$\left(\frac{L}{L_\odot}\right) = \left(\frac{M}{M_\odot}\right)^2$$



Exoplanets

Planetary transits or eclipses. Light curves give radii, orbit size, as before. But need spectrum of star to form mass function and constrain planet mass (later we'll see how star's spectrum can also give its mass).



Planets around other stars

Test solar system formation process

Possibility of life on other planets

Techniques:

Direct detection (images)

Transit of star by planet

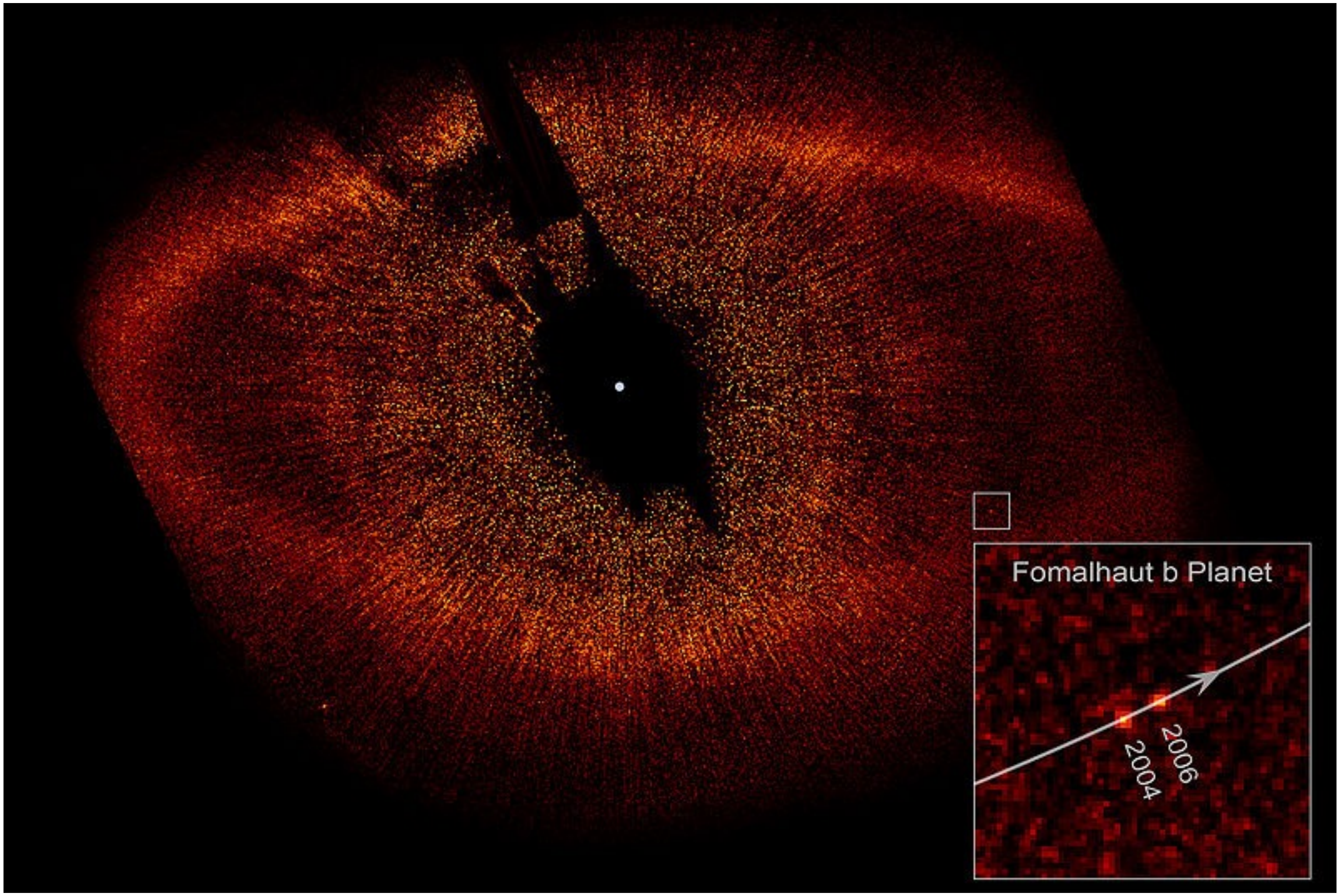
Microlensing

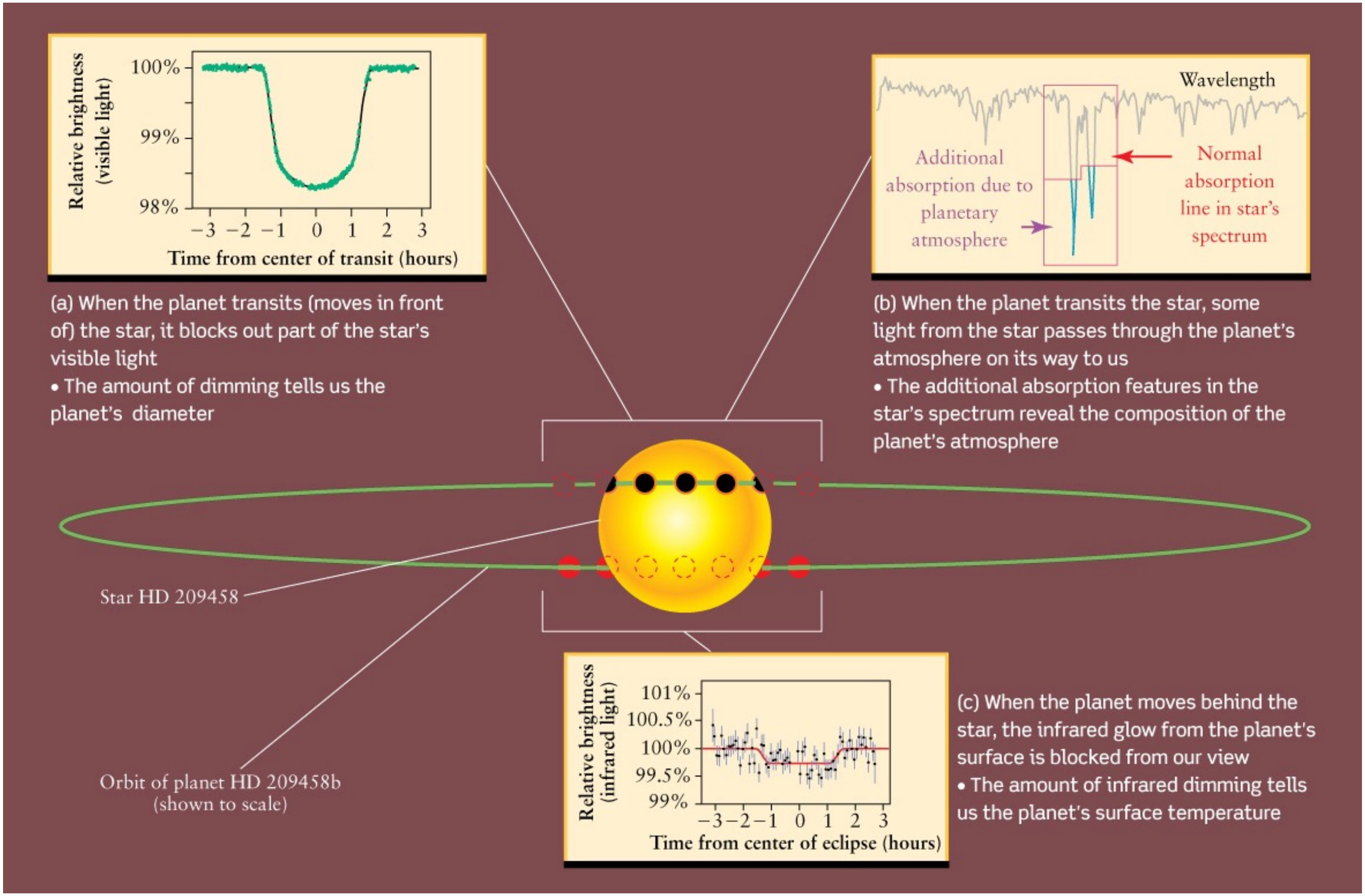
Detection of star's wobble by spectroscopy

Detection of star's wobble by imaging

Detection of radio bursts

Direct Imaging



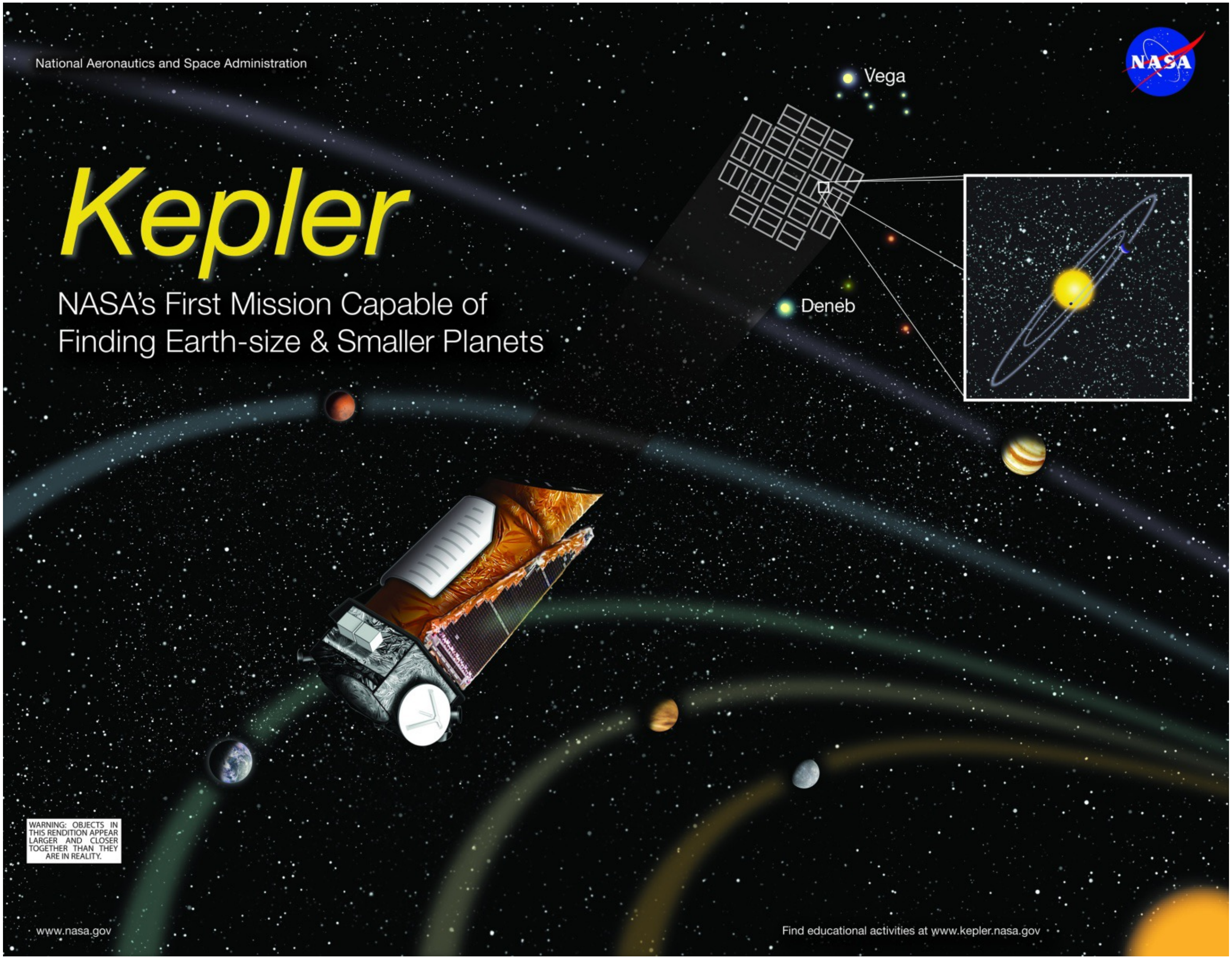


Dims by about 1-2% during the transit

Requires precision photometry, better done from space

Can extract info on planet's atmosphere, size, temp!

Kepler transit mission – 3000+ candidates, confirmed list smaller.



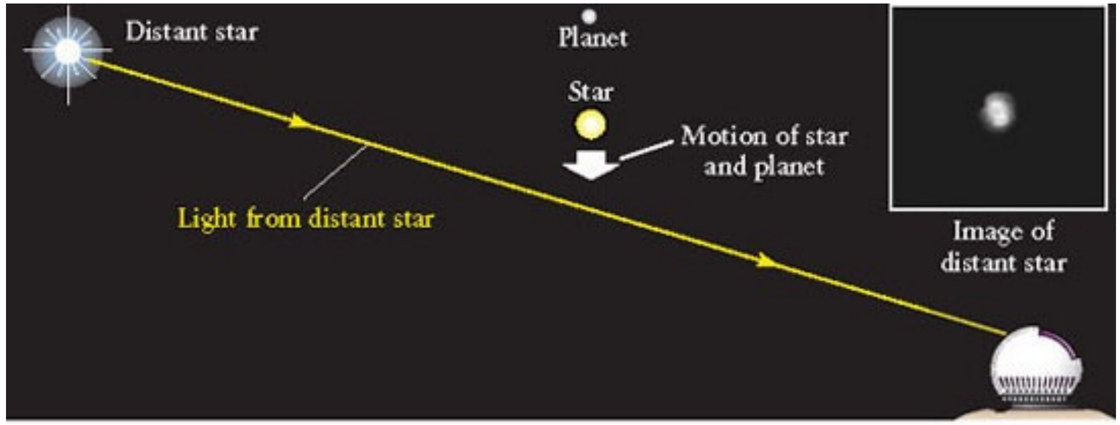
also CoRoT

Gravitational microlensing

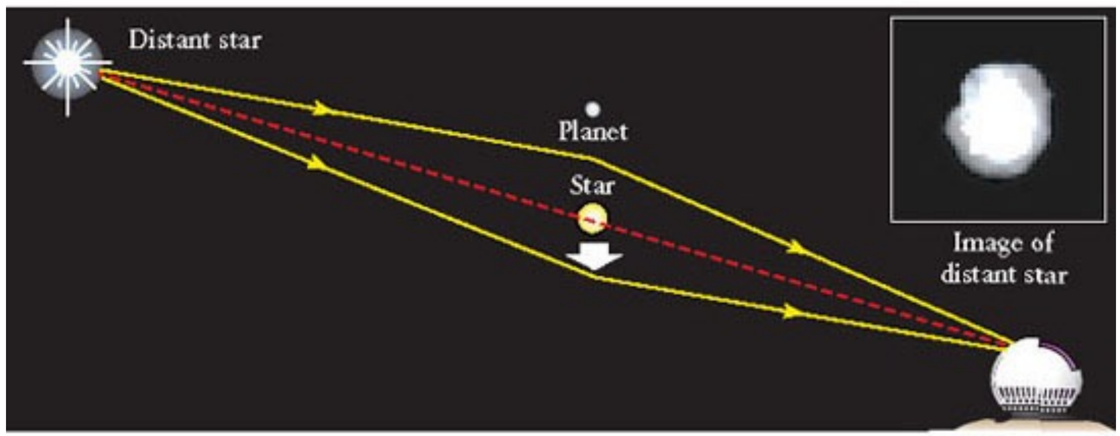
If two stars line up, one near and one far, the light from the background star will bend around the foreground star (due to gravity)

A planet around the foreground star will cause an intense amplification if passing close to the line of sight

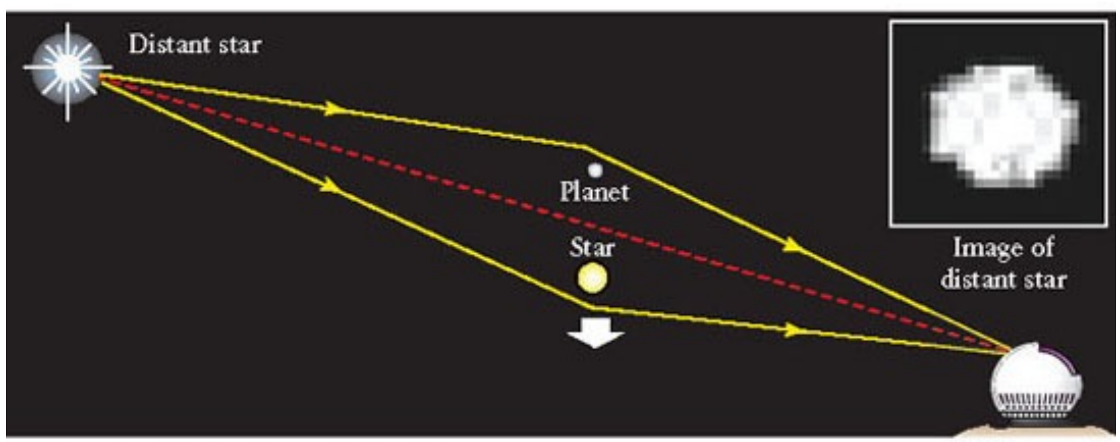
only 15 planets found this way



(a) No microlensing



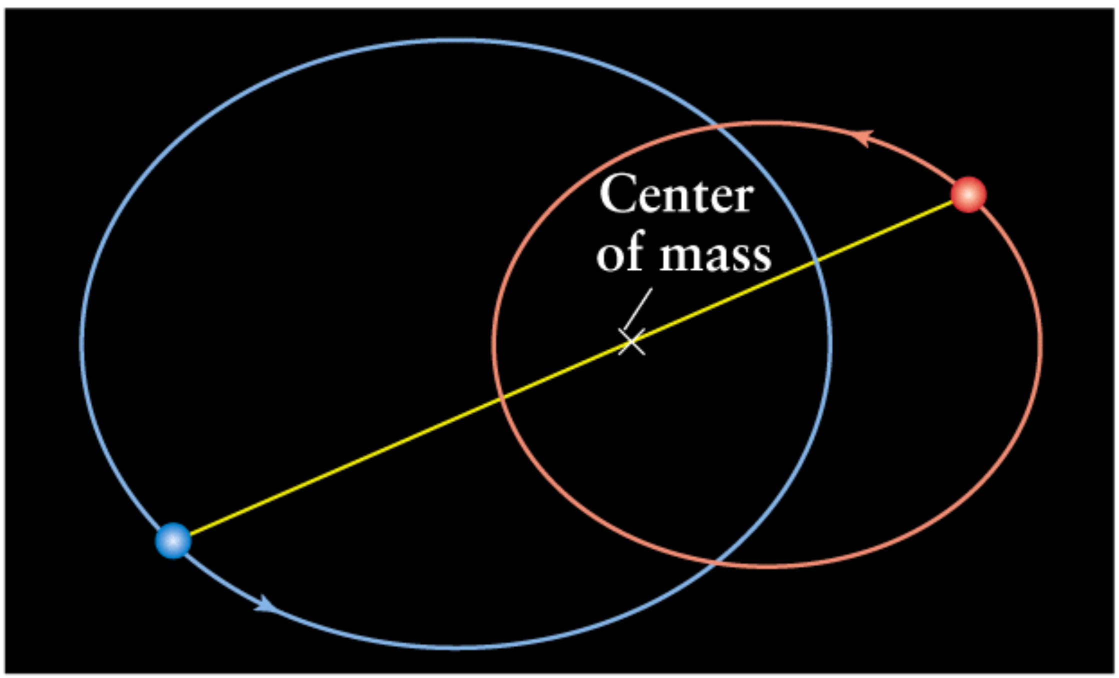
(b) Microlensing by star



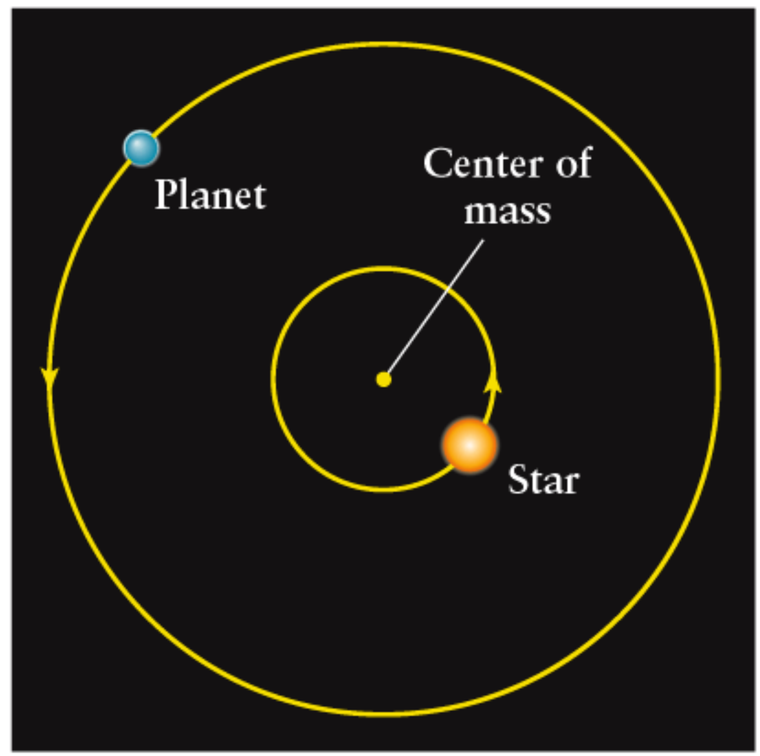
(c) Microlensing by star and planet

Detecting a star’s wobble

Idea: a planet and its star both orbit around their common center of mass, staying on opposite sides of this point. Creates a "wobble".

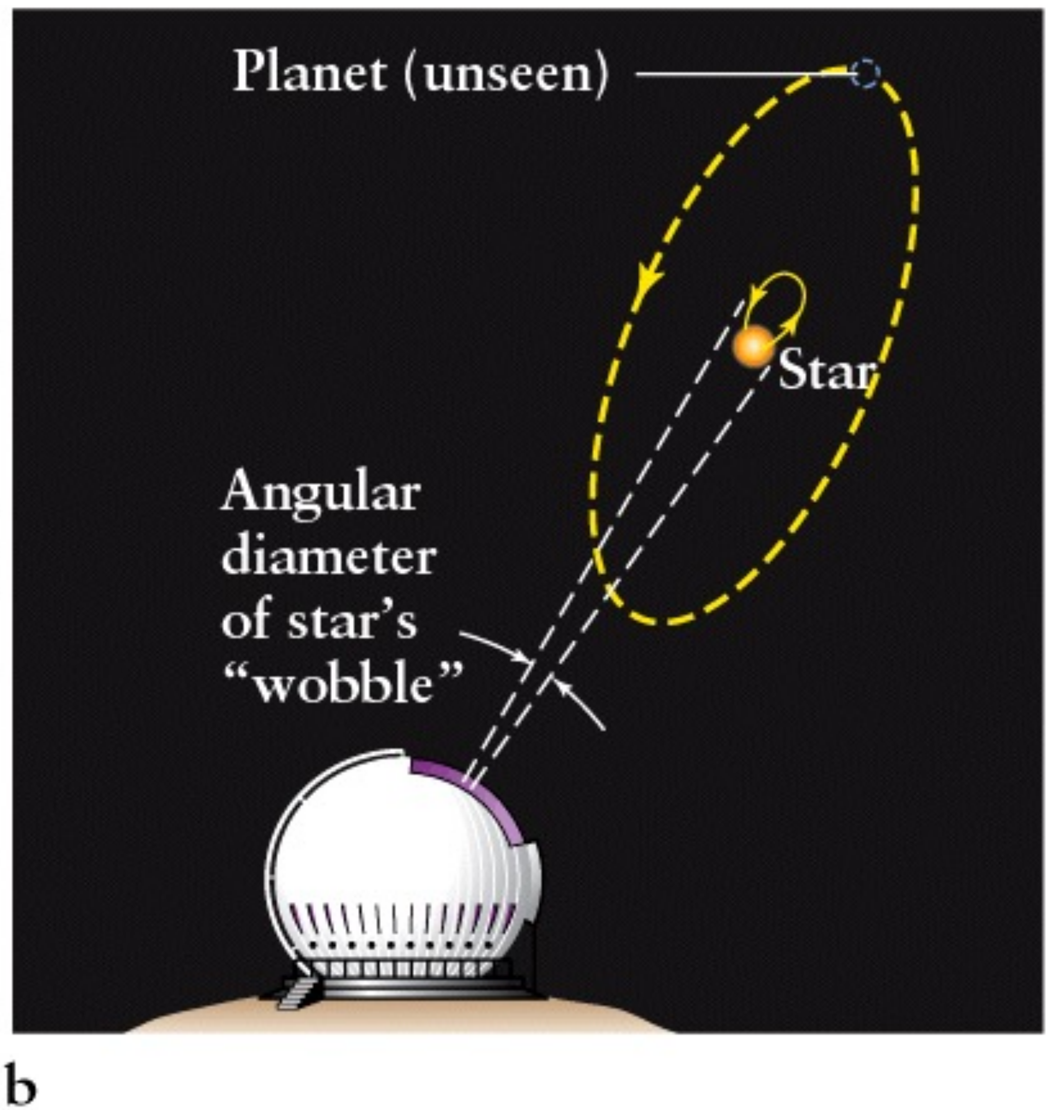


b

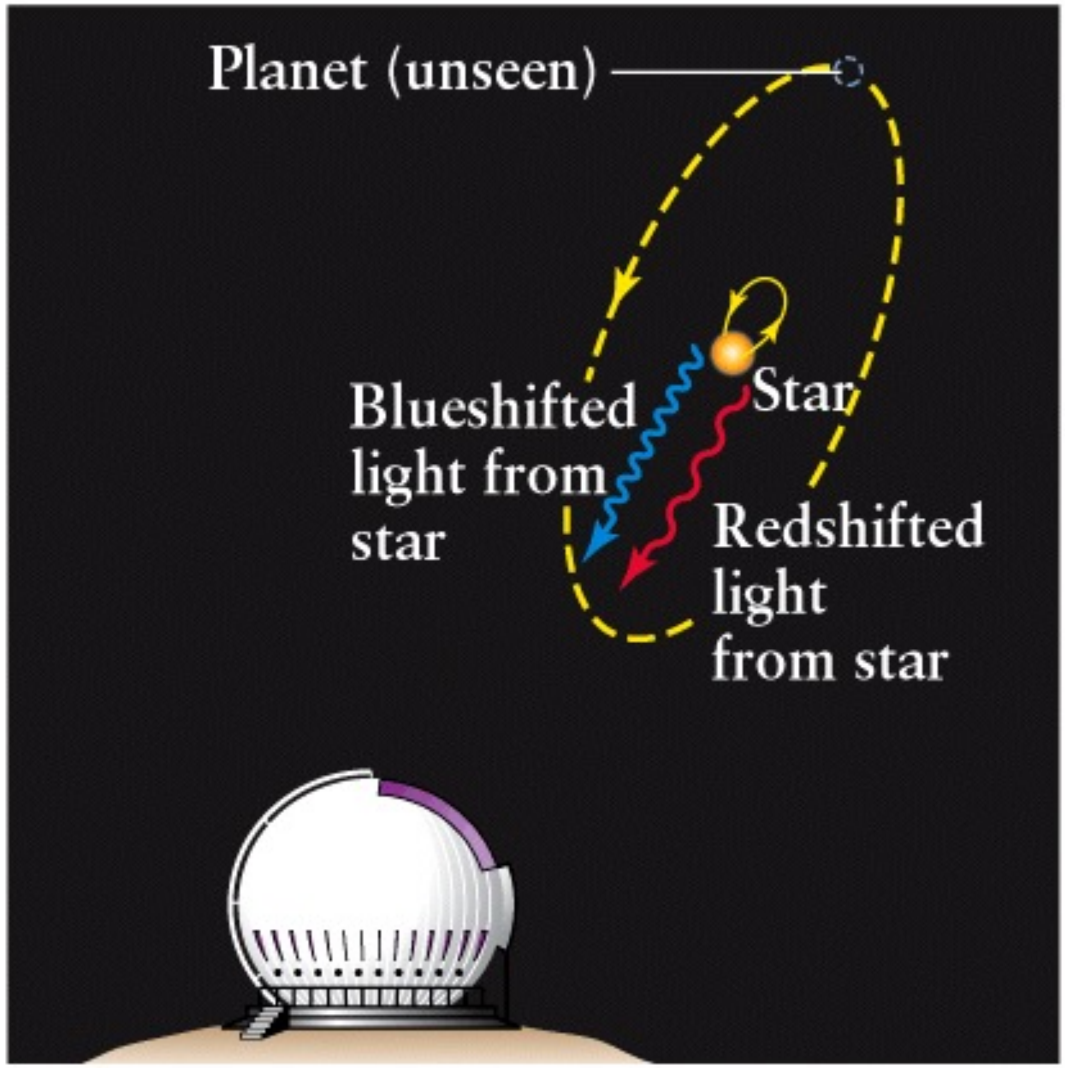


a

A wobbling star might be seen by careful observations, called “astrometry”:



Most successful method: Use the Doppler shift of star’s spectral lines due to its radial (= back and forth) motion:



c

The massive star is closer to center of mass, and moves more slowly than the planet, but it does move!

Worksheet #5: Sun and Jupiter orbit their common center of mass every 11.86 years.

Note $M_J / M_{Sun} = 1/1047$

What is the orbital speed of the Sun?

What is the astrometric displacement for a similar system at a distance of 10 pc?

Solution: First calculate the semi-major axis of the Jupiter's orbit

$P^2 = a^3$ so $a = 5.2$ AU $= 780 \times 10^9$ m

$M_s r_s = M_j r_j$ so $r_s = 740 \times 10^6$ m

Angular size $= 2 \times 740 \times 10^6$ m$/10 \times 3.09 \times 10^{16}$ m

$= 4.8 \times 10^{-9}$ radians $\times$ 206265000 mas/rad

$= 0.99$ mas

Calculate orbital speed of Sun assuming Jupiter is only planet:

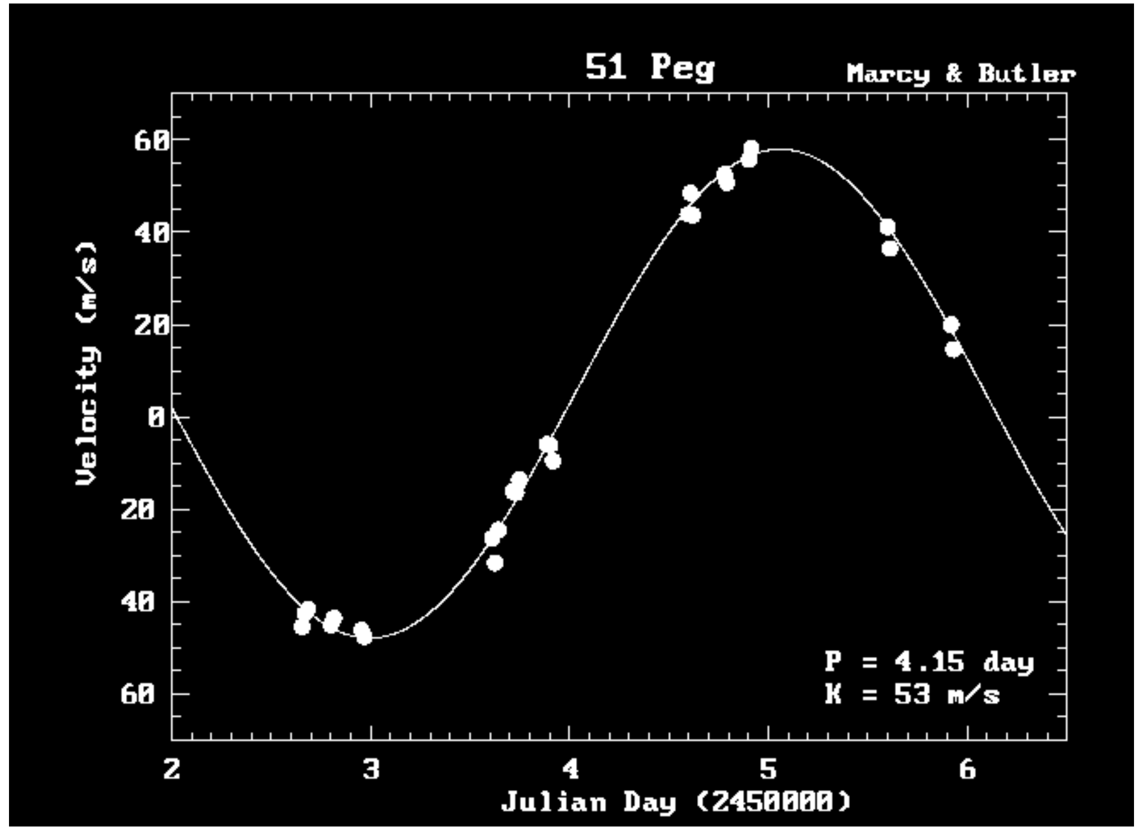
Moves in circular orbit of radius 742,000 km

How much Doppler shift? Consider H-alpha absorption line, at rest wavelength 656 nm:

$$V = \frac{2\pi r}{P} = \frac{2\pi(742{,}000\ \text{km})}{11.86\ \text{years}} = 12.5\ \text{m/s}$$

$$\Delta\lambda = \frac{V}{c}\lambda_0 = \frac{12.5\ \text{m/s}}{3\times 10^8\ \text{m/s}} 656\ \text{nm} = 2.7\times 10^{-5}\ \text{nm}$$

51 Pegasi



Michel Mayor & Didier Queloz at Geneva Observatory observed wobble in 1995

Sun-like star 15 pc distance

Wobble was 53 m/s, period 4.15 days

Implied a planet with 0.5 Jupiter mass orbiting at 0.05 AU!

First planet found around sun-like star

Michel Mayor & Didier Queloz
Nobel Prize 2019

Selection effects

Doppler wobble biased towards massive planets close to their star (leads to larger velocities and shorter periods). Now getting close to Earth-mass planets.

Limited by the orbital speed sensitivity (few m/s but always improving) and length of orbital period: for more than several year periods, hard to tell if motion is periodic

Inclination of binary orbit unknown (unless transits observed). More likely to be close to edge-on for detection. If not, wobble is larger than measured and so is planet.

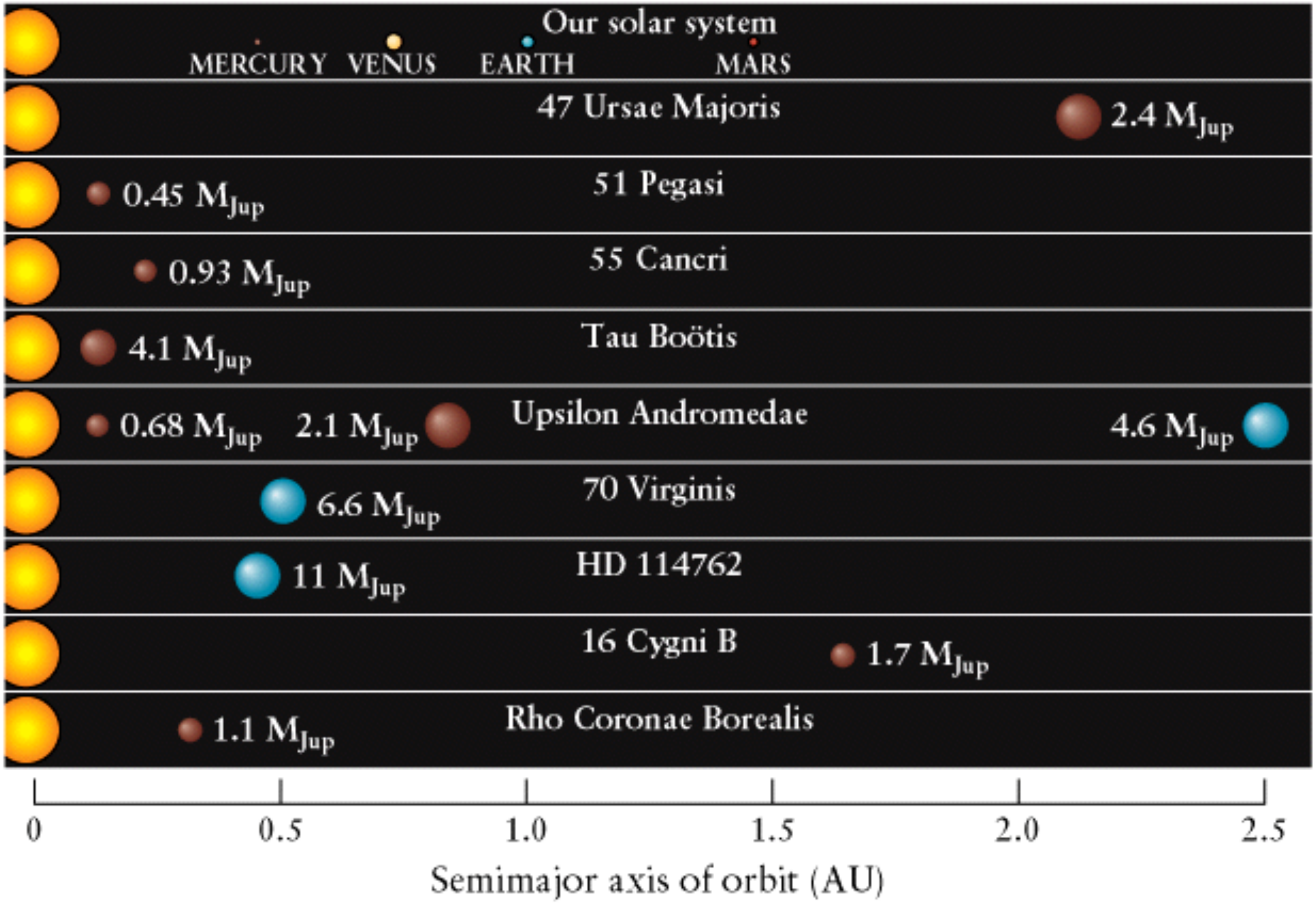
Characteristics of detections

Some hot Jupiters on small orbits (migration or formation?)

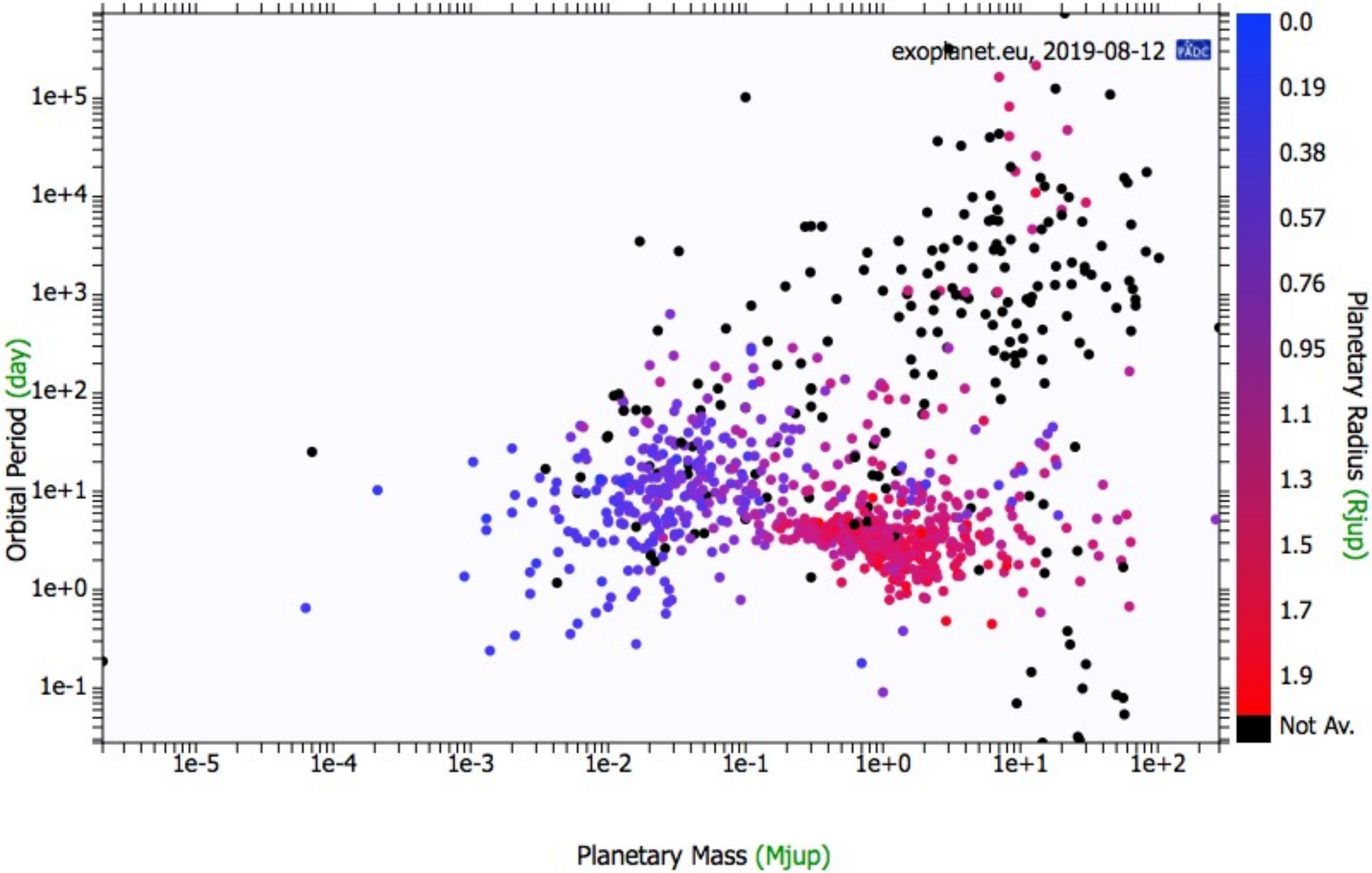
Some with very elliptical orbits (in the Solar System this is a sign of perturbation => supports migration idea?)

Rare around low-mass stars (smaller disks thus less material)

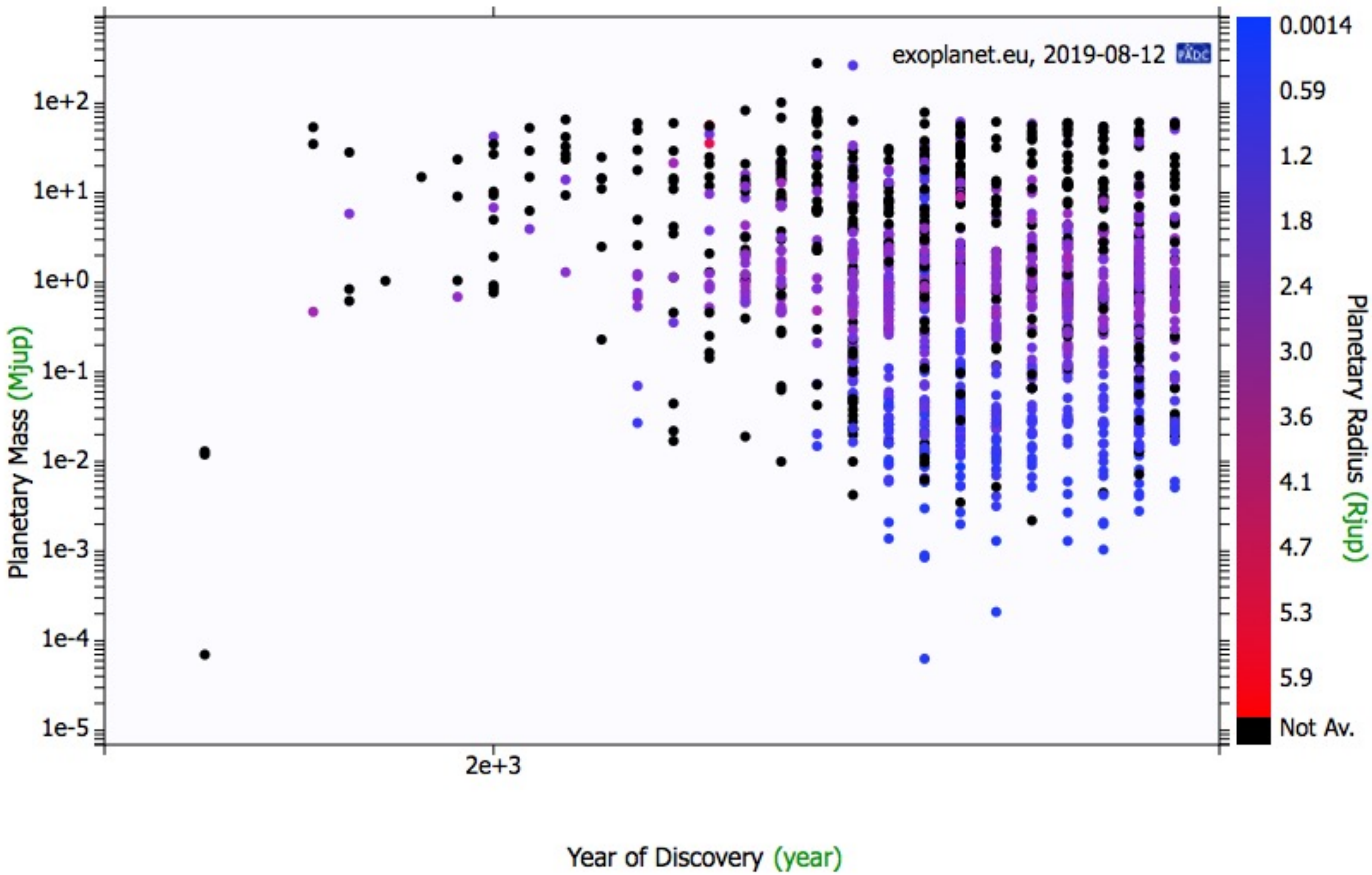
Rare around metal-poor stars

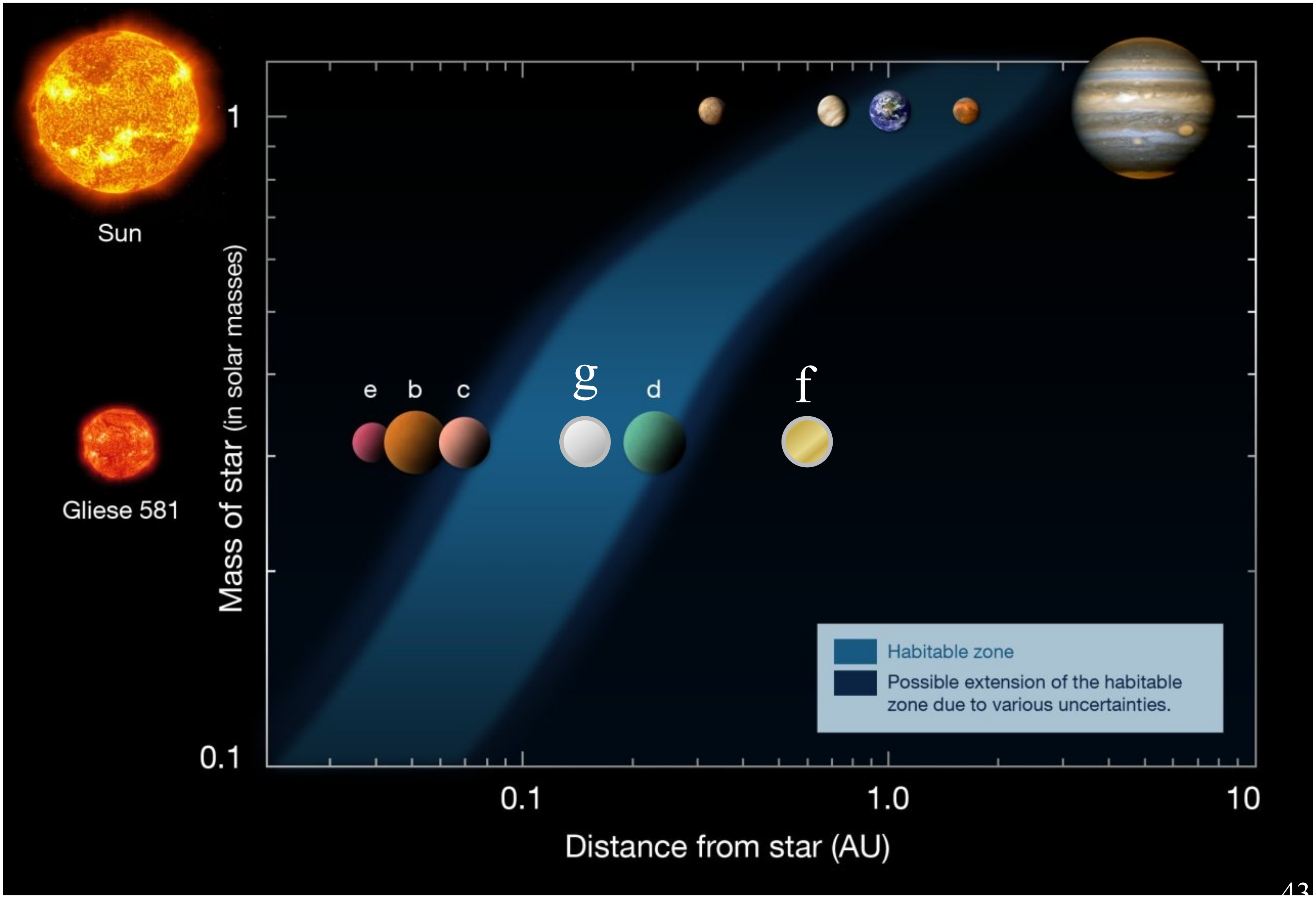


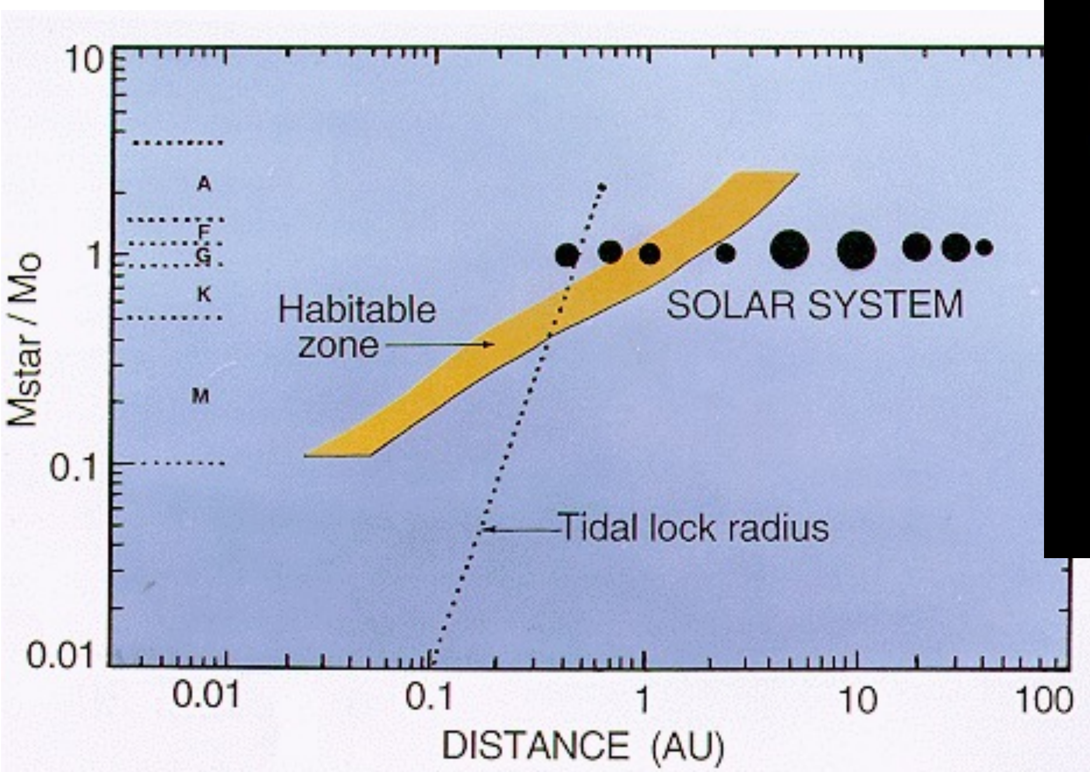
count: 4472 planets! What about these "hot Jupiters"?



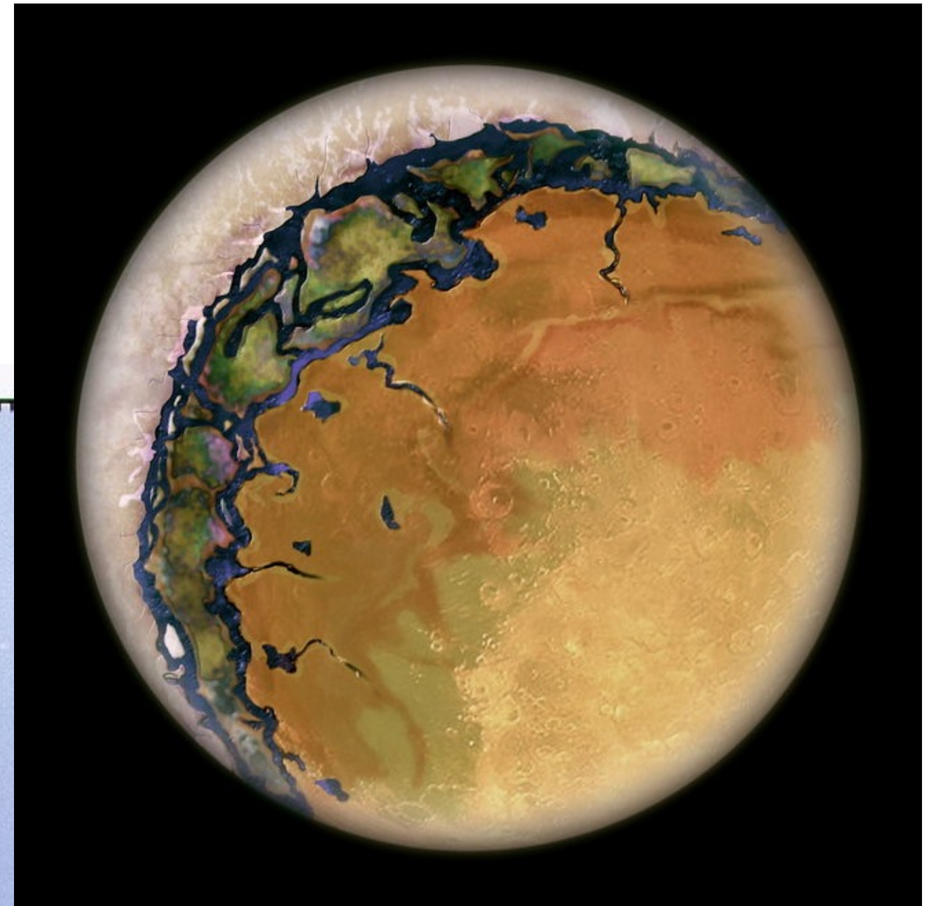
exoplanet.eu, 2019-08-12
1e+5
1e+4
1e+3
1e+2
1e+1
1e+0
1e-1
Orbital Period (day)
1e-5
1e-4
1e-3
1e-2
1e-1
1e+0
1e+1
1e+2
Planetary Mass (Mjup)
0.0
0.19
0.38
0.57
0.76
0.95
1.1
1.3
1.5
1.7
1.9
Not Av.
Planetary Radius (Rjup)

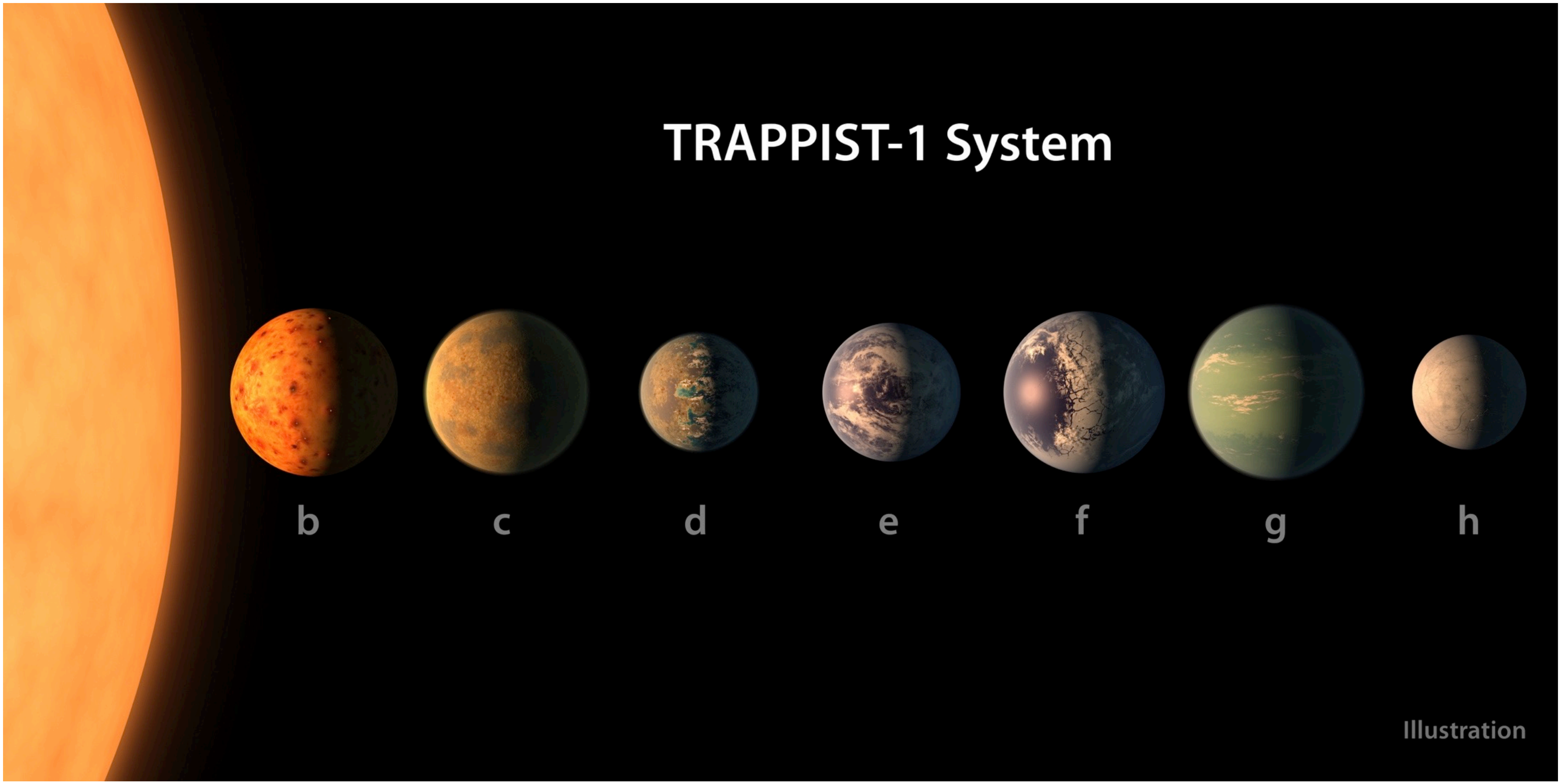

exoplanet.eu, 2019-08-12
1e+2
1e+1
1e+0
1e-1
1e-2
1e-3
1e-4
1e-5
Planetary Mass (Mjup)
2e+3
Year of Discovery (year)
0.0014
0.59
1.2
1.8
2.4
3.0
3.6
4.1
4.7
5.3
5.9
Not Av.
Planetary Radius (Rjup)

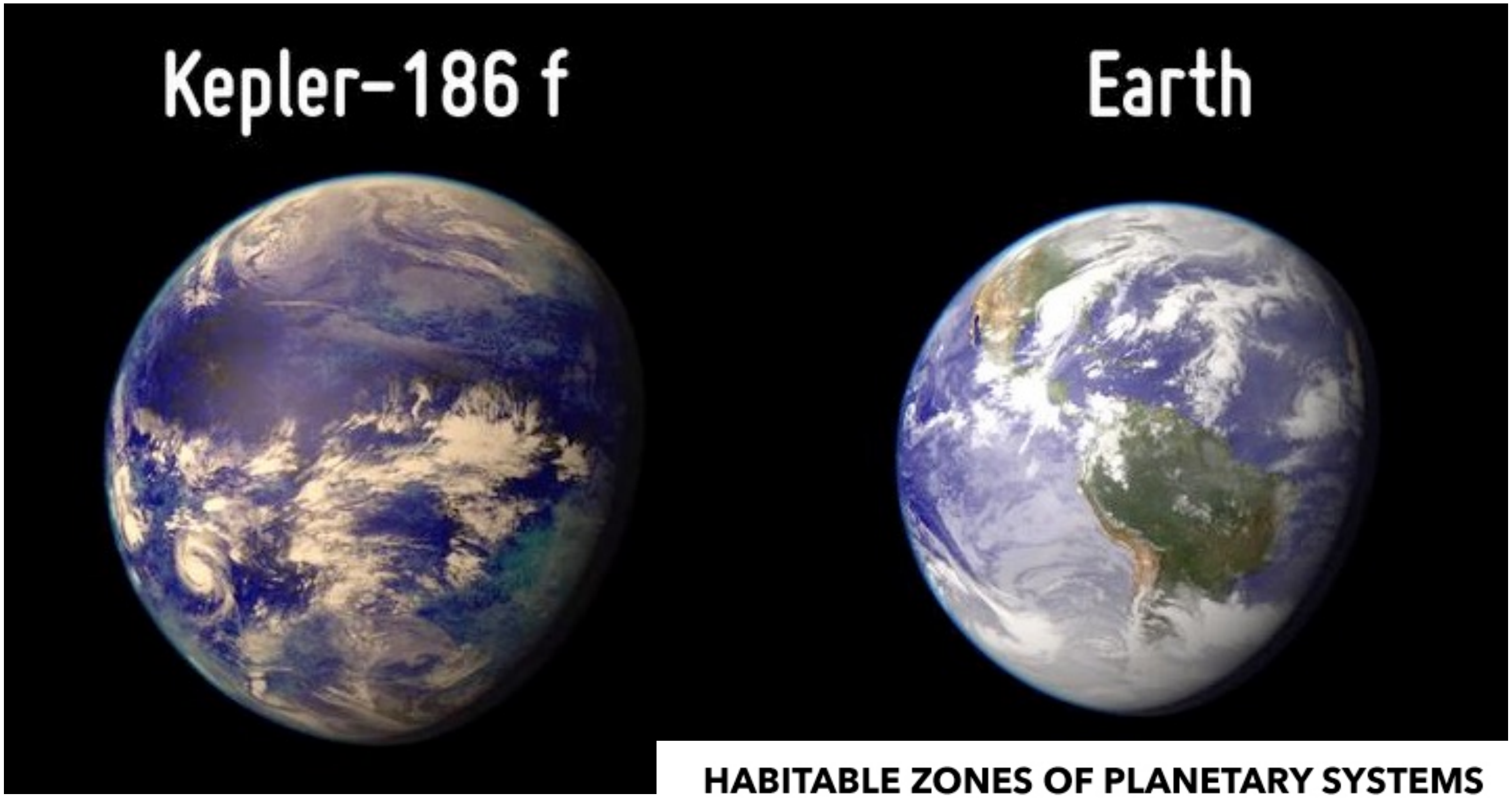

Sun
Gliese 581
1
0.1
Mass of star (in solar masses)
e
b
c
g
d
f
Habitable zone
Possible extension of the habitable zone due to various uncertainties.
0.1
1.0
10
Distance from star (AU)

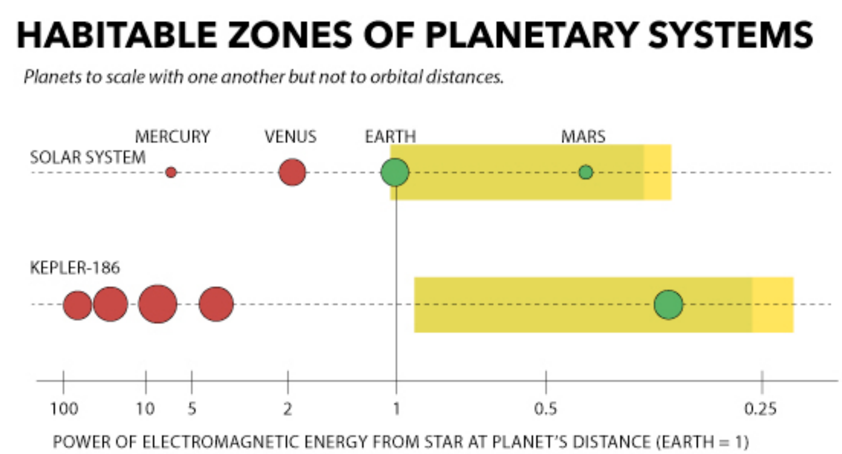


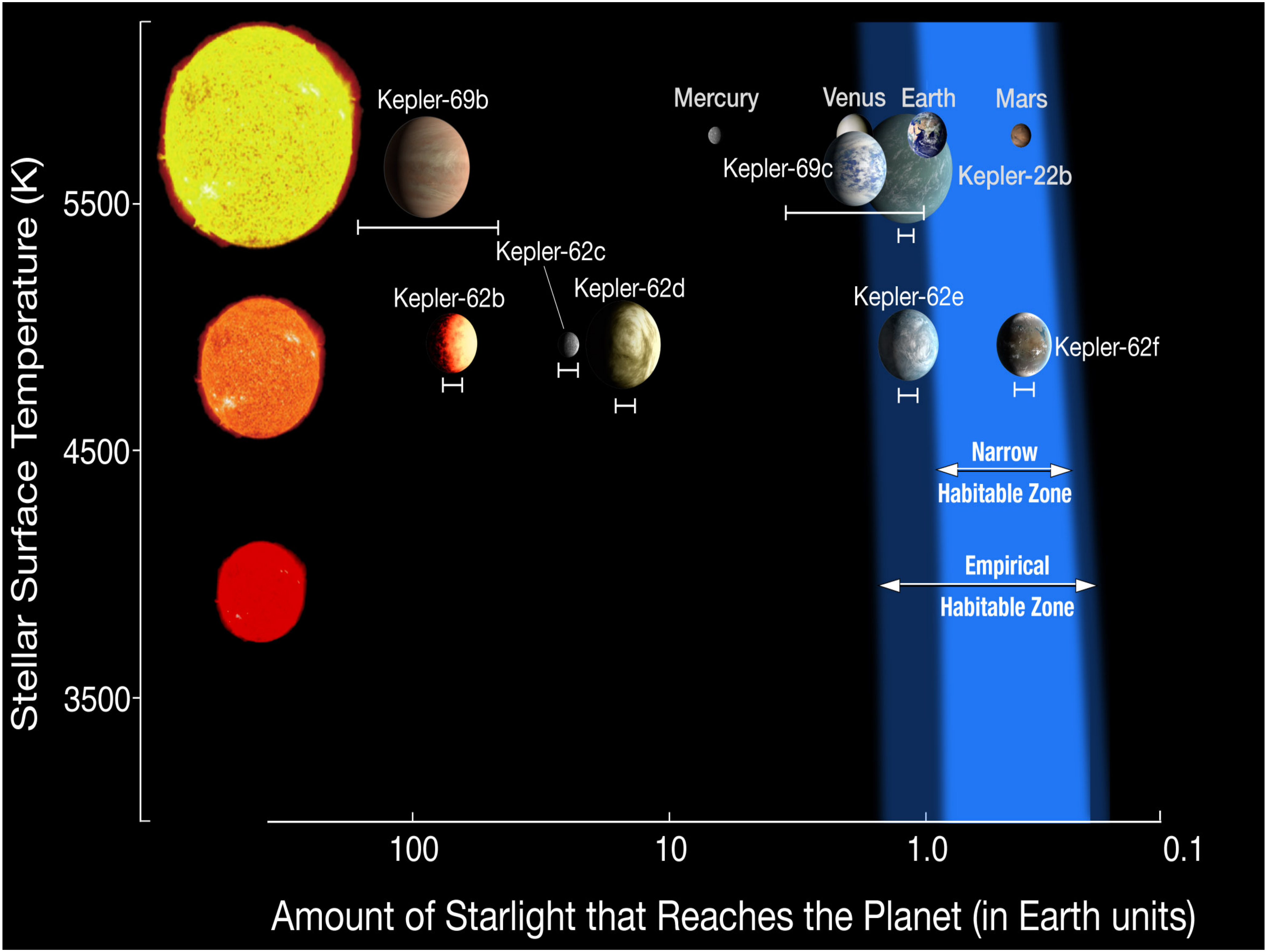
10
1
0.1
0.01
Mstar / Mo
A
F
G
K
M
Habitable
zone
SOLAR SYSTEM
Tidal lock radius
0.01
0.1
1
10
100
DISTANCE (AU)

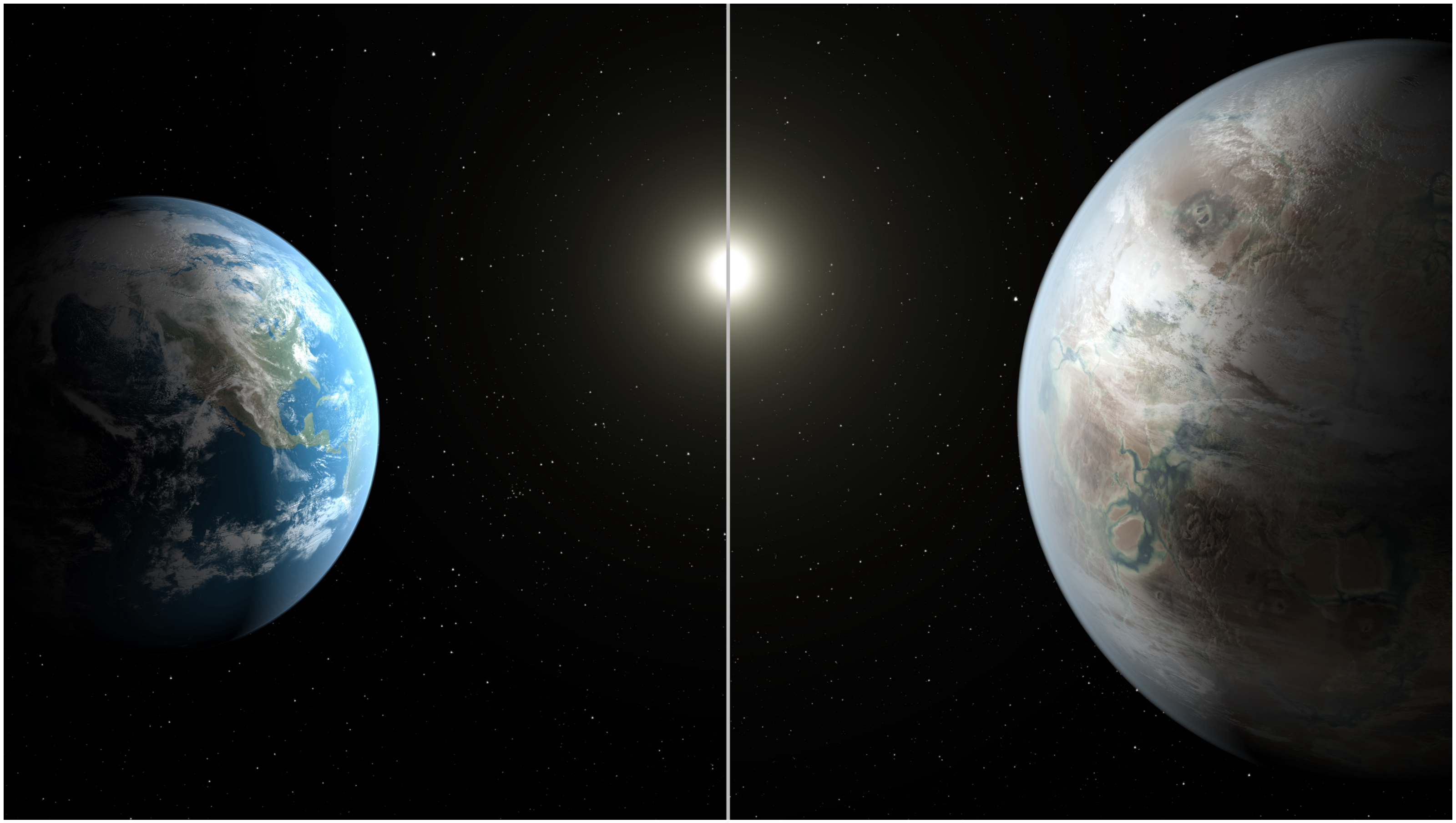



TRAPPIST-1 System
b
c
d
e
f
g
h
Illustration


Kepler-186 f
Earth


HABITABLE ZONES OF PLANETARY SYSTEMS
Planets to scale with one another but not to orbital distances.
SOLAR SYSTEM
MERCURY
VENUS
EARTH
MARS
KEPLER-186
100
10
5
2
1
0.5
0.25
POWER OF ELECTROMAGNETIC ENERGY FROM STAR AT PLANET'S DISTANCE (EARTH = 1)


Stellar Surface Temperature (K)
5500
4500
3500
Kepler-69b
Mercury
Venus
Earth
Mars
Kepler-69c
Kepler-22b
Kepler-62c
Kepler-62d
Kepler-62b
Kepler-62e
Kepler-62f
Narrow
Habitable Zone
Empirical
Habitable Zone
100
10
1.0
0.1
Amount of Starlight that Reaches the Planet (in Earth units)



Detecting Radio Bursts from Exoplanets

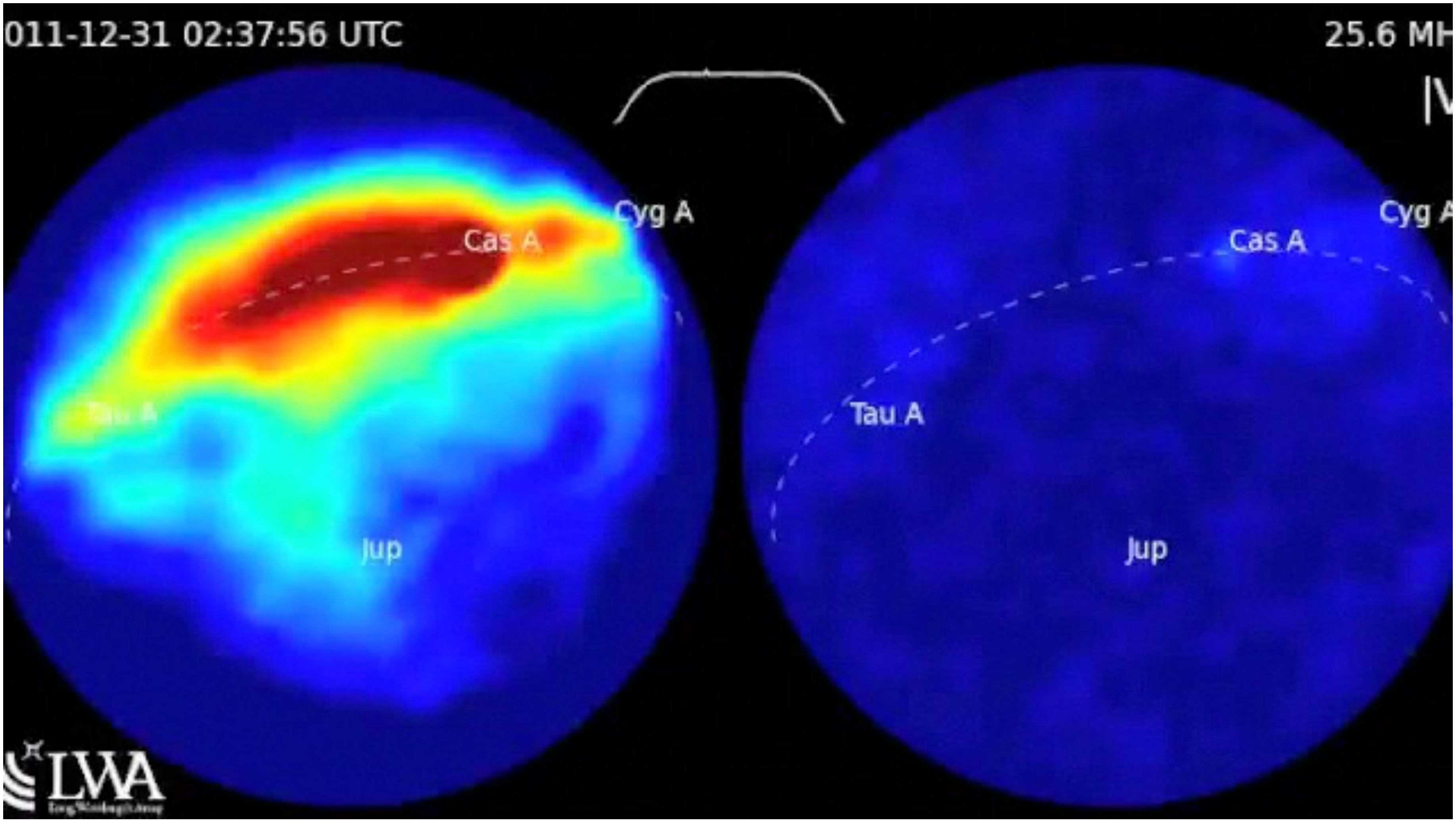
Suitability of the LWA1

Observations to date

Near future: Owens Valley

Farther future: the LWA swarm

Emission from Jupiter



Also, sort the following table out on your own from the physics and geometry involved in each type of event:

Type of binary	**Observations performed (or needed)**	**Parameters determined**
Visual	a) Apparent magnitudes and π	Stellar luminosities
	b) P, a, and π	Semi-major axis (a)
	c) Motion relative to CM	Mass sum ($M+m$)
Spectroscopic	a) Single velocity curve	Mass function f(M,m)
	b) Double velocity curve	Mass ratio (M/m)
Eclipsing	a) Shape of light curve eclipses	Orbital inclination (i)
	b) Relative times between eclipses	Relative stellar radii ($R_{l,s}/a$)
	c) Light loss at eclipse minima	Orbital eccentricity (e)
		Surface temperature ratio (T_l/T_s)
Eclipsing/spectroscopic	a) Light and velocity curves	Absolute dimensions (a, r_s, r_l)
		e and i
	b) Spectroscopic parallax + apparent magnitude	Distance to binary
		Stellar luminosities
		Surface temperatures (T_l, T_s)