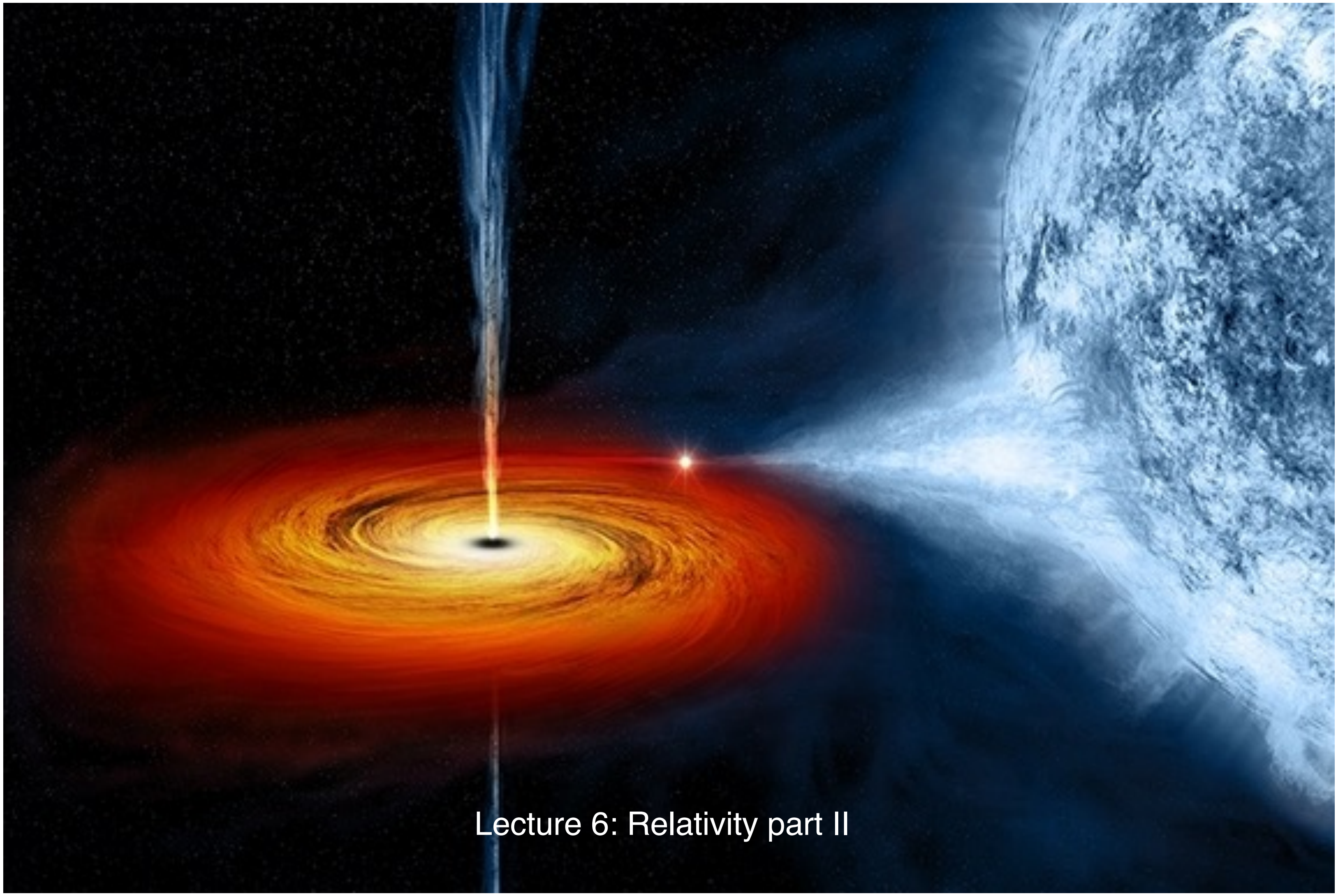
Astronomy 421



Lecture 7 – Key Concepts:

Relativistic beaming

Superluminal Motion

Relativistic Doppler shift

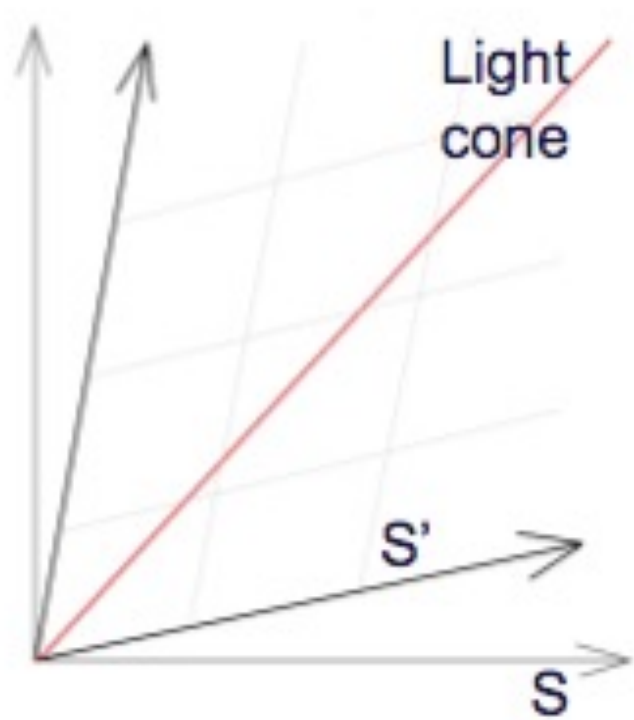
Relativistic dynamics

Spacetime diagram and the Lorentz transformations

Changing from one reference frame to another via the Lorentz transformations will:

a) Affect the time coordinate (time dilation)
b) Affect the space coordinates (length contraction)

This is leading to a distortion of the spacetime diagram.



Spacetime

- Two side-by-side observers agree on all space and time measurements
  - Share same spacetime
- Two observers in relative motion disagree on space and time measurements
  - But always same ratio!
    - Differences imperceptible at low speeds
    - Important at speeds near *c (relativistic speeds)*

Observers in relative motion experience space and time differently, but speed of light is always constant!

Time Dilation Animated

Time between ‘ticks’ = distance / speed of light

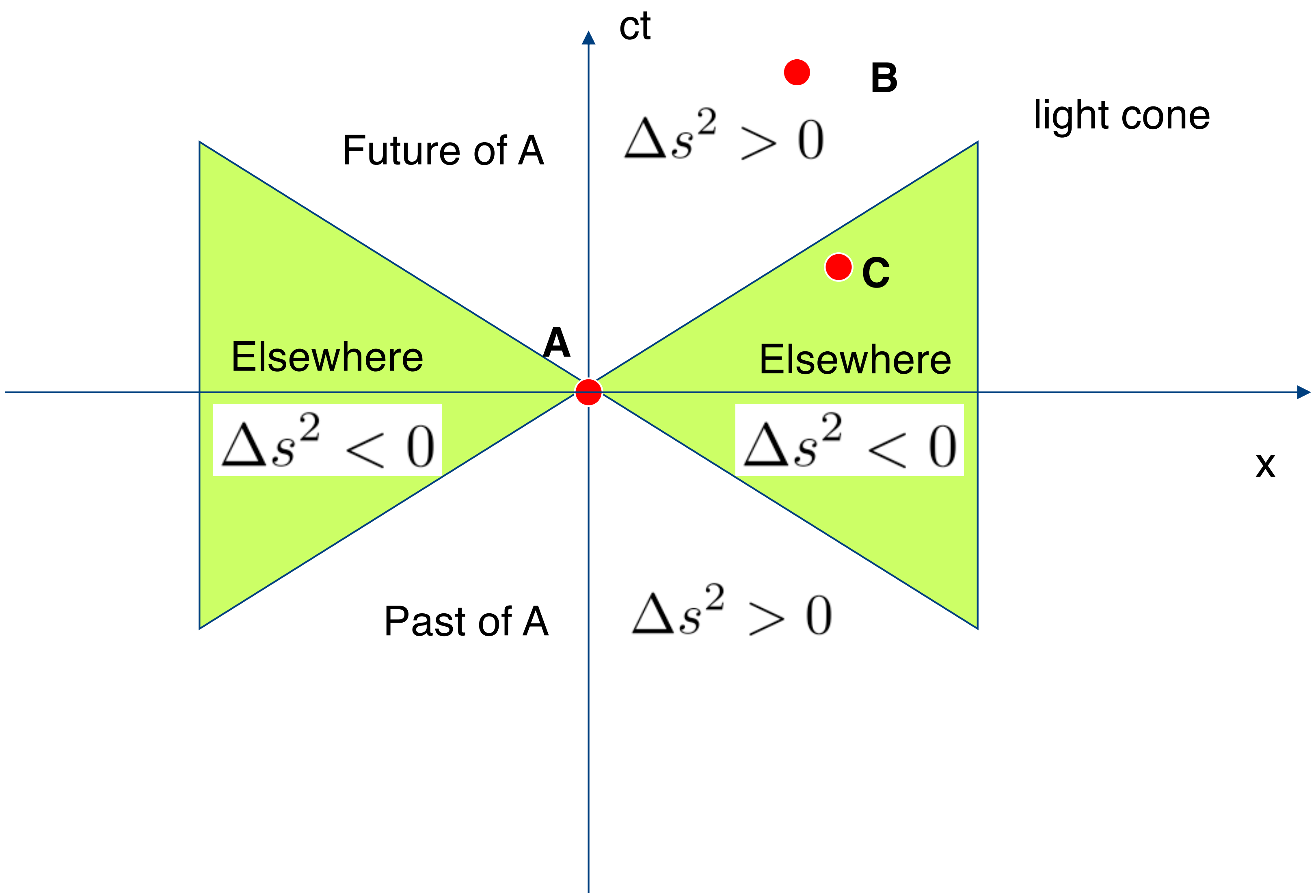
Light in the moving clock covers more distance…

…but the speed of light is constant…

…so the clock ticks slower!

Moving clocks run more slowly!

Past, future and elsewhere



Causality

Events A and B:

- A can communicate information to B by sending a signal at (or less than) the speed of light
- The temporal order of A and B cannot be changed by changing the reference frames
  - **A and B are causally connected**

Events A and C:

- Any communication between A and C must happen at a speed *faster* than the speed of light
- The temporal order of A and C can be changed by changing the reference frame.
  - **A and C are causally disconnected**

Relativistic Mass

There is an increase in the effective mass of an object moving at relativistic speeds given by:

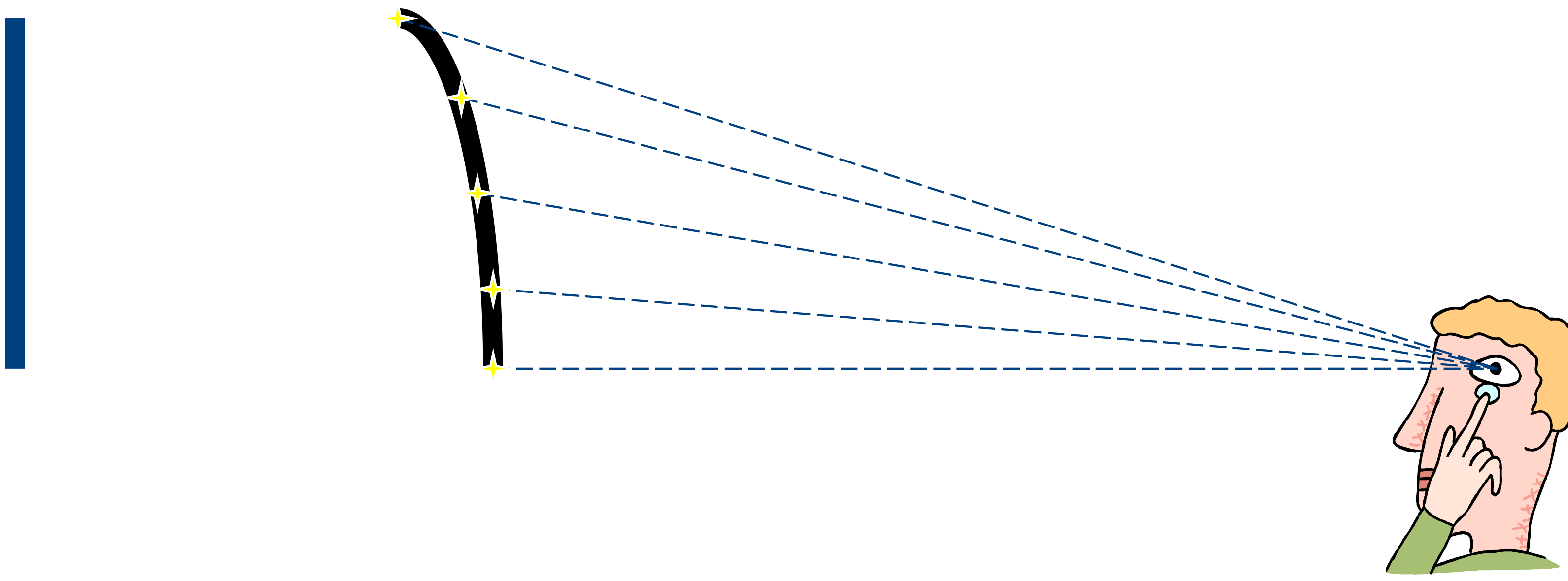
$m = \gamma m_0$ where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

you have to reach 0.14c to change the mass by 1%

at 0.99c the mass is 7.14 times greater than rest mass

Lorentz Transformations



Light from the top of the bar has further to travel.

It therefore takes longer to reach the eye.

So, the bar appears bent.

Weird!

Velocity transformations

Recall Lorentz coordinate and velocity transformations:

$$x = \gamma(x' + ut')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{ux'}{c^2}\right)$$

$$x' = \gamma(x - ut)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{ux}{c^2}\right)$$

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2}$$

$$v'_y = \frac{v_y\sqrt{1 - u^2/c^2}}{1 - uv_x/c^2}$$

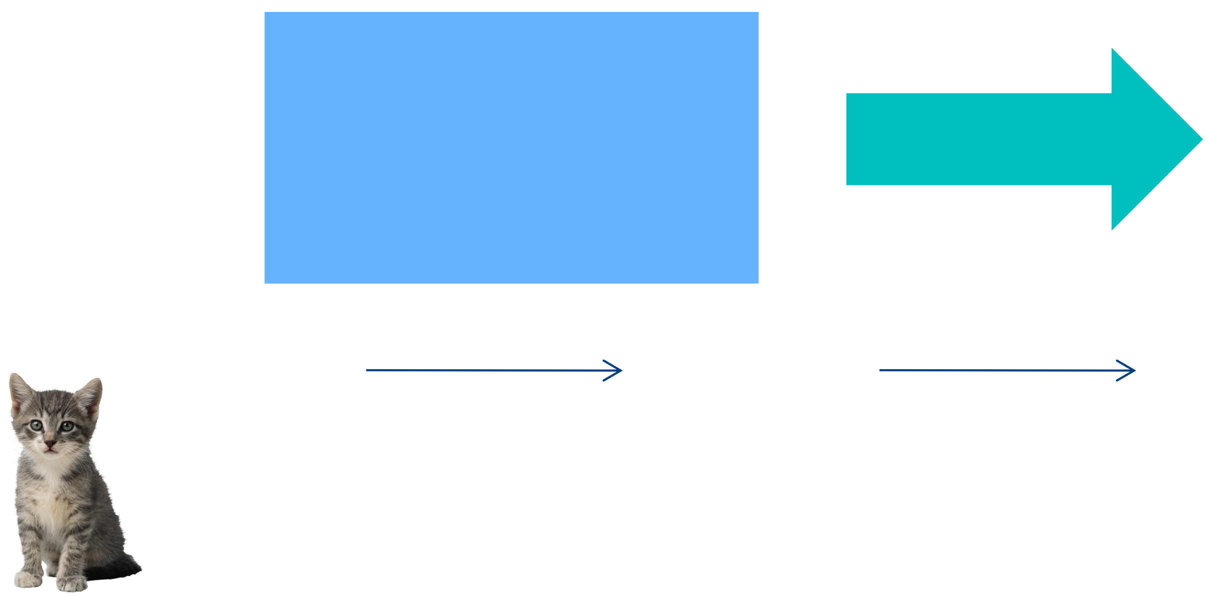
$$v'_z = \frac{v_z\sqrt{1 - u^2/c^2}}{1 - uv_x/c^2}$$

To get inverse transforms, just replace u with –u.

Relativistic Velocity Addition

Ship moves away from a cat at $0.5c$ *and fires a rocket with velocity (relative to ship) of $0.5c$*

How fast (compared to the speed of light) does the rocket move relative to the cat?



Relativistic Velocity Addition

Classically: $V = v_1 + v_2$

Relativistically:

$$V = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

Ship moves away from cat at 0.8c (as measured in the rest-frame of the cat)

Relativistic Velocity Addition

Classically: $\boldsymbol{V = v_1 + v_2}$

Relativistically:

$$V = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

Ship moves away from you at 0.5*c and fires a rocket with velocity (relative to ship) of 0.5c*

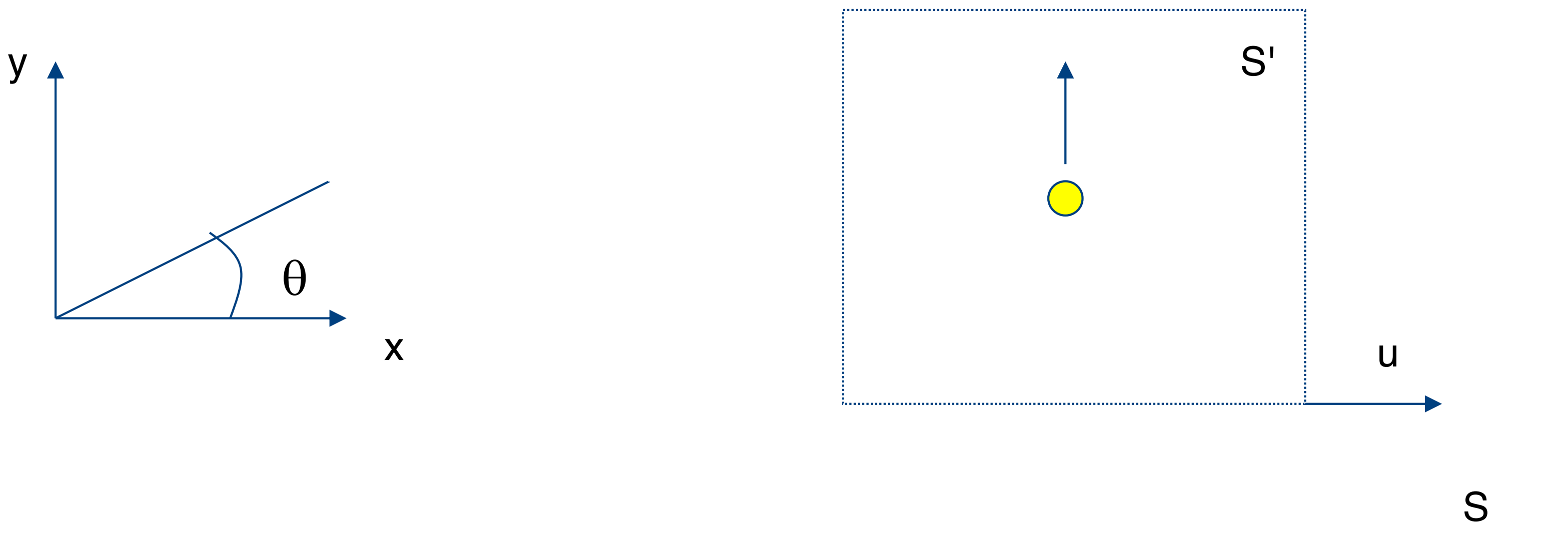
You see rocket move at 0.8c

No massize object can be accelerated to the speed of light!

If instead the ship fires a laser at speed c, what speed do you measure for the light?

Relativistic beaming or the 'headlight effect'.

If a light source emits isotropically in its rest frame (S'), then radiation is beamed along the direction of motion in a frame S where the source moves at large speed.



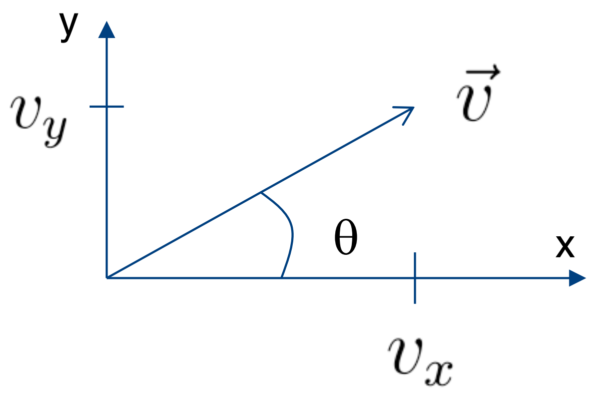
Light source in frame S' moving with speed u along x.
Consider a ray emitted along the y-axis in S'. Then
$v_x'=v_z'=0,\ v_y'=c$

In the S frame:

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = u \qquad (\neq 0)$$

$$v_y = \frac{v'_y\sqrt{1 - u^2/c^2}}{1 + uv'_x/c^2} = c\sqrt{1 - u^2/c^2}$$

$$v_z = \frac{v'_z\sqrt{1 - u^2/c^2}}{1 + uv'_x/c^2} = 0$$

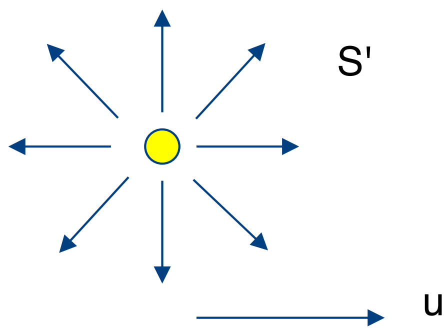


$$\sin\theta = \frac{v_y}{v}$$

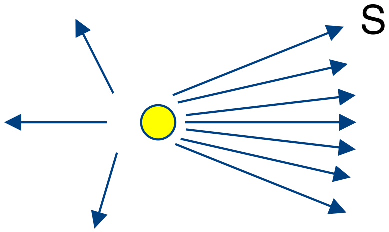
In frame S:

$v_x > 0, v_y > 0,$ so $\theta < 90^{\circ}$ i.e. v not along the y axis

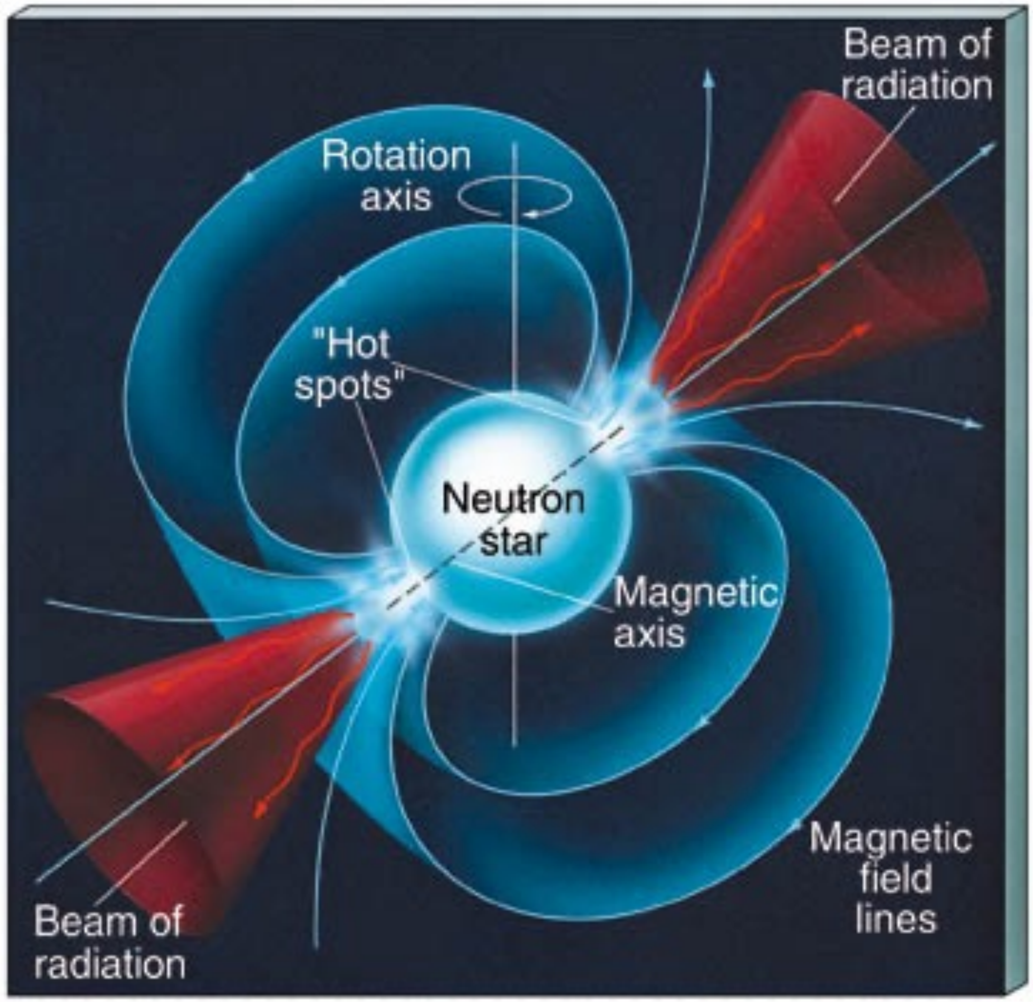
$$\sin\theta = \frac{c\sqrt{1-u^2/c^2}}{c} = \frac{1}{\gamma}$$



transforms to



Example: Neutron star emission. The radiation is said to be *beamed*, or *relativistically boosted.*



Superluminal motion

Pearson et al. 1981

constant expansion observed at

rate = $\Delta\theta$/year = 0.76 ± 0.04 mas/year

z = 0.158 so D = 940 Mpc

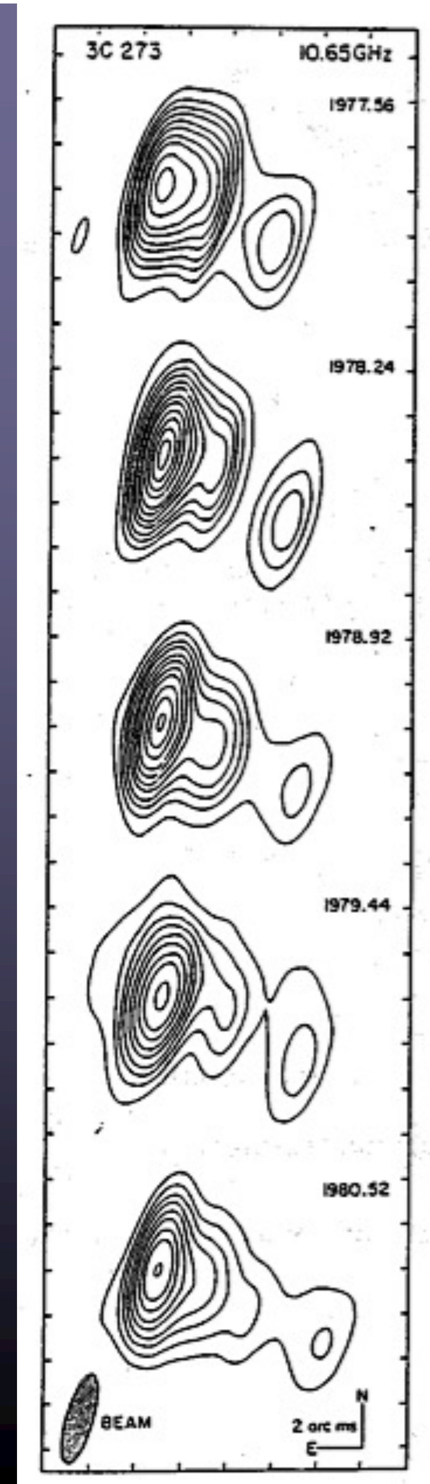
assuming H_0 = 50 km sec^{-1} Mpc^{-1}.

1 mas = 10^{-3} arcsec = 4.85 x 10^{-9} radians

d = D$\Delta\theta$ so the apparent transverse velocity, or rate = d/year

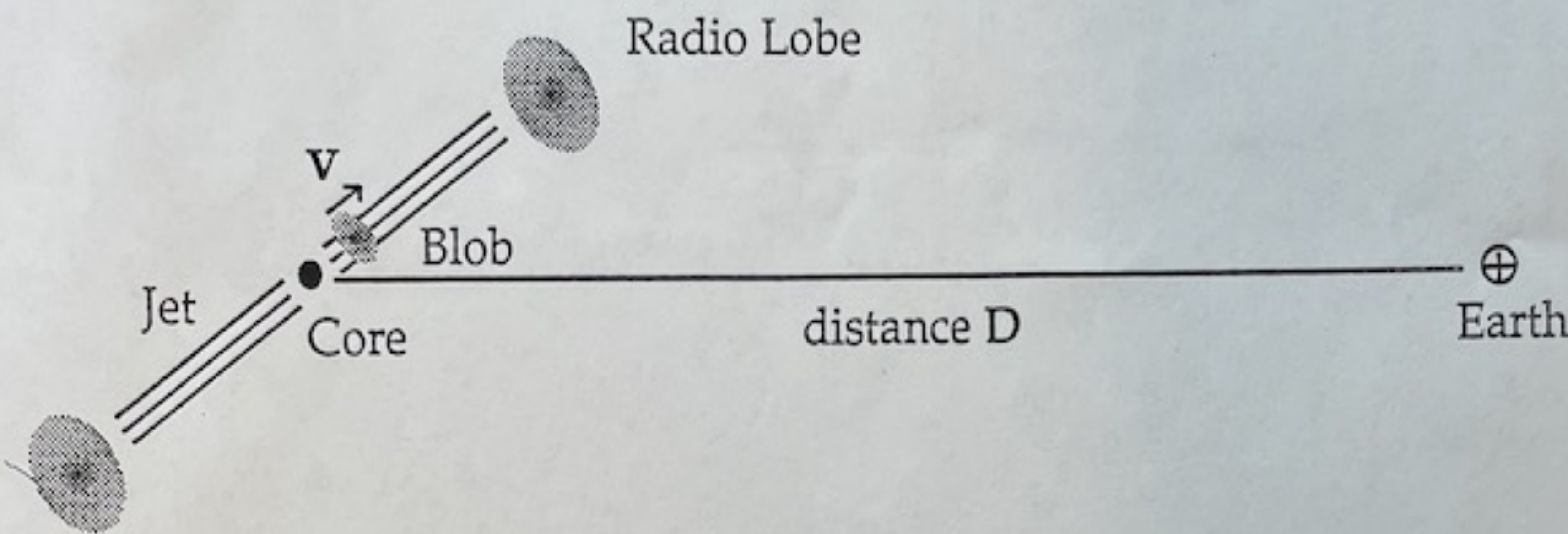
= 10 lt-years/year

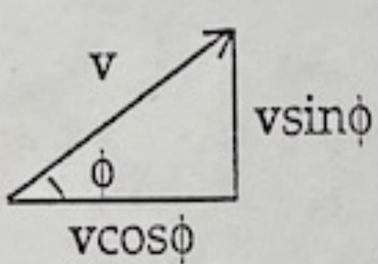
= 10 c [!!]



Superluminal Motion

Observed in Quasar Radio Jets like 3C-273





Suppose that at $t = 0$, the blob is at the quasar and at $t = t'$ the blob is along the jet a distance r

Radiation sent at Blob	Received at Earth
$t = 0$	D/c
$t = t'$	$(D - x)/c + t'$

so

$$\Delta t = t' - x/c$$

$$\Delta t = t' (1 - v/c \cos\phi)$$

$$\text{Apparent transverse velocity} = \frac{d}{\Delta t} = \frac{v\sin\phi}{1 - v/c\cos\phi}$$

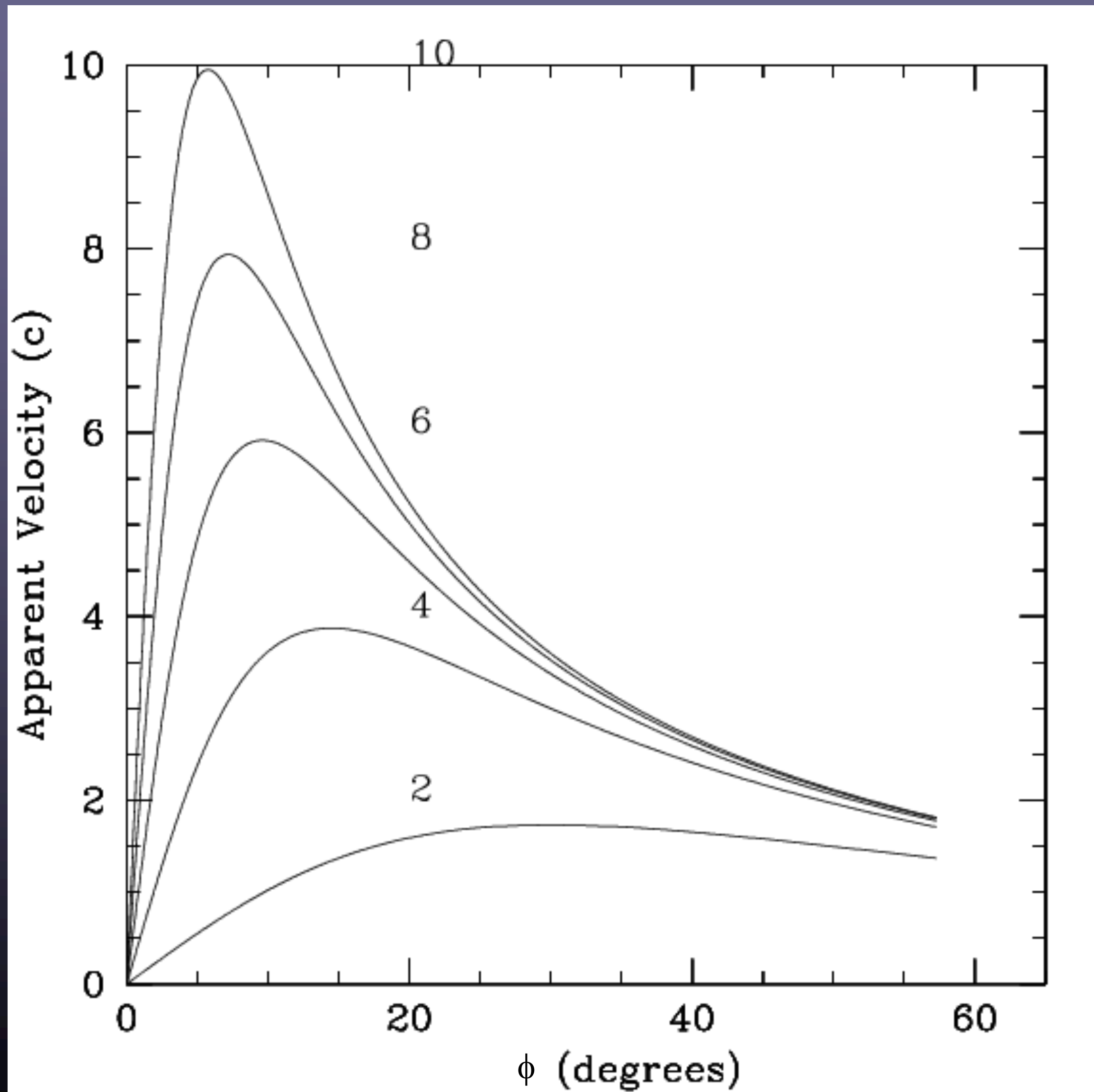
For $v/c \sim 1$, and small ϕ, $\sin\phi \approx \phi$, and $1 - \cos\phi \approx 1/2\phi^2$ so

$$\frac{d}{\Delta t} \approx \frac{c\phi}{1/2\,\phi^2}$$

$$\approx \frac{2c}{\phi} \gg c \quad [!!]$$

Apparent Velocity as a function of angle for γ=2 to 10

$$\frac{v\sin\phi}{1 - v/c\,\cos\phi}$$



The University of New Mexico

Relativistic doppler shift

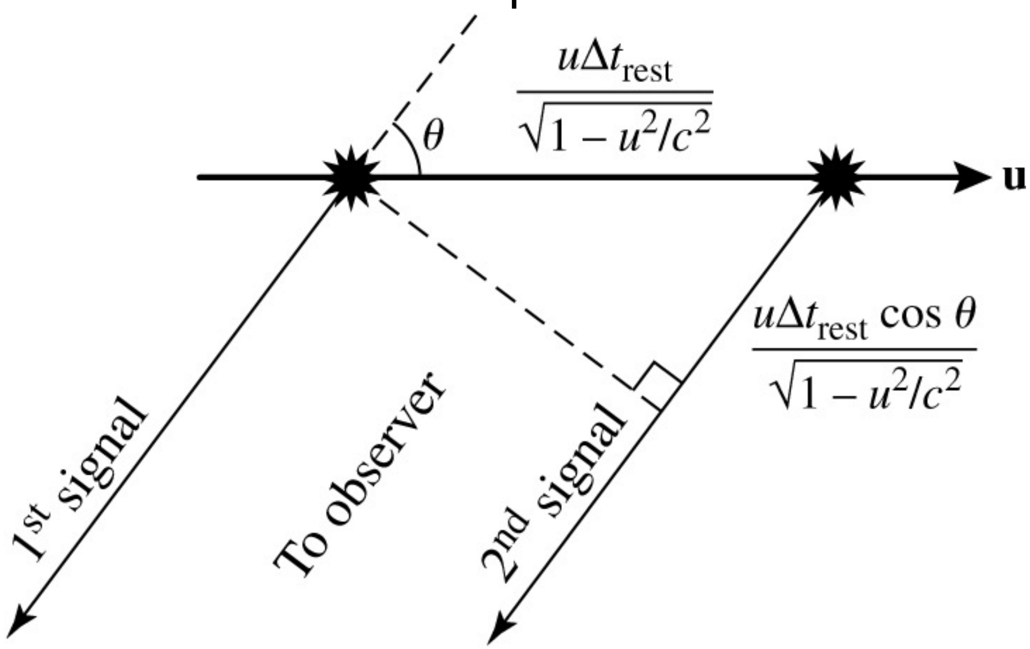
Classical (low velocity) doppler shift, e.g., sound source moving through a medium:

$$V_{los} = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} c = \frac{\Delta\lambda}{\lambda_{rest}} c$$

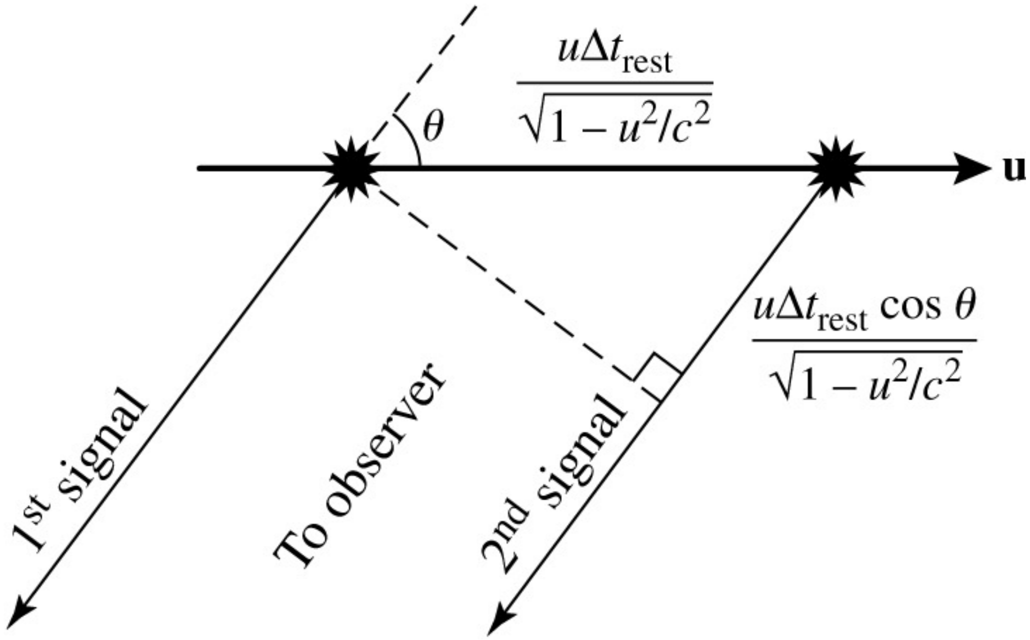
Due to relative motion of source and observer

Relativistic case: must take time dilation into account! Also, let's consider observer at any angle to direction of motion.

Light source emitting signals every Δt_{rest} as measured by clock at rest wrt source. For stationary observer, this interval is $\gamma\Delta t_{rest}$. distance traveled is speed of source *u* times $\Delta t'_{rest}$ ($> \Delta t_{rest}$).



But second signal must also travel an extra distance, which is given by $u\cos\theta\ \gamma\Delta t_{rest}$. So extra time traveled is $u/c\cos\theta\ \gamma\Delta t_{rest}$



In observer's frame, this time interval is

$$\Delta t' = \frac{\Delta t_{rest}}{\sqrt{1 - {u^2}/{c^2}}}$$

But also, 2nd signal travels extra distance (in observer's frame)

$$\frac{u\Delta t_{rest} \cos\theta}{\sqrt{1 - {u^2}/{c^2}}}$$

So the time interval between emission of light signals in observer's frame:

$$\Delta t_{obs} = \frac{\Delta t_{rest}}{\sqrt{1 - u^2/c^2}} \left[1 + \frac{u}{c}\cos\theta\right]$$

Time dilation

Time it takes light to travel the extra length

We are free to set Δt_{rest} as the period of a wave in the source's frame.

Then

$$\nu_{rest} = \frac{1}{\Delta t_{rest}}, \quad \nu_{obs} = \frac{1}{\Delta t_{obs}}$$

$$\Rightarrow \nu_{obs} = \frac{\nu_{rest}\sqrt{1 - u^2/c^2}}{1 + (u/c)\cos\theta}$$

$$\nu_{obs} = \frac{\nu_{rest}\sqrt{1 - u^2/c^2}}{1 + v_r/c}$$

v_r = radial velocity = $u\cos\theta$

For motion directly away ($v_r = u$) or toward ($v_r = -u$)

$$\Rightarrow \sqrt{1 - u^2/c^2} = \sqrt{(1 - v_r/c)(1 + v_r/c)}$$

$$\nu_{obs} = \nu_{rest}\sqrt{\frac{1 - v_r/c}{1 + v_r/c}}$$

NB: There is also a transverse Doppler shift due to time dilation alone.

NB: This cannot be used for cosmological (due to expansion of universe) redshifts. That redshift is not due to motion through space.

Extragalactic astronomers commonly use:

The redshift parameter $$z \equiv \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}}$$

Since $c = \lambda\nu$

$$z = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} - 1$$

$$\frac{v_r}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

Also

$$z + 1 = \frac{\Delta t_{obs}}{\Delta t_{rest}}$$

If we observe brightness of quasar to vary over Δt_{obs} , is variation timescale shorter or longer in the rest frame of the quasar?

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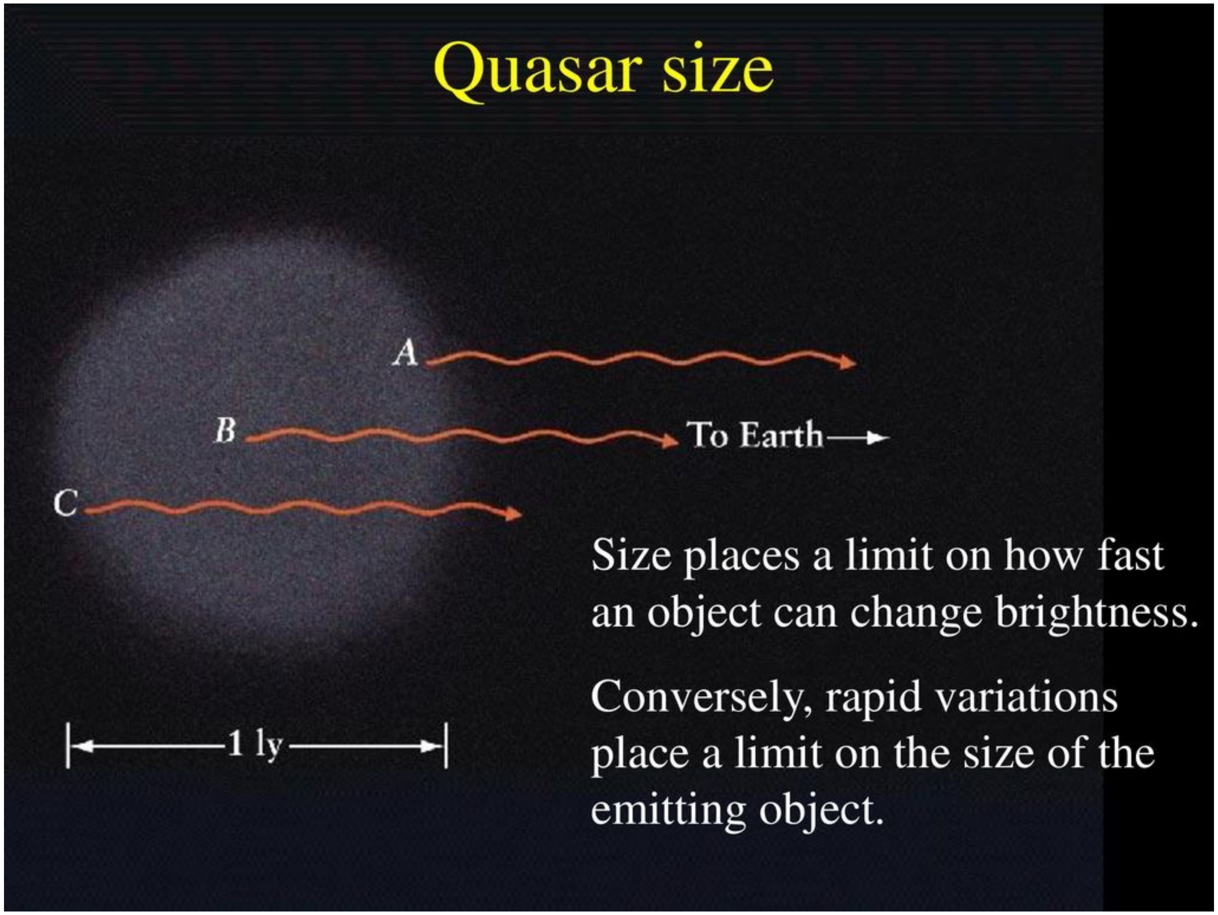
If we observe brightness of quasar to vary over Δt_{obs}, is variation timescale shorter or longer in the rest frame of the quasar?



Relativistic Dynamics

Momentum

If momentum is conserved in all inertial reference frames, we can't have $\vec{p} = m\vec{v}$. Instead, we need:

$$\vec{p} = \gamma m \vec{v}$$

Where $\vec{v}$ is particle's velocity relative to observer. (Derived at end of C+O 4.4 but not responsible for it).

Energy

Kinetic energy

$$KE = \int F dx = \int \frac{dp}{dt} dx$$

With $\vec{p} = \gamma m \vec{v}$, you can show (C&O ch 4.4)

$$KE = mc^2(\gamma - 1)$$ (reduces to $\frac{1}{2}mv^2$ for $v \ll c$)

Note: γmc^2 depends on speed

mc^2 independent of speed

mc^2 called *rest energy*

$$E = \gamma mc^2$$

is total relativistic energy (*KE = E – E_{rest}*)

With $E = \gamma mc^2$ $\vec{p} = \gamma m \vec{v}$ it can be shown (prob. 4.19) that:

$$E^2 = p^2c^2 + m^2c^4$$

What is E for photons?

Twin Paradox

Suppose there are two twins, Al and Bill age 10. Al goes to summer camp 25 light-years away. If he travels at 0.9999c then it takes 25 years each way and Bill is age 60 when Al gets back. But Al is only 10 and a half because time for him was moving slower. But from Al's point-of-view Bill was the one moving so how did Bill get so old?