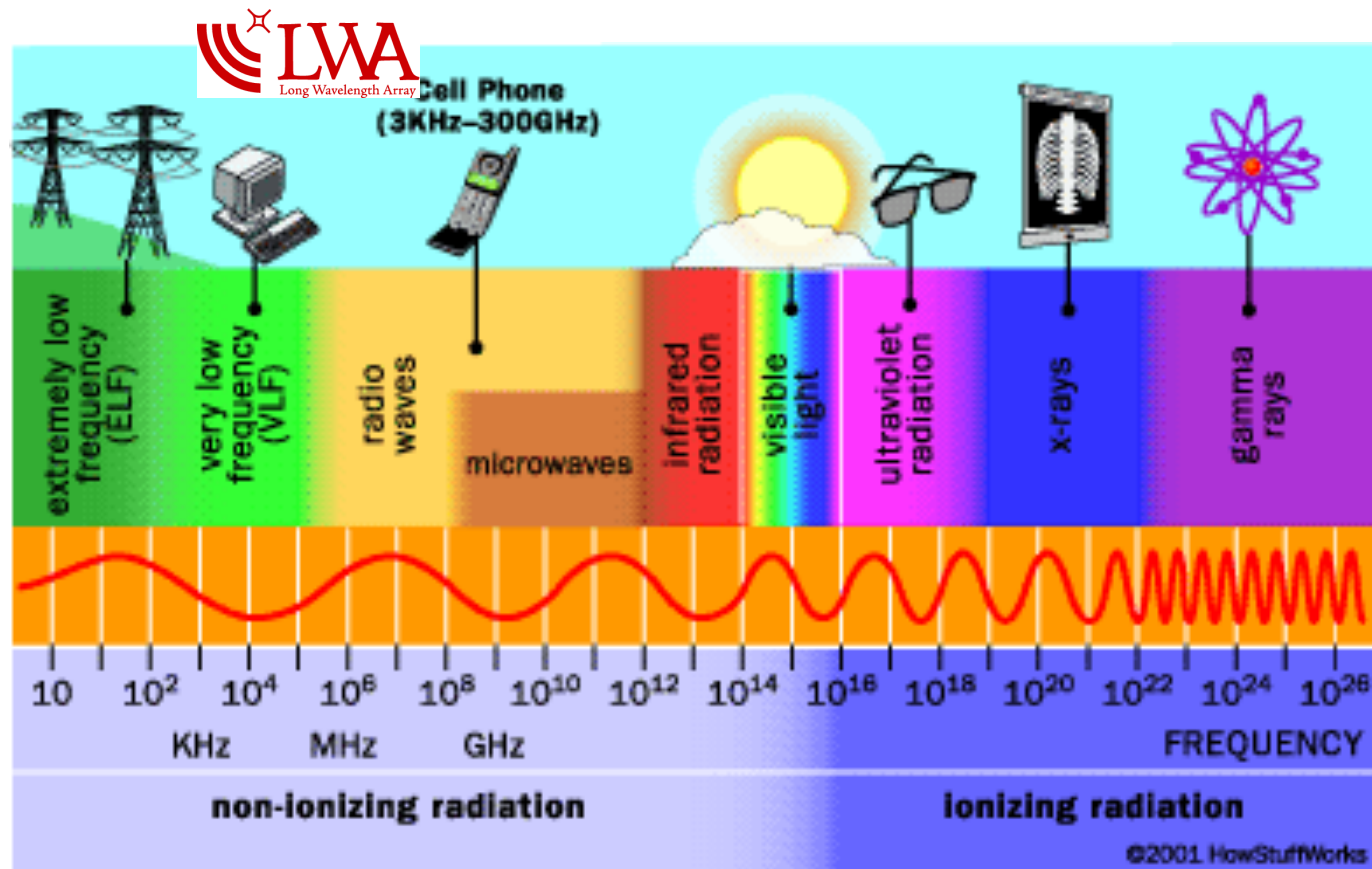# Worksheet #2

Problem: How long until the Sun collides with another star? Assume the Sun is moving at 220 km/s and the density of stars in the stellar neighborhood stays constant at 0.125 stars/cubic parsec.

How does this compare to the age of the Universe? How likely is it that the Sun will collide with another star?

Astronomy 421

Radiation

THE ELECTROMAGNETIC SPECTRUM

Penetrate Earth's Atmosphere

| | | N | | Y | | N | | Y |

Radiation Type	Gamma Ray	X-ray	Ultraviolet	Visible	Infrared	Microwave	Radio
Wavelength (m)	10^{-12}	10^{-10}	10^{-8}	5×10^{-6}	10^{-5}	10^{-1}	10^{3}
About the Size of	Atomic Nuclei	Atoms	Molecules	Protozoans	Pinpoint	Honey Bee	Humans	Buildings

Short wavelength
High energy
High frequency

Long wavelength
Low energy
Low frequency

Radiation

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Radiation Outline

- Blackbody radiation (thermal radiation), Wien's Law, Stefan Boltzmann Law

- Emergent flux, luminosity and incident flux

- Magnitudes, Colors

- Kirchhoff's Laws

- Photons and wave-particle duality

- H atom and quantized energy levels => spectral lines

- Heisenberg Uncertainty Principle

- Pauli Exclusion Principle

Blackbody Radiation

True blackbody is an idealization, but many cosmic objects radiate roughly like one (e.g., stars, the Universe).
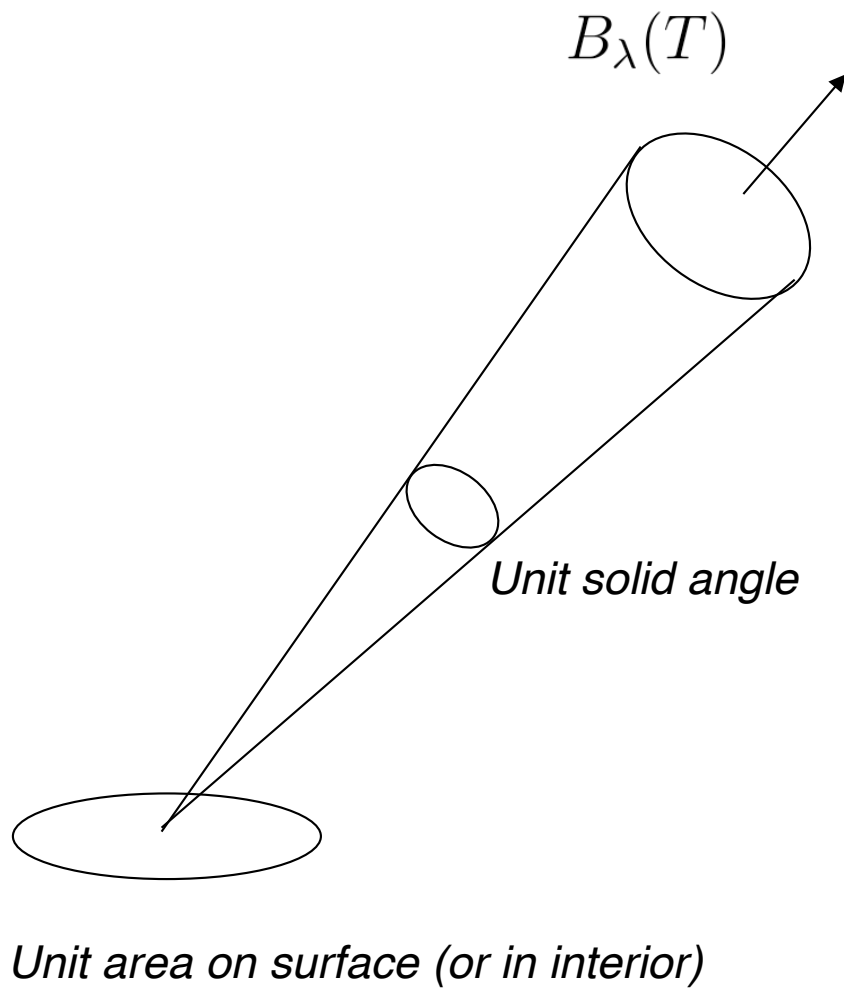
Planck's Radiation Law

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

mks Units: W m^{-2} m^{-1} sr^{-1}

$B_\lambda(T)$ is *monochromatic specific intensity.*

Describes spectrum of opaque body of temperature *T*, in thermal equilibrium with (at the same temperature as) surroundings.

"Specific Intensity" => per unit solid angle. Often just called "intensity".

$B_\lambda(T)$

Unit solid angle

Unit area on surface (or in interior)

$B_\lambda(T)$

is energy emitted per second (W)

into a unit solid angle (sr^{-1})

per unit area (m^{-2})

per wavelength interval (m^{-1})
(between λ and $\lambda+d\lambda$)

Depends only on *T*, and is isotropic (other types of radiation may not be isotropic. Then need to worry about angular dependence of intensity).

- Broad range of wavelengths
- Steep rise, long tail
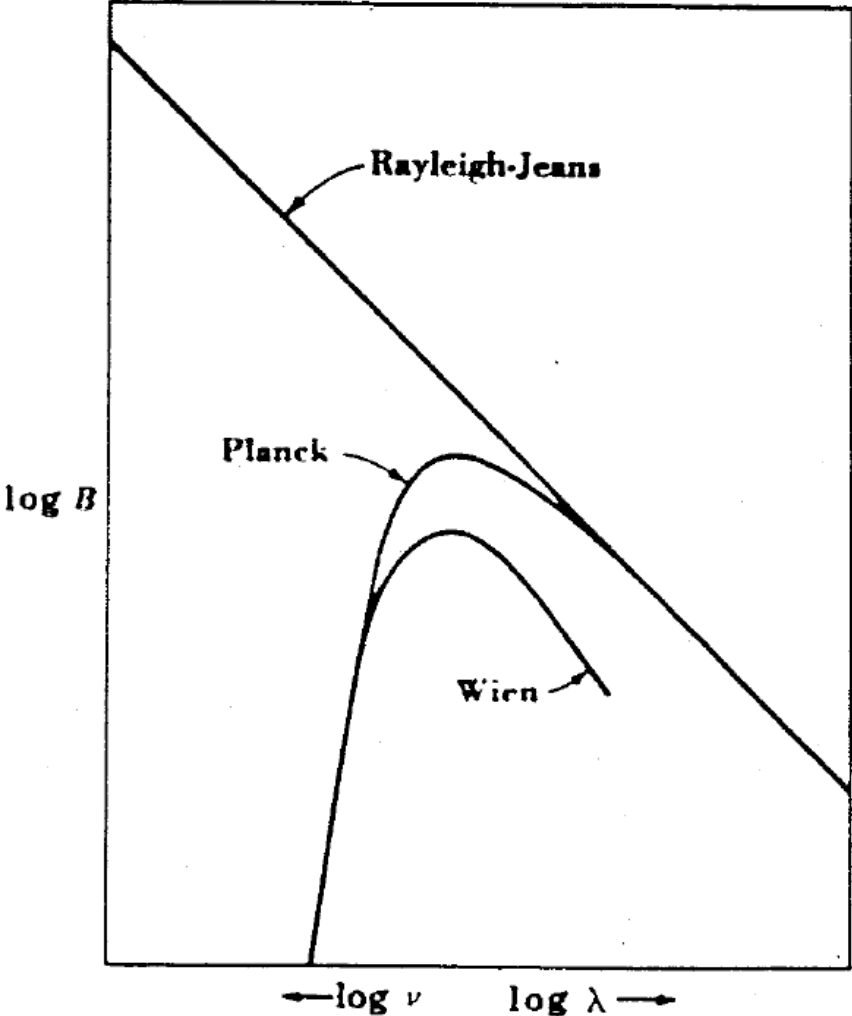- Dramatic difference for *T* changing by a factor of 2 (Intensity up by factor 16)
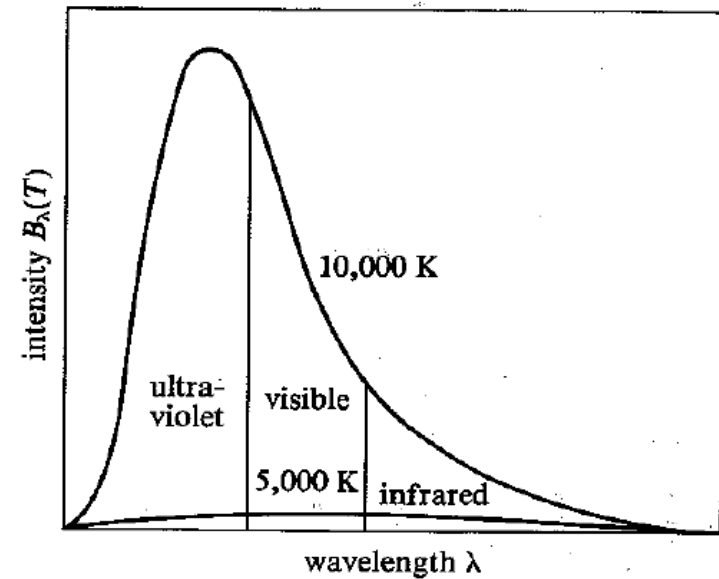
Often intensities and wavelengths span so many factors of 10, makes sense to use log-log plot.
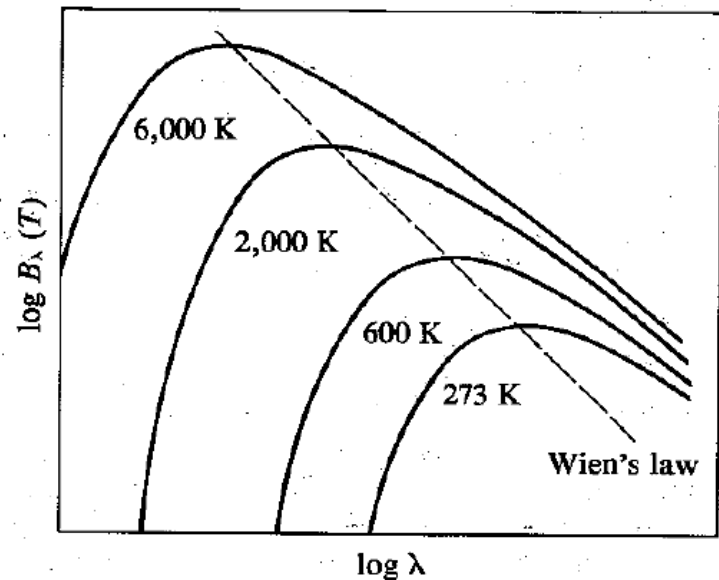
Need to be familiar with different parts of the EM spectrum; IR, UV etc.

It is useful to introduce approximations for short
wavelengths (Wien) and long wavelengths
(Rayleigh-Jeans).

The Wien Limit and Wien's Law: short wavelength regime

$$\lambda << \frac{hc}{kT} \Rightarrow \frac{hc}{kt\lambda} >> 1$$

$$e^x - 1 \approx e^x \qquad x >> 1$$

Thus
$$\boxed{B_\lambda(T) = \frac{2hc^2}{\lambda^5} e^{-hc/\lambda kT}}$$
for $\lambda << \dfrac{hc}{kT}$

Wien's radiation formula

Where is Wien's approximation to $B_\lambda(T)$ maximum for a given *T*?

$$\frac{dB_\lambda(T)}{d\lambda} = 0 \Rightarrow 2hc^2 e^{-hc/kT\lambda}\left(-\frac{5}{\lambda^6} + \frac{hc}{\lambda^7 kT}\right) = 0$$

$$\frac{5}{\lambda^6} = \frac{hc}{\lambda^7 kT}$$

$$\lambda_{max} = \frac{hc}{5kT} = \frac{0.0028776}{T} \text{ m}$$

NB:

1. *For x=* $\dfrac{hc}{\lambda_{max} kT} = 5 \; (> 1)$ *so the Wien limit is an OK approximation.*

2. *Using the full Planck's radiation law we get*

$$\lambda_{max} = \frac{0.0028979}{T} \text{ m}$$

This is Wien's Law

An alternative form of the radiation law:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{(e^{h\nu/kt} - 1)} \qquad \text{W m}^{-2} \text{ sr}^{-1} \text{ Hz}^{-1}$$

Again, let's look where $B_\nu(T)$ is maximum, considering $\dfrac{h\nu}{kT} >> 1$

$$\frac{dB_\nu(T)}{d\nu} = 0 \Rightarrow \frac{2h}{c^2} e^{-h\nu/kT} \left(3\nu^2 - \frac{h\nu^3}{kT}\right) = 0$$

$$\nu_{max} = \frac{3kT}{h} \Rightarrow \lambda_{max} = \frac{hc}{3kT} \neq \frac{hc}{5kT} \qquad \textit{WHAT??!}$$

$B_\nu(T)$ has a maximum at a different wavelength from $B_\lambda(T)$?

How can this be, since they are alternate ways of expressing the same thing???

$B_\nu(T)$ and $B_\lambda(T)$ are different functions and not what you measure!

When you make a measurement, it is always over some bandwidth or frequency interval. Measurements must agree! How do such intervals relate?

$$\lambda = \frac{c}{\nu}$$

$$\boxed{d\lambda = -\frac{c}{\nu^2}d\nu}$$

So $B_\lambda d\lambda = B_\nu d\nu$ which can be easily shown.

Rayleigh-Jeans Law: Long wavelength regime

Valid when $\quad \lambda >> \dfrac{hc}{kT} \quad$ *or, when (photon energy)/(characteristic atom KE) <<1*

In this limit: $\qquad x << 1 \quad \lim e^x = 1 + x$

$$B_\lambda(T) \propto \frac{1}{e^x - 1} \propto \frac{1}{x} = \frac{\lambda kT}{hc}$$

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{\lambda kT}{hc} \Rightarrow B_\lambda(T) = (\frac{2c}{\lambda^4})kT$$

Rayleigh-Jeans Law: Applies to radio astronomy of thermal sources.

At λ=10 cm, how low would T have to be for the RJ law to be invalid?

Worksheet 3

Rayleigh-Jeans Law: Long wavelength regime

Valid when $\quad \lambda >> \dfrac{hc}{kT} \quad$ *or, when (photon energy)/(characteristic atom KE) <<1*

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\left(e^{h\nu/kt} - 1\right)} \qquad \text{W m}^{-2}\,\text{sr}^{-1}\,\text{Hz}^{-1}$$
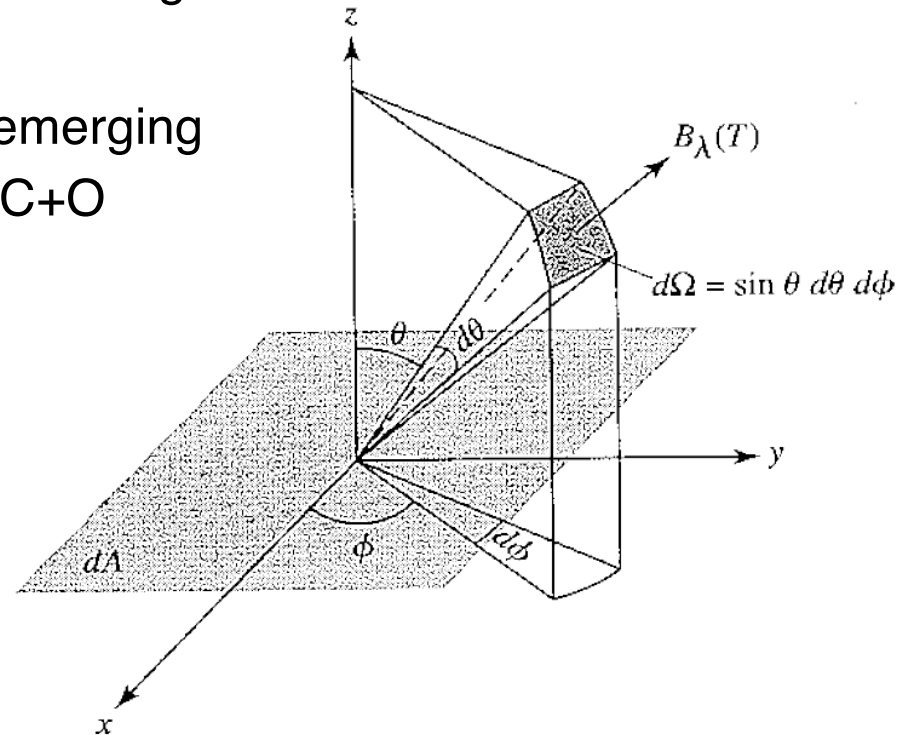
Rayleigh-Jeans Law: Applies to radio astronomy of thermal sources.

Note 1 Jy = 10^-26 W m^{-2} Hz^{-1}

In practice we don't measure intensity, but flux.

Flux is defined as energy per unit area per unit time, or the specific intensity integrated over solid angle and wavelength.

We define the "emergent flux" as the flux emerging through the surface of a radiating object. C+O call this "surface flux".

$$F_e = \int_\Omega \int_0^\infty B_\lambda \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda =$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \int_0^\infty \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k \, T} - 1} d\lambda$$

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Substitute $\quad x = \dfrac{hc}{\lambda kT}$

$$F_e = \pi 2hc^2 \left(\dfrac{kT}{hc}\right)^4 \underbrace{\int_o^\infty \dfrac{x^3}{e^x - 1} dx}_{\pi^4/15}$$

$$\boxed{F_e = \sigma T^4}$$ mks units W m^{-2}

Stefan-Boltzmann Law
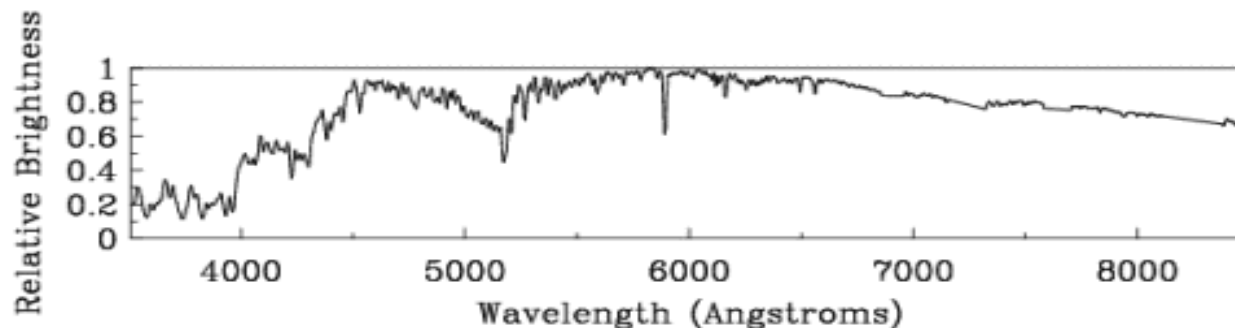
Luminosity

$$L = \int_A F_e \; dA$$

If F_e = constant over A, for a spherical blackbody:

$$\boxed{L = 4\pi R^2 \sigma T^4}$$

Since stars are not perfect blackbodies, define an 'effective' temperature, T_e

$$L = 4\pi R^2 \sigma T_e^4$$

(the temperature of a blackbody of the same luminosity)

Incident flux, or 'radiant flux':

At a distance r from the star, the luminosity is spread over a sphere of area $4\pi r^2$

$$F_i = \frac{L}{4\pi r^2} = F_e \, \frac{R^2}{r^2}$$

This tells you how bright a star appears.

Monochromatic Luminosity

$$L_\lambda d\lambda = \int_A dA \int_\Omega d\Omega \, B_\lambda d\lambda$$

$$= 4\pi^2 R^2 B_\lambda d\lambda$$

Monochromatic Incident Flux

$$F_{i,\lambda} d\lambda = \frac{L_\lambda d\lambda}{4\pi R^2} = \pi B_\lambda \frac{R^2}{r^2} d\lambda \qquad \text{(at distance } r \text{ from star)}$$

Magnitudes

Sometimes, astronomers use (ancient) magnitudes to measure incident flux.

Definition: difference of 5 magnitudes between magnitudes of 2 stars corresponds to smaller magnitude star being 100 times brighter.

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}$$

or

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right)$$

m is <u>apparent</u> magnitude, or how bright the star appears to be.

We define the absolute magnitude *M* as the magnitude a star would have at d=10pc:

$$100^{(m-M)/5} = \frac{F_{10}}{F} = \left(\frac{d}{10\mathrm{pc}}\right)^2$$

Where F_{10} is the flux received if the star would be at 10 pc distance. Absolute magnitude is related to luminosity and is intrinsic to the star.
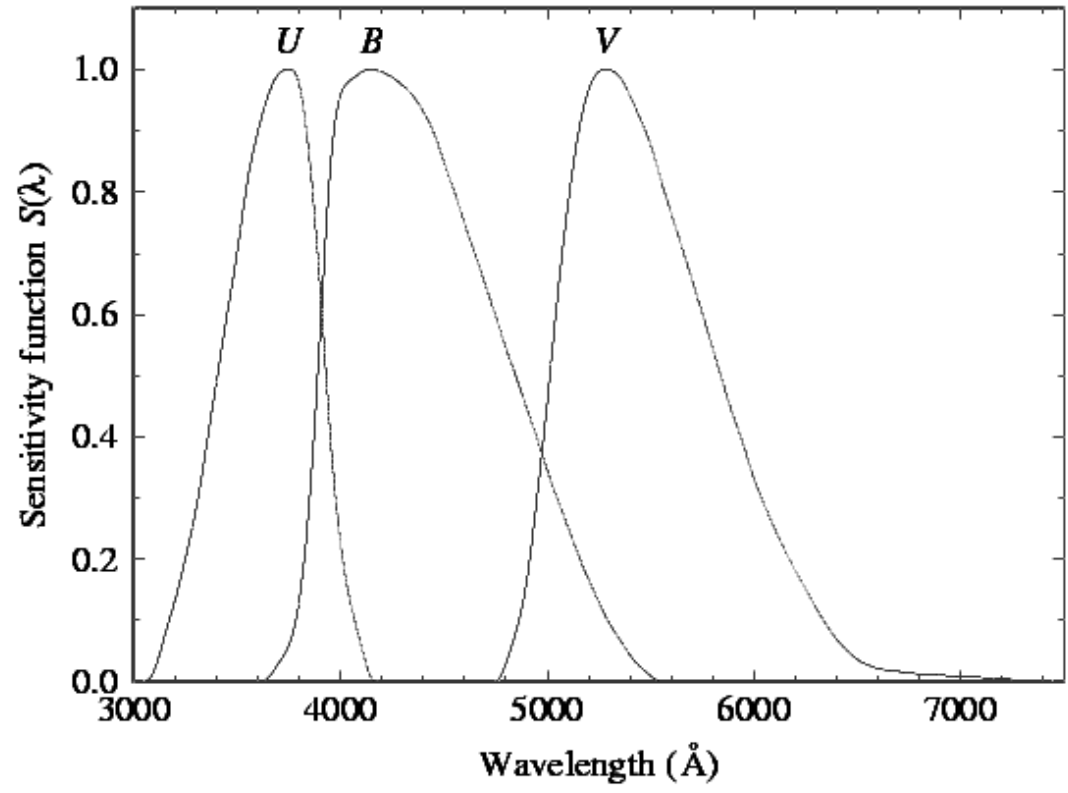
$$\Rightarrow d = 10^{(m-M+5)/5} \ \mathrm{pc}$$

or *the distance modulus:*

$$\boxed{m - M = 5\log_{10}(d) - 5}$$
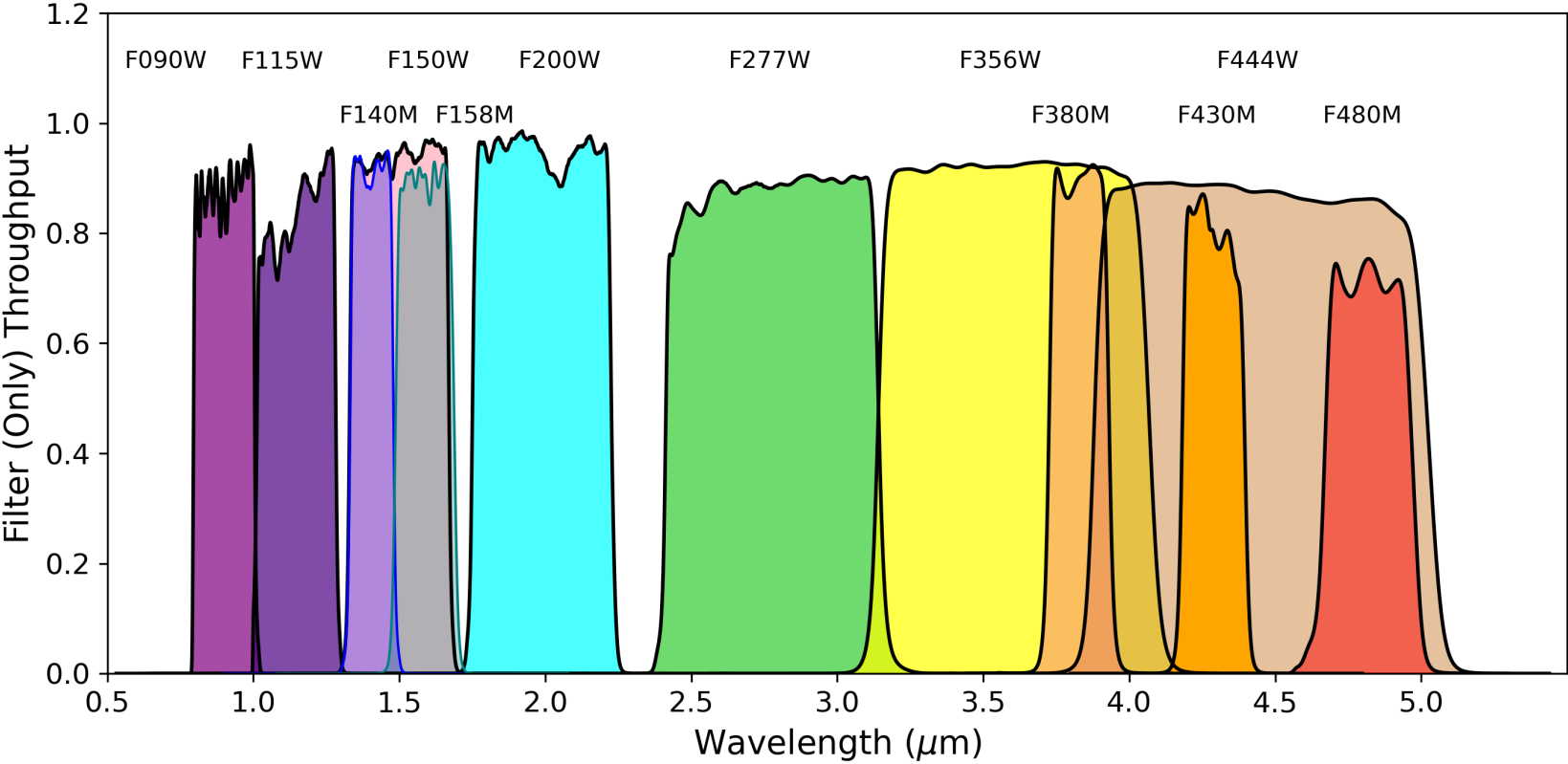
So far, referring to *bolometric magnitudes,* i.e., measure over all wavelengths. Also written as m_{bol} and M_{bol}. Not what is typically measured!

If we want to know how the flux varies with wavelength, then we will measure brightness through filters.

Example: the UBV system (there are others).

JWST Transmission filters: 5000 A -> 50000 A

The Color Index

Flux usually is measured through color filters, e.g., U, B or V filters. Flux through e.g., B filter is called *B magnitude* written *B* or m_B.

$$U - B \propto -\log \frac{F_U}{F_B}$$

$$B - V \propto -\log \frac{F_B}{F_V}$$

are *color indices* of a star. Smaller *B-V* => bluer star. Color indices indicate temperature.

A blackbody temperature that reproduces a star's *B-V* or *U-B* is called a "color temperature".

More Precisely

Relation between apparent magnitude and incident flux, e.g. for U:

$$U = -2.5 \, log_{10}(\int_0^\infty F_\lambda S_U \, d\lambda) + C_U$$

F_λ is monochromatic incident flux at λ.

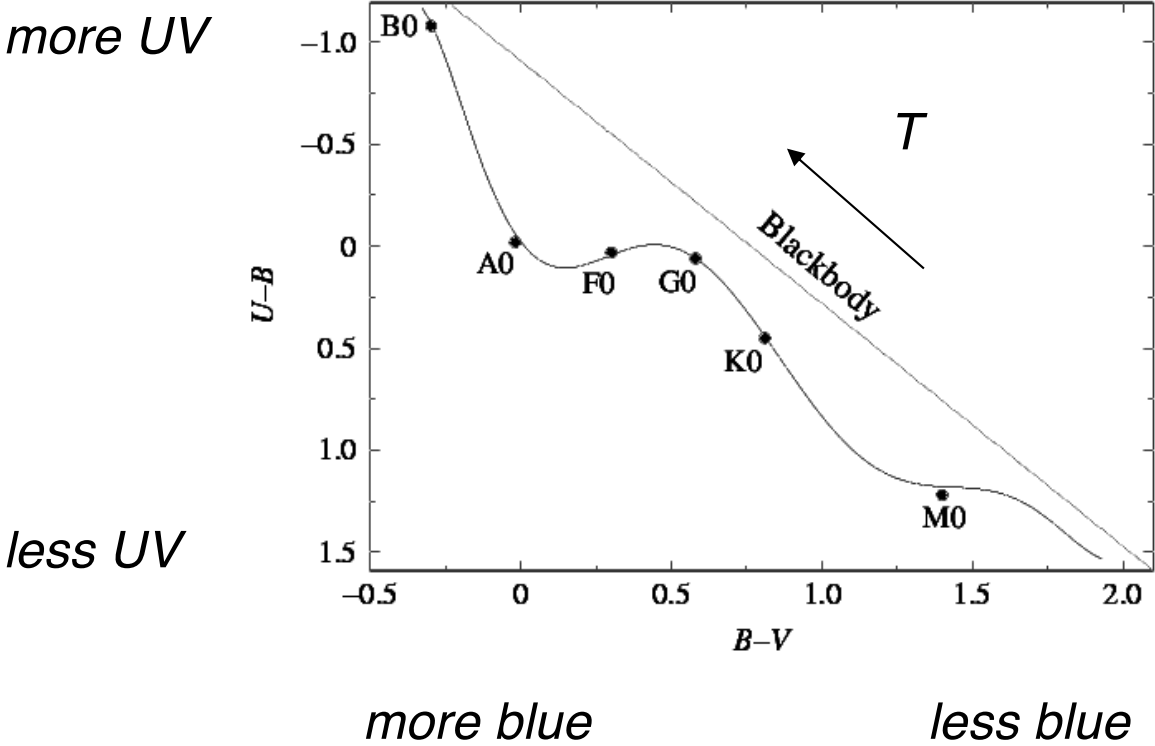S_U is "sensitivity function" of U filter. Function of λ.

C_U is a constant.

C_U, C_B, C_V chosen such that U=B=V=0 for Vega (arbitrary).

$$U - B = -2.5 \, log_{10}(\frac{\int_0^\infty F_\lambda S_U \, d\lambda}{\int_0^\infty F_\lambda S_B \, d\lambda}) + C_U - C_B$$

$$C_{U-B} \equiv C_U - C_B$$

Color-color diagram

more UV

less UV

more blue   *less blue*

Stars are not true blackbodies!

More on Bolometric Magnitudes

$$m_{Bol} = -2.5 \, log_{10}(\int_0^\infty F_\lambda d\lambda) + C_{Bol}$$

(perfect bolometer: $S_\lambda = 1$ for all λ)

"Bolometric correction":

$$BC = m_{Bol} - V = M_{Bol} - M_V$$

C_{bol} was chosen so that $BC < 0$ for all stars (but some supergiants found with $BC > 0$).