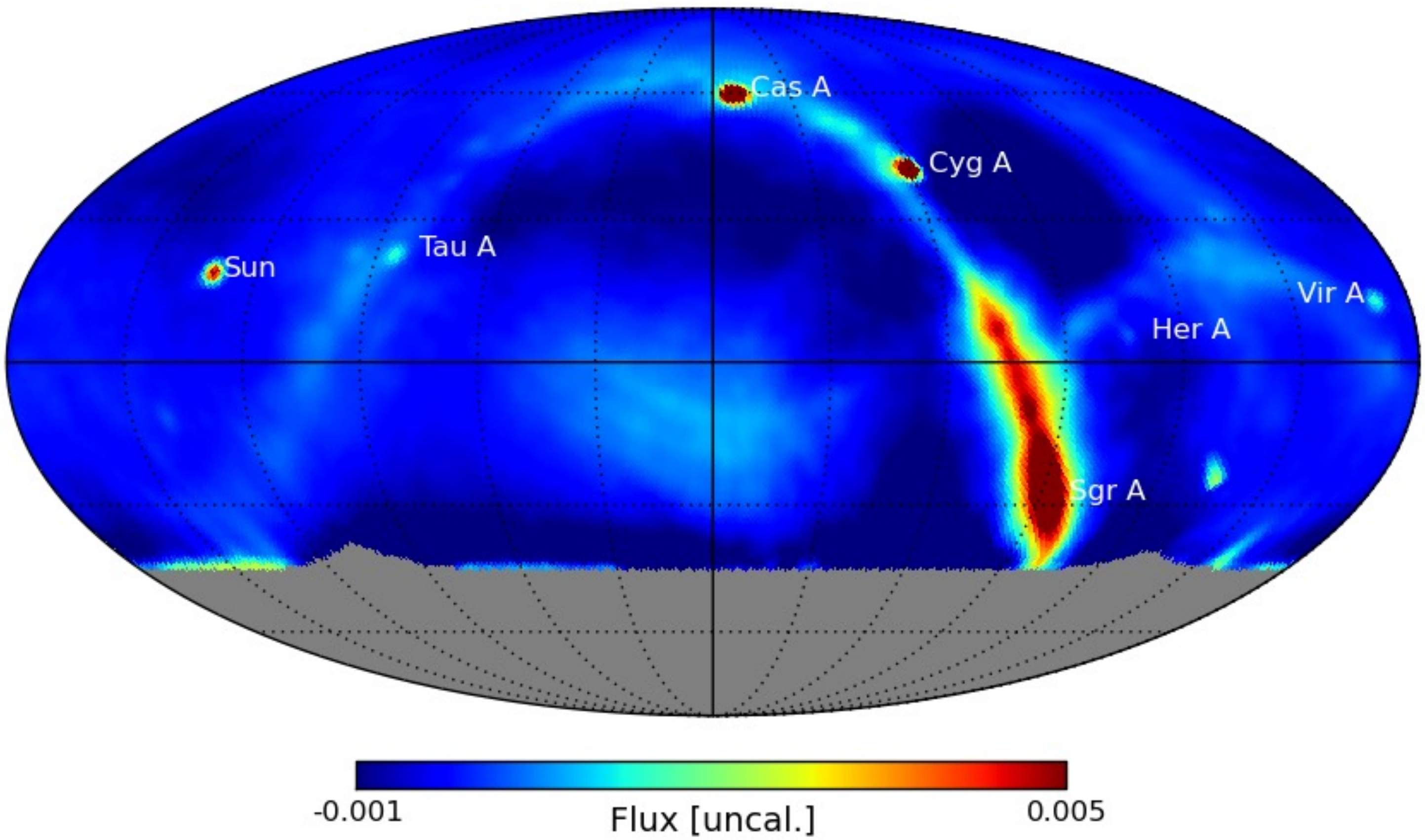
Astronomy 421 Concepts of Astrophysics I



G.B. Taylor Fall 2022

Course Logistics

Goals:
- Improve knowledge of astrophysics
- develop research skills

Main Areas of Study:
- Orbital Mechanics
- Radiation and Matter
- Relativity
- Stars
- Stellar Remnants (including Black Holes)
- GRBs, FRBs, TDEs, TLAs, …

- Lecture topics and reading can be found on the syllabus
- Problem solving (homeworks and in class)
- Peer review exercise
- Term paper (and presentation) will be a significant part of the course (see handout)

Please review material in Ch 1 of Carroll +Ostlie (The Celestial Sphere) but it won’t be in lectures, homeworks or tests.

HW 1 due Thursday, September 1

Astrophysics Talks at UNM

- Thursdays 2-3PM in room 3205 (see schedule on dept web pages)
  Astronomy specific seminars
  Great opportunity to learn more about a wide range of topics in astrophysics
  Free Donuts!

- Fridays 11am – Noon NRAO Colloquium Series by Video from Room 3205
  Astronomy specific Colloquia

- Friday 3:30-4:30 pm Colloquia at PandA (see schedule on dept web pages)
  Physics and Astronomy Colloquia

Other Opportunities

UNM runs a University Radio Observatory – The Long Wavelength Array

- You can get telescope time on this world-class facility

APS 4 Corners Meeting Oct 14&15 at UNM, Abstract Deadline Sept 16

ASTRON/JIVE summer student program

- graduate and advanced undergrads
- 10-12 weeks in the Netherlands
- Application deadline is Feb. 1

NRAO Summer Student program

- undergrads and grads
- 10-12 weeks in Socorro (!!!), Green Bank, or Charlottesville
- Application deadline Feb. 1

NRAO Synthesis Imaging Workshop

8 days of Aperture Synthesis Techniques

See me for more details

Backgrounds

Prof. Greg Taylor, PhD UCLA 1991 "Cluster Magnetic Fields"

Employment: NRAO predoctoral Fellow
Postdocs at Arcetri and Caltech
NRAO Socorro staff scientist
UNM Professor and Director of the LWA

Research Areas: Galactic and Extragalactic Astronomy, Cosmic Explosions, Radio Interferometry Instrumentation and Techniques

Your Background?

Mechanics - Outline

- Kepler’s Laws
- Newtonian Mechanics
- Energy, Potential Energy, Kinetic Energy, Escape Velocity
- Newtonian Explanation of Kepler’s Laws
- Virial Theorem
- Tidal Force, Roche Limit
- Collision Physics

Celestial Mechanics – (C+O Chapter 2)

Kepler's laws:

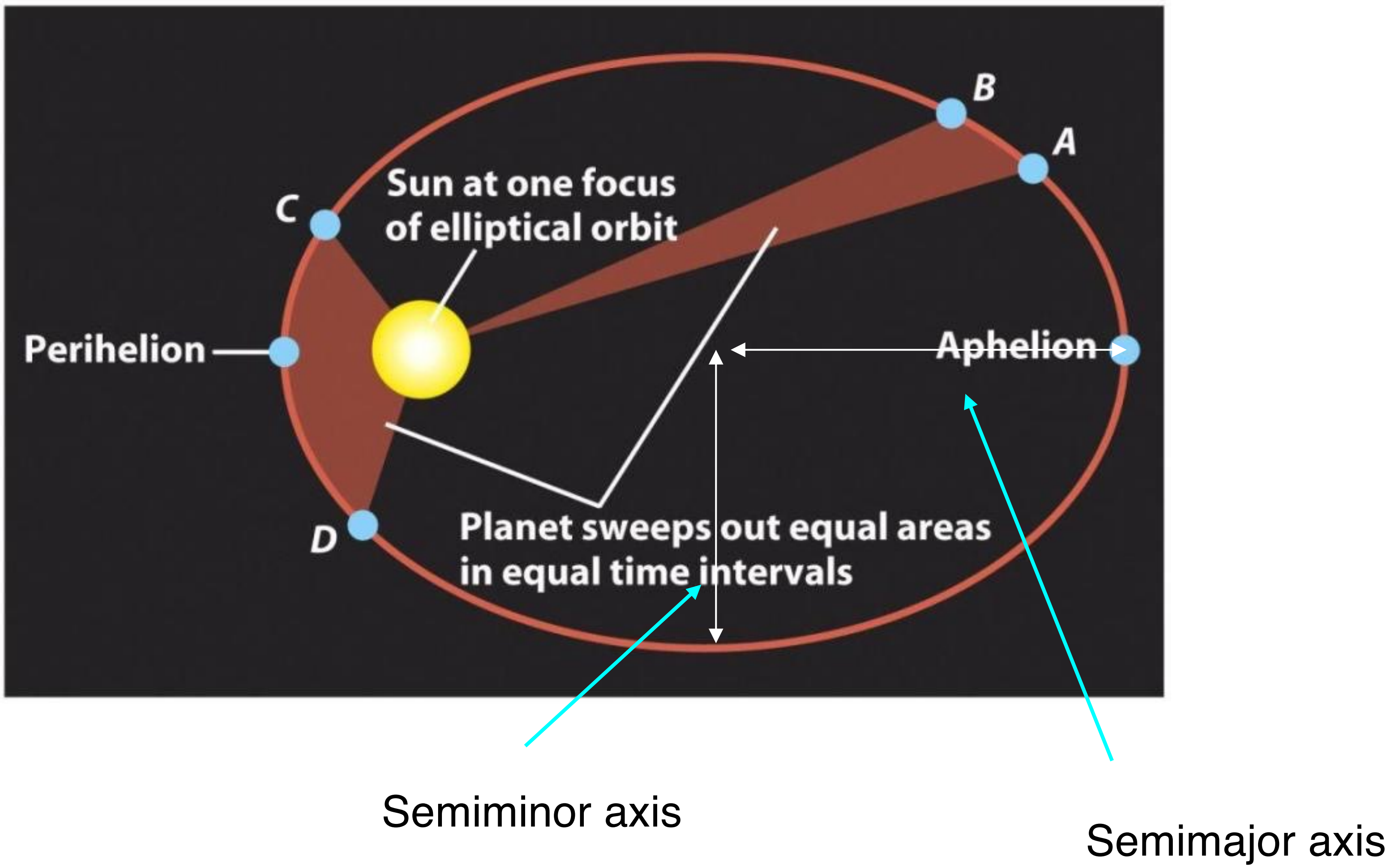
1. A planet orbits the Sun in an ellipse, with the Sun at one focus of the ellipse.

2. A line connecting a planet to the Sun sweeps out equal areas in equal time intervals.

3. The square of the orbital periods of a planet is proportional to the cube of the semi-major axis of its elliptical orbit.

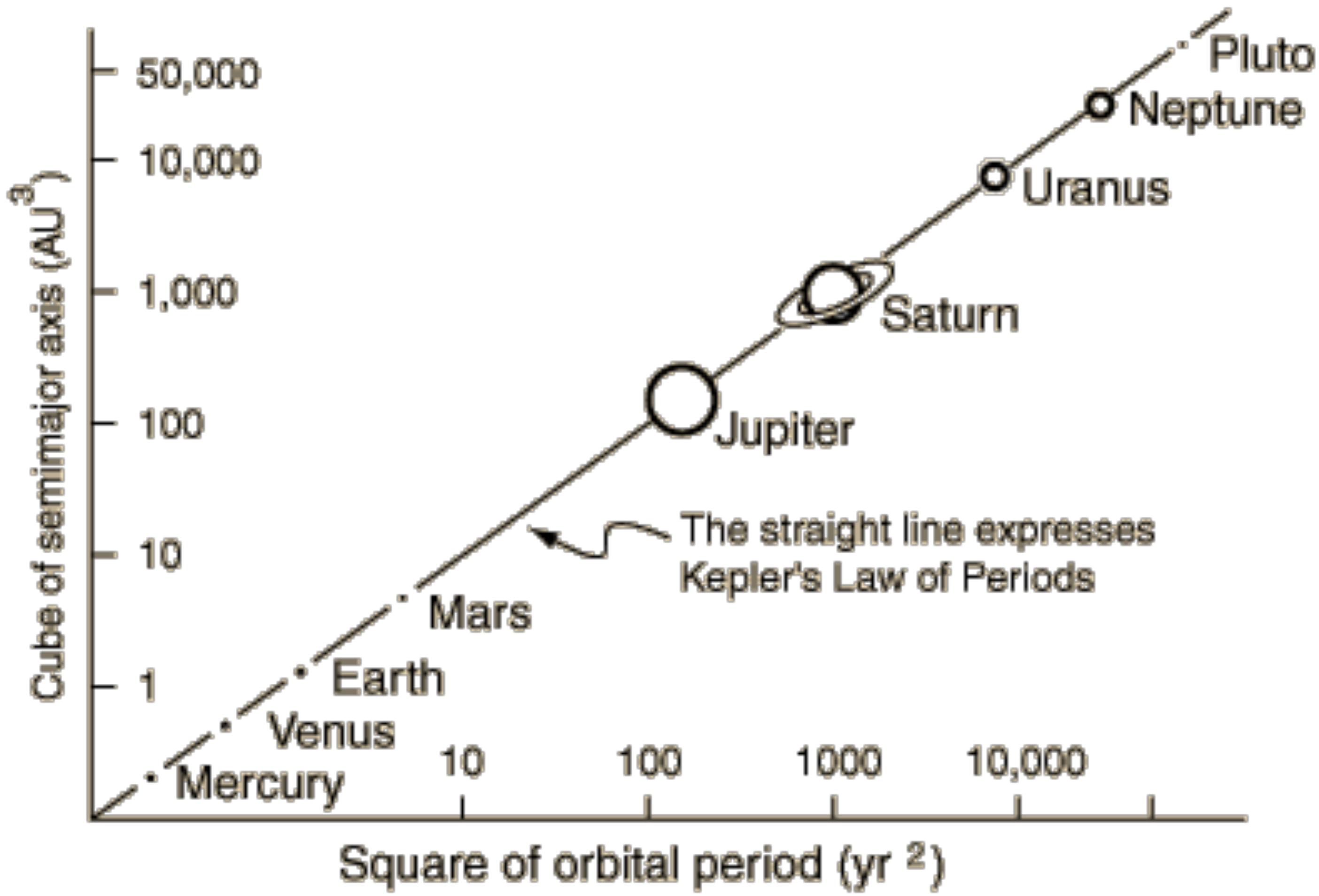
These are kinematical relations - describe the motions without reference to the dynamical forces. We shall see that they can be derived and refined from Newton's law of gravitation.

Kepler's first and second laws illustrated



Kepler's third law illustrated

- $P^2 \propto a^3$

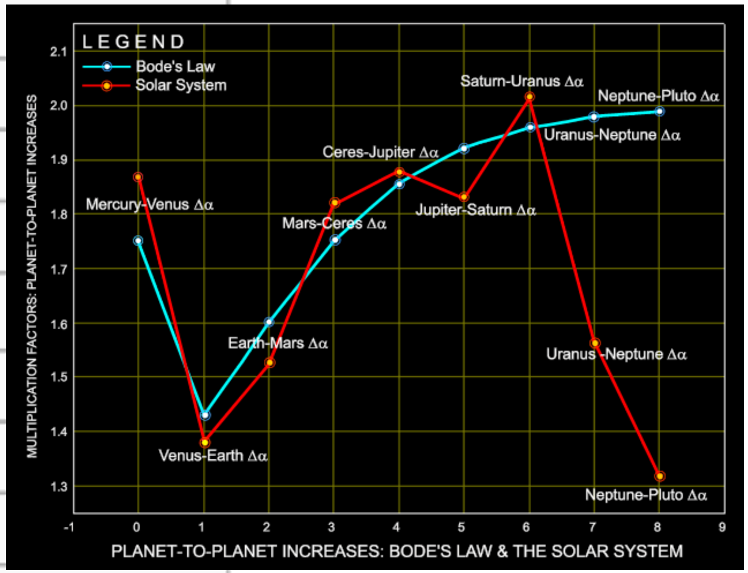


Bode's Law

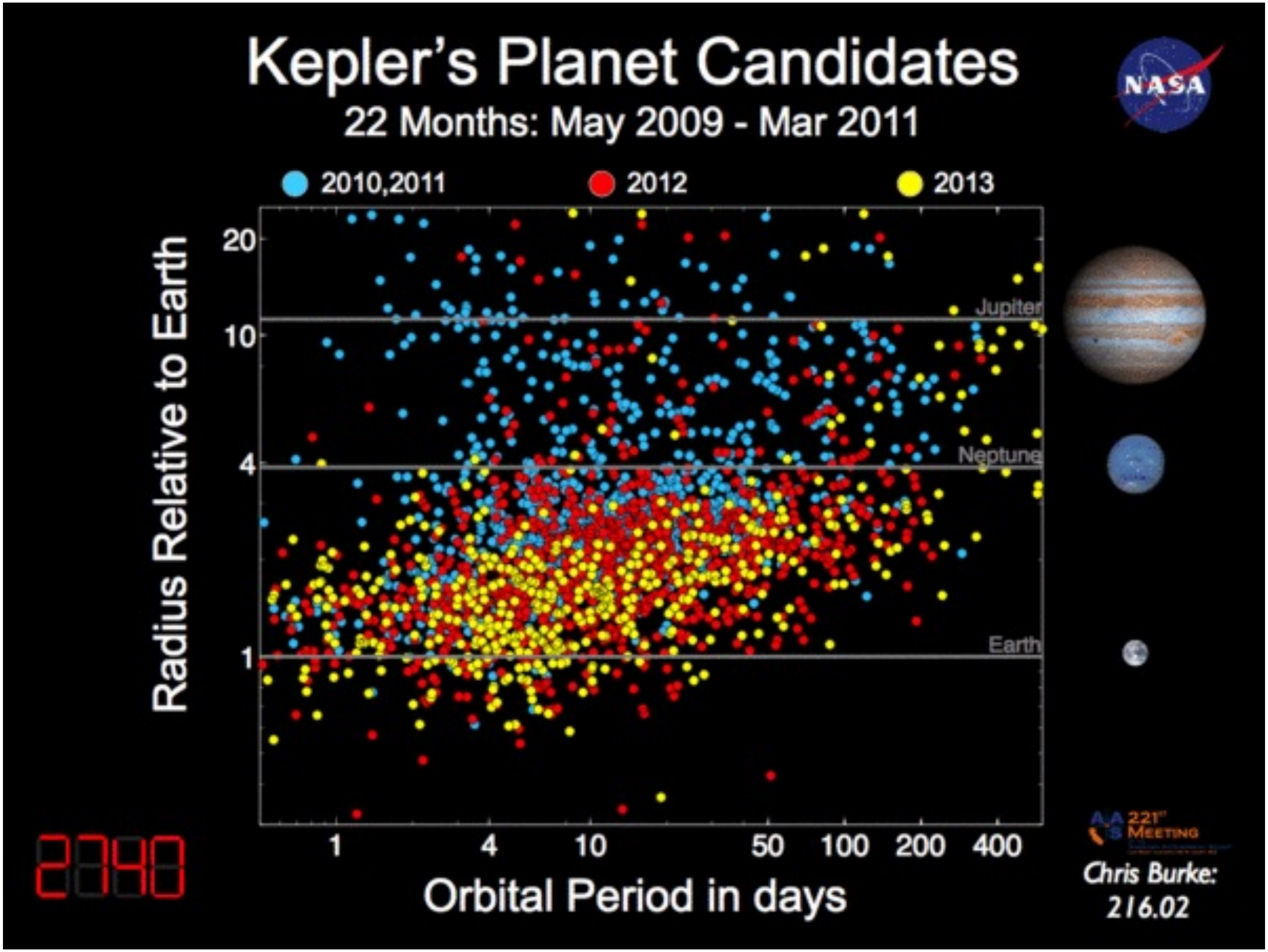
$a = 0.4 + 0.3 * 2^m$ where m = 0, 1, 2, ... 9

Devised by Bode in 1772, no longer accepted

Planet	k	T-B rule distance (AU)	Real distance (AU)	% error (using real distance as the accepted value)
Mercury	0	0.4	0.39	2.56%
Venus	1	0.7	0.72	2.78%
Earth	2	1.0	1.00	0.00%
Mars	4	1.6	1.52	5.26%
Ceres[1]	8	2.8	2.77	1.08%
Jupiter	16	5.2	5.20	0.00%
Saturn	32	10.0	9.54	4.82%
Uranus	64	19.6	19.2	2.08%
Neptune	128	38.8	30.06	29.08%



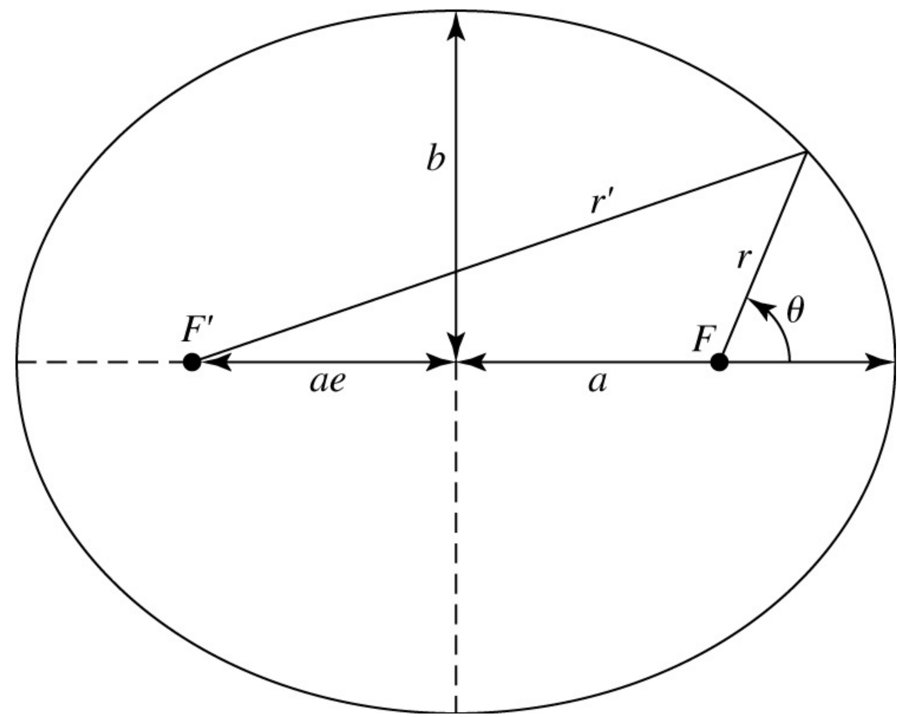
Modern View



Kepler's Planet Candidates
22 Months: May 2009 - Mar 2011
2010,2011
2012
2013
Radius Relative to Earth
20
10
4
1
Jupiter
Neptune
Earth
1
4
10
50
100
200
400
Orbital Period in days
2740
NASA
Chris Burke:
216.02

Geometry of elliptical orbits

An ellipse is a set of points that satisfies $r + r' = 2a$



a = semimajor axis

b = semiminor axis

e = eccentricity

= (distance between foci)/(major axis)

A bit of trig will show that

$$b^2 = a^2(1 - e^2)$$

In polar coordinates, the equation for an ellipse becomes

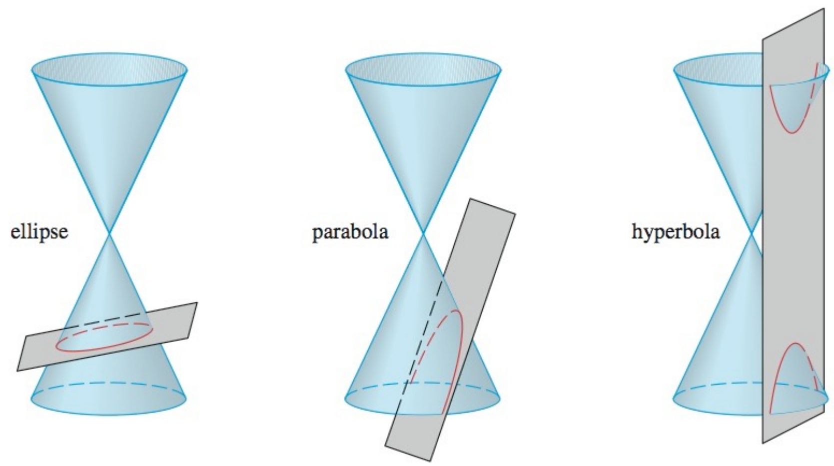
$$r = \frac{a(1 - e^2)}{1 + e\cos\theta} \qquad (0 \leq e < 1)$$

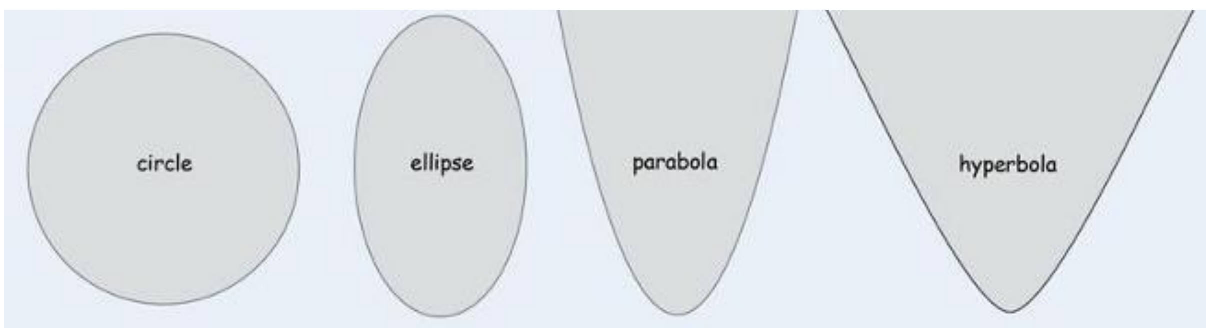
(see C+O 2.1)

Area of ellipse:

$$A = \pi ab$$

More generally, orbital motions follow *conic sections,* depending on total energy. A conic section is the surface formed by cutting a cone with a plane.





Compare *closed* and *open* curves.

Curves with e = 1 are *parabolas* $r = \dfrac{2p}{1+\cos\theta}$

where p is the distance of closest approach to the focus (at angle 0).

Curves with e >1 are *hyperbolas* $r = \dfrac{a(e^2-1)}{1+e\cos\theta}$

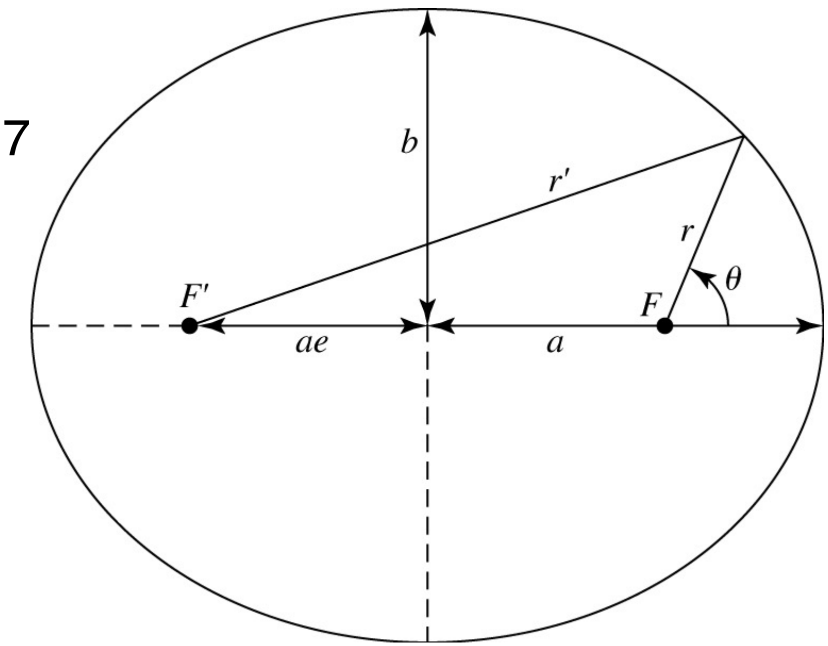
Curves with $0 \leq$ e < 1 are *ellipses*

Curves with e = 0 are *circles*

Example 2.1.1

- Determine the variation in distance of Mars from the focus through its orbit.

- Semimajor axis of Mars's orbit = 1.5237
- e=0.0934



Furthest distance from focus is $a + ae = a(1 + e) =$ 1.67 AU. Closest distance is $a - ae = a(1 - e) =$ 1.38 AU.

Now, let's move onto Newton's description!

Newtonian Mechanics

Velocity: speed and direction. If $\vec{x}$ is a position vector, then

$$\vec{v} \equiv \frac{d}{dt}\,\vec{x}$$

Acceleration: rate of change of velocity:

$$\vec{a} \equiv \frac{d}{dt}\,\vec{v}$$

If $|\vec{v}|$ is constant, can $\vec{a}$ be non-zero?

Newtonian Mechanics

Velocity: speed and direction. If $\vec{x}$ is a position vector, then

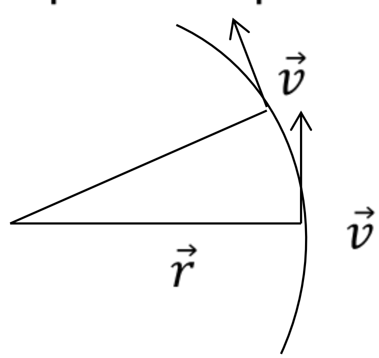
$$\vec{v} \equiv \frac{d}{dt}\,\vec{x}$$

Acceleration: rate of change of velocity:

$$\vec{a} \equiv \frac{d}{dt}\,\vec{v}$$

If $|\vec{v}|$ is constant, can $\vec{a}$ be non-zero?

Yes! Simple example is uniform circular motion:



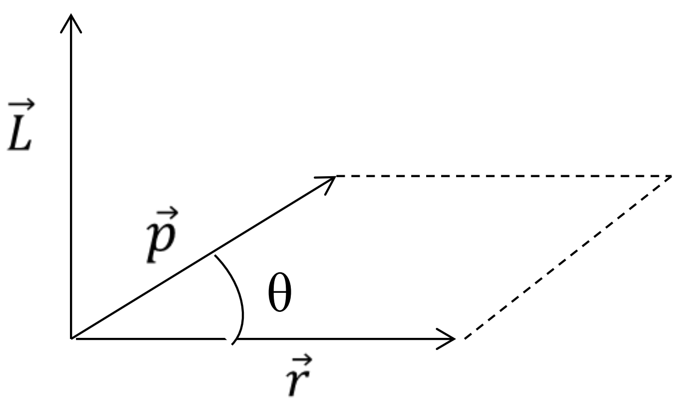
$$a = \frac{v^2}{r}$$ (centripetal acceleration)

Direction of $\vec{a}$?

Linear momentum

$$\vec{p} \equiv m\vec{v}$$

Angular momentum



$$\vec{L} \equiv \vec{r} \times \vec{p}$$

$$|\vec{L}| = |\vec{r}||\vec{p}| \sin\theta$$

$\vec{L} \perp \vec{r}, \vec{p}$ plane

Newtonian Mechanics

Newton's laws:

1. An object at rest will remain at rest and an object in motion will remain in motion in a straight line at a constant speed unless acted upon by an external force.

   => Conservation of linear momentum in absence of unbalanced force.

$$\frac{d\vec{p}}{dt} = 0$$

The law of inertia

2. Force is the change of momentum with time.

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

First law is thus a special case of second.

3. The law of action-reaction, or conservation of total linear momentum. In any interaction, of e.g., two particles, the changes in momentum obey

$$\Delta \vec{p}_1 = -\Delta \vec{p}_2$$

If time interval of interaction is Δt , then the rate of momentum change is

$$\frac{\Delta \vec{p}_1}{\Delta t} = -\frac{\Delta \vec{p}_2}{\Delta t}$$

$$\Delta t \rightarrow dt \qquad \frac{d\vec{p}_1}{dt} = \frac{-d\vec{p}_2}{dt}$$

$$\boxed{\vec{F}_1 = -\vec{F}_2}$$

Claim: $\vec{L}$ is conserved for central forces ($\vec{F} \parallel \vec{r}$):

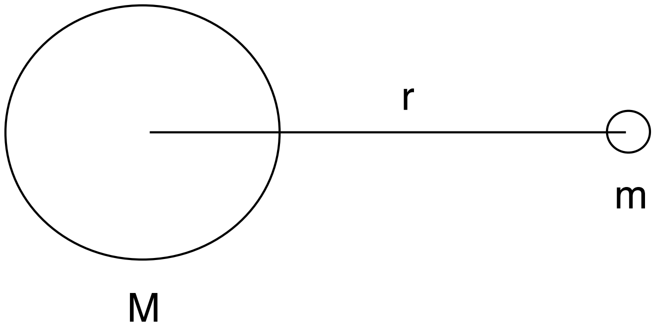
$$\dot{\vec{L}} = \frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \underbrace{\vec{v} \times m\vec{v}}_{=0} + \underbrace{\vec{r} \times \vec{F}}_{=0 \text{ for central forces. Why?}}$$

When $\dot{\vec{L}} \neq 0$ a torque exists

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Newton's Law of Gravitation



$$F = \frac{GMm}{r^2}$$

or

$$F = \frac{GMm}{\mu\left(\frac{a}{a_0}\right) r^2}$$

Milgrom

(see C+O 2.2 for derivation, from Kepler III and centripetal force)

For point masses, this attractive force is a central force. Still true for spherically symmetric bodies (see C&O example 2.2.1).

$$F = mg \Rightarrow g = \frac{GM}{r^2}$$

Gravitational acceleration

Note that *g* is independent of *m*.

Gravitational potential energy:

Energy (or work) necessary to raise an object against a gravitational force is given by:

$$U_f - U_i = \Delta U = \int_{r_i}^{r_f} \boldsymbol{F} \cdot d\boldsymbol{r}$$

Only changes in *U* relate to *F*. Zero point of *U* arbitrary. So define *U* to be zero at infinity. Then plug in in for ***F***, and integrating, we find *U* at any other *r* to be:

$$U = -\frac{GMm}{r}$$

For $\vec{F} = \vec{F}(r)$, $\vec{F} = -\frac{\partial U}{\partial r}\hat{r}$

In 3-D, generally $\vec{F} = -\vec{\nabla} U$

Kinetic energy

Likewise we can use Newton 2 and integrate the RHS, $\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$

$$\int_{r_i}^{r_f} \frac{d\vec{p}}{dt} \bullet d\vec{r} = \int_{t_0}^{t_1} m \frac{d\vec{v}}{dt} \bullet \vec{v} dt = \int_{t_0}^{t_1} m\left(\vec{v} \bullet \frac{d\vec{v}}{dt}\right) dt$$

$$= \int_{t_0}^{t_1} m \frac{d\left(\frac{1}{2}v^2\right)}{dt} dt = \int_{v_0}^{v_1} md\left(\frac{1}{2}v^2\right) = \left.\frac{1}{2}mv^2\right]_{v_0}^{v_1} = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2$$

which is change in kinetic energy between the times t_1 and t_0.
Call these the kinetic energies K_1 and K_0 respectively.

Total energy

From definitions of *KE* and *U*, any change in *U* is balanced by opposite change in *KE:*

$$U_0 - U_1 = K_1 - K_0$$

and thus the total energy *E* is constant:

$$E = K_0 + U_0 = K_1 + U_1$$

If we consider the mass *M* to be fixed, then the only KE is due to the mass, *m*. This is fine for, e.g. satellites orbiting the Earth, but we will generalize this later.

$$\boxed{E = \frac{1}{2}mv^2 - \frac{GMm}{r}}$$

Summarizing:

Gravitational potential energy

$$U = -\frac{GMm}{r}$$

Kinetic energy

$$KE = \frac{1}{2}mv^2$$

Total energy

Any change in *U* balanced by opposite change in *KE*: total energy of *m*

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

A consequence of energy conservation: **Escape velocity**

If mass *m* moves to $r \longrightarrow \infty$ and slows to *v=0* (i.e. "escapes"), then *KE* = *U* = 0, so *E* = 0 at $r \longrightarrow \infty$. Then

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = 0$$ everywhere.

In this case, $v = v_{\text{escape}}$

$$\frac{1}{2}mv_{esc}^2 = \frac{GMm}{r}$$

Note: independent of m.

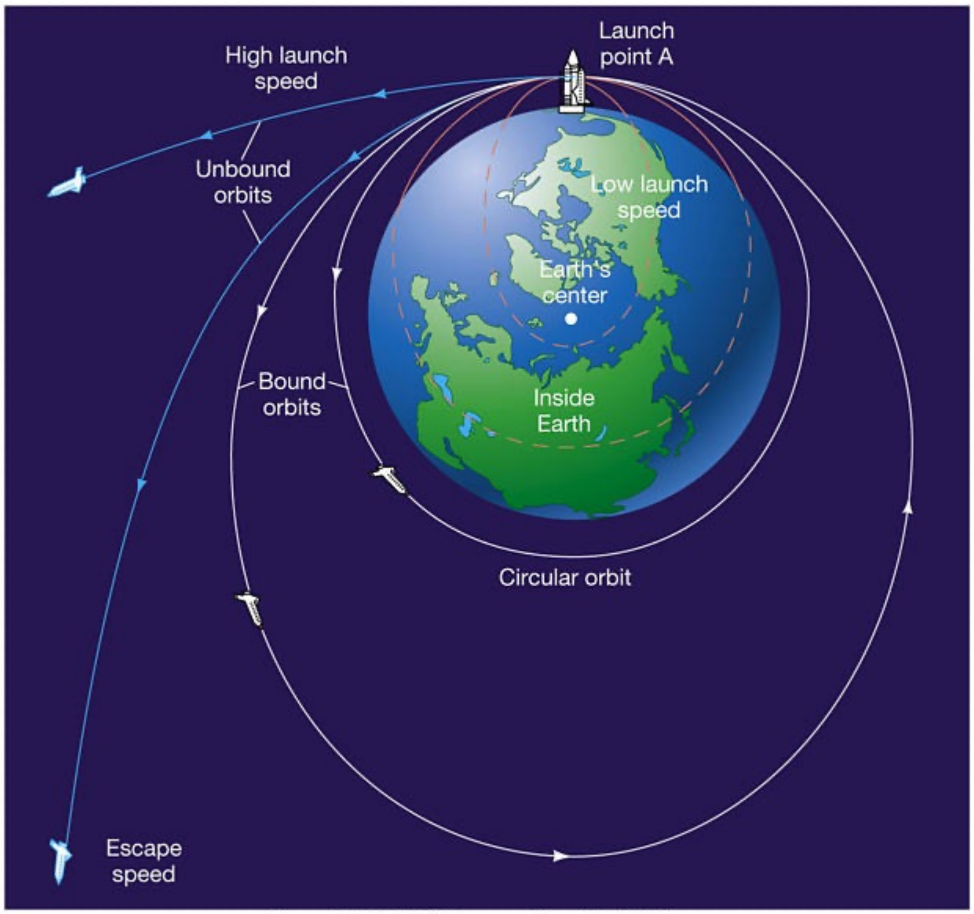
$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

What is r physically? Does it have to be the radius of a planet?

Worksheet #1

Escape Velocity problem: What velocity is needed for a rocket to escape the gravitational influence of the Earth?

M_Earth = 5.976×10^{24} kg
R_Earth = 6378 km



Artemis 1