Astronomy 421 – Problem set 7

Due Tuesday, Nov. 15

1. The most readily observed white dwarf in the sky is in the constellation of Eridanus. Three stars make up the 40 Eridani system: 40 Eri A is a 4^{th} magnitude star similar to our Sun; 40 Eri B is a 10^{th} magnitude white dwarf; and 40 Eri C is an 11^{th} magnitude red M5 star. The following two problems only deal with the 40 Eri B and C since A is separated from them by 400 AU.
   a. The period of the 40 Eri B and C system is 247.9 years. The system's parallax is 0.201" and the true angular extent of the semimajor axis of the reduced mass is 6.89". The ratio of distances of 40 Eri B and C from the center of mass is $a_B/a_C = 0.37$. Find the masses of the two stars in solar masses.
   b. The absolute bolometric magnitude of 40 Eri B is 9.6. Determine its luminosity in solar luminosities.
2. Use the information presented in problem (1) to determine:
   a. The effective temperature of 40 Eri B is 16,900 K. Calculate its radius, and compare your answer to the radii of the Sun, Earth, and Sirius B.
   b. Calculate the average density of 40 Eri B, and compare your result with the average density of Sirius B. Which is more dense and why?
   c. Calculate the product of the mass and volume of both 40 Eri B and Sirius B. Is there a departure from the mass-volume relation? What might be the cause?
3. Estimate the ideal gas pressure and the radiation pressure at the center of Sirius B, using 3×10^7 K for the central temperature. Compare these values with the estimated central pressure (Eq. 16.1).
4. Suppose that the Sun were to collapse down to the size of a neutron star (10-km radius). (a) Assuming that no mass is lost in the collapse, find the rotation period of the neutron star. (b) Find the magnetic field of the neutron star. Even though our Sun will not end its life as a neutron star this shows that conservation of angular momentum and magnetic flux can easily produce pulsar-like rotation speeds and magnetic fields.
5. Determine the minimum rotation period for a 1.4 Msun neutron star (the fastest it can spin without flying apart). For convenience, assume the star remains spherical with a radius of 10 km.
6. During a glitch, the period of the Crab pulsar decreased by $\Delta P \sim 10^{-8} P$. If the increased rotation speed was due to an overall contraction of the neutron star, find the change in the star's radius. Assume that the Crab pulsar is a rotating sphere of uniform density with an initial radius of 10 km.
7. The Geminga pulsar has a period of 0.237 s and a period derivative of Pdot = 1.1×10^{-14}. Assuming that t = 90 degrees, estimate the magnetic field strength at the pulsar's poles.
8. Leadville, Colorado, is at an altitude of 3.1 km above sea level. If a person lives there for 75 years (as measured by an observer far from Earth), how much longer would gravitational time dilation allowed that person to live if he or she had instead lived in Santa Cruz, California (at sea level)? Should we all move to the coast to live longer?
9. Estimate the radius of curvature of a horizontally travelling photon at the surface of a 1.4 Msun neutron star, and compare the result with the 10 m radius of the star. Can general relativity be neglected when studying neutron stars?
10. By combining gravitation, thermodynamics, and quantum mechanics, Stephen Hawking calculated the temperature for a nonrotating black hole to be: $T_{\mathrm{H}} = \dfrac{\hbar c^3}{8\pi G M k_{\mathrm{B}}}$
    a. Verify the expression was the right units
    b. If a primordial black hole starts with a mass of 1.7×10^{11} kg, it would be ending its life now. Compute the temperature of such a primordial black hole.
    c. What portion of the electromagnetic spectrum would this blackbody temperature correspond to?
    d. What would be the radius of a sphere with the density of water for a mass of 1.7×10^{11} kg?
    e. Compute the temperature of a 10 Msun black hole.