Astronomy 421 - Problem Set 5

Due Thurs Oct 20

1. Show that the equation for hydrostatic equilibrium (10.6) can also be written in terms of the optical depth, τ, as

$dP/d\tau = g/\kappa$

This form of the equation for hydrostatic equilibrium is often useful in modeling stellar atmospheres.

2. Assuming that 10 eV could be released by every atom in the Sun through chemical reactions, estimate how long (in years) the Sun could shine with its current luminosity through chemical processes alone. For simplicity, assume that the Sun is composed entirely of hydrogen. Is it possible that the Sun's energy source is entirely chmical? Why or why not?

3. By invoking the virial theorem, make a crude estimate of the "average" temperature for the Sun. Is your result consistent with other estimates obtained in Chapter 10? Why or why not?

4. Estimate the hydrogen-burning lifetimes of stars near the lower and upper ends of the main sequence. The lower end occurs near 0.072 $M_\odot$, with $\log_{10} T_e = 3.23$ and $\log_{10}(L/L_\odot) = -4.3$. On the other hand, an 85 $M_\odot$ star near the upper end of the main sequence has an effective temperature and luminosity of $\log_{10} T_e = 4.705$ and $\log_{10}(L/L_\odot) = 6.006$. Assume that the low mass star is entirely convective (rather than just the inner 10%) so that all of the star is available for nuclear burning.

5. Explain what is meant by hydrostatic equilibrium. Why is this an important assumption in modeling the interior of stars?

6. In this problem we will use results from the stellar interiors computer code, STATSTAR, which is described in Carroll and Ostlie Section 10.5 and Appendix L. Please read these to become familiar with how these models are calculated. The inputs are the mass, luminosity, effective temperature, and mass fraction of hydrogen, helium, and "metals" (sum of all elements heavier than helium). The code uses the basic differential equations of stellar structure (summarized in 10.5), as well as equations for the energy generation rate, and equation of state, and approximations for the opacity, to calculate the stellar structure as a function of radius. Notice from the results below that because of all the approximations in the code it doesn't reproduce the properties of a 1 $M_\odot$ star exactly.

You will need to download the fortran source code statstar.f from the class web page and compile it. On a Mac this is done with the following line:

gfortran -o runstatstar statstar.f

We will use a model with typical abundances to find out how various parameters scale with the mass of the star for H burning stars on the Main Sequence, up to 10 $M_\odot$. The mass fractions for this model are X=0.7, Y=0.29, and Z=0.01 (typical stellar abundances). Below are the luminosity, radius, and effective temperature. You will need to run STATSTAR in order to find the radius, central temperature and the energy generation rate (ϵ) for five masses.

M ($M_\odot$)	L ($L_\odot$)	T_{eff}	R ($R_\odot$)	T_c (K)	ϵ (J s^{-1} kg^{-1})
0.5	0.018065	2196.04			
1.0	0.746150	5233.10			
2.0	20.297537	10765.12			
4.0	320.644369	17372.69			
10.0	6508.785848	27812.81			

(a) Make log-log plots of these quantities vs. mass and fit a straight line by eye to find their rough power law dependence on mass. If the data suggest a straight line fit is not appropriate, try fitting straight lines for a lower mass range and an upper mass range. You can also use fitting software if you are familiar with it but fitting by eye will do.

(b) For dependences we discussed in class (namely radius, luminosity), how do these compare (steeper, shallower, comparable)?

(c) For the highest masses, does the ϵ vs. M dependence make sense to you given the L vs. M dependence (remember our assumption that energy generated = energy radiated)? A lot of the strange things you find are due to the limitations of the code. This is an exploratory problem. Just see what you find out!