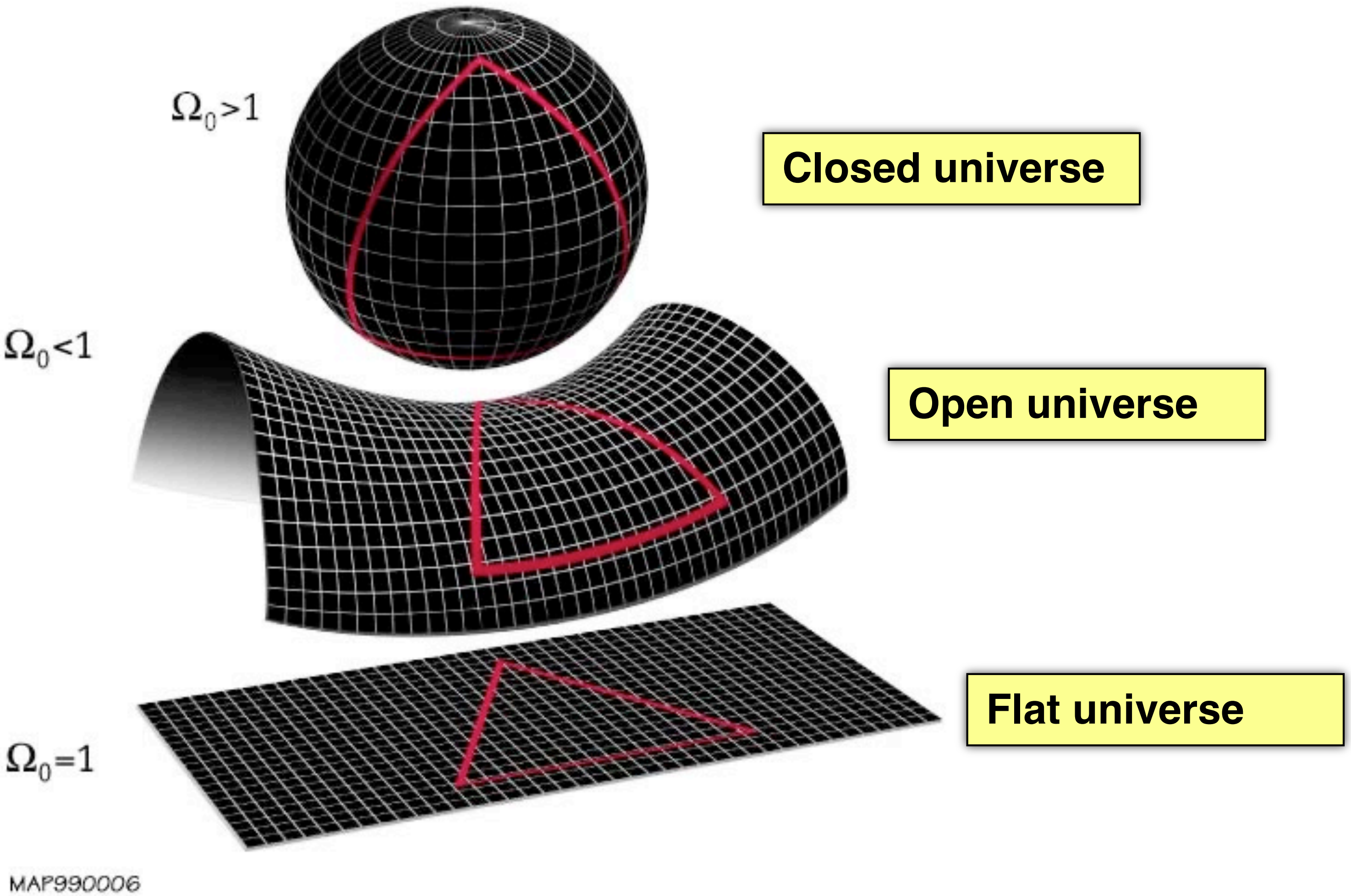
- We define the *density parameter* $$\Omega_0 = \frac{\rho_0}{\rho_c}$$



The critical density

- The critical density can be estimated by looking at the total energy of an expanding region, assuming the total energy should remain constant.

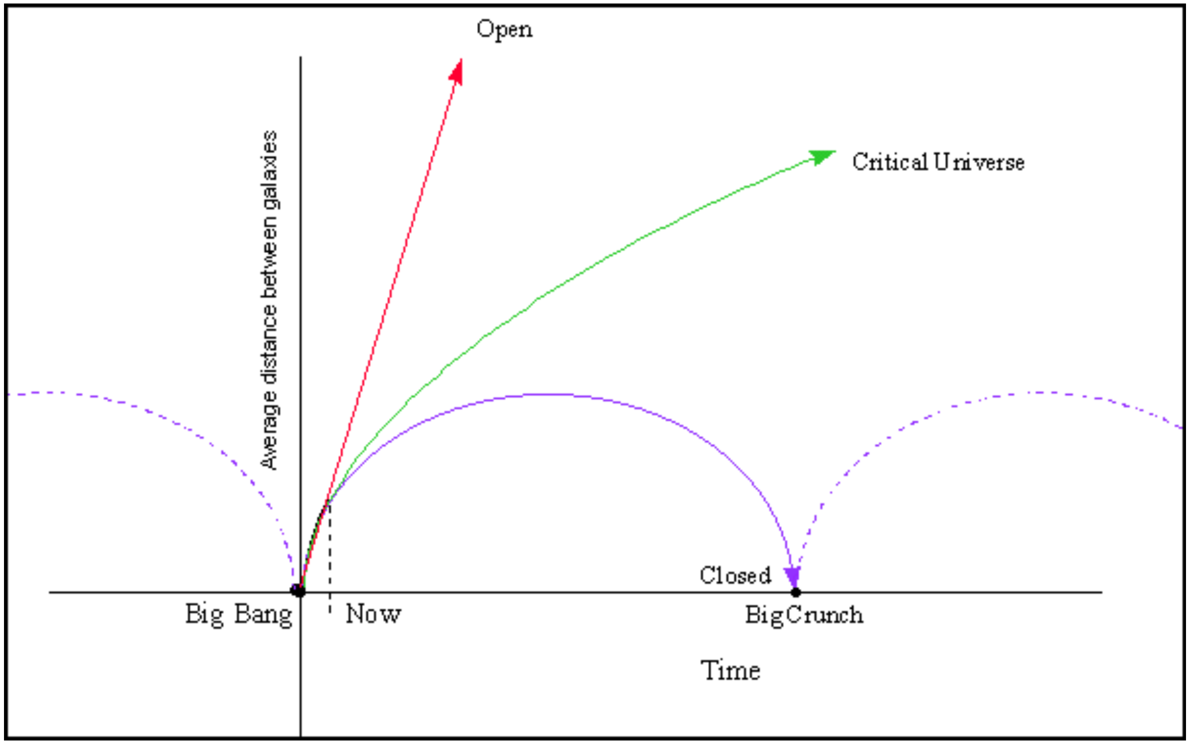
- This leads to $$\rho_c = \frac{3H_0^2}{8\pi G}$$

with a numerical value of about 1×10^{-26} kg m^{-3}.

- If the pressure of matter is low (usually the case), then the future of the Universe is governed by the density ρ_0.

- If $\rho_0 < \rho_c$ ($\rho_c \propto H_0^2$): universe expands forever (open).
- If $\rho_0 > \rho_c$: universe will collapse back (closed).

- This will then determine the geometry of the Universe. Thus, there is a direct link between the shape of the Universe and its fate!



A flat universe

- By measuring the spot sizes in CMB images, it appears that space is flat, or very nearly so. This would mean $\Omega_0 = 1$.

- Problem: the mass density of the universe is measured to be 2.4×10^{-27} kg/m^3 (including dark matter). The critical density from theory is 1×10^{-26} kg/m^3. This matter density parameter, Ω_m, is

$$\Omega_m = \frac{\rho_m}{\rho_c} = 0.24$$

- But total Ω_0 is 1: we have missing density.

Missing density

- Must be some energy we cannot detect via gravitational effects, and it cannot emit detectable radiation.

- Dark energy density parameter:

and $$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$$

- Therefore, dark energy accounts for 76% of the Universe's mass/energy density.

$$\Omega_0 = \Omega_m + \Omega_\Lambda = 1$$

The cosmological constant Λ

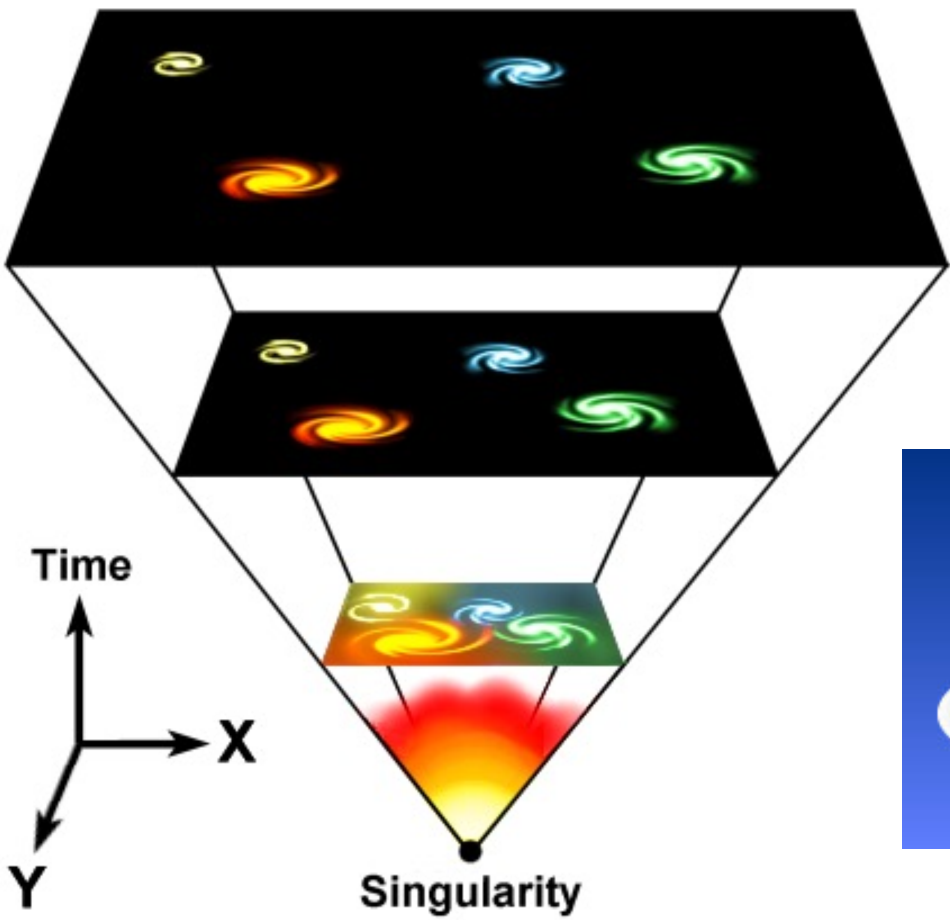
- Λ notation comes from Einstein's idea of *cosmological constant*, a fudge factor he invoked to force a static universe – before Hubble's observation of expanding universe.

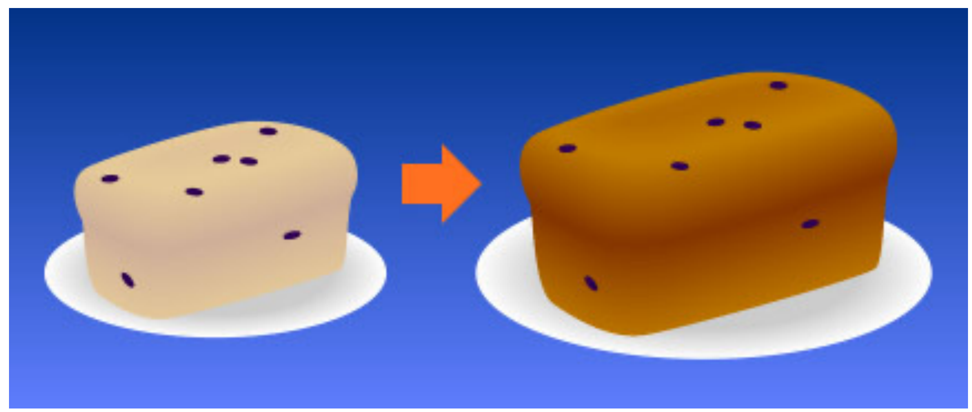
- The cosmological constant has negative pressure equal to its energy density and will therefore cause the expansion of the universe to accelerate (energy must be lost in order for expansion to occur).

- Λ may not be "constant", could vary with time.

- Depends on the currently unknown nature of dark energy.

Expansion of space





What is the future of the Universe?

- Density is part of the destiny: Universe contains matter, radiation, and dark energy – which is more important? Depends on *when*!

- Radiation dominated, $\rho_{rad} \propto t^{-4}$.

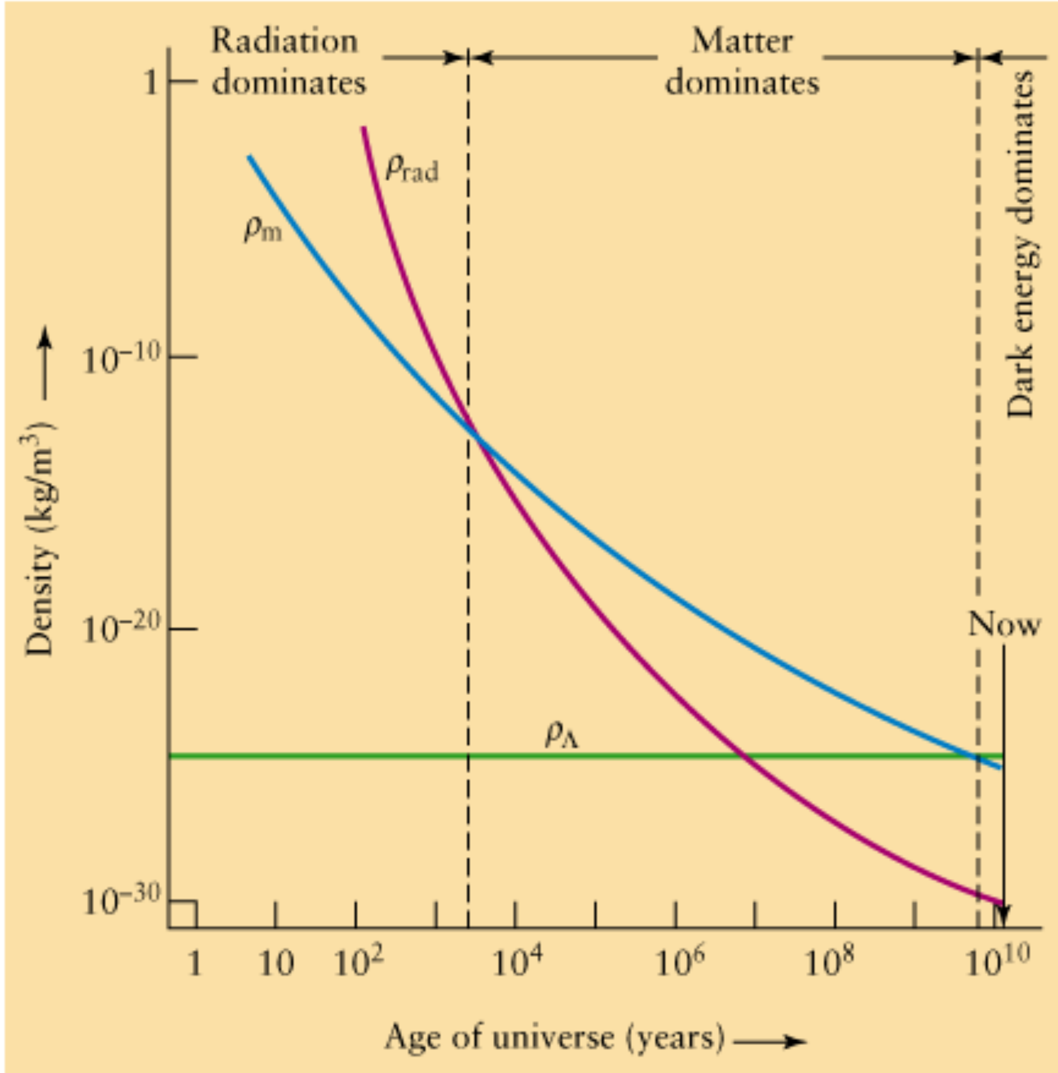
$R(t) \sim t^{1/2}$

- Matter dominated, $\rho_m \propto t^{-3}$.

$R(t) \sim t^{2/3}$

- Dark Energy dominated, $\rho_\Lambda \propto$ constant

$R(t) \sim e^{Ht}$ where $H = \mathrm{sqrt}(\Lambda/3)$



Evidence is we currently live in a dark-energy dominated universe. Since about 5 billion years go, ρ_Λ has been greater than ρ_m.

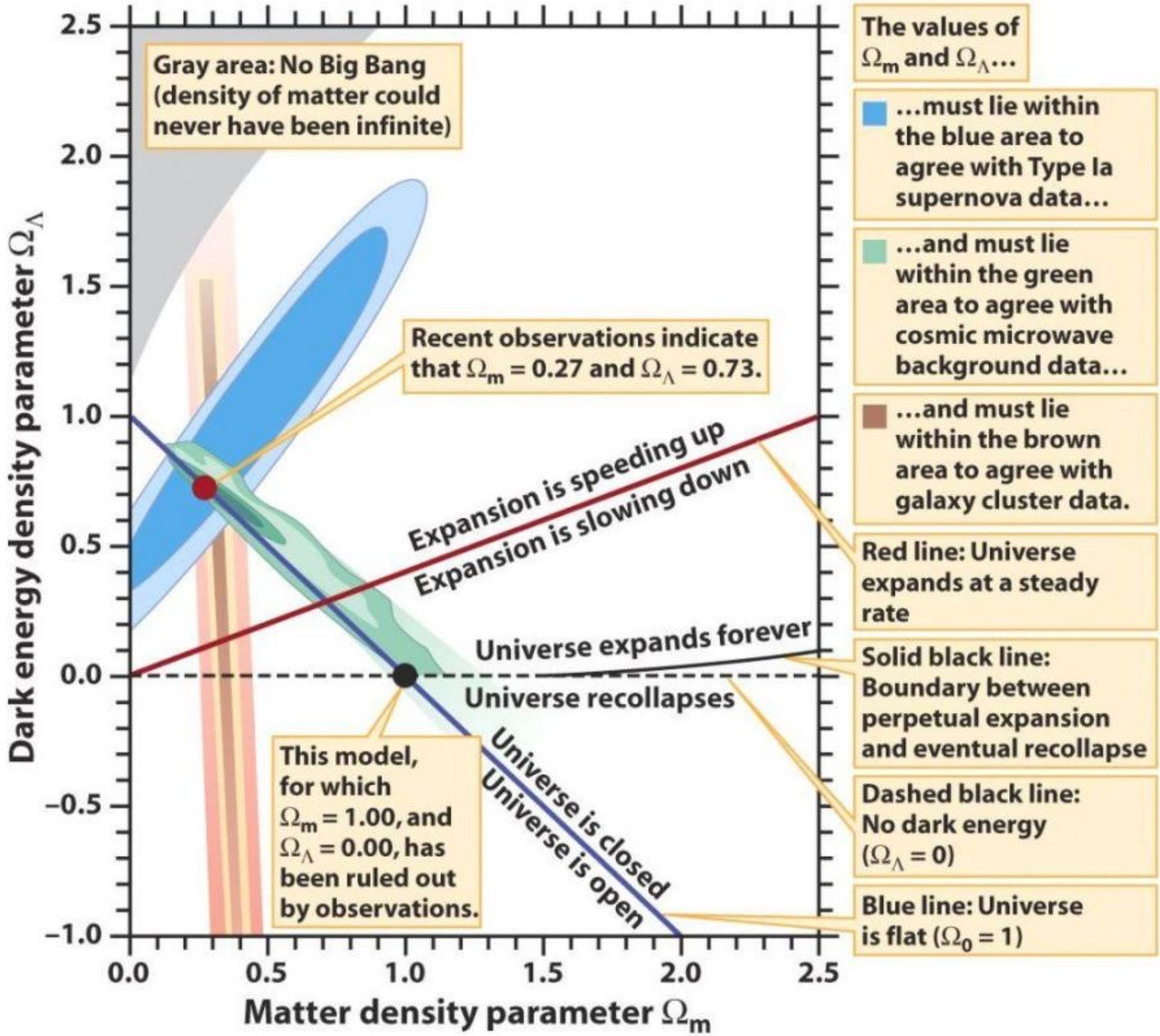
Consider the density of the universe

(Any kind of density: radiation, baryonic and dark matter)

- High-density:
  - Enough matter to slow and eventually stop the expansion
  - Universe turns around and collapses in a *Big Crunch*

- Low-density or Flat:
  - Not enough matter to stop the expansion, keeps expanding forever
  - Keeps getting cooler, ending in a *Big Chill*

- But as we have learned, we live in a dark energy dominated universe. Density is not the only key to destiny.



Gray area: No Big Bang (density of matter could never have been infinite)
Recent observations indicate that $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$.
Expansion is speeding up
Expansion is slowing down
Universe expands forever
Universe recollapses
Universe is closed
Universe is open
This model, for which $\Omega_m = 1.00$, and $\Omega_\Lambda = 0.00$, has been ruled out by observations.
The values of Ω_m and Ω_Λ...
...must lie within the blue area to agree with Type Ia supernova data...
...and must lie within the green area to agree with cosmic microwave background data...
...and must lie within the brown area to agree with galaxy cluster data.
Red line: Universe expands at a steady rate
Solid black line: Boundary between perpetual expansion and eventual recollapse
Dashed black line: No dark energy ($\Omega_\Lambda = 0$)
Blue line: Universe is flat ($\Omega_0 = 1$)
Dark energy density parameter Ω_Λ
Matter density parameter Ω_m
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.5
1.0
1.5
2.0
2.5

Summary

- So far we have the following parts describing the Universe (all included in the Big Bang model):
  - On large scales the Universe is uniform (Cosmological Principle).
  - It is currently expanding (Hubble Law), and accelerating.
  - It used to be denser and hotter in the past.

  - What is the fate of the Universe? Depends on
    - The overall shape of the Universe, and on
    - The density of matter and energy.

Shortcomings of Big Bang

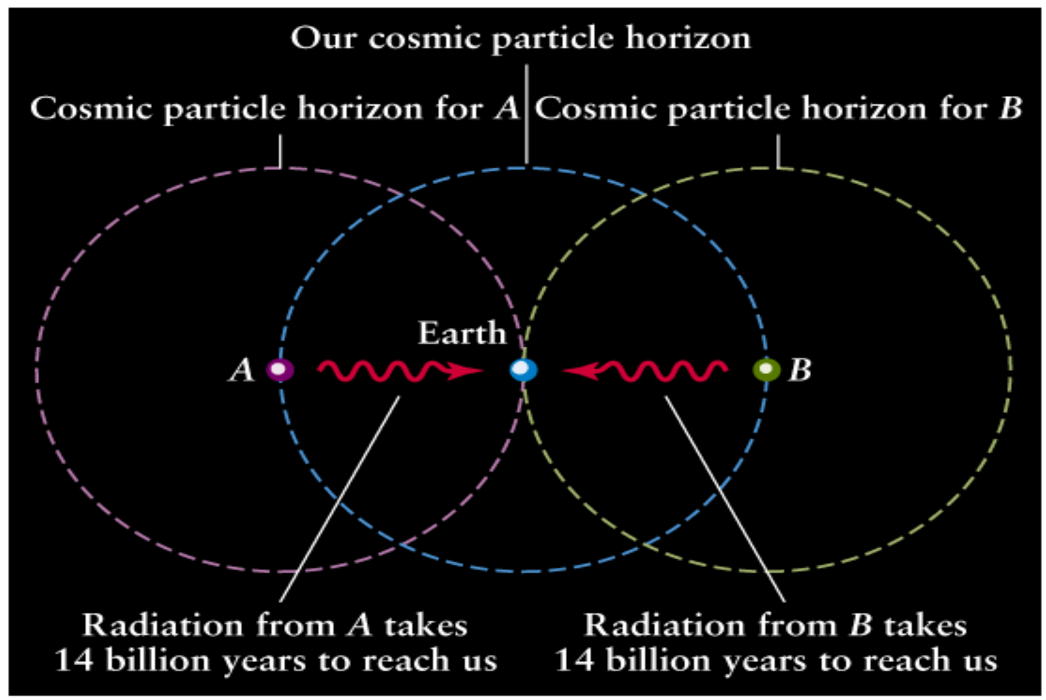
- The simplest Big Bang theory has a few shortcomings, e.g.

  - The horizon problem
  - The flatness problem
  - The relic problem
  - Structure formation problem

- No theory complete without addressing those problems!

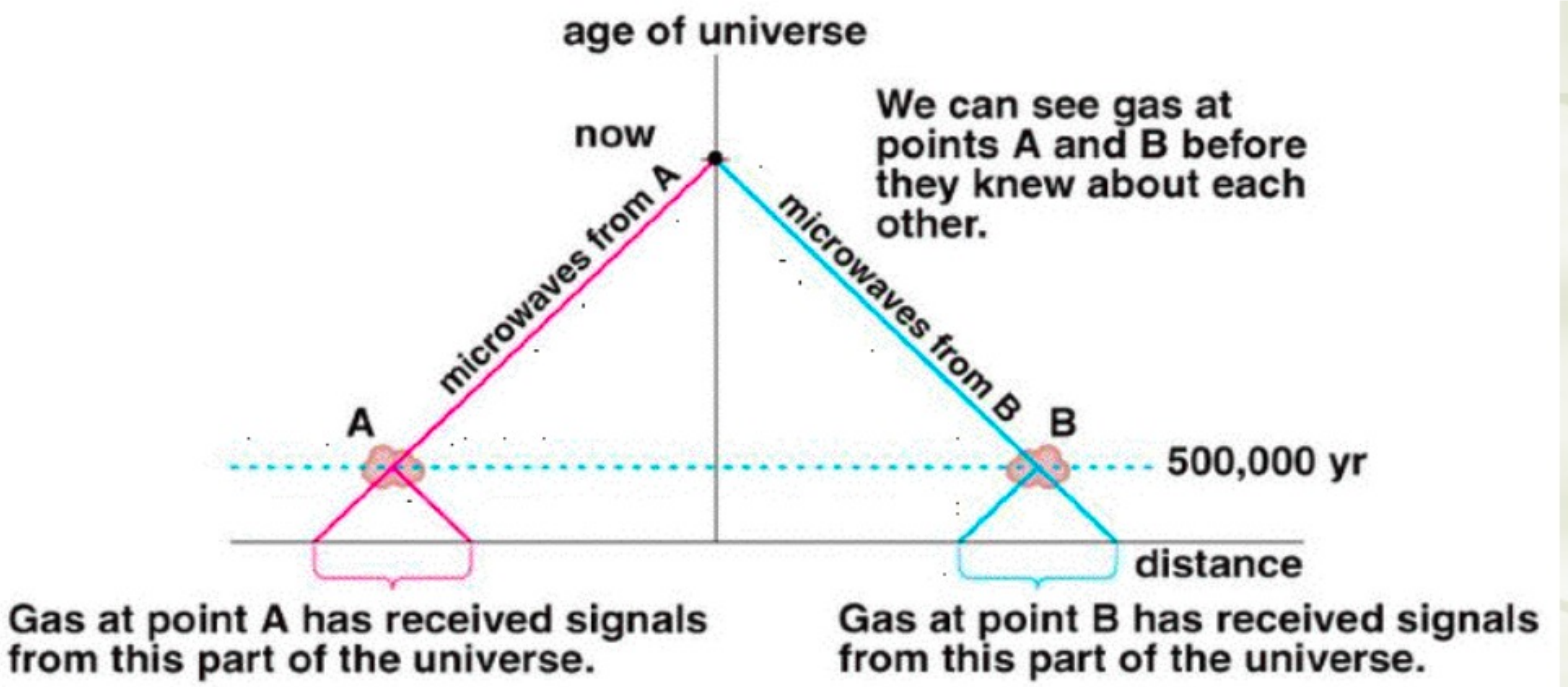
1. The horizon problem

- If the Universe is 14Gyrs old, we can only see things 14 billion ltyrs away
- Two 'opposite' locations of the horizon would be 28 billion ltyrs apart
- Nothing travels faster than c, so there could have been no heat transfer between the two regions - yet they are in thermal equilibrium?



The Universe is too large!

The horizon problem



2. The flatness problem

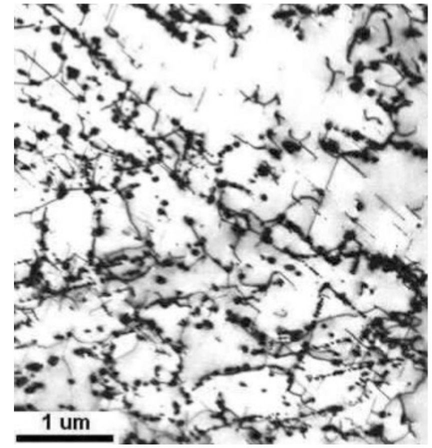
- The present Universe has very low curvature (nearly flat since Ω_0 is close to 1 today), thus there is enough energy density (in the form of matter and the cosmological constant) to provide a critical density.

- The Einstein equations predict that any deviation from flatness in a Universe filled with matter or radiation will only grow with the expansion.

- Small deviation from flatness now => immensely small in the early Universe.

Any deviation early on would have caused a very fast collapse or chill (and no time for structure formation).

3. The relic problem

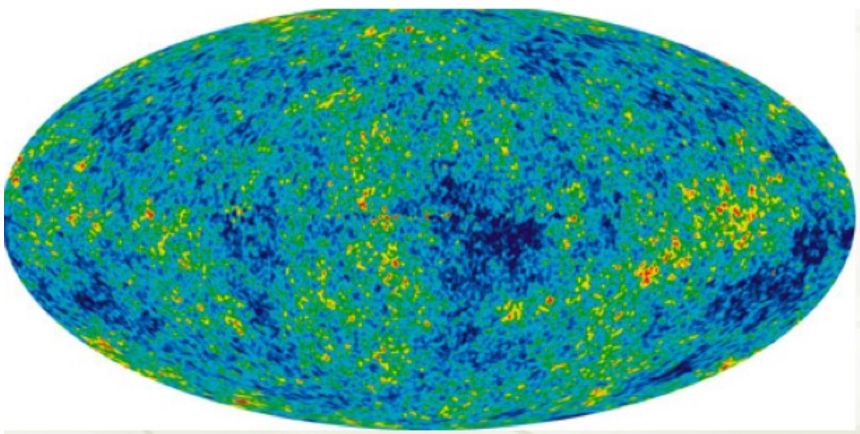


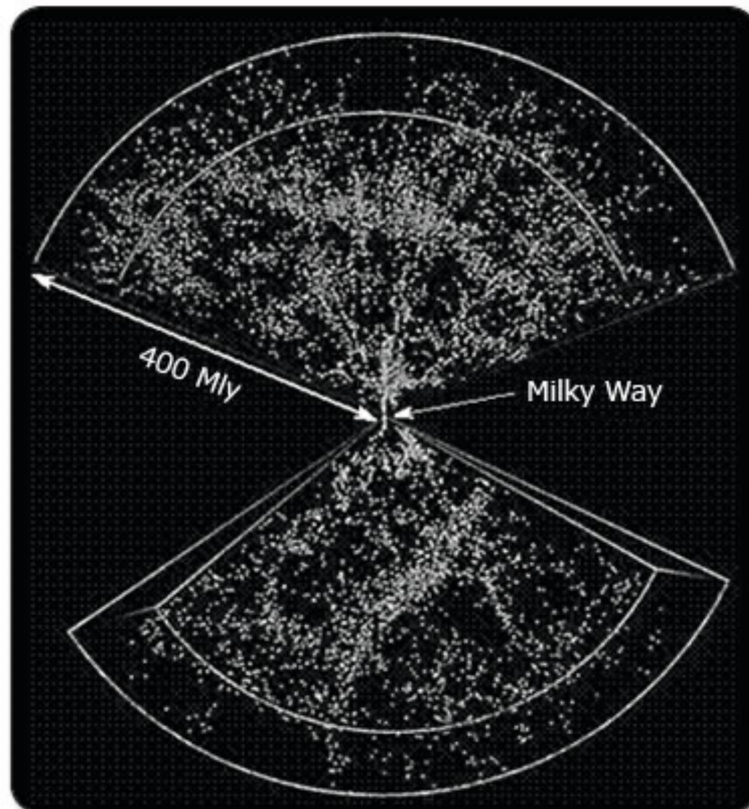
- Analogy: consider cooling of a liquid like water
- Once liquid reaches freezing point:
  - freezing does not occur smoothly and uniformly
  - starts at certain locations, and the crystals starts growing
  - when crystals merge to form solid, there will be dislocations where individual crystals meet
- The process of freezing is called a *phase transition,* that is when matter is changing from one phase to another.
- This could produce exotic structures that we call *topological effects*
  - domain walls (2d sheet-like structures)
  - cosmic strings (1d string-like structures)
  - Magnetic monopoles
  - None of these have been seen in the observable universe (good limits from CMB data: strings would gravitationally lens the background)

4. The structure problem

- Structure in the universe (galaxies, clusters of galaxies etc) came from inhomogeneities in the early universe.
- We see those same inhomogeneities in the CMB maps
- How was this coherence achieved?
- Where did these structures come from?
- Why do galaxies on opposite sides of

the Universe have the same properties?

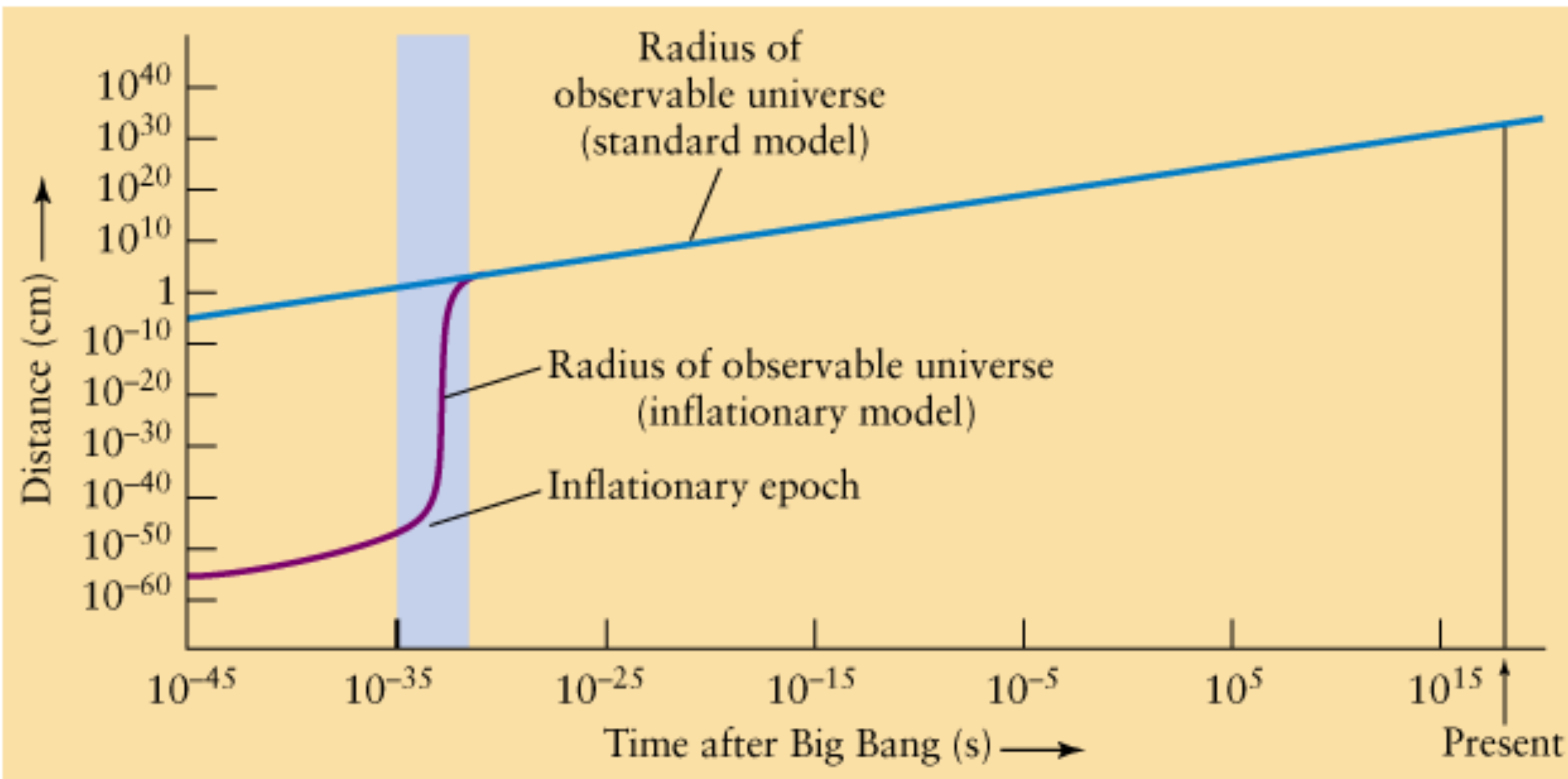




Inflation

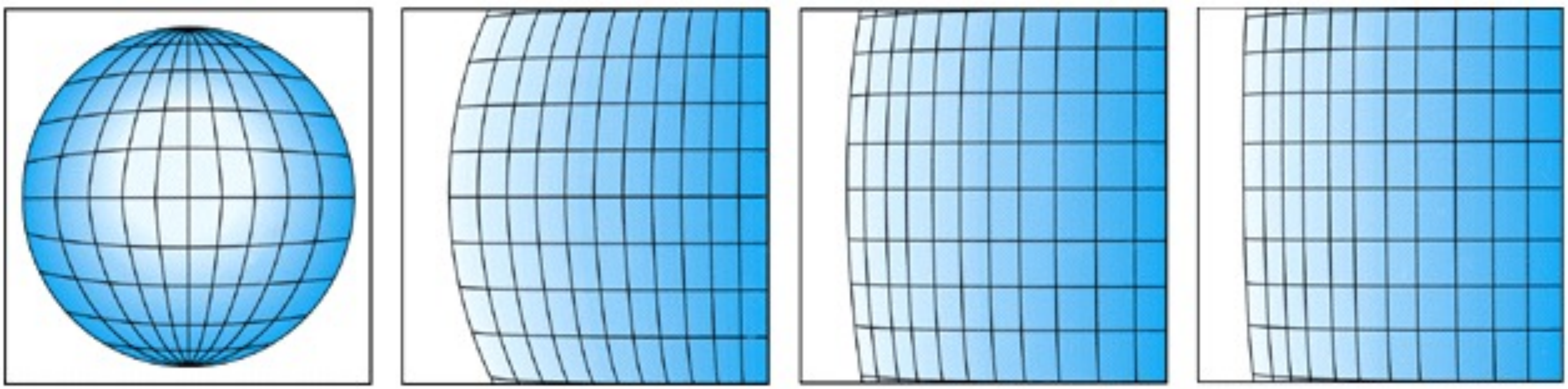
- Proposed in 1980's by Alan Guth: a brief period of extremely rapid expansion when Universe was 10^{-35} s old.

- Lasted 10^{-24} s, during which the Universe inflated by a factor of 10^{50}.



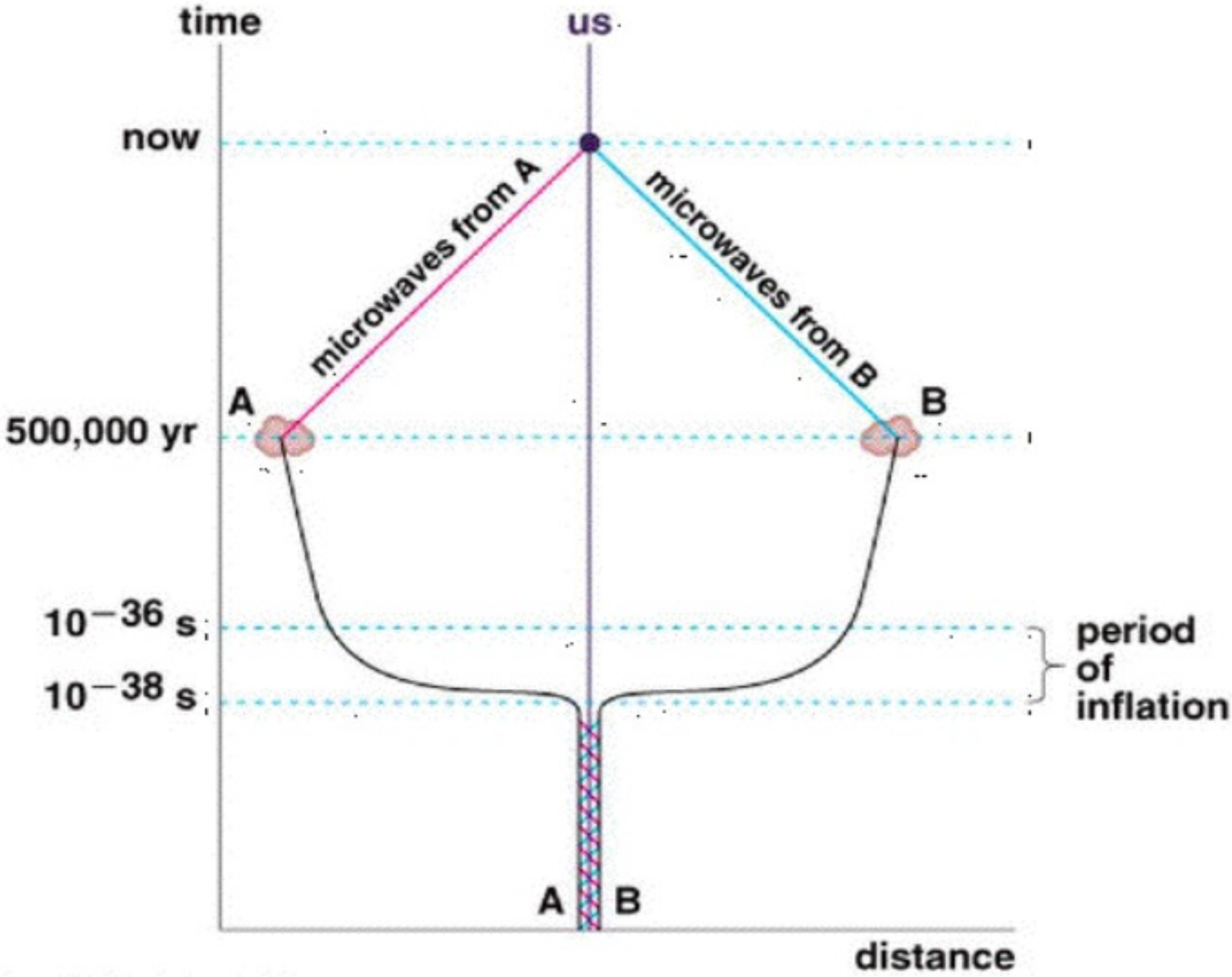
Solves the horizon, flatness, relic, and structure problems

- Solves the horizon problem :
  - vacuum pressure accelerates space so much that a photon could traverse much more space than it could in a spacetime filled with matter.
  - With inflation, light could have crossed the whole Universe before the radiation dominated era.
- Solves flatness problem:
  - enormous inflation produces locally flat geometry:



Solves Relic problem: relics are spread across huge volume making them rare

Solve structure problem: quantum fluctuations are grown during inflation



time
us
now
microwaves from A
microwaves from B
A
B
500,000 yr
10^{-36} s
10^{-38} s
period of inflation
A
B
distance

How does inflation work?

- Need to consider particle physics: how do particles interact at very high energies (temperatures)
- Consider binding energy, or *binding temperature* (matter 'melts', or unbinds)
- Typical sizes/binding energies
  - Atoms ~10^{-10} m, ~10^{3} K
  - Nuclei ~10^{-14} m, ~10^{10} K
  - Protons & Neutrons ~10^{-15} m, 10^{11} K
  - Quarks ~10^{-18} m, 10^{13} K

Getting to the temperatures and scale sizes of the early Universe.

table 29-1 The Four Forces

Force	Relative strength	Particles exchanged	Particles on which the force can act	Range	Example
Strong	1	gluons	quarks	10^{-15} m	holding protons, neutrons, and nuclei together
Electromagnetic	$1/137$	photons	charged particles	infinite	holding atoms together
Weak	10^{-4}	intermediate vector bosons	quarks, electrons, neutrinos	10^{-16} m	radioactive decay
Gravitational	6×10^{-39}	gravitons	everything	infinite	holding the solar system together

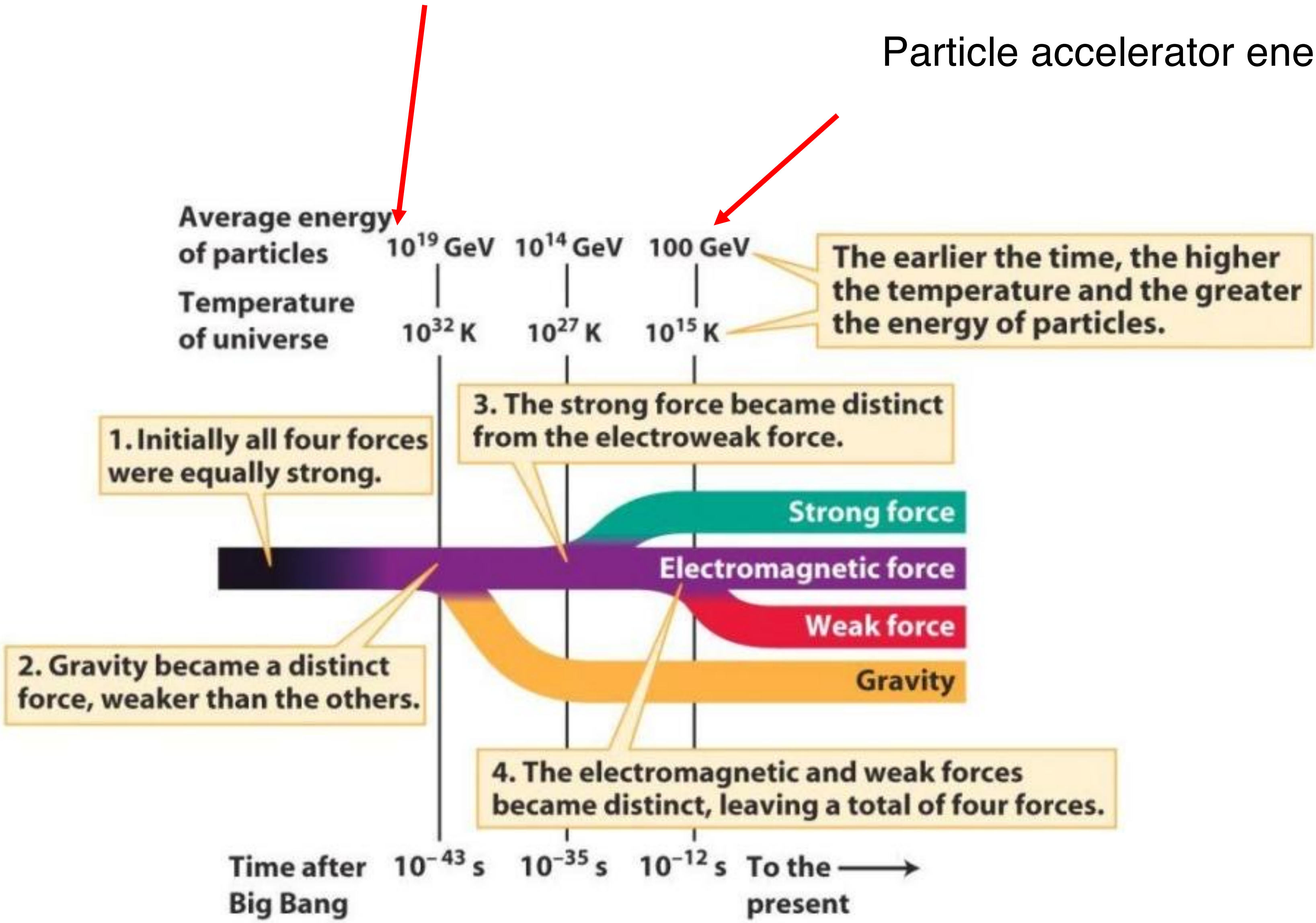
Mediates the forces

At high energies these mediators acts the same: the forces are *unified* into one.

Grand Unified Theories (GUT): strong, electromagnetic and weak forces unified above 10^{14} GeV.

Theory Of Everything (?) hard to test.

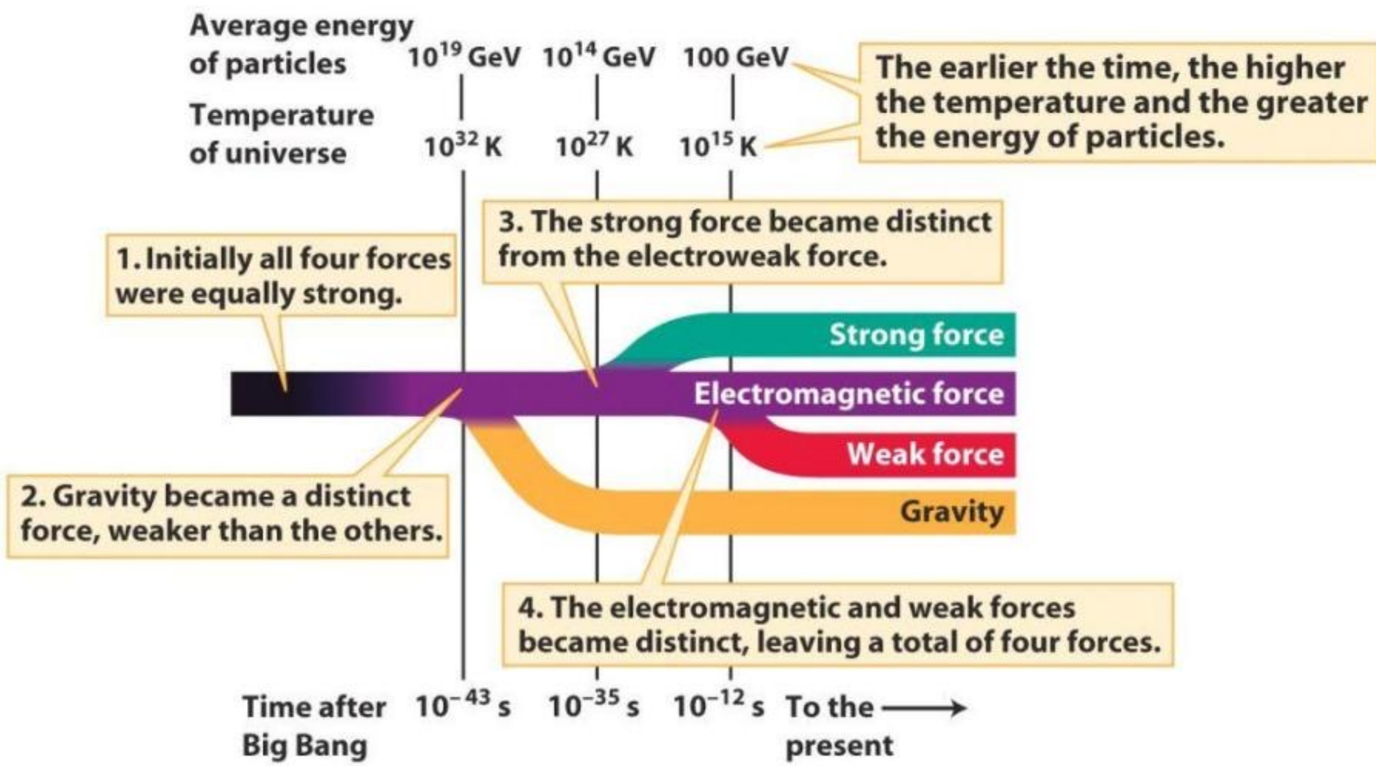
Particle accelerator energies



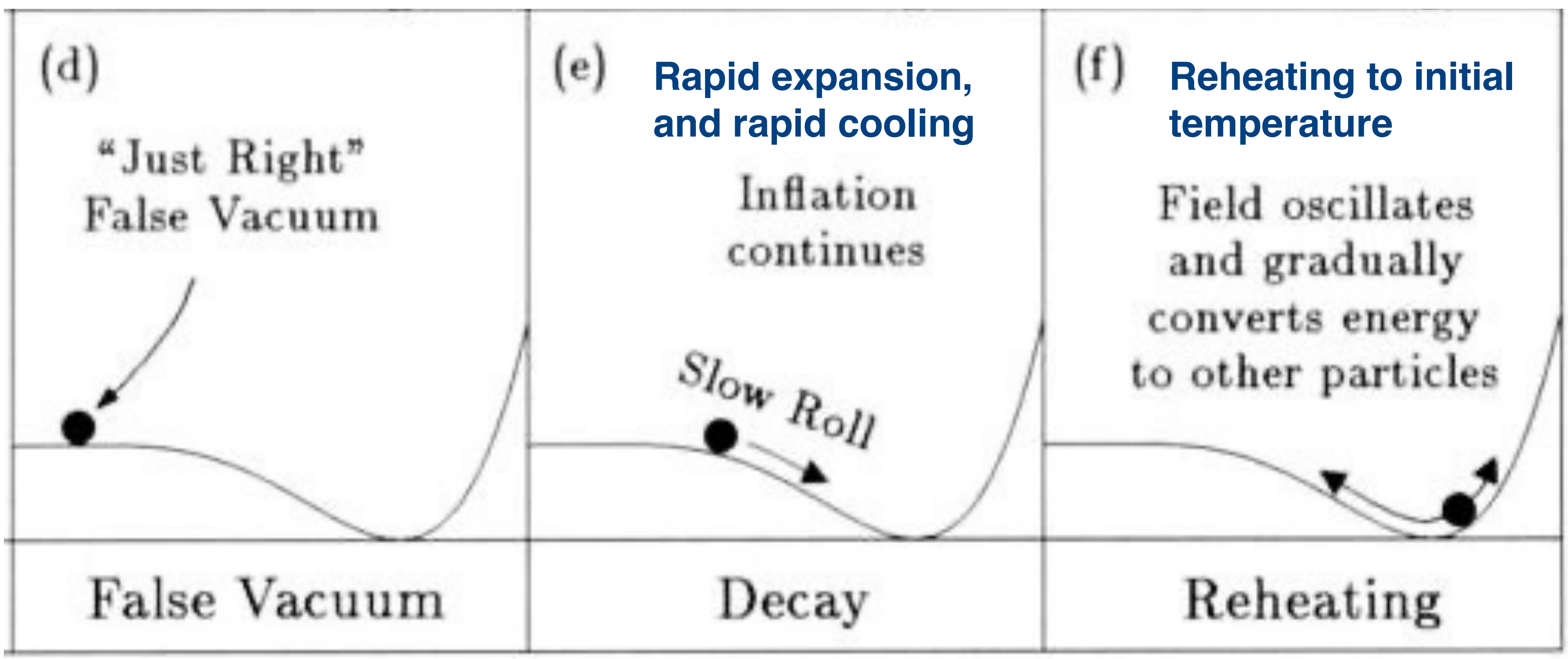
The first three minutes

- The *Planck epoch* occurred within the first 10^{-43} s. Need a quantum gravity theory to address the physics here.

- At t = 10^{-43} s, there was a symmetry breaking: force of gravity 'froze out'.



- Next: At this point the Universe was in a false vacuum.
- Inflation was caused by energy released by dynamic vacuum of space as the strong force separates from weak and electromagnetic.
- Universe moved from false vacuum (higher energy state) to lower energy state, releasing energy causing expansion.
- At this point we have an enormous, hot, expanding universe, which is also homogeneous and nearly flat.



What’s the Matter?

- Inflation helps to explain where matter and radiation come from.

- Heisenberg uncertainty principle for mass and time:

$$\Delta m \times \Delta t = \frac{h}{2\pi c^2}$$

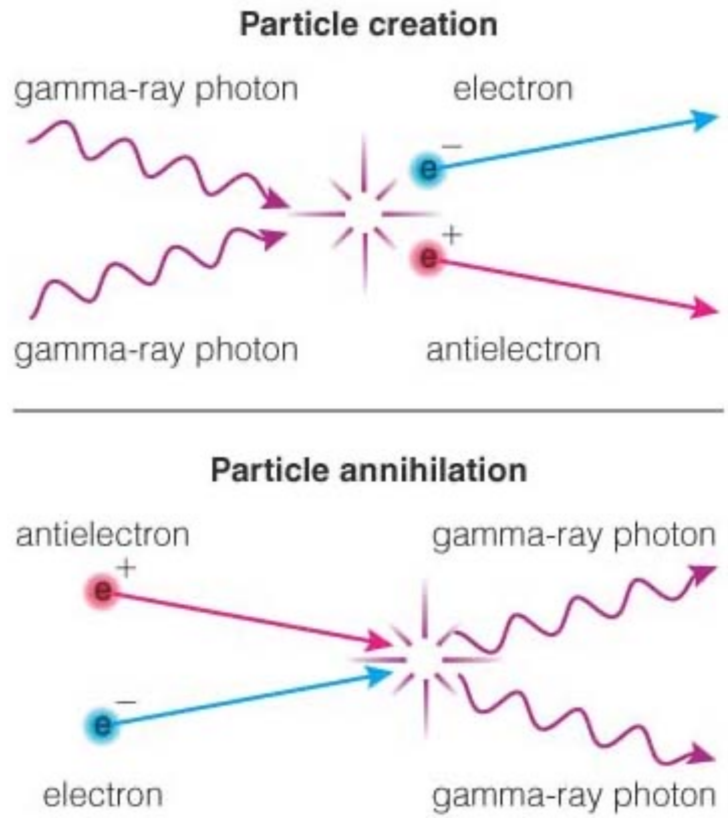
- This implies that in a short time interval we cannot say how much matter exist in a particular location. In this time interval, matter can be created and destroyed.

- The greater amount of matter, the shorter time it can exist before being destroyed.

- Example: an electron-positron pair (matter-antimatter: just opposite charges).

- Virtual pairs can become real pairs, we call it pair production:

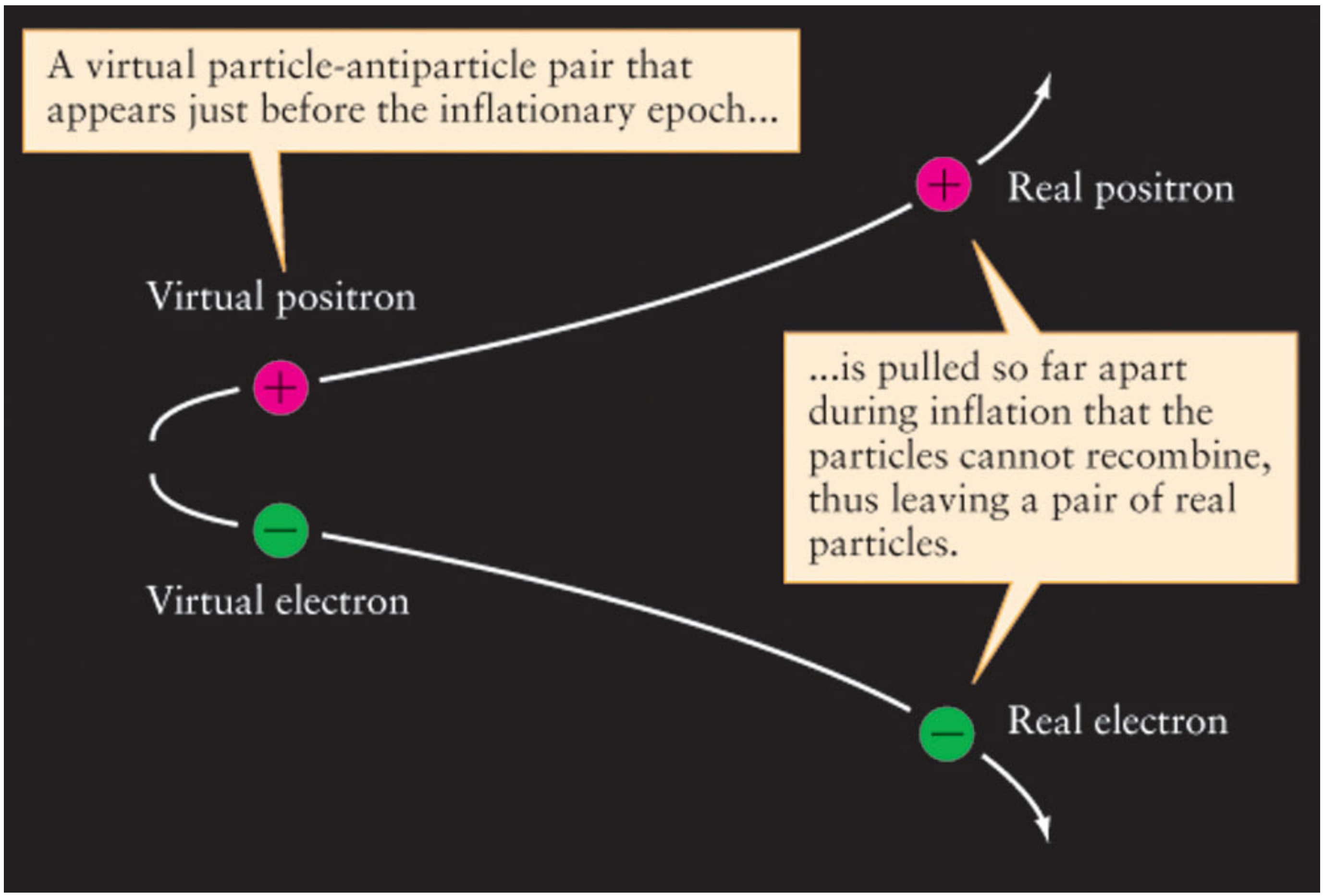
- Example: an electron-positron (electron-antielectron) creation and annihilation.



- Reactions like these constantly converted photons to particles and vice versa in the early Universe.

Real particles from virtual

- During inflation the expansion of the Universe was very fast, and the virtual pair could not get back together within the short time interval.



Electroweak era and particle era

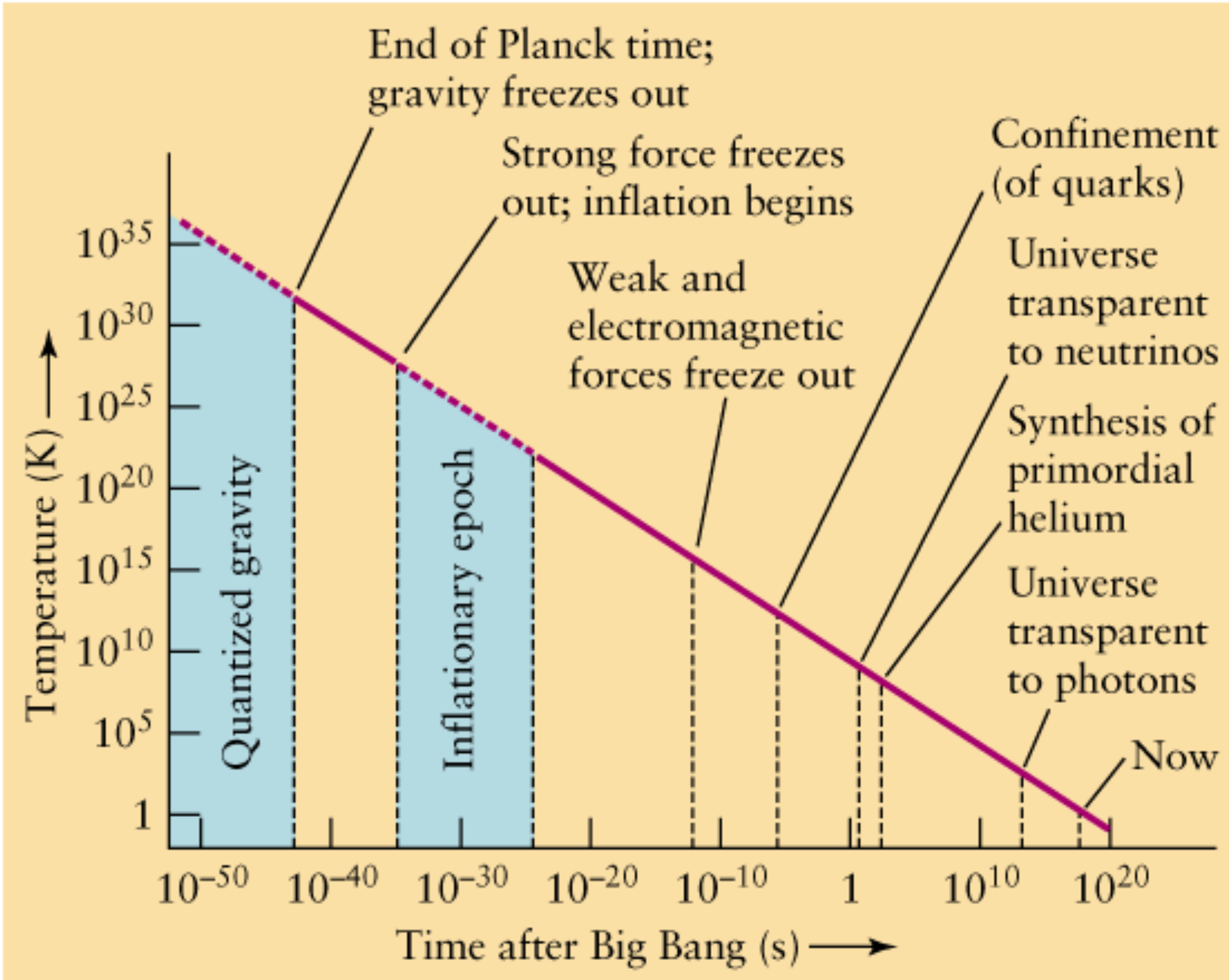
- After the inflation, the Universe is a uniform soup of elementary particles (including quarks), and radiation dominates.
  - Photons are energetic enough to continue the process of annihilation and pair production.
  - There is roughly equal amount of matter and antimatter.
  - The total number of photons roughly equal to total number of particles (*the electroweak era*).

- As the Universe expanded, gamma-ray photons became more redshifted. This is the beginning of the *particle era*.
  - Photons could still turn into various elementary particles

- In the end of the particle era, photons are not energetic enough to produce new pairs, but particles and antiparticles could still collide to produce photons.
  - All quarks will now combine to form protons and neutrons.

Baryon Asymmetry Problem

- If there would have been the same number of protons and antiprotons at the end of the particle era, all the pairs would have annihilated.
  - Would have left only photons.
- Obviously, the Universe does contain matter, so there must have been a few more protons than antiprotons.
  - For every billion antiproton and protons, annihilating each other at the end of the particle era there must have been a single proton left over.
- This slight excess of matter over antimatter is what makes up the matter in the Universe today.



…so how was the AMOUNT of the elements determined after neutrons and protons were formed?

Structure formation

- At t = 500,000 years, density of matter slightly higher in regions now containing galaxies (0.5%).
- These regions of space expanded more slowly than surrounding regions.
- Subsequently, the relative over-density grew, and at t = 15 Myrs these regions of space were ~5% denser.
- Gradual growth continued as Universe expanded, and 0.3 Gyrs after Big Bang inner portions of galaxies began to assemble

Two scenarios

- Matter density important for gravitational formation, but most matter is *dark matter.*

- Different flavors of non-baryonic dark matter:
  - Hot dark matter (HDM): relativistic velocities, e.g., neutrinos
  - Cold dark matter (CDM): slower, clump together (no known particles, WIMPS suggested)

- HDM particles will escape from small mass concentrations, large structures form first. Slow process, galaxies form very late. Not what we observe!

Standard model

- Favored model is the *hierarchical structure formation*, it is a natural outcome of Cold Dark Matter (CDM models).
- Structure begins to form early, it forms bottom-up and galaxies form before galaxy clusters.
- Better description of what we see in the Universe.

Jeans' analysis of gravitational stability

- Which ripples will collapse depends on the gravity/pressure balance

- If gravity wins => collapse,
  if pressure wins => oscillations (sound waves)

Cooling lowers pressure, triggers collapse

Applies to both star formation and galaxy formation!

When does gravity win?

- N molecules of mass m in box of size L at temp T.
- Total mass $$M = Nm \sim L^3 \rho$$
- Gravitational energy $$U_G = -\frac{GMm}{L}$$
- Thermal energy $$U_T = NkT$$
- Ratio $$\frac{U_G}{U_T} \sim \frac{GMm}{LNkT} \sim \frac{G\rho L^3 m}{LkT} = \left(\frac{L}{L_J}\right)^2$$
- Jeans Length $$L_J \sim \left(\frac{kT}{G\rho m}\right)^{1/2}$$

Gravity wins when $L > L_J$: large regions collapse, small regions oscillate

- Jeans length (smallest size that collapses)

$$L_J \sim \left(\frac{kT}{G\rho m} \right)^{1/2}$$

=> Large, cool, dense regions collapse

- Jeans mass (smallest mass that collapses)

$$M_J \sim \rho L_J^3 \sim \rho \left(\frac{kT}{G\rho m} \right)^{3/2} \propto \frac{T^{3/2}}{\rho^{1/2}}$$

- Need cool, dense regions to collapse to form stars, BUT galaxy-mass regions can collapse sooner

Conditions at decoupling

- $\rho = 1.4\text{x}10^{-19}$ kg/m^3, T=3000 K

$$L_J \sim \left(\frac{kT}{G\rho m}\right)^{1/2}$$

- $M_J = L_J^3 \rho$

Worksheet 15

Conditions at decoupling

- $\rho = 1.4\text{x}10^{-19}$ kg/m^3, T=3000 K

$$L_J \sim \left(\frac{kT}{G\rho m}\right)^{1/2}$$

$\Rightarrow$ Jeans Length = 50 pc

$\Rightarrow$ Jeans Mass = $3\text{x}10^5$ M_{sun}

- More than a star, less than a galaxy, close to a globular cluster mass.

Globular clusters hold the oldest stars!