Relativistic Doppler Equation

$z = \Delta\lambda/\lambda$

$$1 + z = \sqrt{\frac{1 + \beta}{1 - \beta}} \quad \text{or} \quad \beta = \frac{(1 + z)^2 - 1}{(1 + z)^2 + 1}$$

Where $\beta = v/c$

Relativistic Summary

- time dilation : $t = t_0\gamma$
- length contraction: $L = L_{0/}\gamma$
- mass increases: $m = \gamma\, m_0$
- where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- 0.14c --> $\gamma = 1.01$
- 0.99c --> $\gamma = 7.14$
- 0.998c --> $\gamma = 15$

Key results from special relativity

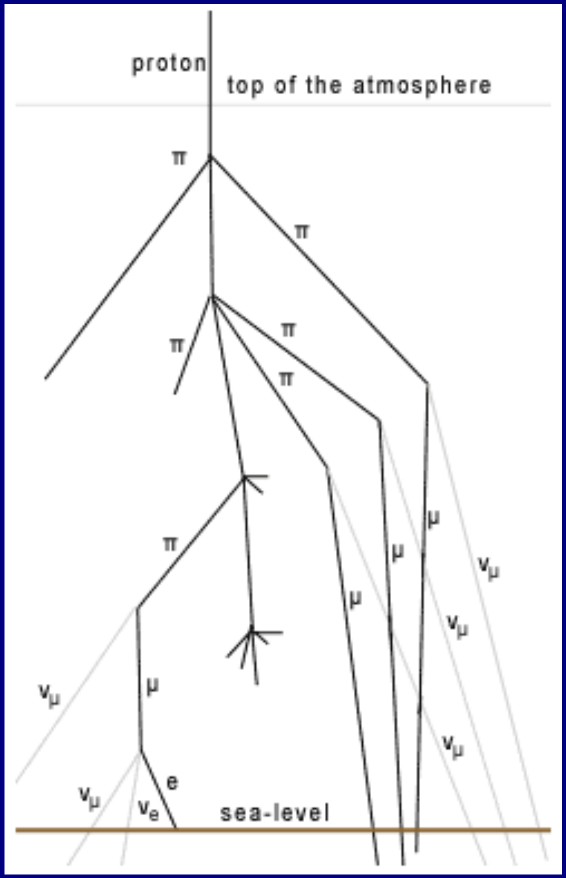
- Speed depends on time and distance, thus space and time are affected by relativity.

- Length contraction: the length you measure of an object, depends on the motion of the object relative to you.

- Time dilation: the time you clock between two events depends on the motion of the event relative to you.

Worksheet #10

Suppose a muon is created 5 km up in the atmosphere. If it is moving at 0.998c and has a lifetime of 2×10^{-6} seconds, can it reach the ground?



Class Project

- Glance over 7 papers (read the abstracts)
- Choose 1 paper to read carefully
- Prepare a talk using powerpoint as if you were a University professor proposing to get funding to continue this research

- See handout for details

Black Holes

Credit: Interstellar

What about gravitation?

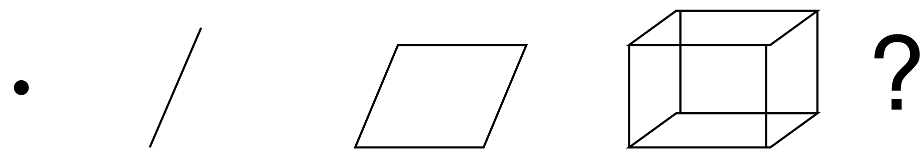
- What is gravity? Newton's picture is a weird force that somehow reaches across huge distances of space, keeping planets in their orbits.

- Einstein removed the question of how this force would act at a distance - With general relativity he explained gravity in terms of structure of space and time.

Consider this: two persons traveling along a straight line at Earth, in opposite directions, will meet. This is attributed to the curvature of the Earth.

Spacetime: 4 dimensions

- Curved space is hard to visualize in 3D. Let's start with the dimensions (0-4):



- If the 4th dimension is time, how would you look? You would be stretched through time.

- Why do observers disagree about time and distances? Compare showing someone a real copy of an object (3D) and a photograph (2D).
  - A 2D picture of an object may look very different, depending on how you took the photo.
  - If you believe the 2D photo reflects reality, you will argue endlessly about how objects look.
  - Spacetime is 4D, but we can only see 3D 'pictures': why we get different results.

Spacetime

- A 4D Pythagorean theorem describes the *interval* between two events
  - (time is seen as intimately related to space).

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

- Events happen in both space *and* time.

- Motion affects rulers and clocks (time dilation and length contraction), and motion is caused by acceleration.

- **Thus, "gravity" must affect the shape of space, and flow of time.**

Black Holes and General Relativity

General Relativity: Einstein's description of gravity (extension of Newton's). Published in 1915. It begins with:

The Equivalence Principle

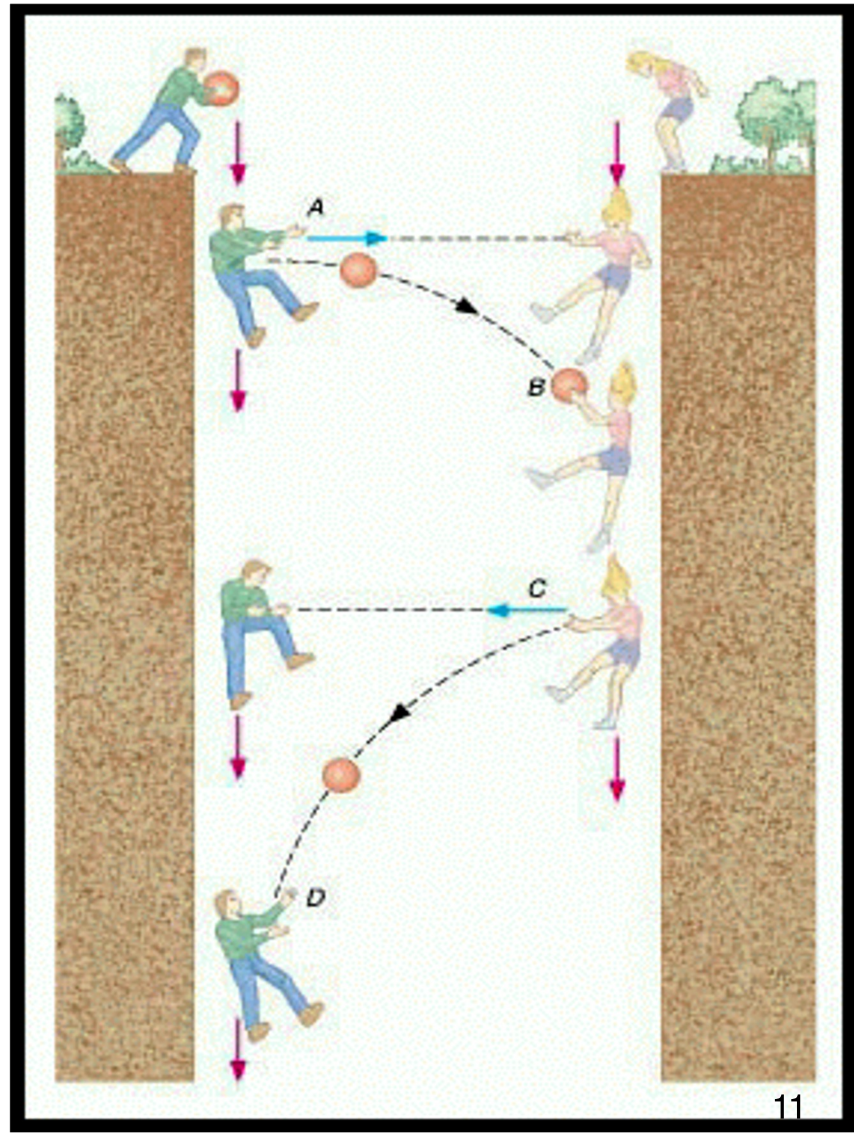
Let's go through the following series of thought experiments and arguments:

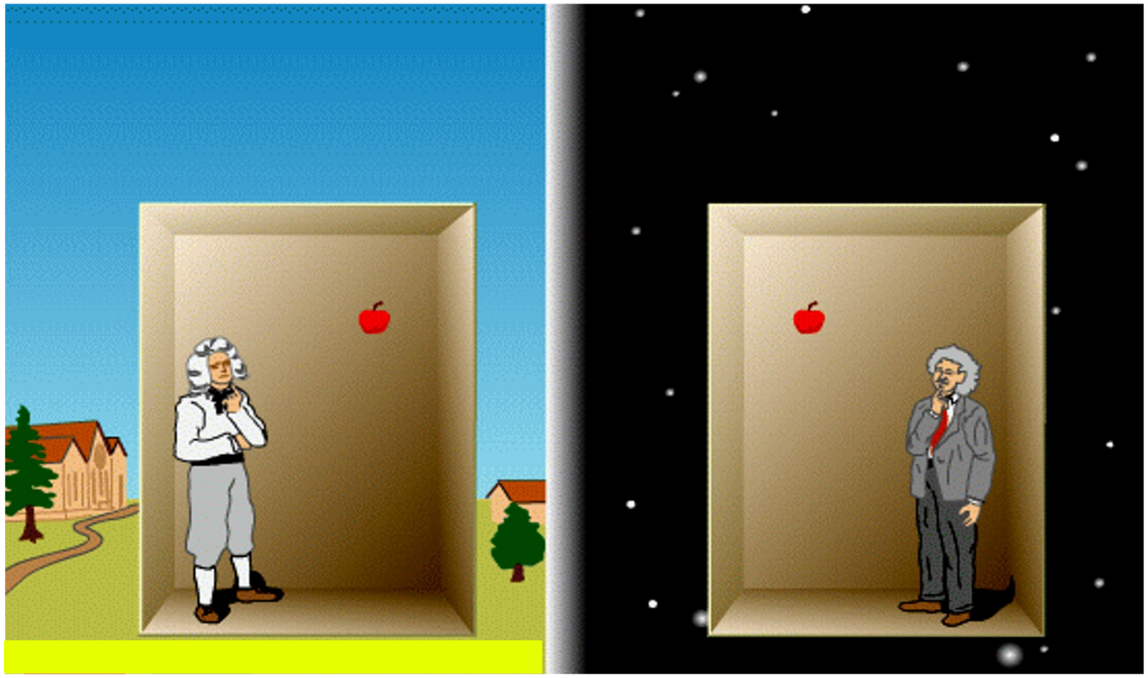
1) Imagine you are far from any source of gravity, in free space, weightless. If you shine a light or throw a ball, it will move in a straight line.

2. If you are in freefall, you are also weightless. Einstein says these are equivalent. So in freefall, the light and the ball also travel in straight lines.

3. Now imagine two people in freefall on Earth, passing a ball back and forth. From their perspective, they pass the ball in a straight line. From a stationary perspective, the ball follows a curved path. So will a flashlight beam, but curvature of light path is small because light is fast (but not infinitely so).

The different perspectives are called frames of reference.

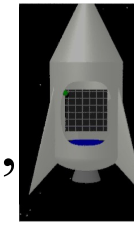




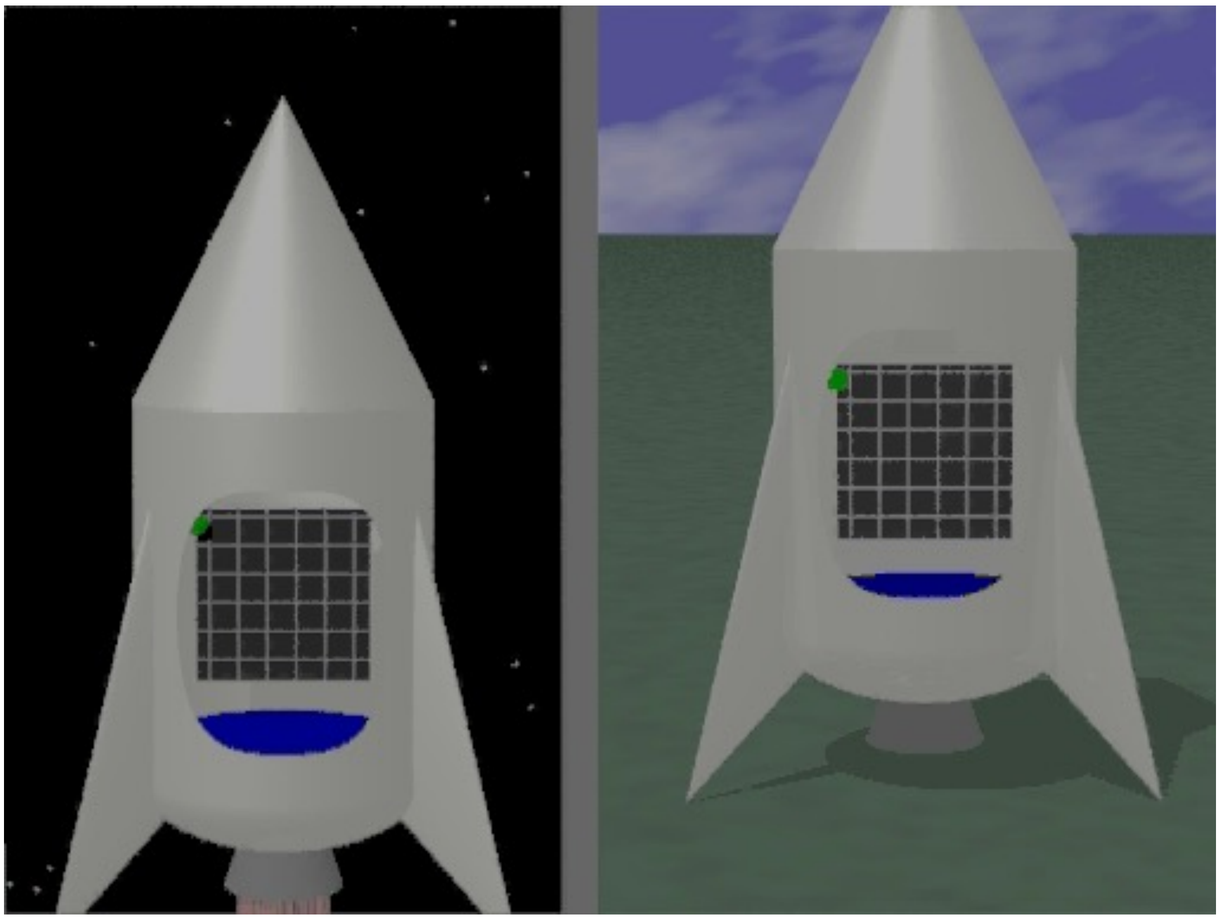
4. Gravity and acceleration are equivalent. An apple falling in Earth's gravity is the same as one falling in an elevator accelerating upwards, in free space.

5. All effects you would observe by being in an accelerated frame of reference you would also observe when under the influence of gravity.

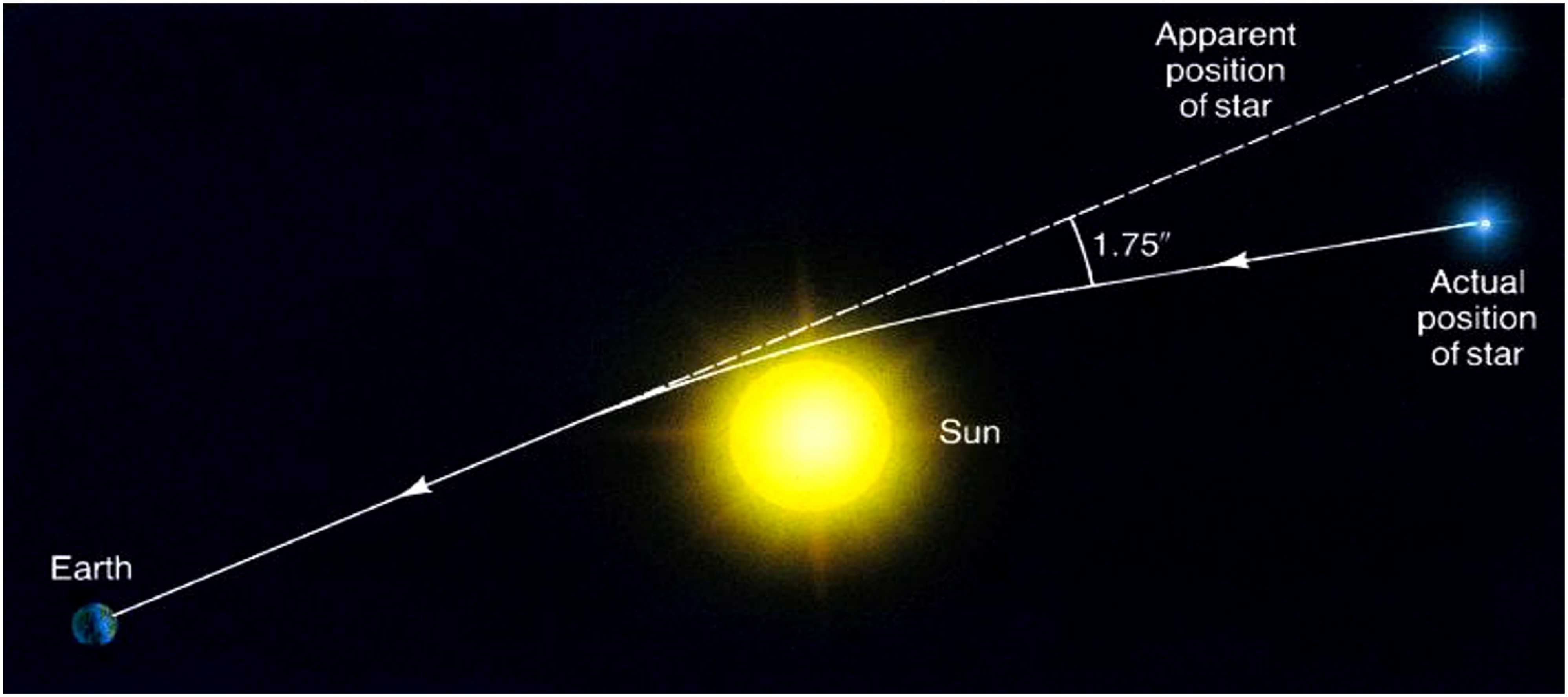
Examples:



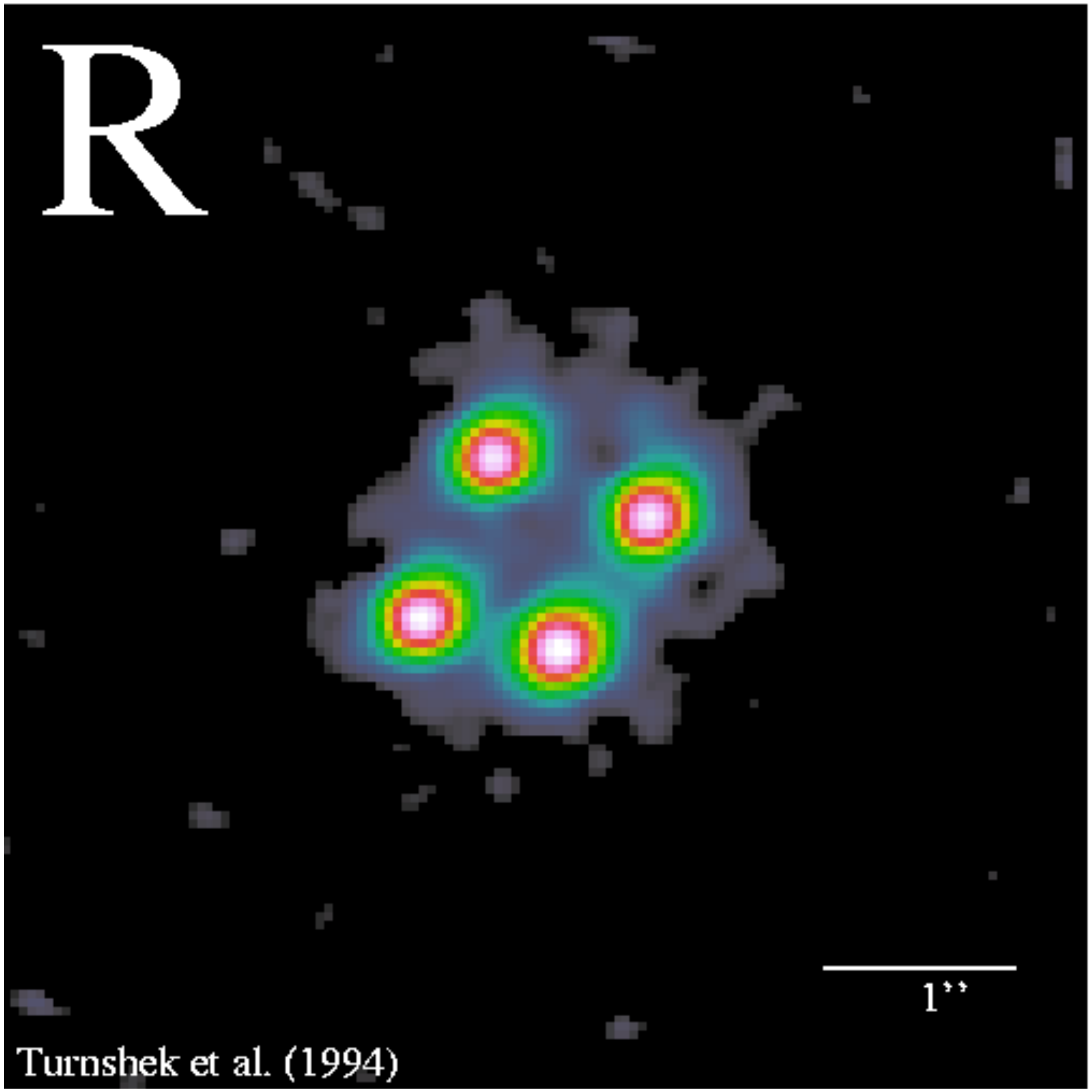
1) Bending of light. If light travels in straight lines in free space, n gravity causes light to follow curved paths.



Observed! In 1919 eclipse by Eddington



<u>Gravitational lensing</u> of a single background quasar into 4 objects



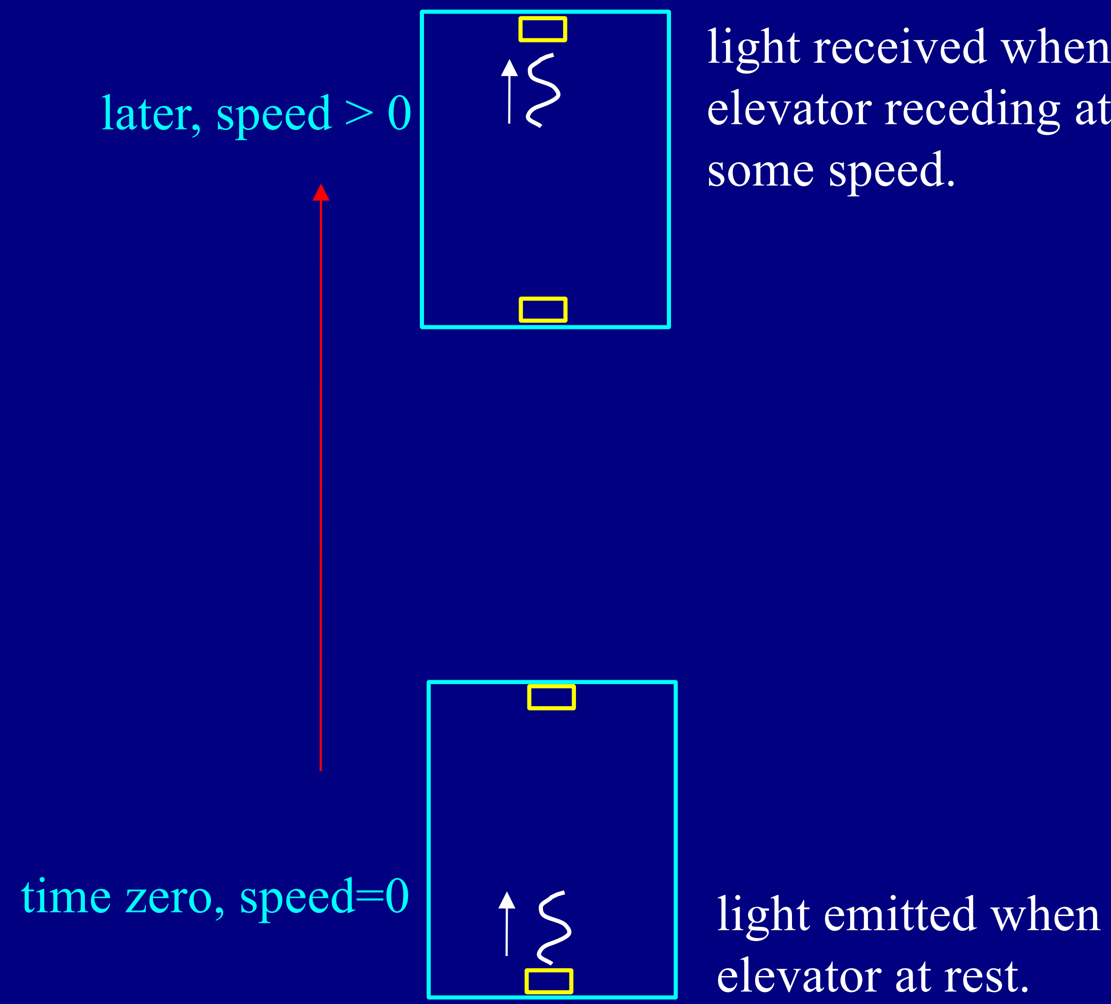
1413+117 the
"cloverleaf" quasar
A 'quad' lens

Gravitational lensing. The gravity of a foreground cluster of galaxies distorts the images of background galaxies into arcs

Saturn-mass
black hole

2. Gravitational Redshift

Consider accelerating elevator in free space (no gravity).

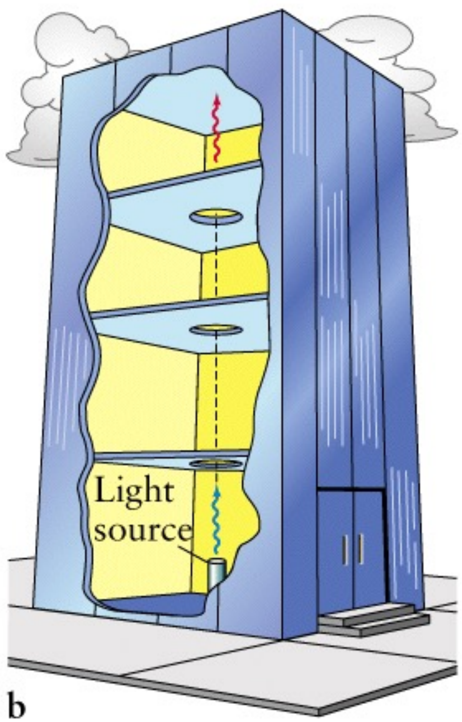


Received light has longer wavelength (or shorter frequency) because of Doppler Shift ("redshift"). Gravity must have same effect! Verified in Pound-Rebka experiment.

2. Gravitational Redshift

If light has to "climb out" of a potential well, its wavelength is shifted to the red. That is, the light loses energy.

Spectral lines emit at specific frequencies, which become redshifted. Thus, time runs more slowly close to a massive object (fewer cycles/s as measured by someone at a distance).



$$\frac{\Delta\lambda}{\lambda_0} = \frac{GM}{c^2 R}$$

G is the universal gravitational constant,

M is the mass of the central object,

R is the distance to the center of the object.

- Let's consider the expression for gravitational redshift:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{GM}{c^2 R}$$

- What happens when R $\rightarrow$ 0, or M/R $\rightarrow \infty$?

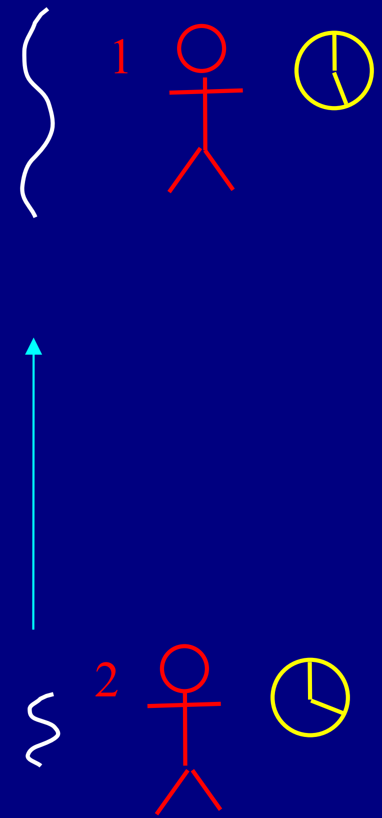
- The photon will be redshifted to infinite wavelength - equivalently zero energy! It's redshifted out of existence!

- Thus light can't escape – a **black hole**.

3. Gravitational Time Dilation

A photon moving upwards in gravity is redshifted. Since

$$\nu = \frac{1}{T}$$

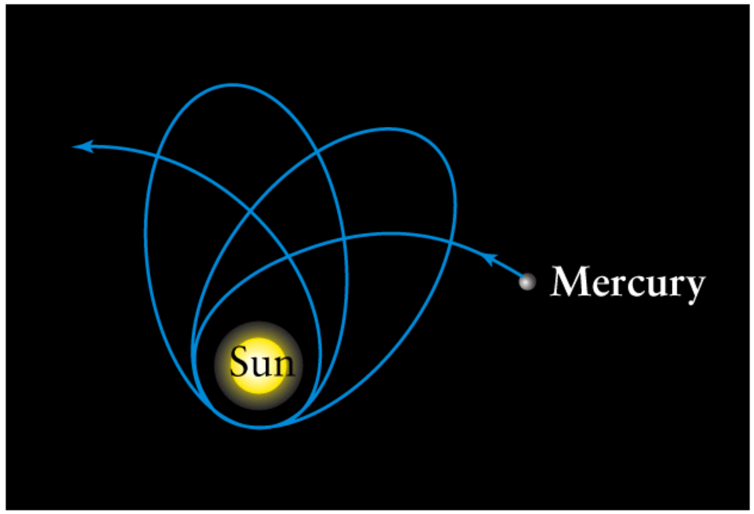


the photon's period gets longer. Observer 1 will measure a longer period than Observer 2. So they disagree on time intervals. Observer 1 would say that Observer 2's clock runs slow!

All these effects are unnoticeable in our daily experience! They are tiny in Earth's gravity, but large in a black hole's.

Observational tests of GR

4. Precession of Mercury's orbit: the advance of perihelion.



Most precession is due to the other planets, but after taking this into account the observations were off by 43 arcsec/century (measurable).

Newton's law of gravity assumes space is flat and time is absolute. In reality, time runs more slowly and space is more curved on the part of the orbit that is closer to the Sun. GR takes this into account.

Discussion Question:

Suppose we start with two atomic clocks and take one up a high mountain for a week. Which is true?

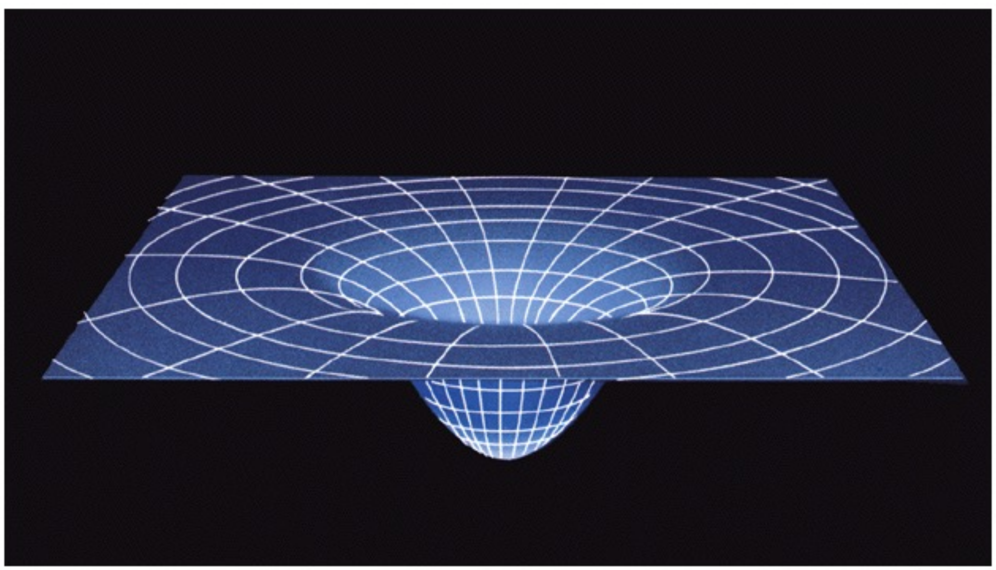
A: The two clocks will show the same amount of time has passed.

B: The mountain clock will be slightly ahead (fast)

C: The mountain clock will be slightly behind (slow)

General relativity

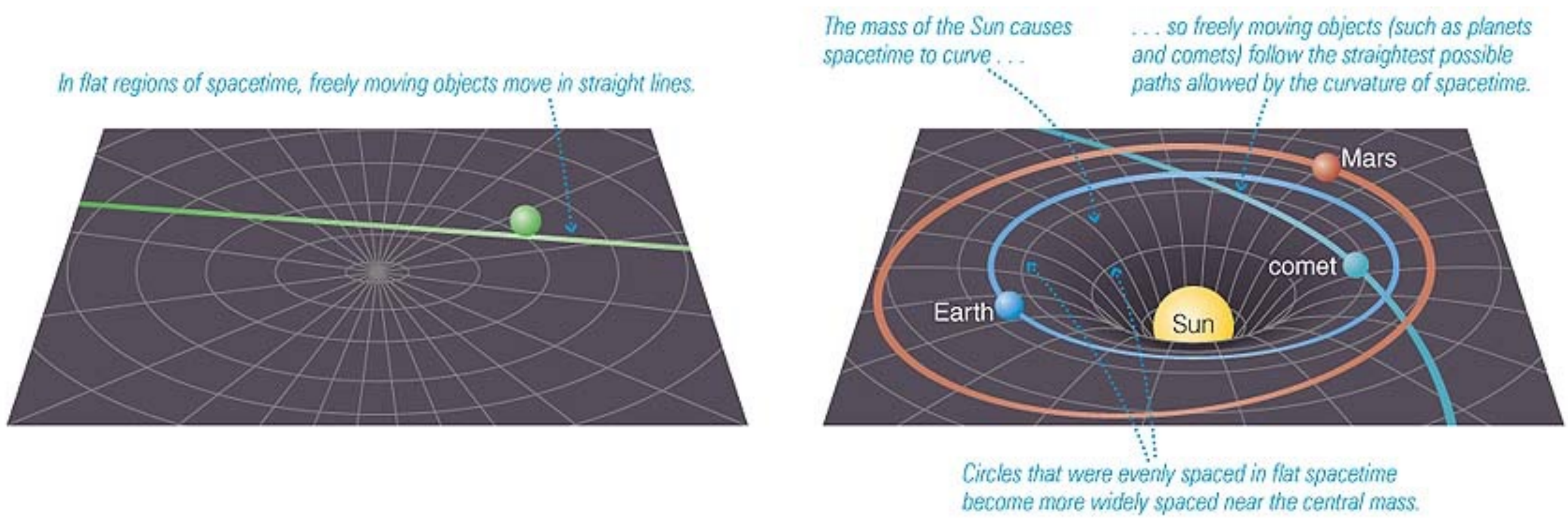
- Main concept: *matter distorts spacetime*.
- Without a large mass, spacetime is flat and objects move naturally along straight lines.
- When a large mass is present, the natural paths of objects are curves or orbits.
- Gravitation is seen as the natural path of an object in whatever curvature is present. (Compare to Newton's mysterious "action at a distance".)



Lines indicate natural paths of objects.

How many dimensions?

The standard picture of distorted spacetime. Massive objects (matter) distorts the spacetime: would be flat without the presence of mass.



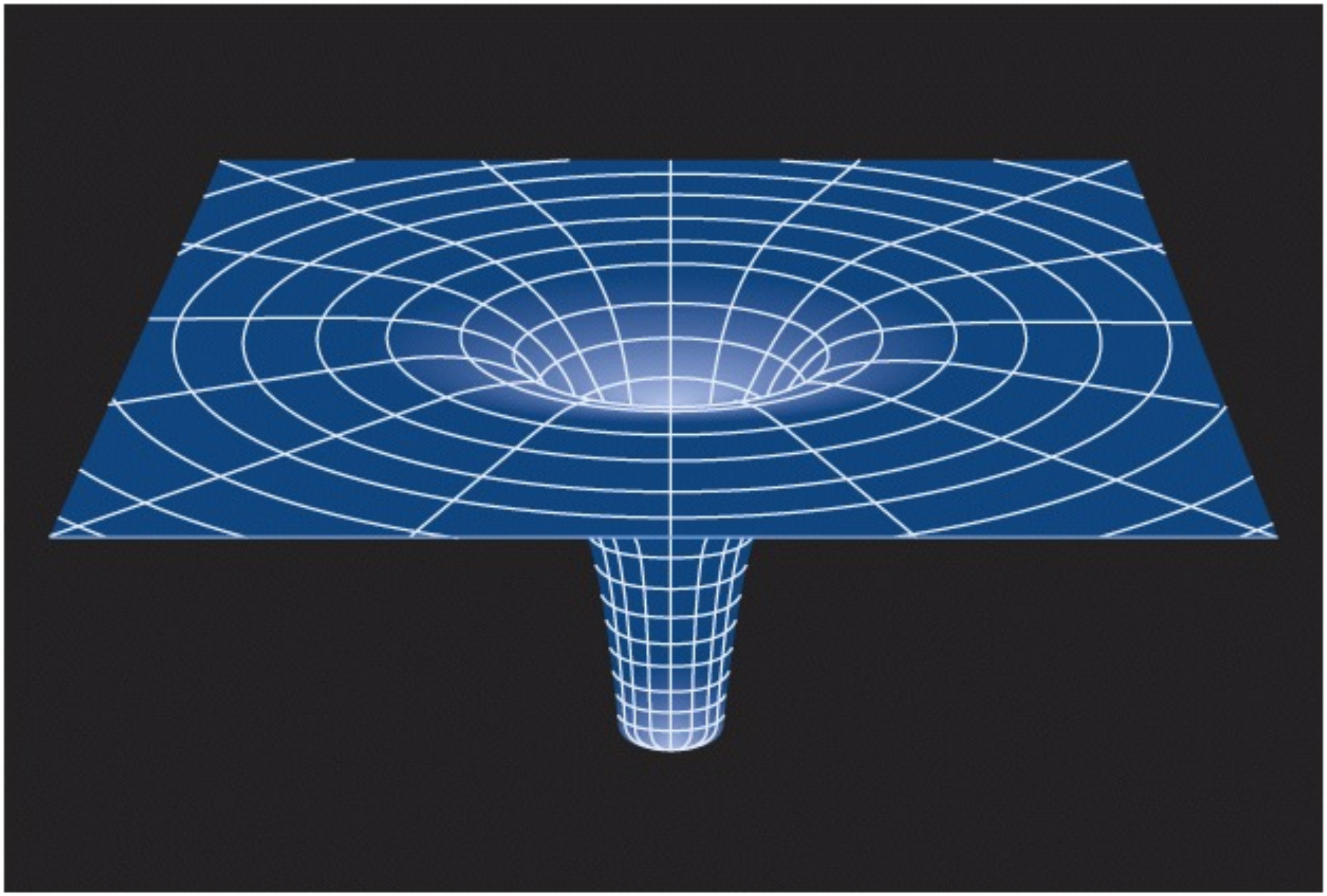
a On a flat rubber sheet, evenly spaced circles all have circumference $2\pi r$.

b The Sun curves spacetime much like a heavy weight curves a rubber sheet.

Lines indicate natural paths of objects.

Light will also bend around massive objects, following these lines.

Highly distorted spacetime near black hole – gravitational “well” becomes infinitely deep.



Escape velocity from a BH

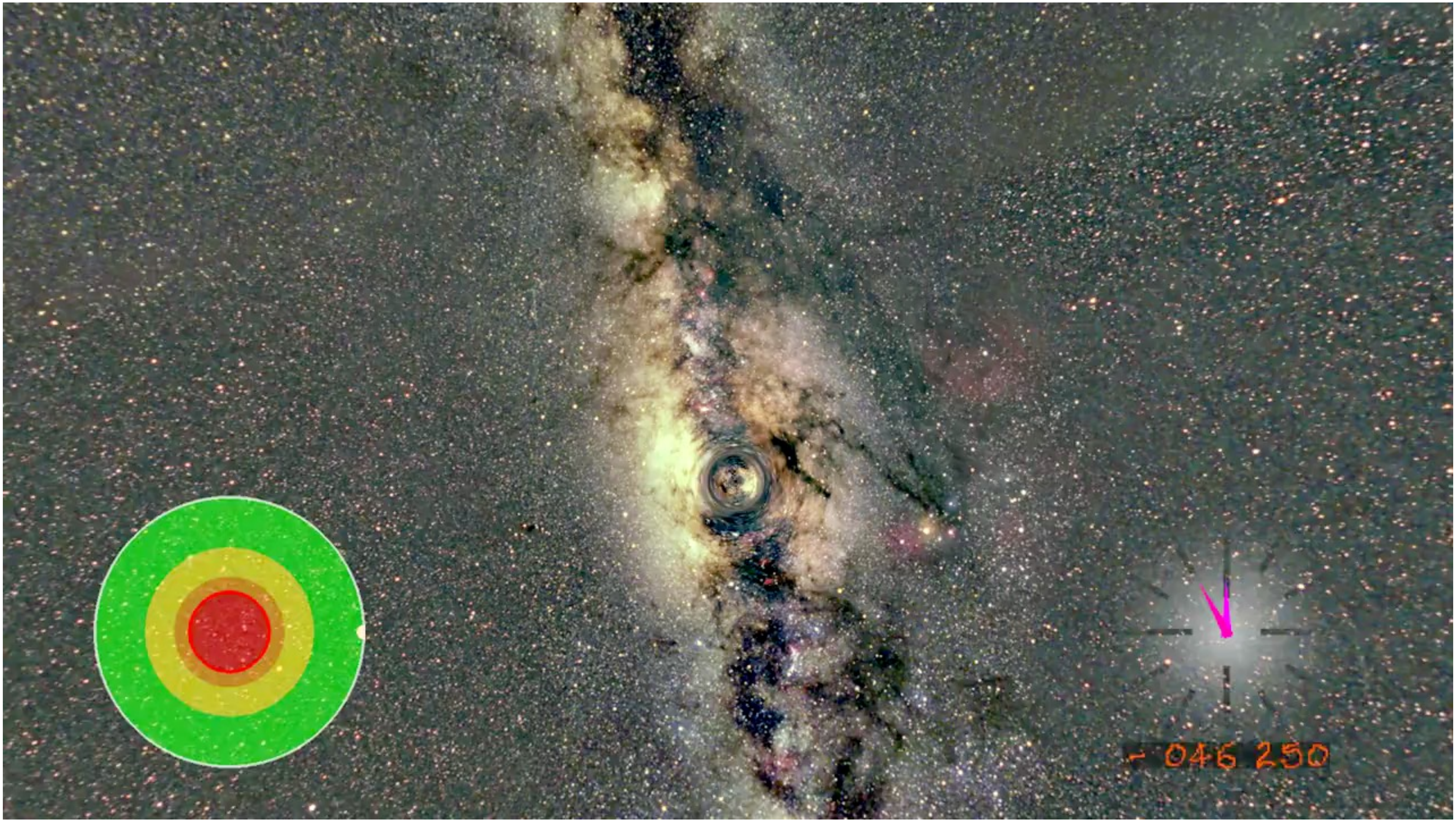
- Another way to look at it is by using the concept of escape velocity. The escape velocity of a black hole is the speed of light (or could even be more) – so nothing escapes, not even light.

$$V_{esc} = \sqrt{\frac{2GM}{R}}$$

- What if escape velocity is equal to the speed of light, c (a black hole)? Solve for R:

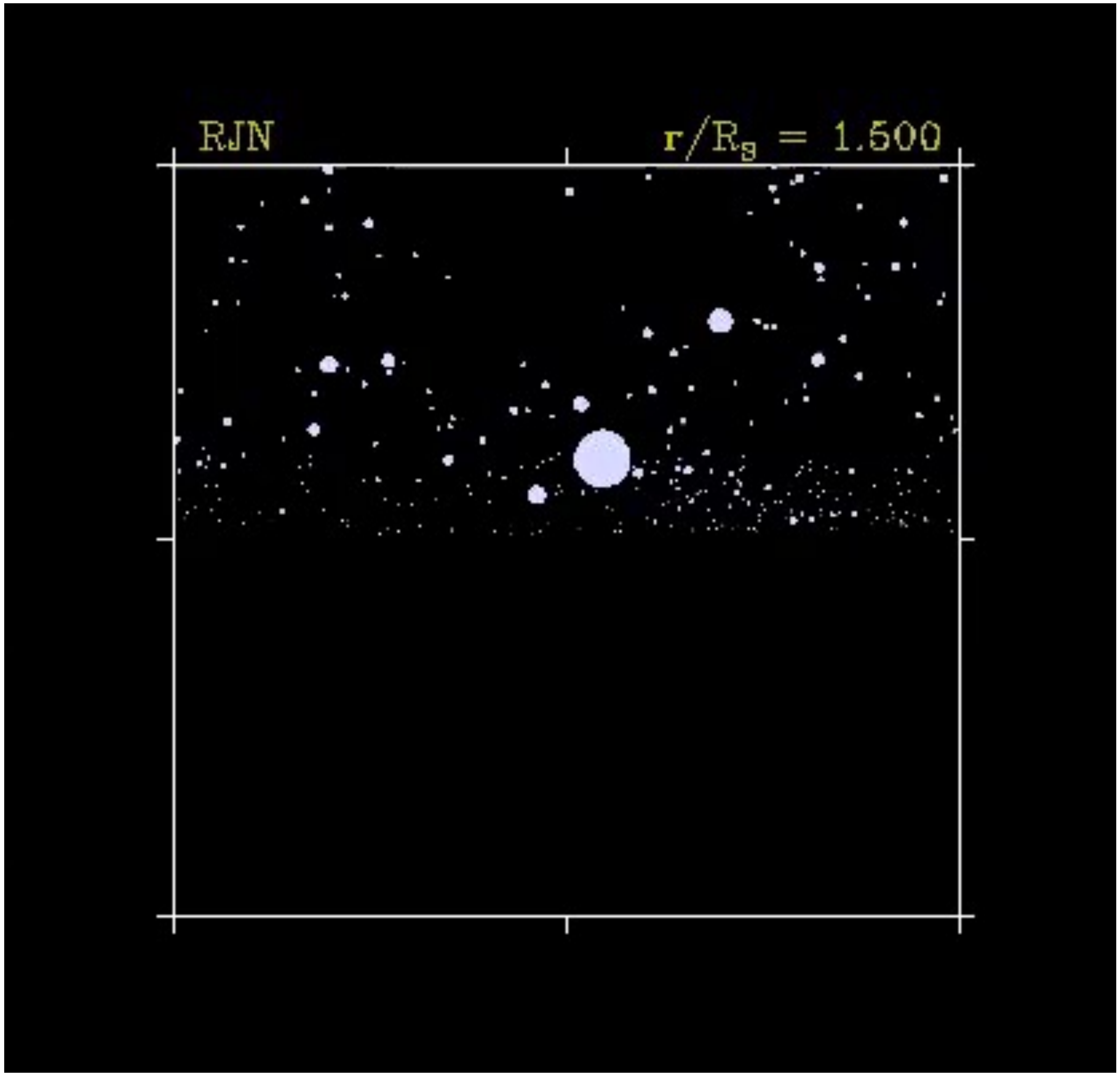
$$R_{Vesc=0} = \frac{2GM}{c^2} = R_{Sch}$$: the *Schwarzschild Radius.*

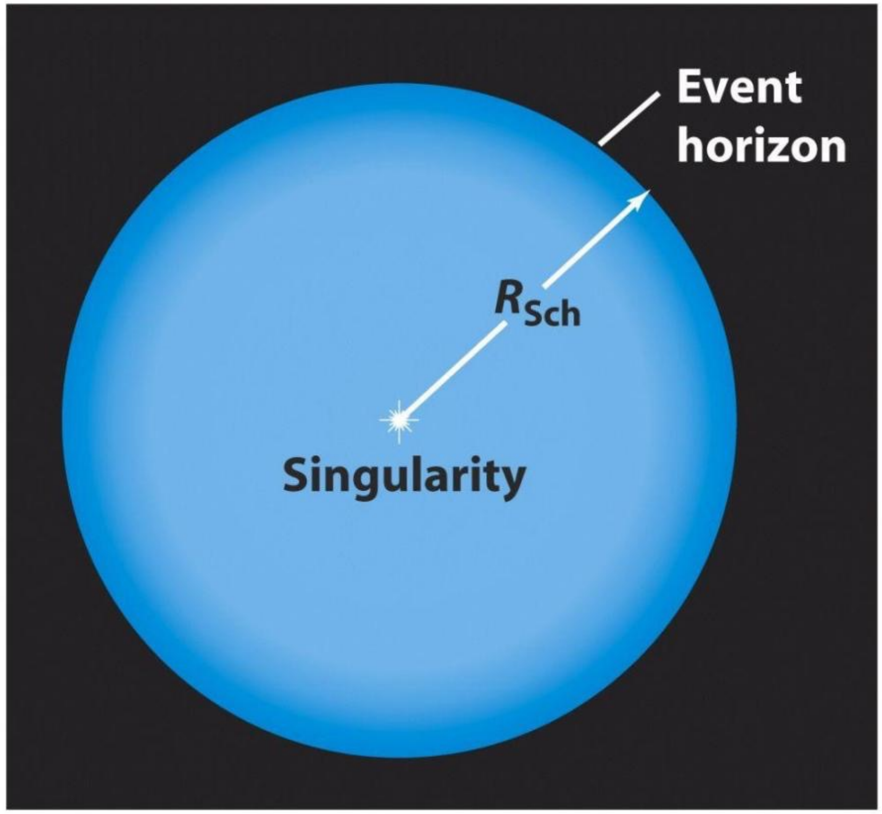
- The surface at the Schwarzschild Radius is called the *event horizon*, and is the boundary between the black hole and the rest of the Universe. Note that it is not a physical surface.



Simulation by Andrew Hamilton

Circling a Black Hole at the Photon Sphere:





What lies within the event horizon? In principle, everything falls into a *singularity*, a point with large mass but no size – therefore infinite density.

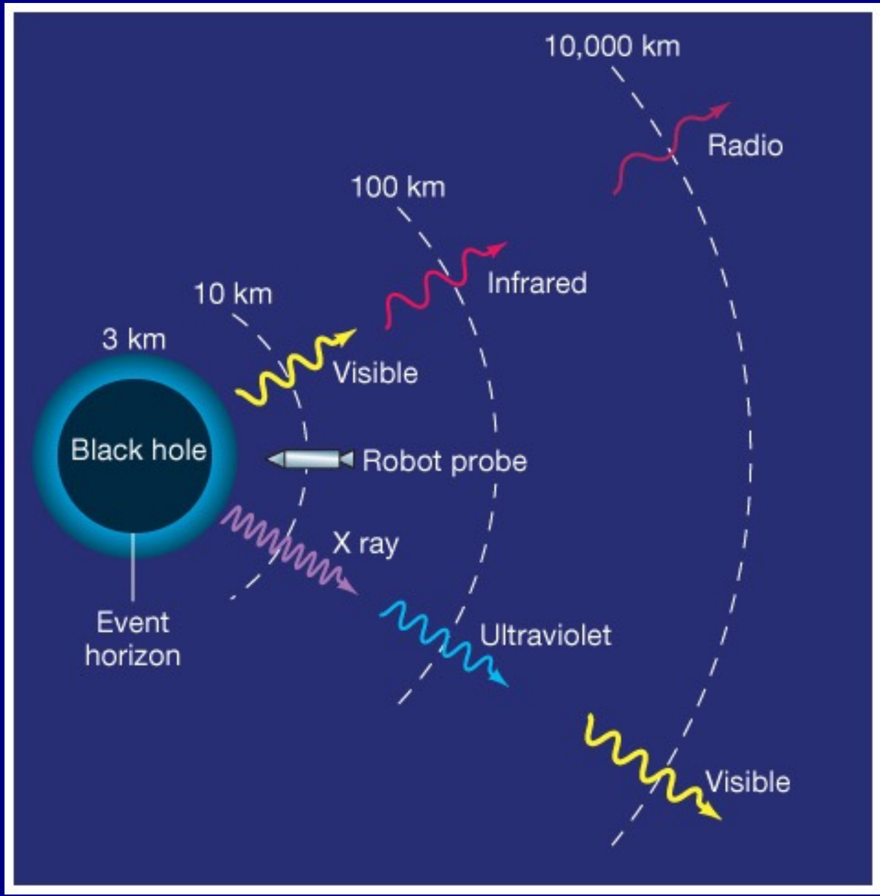
Current physics fails – need theory of *quantum gravity*.

Effects around Black Holes

1) Enormous tidal forces.

2) Gravitational redshift. Example, blue light emitted just outside event horizon may appear red to distant observer.

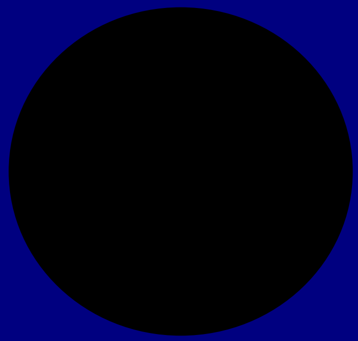
3) Time dilation. Clock just outside event horizon appears to run slow to distant observer. At event horizon, clock appears to stop.



Black Holes have no Hair

Properties of a black hole:

- Mass
- Spin (angular momentum)
- Charge (tends to be zero)



- The “radius” of the black hole is solely dependent upon its mass. For the Earth, it would be the size of a golf ball. For the Sun, it would be about 3 km.

- A useful relation: $R_{Sch} = 3$ km $(M/M_\odot)$

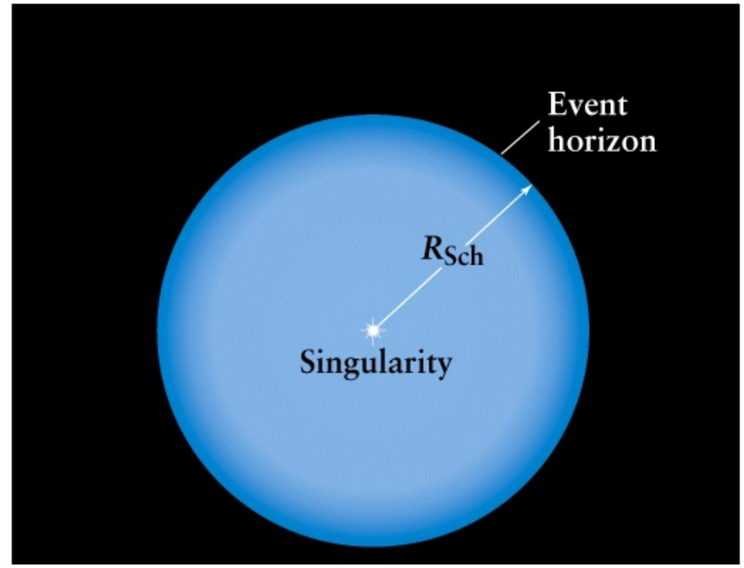
- Question: What would happen to the orbit of the Earth if the Sun became a BH? Nothing!

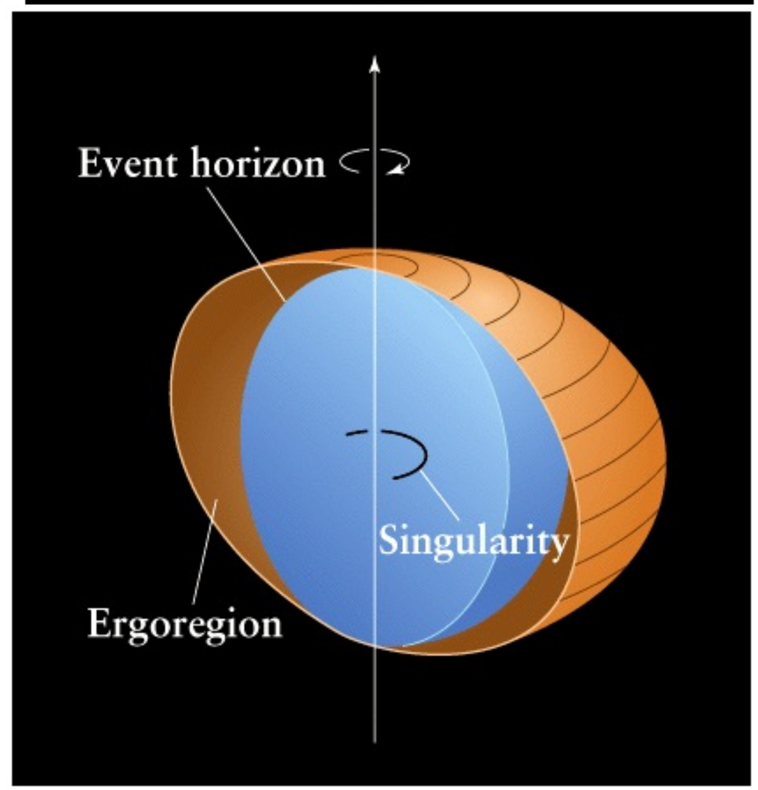
$$F = Gm_1m_2/R^2$$

- The excitement starts only very close to the event horizon, e.g., within 1000 km for a 10 $M_\odot$ black hole.

Rotating black holes

- Rotating black holes have a more complicated structure, containing an ergosphere.
- The ergosphere is a region outside the event horizon, where spacetime is dragged along with the rotation of the black hole.
- In the ergosphere, particles may escape by gaining energy on the expense of the rotation ot the black hole: Penrose process.
- Most black holes should rotate.
- Solution to GR for rotating black holes is called the Kerr metric





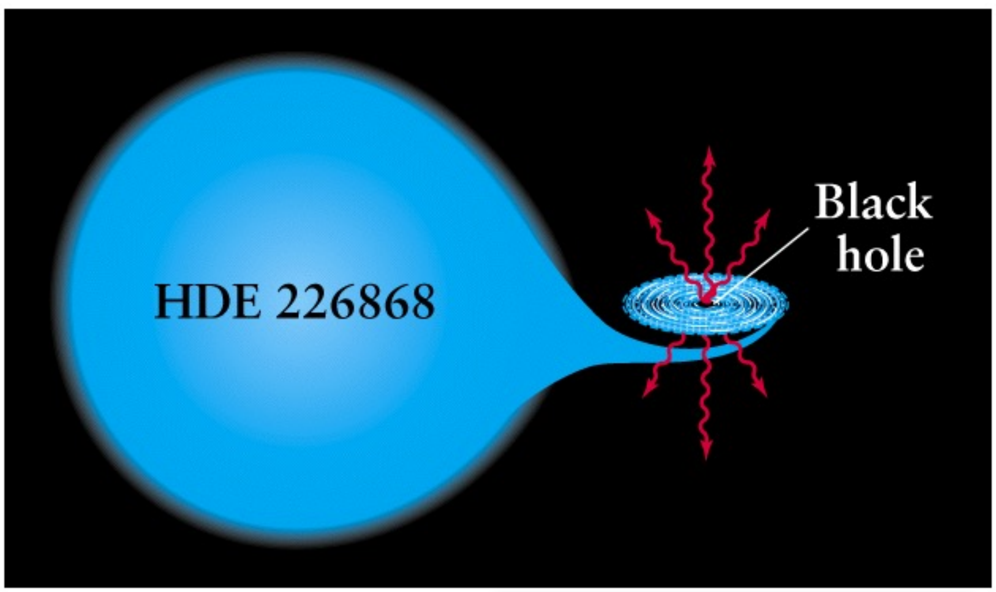
What would happen if you fell into a BH?

- If you went in feet first, the tidal forces would pull your feet off your ankles, then your legs off your torso, etc. From your point of view, this would happen at “normal” rate, and you’d fall through the event horizon (but not enjoy it).

- From the outside, you would appear to take an infinite length of time to reach the event horizon, due to gravitational slowing of time! Light from you would be redshifted.

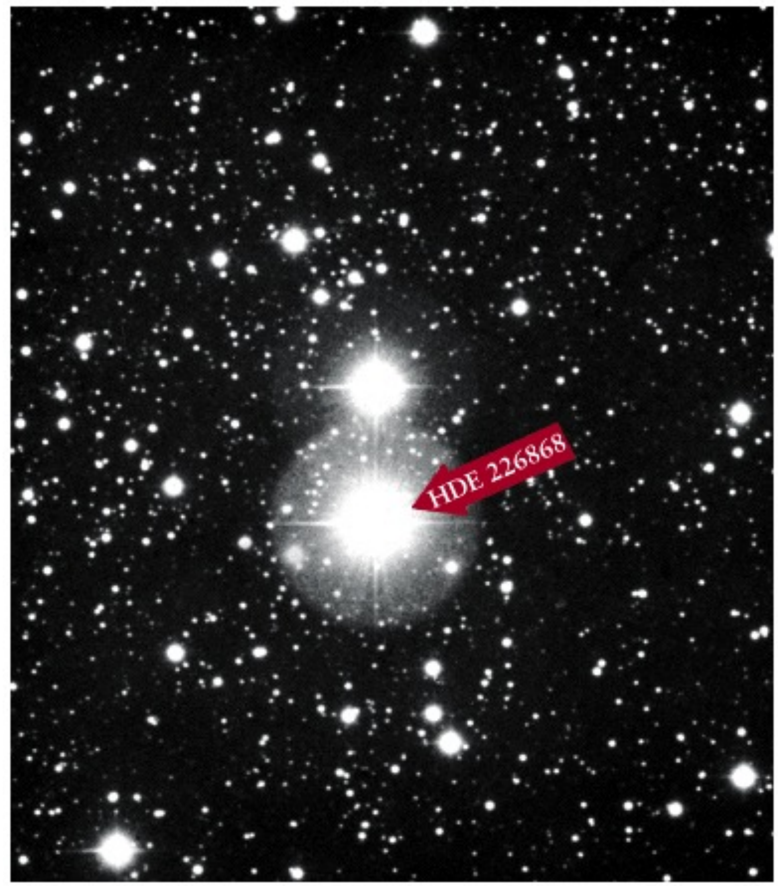
Observations of black holes

- Stellar-size black holes– current theory says that the most massive white dwarf is 1.4 $M_\odot$ and the most massive neutron star is 2 to 3 $M_\odot$.
- If we find a close binary with the collapsed star well above 3 $M_\odot$, this should be a black hole.



a

Cygnus X-1: **7** $M_\odot$

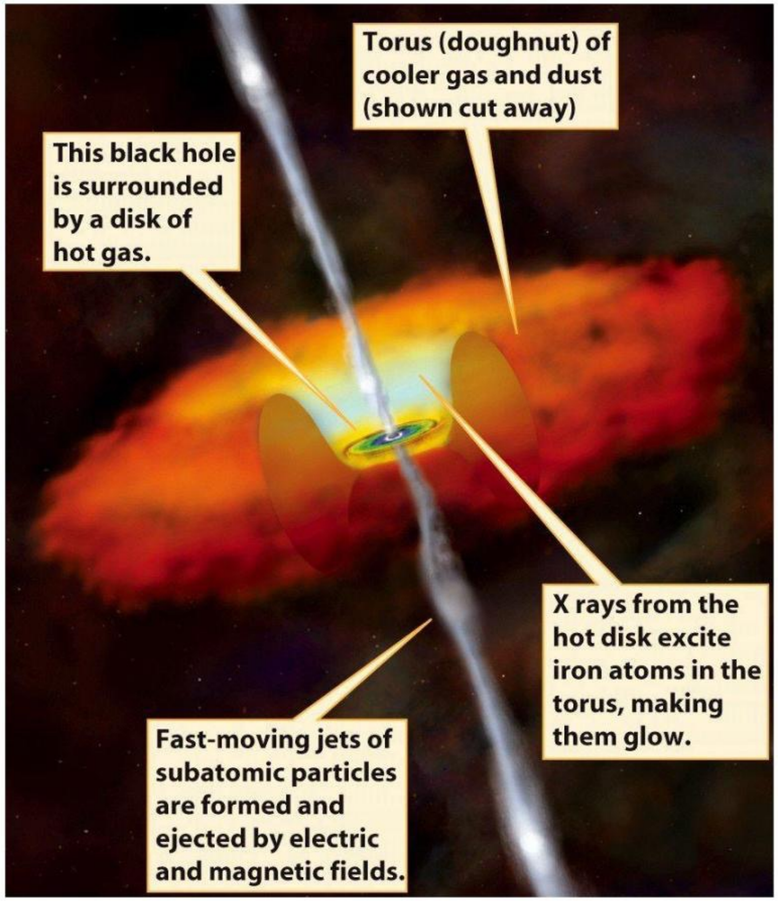


Supermassive black holes

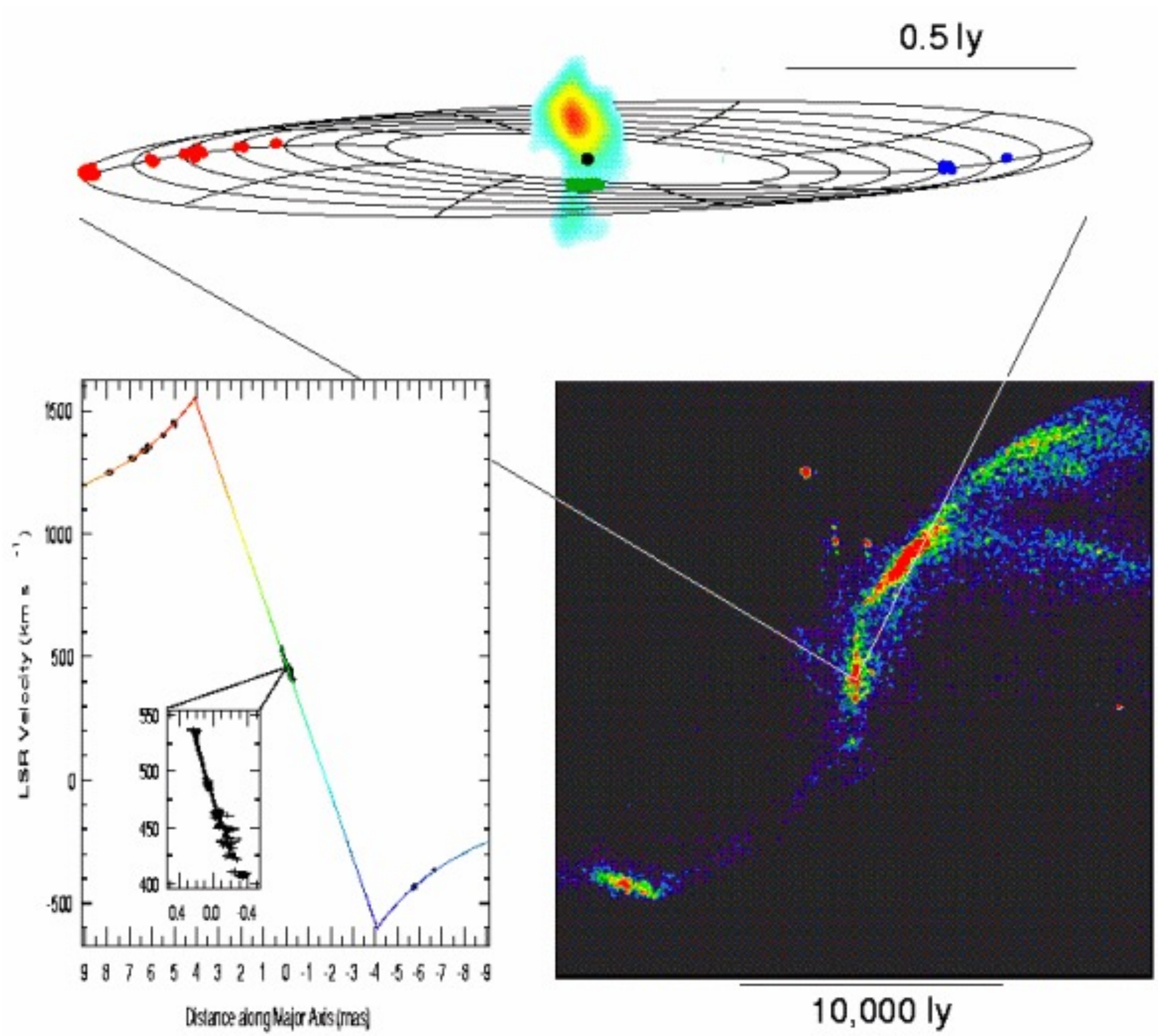
- We find supermassive black holes at the centers of essentially all large galaxies.

- Evidence comes from high orbital speeds of nearby gas or stars. Masses range from $10^6\ M_\odot$ to $10^9\ M_\odot$.

- These objects show themselves by their accretion disks or jets.

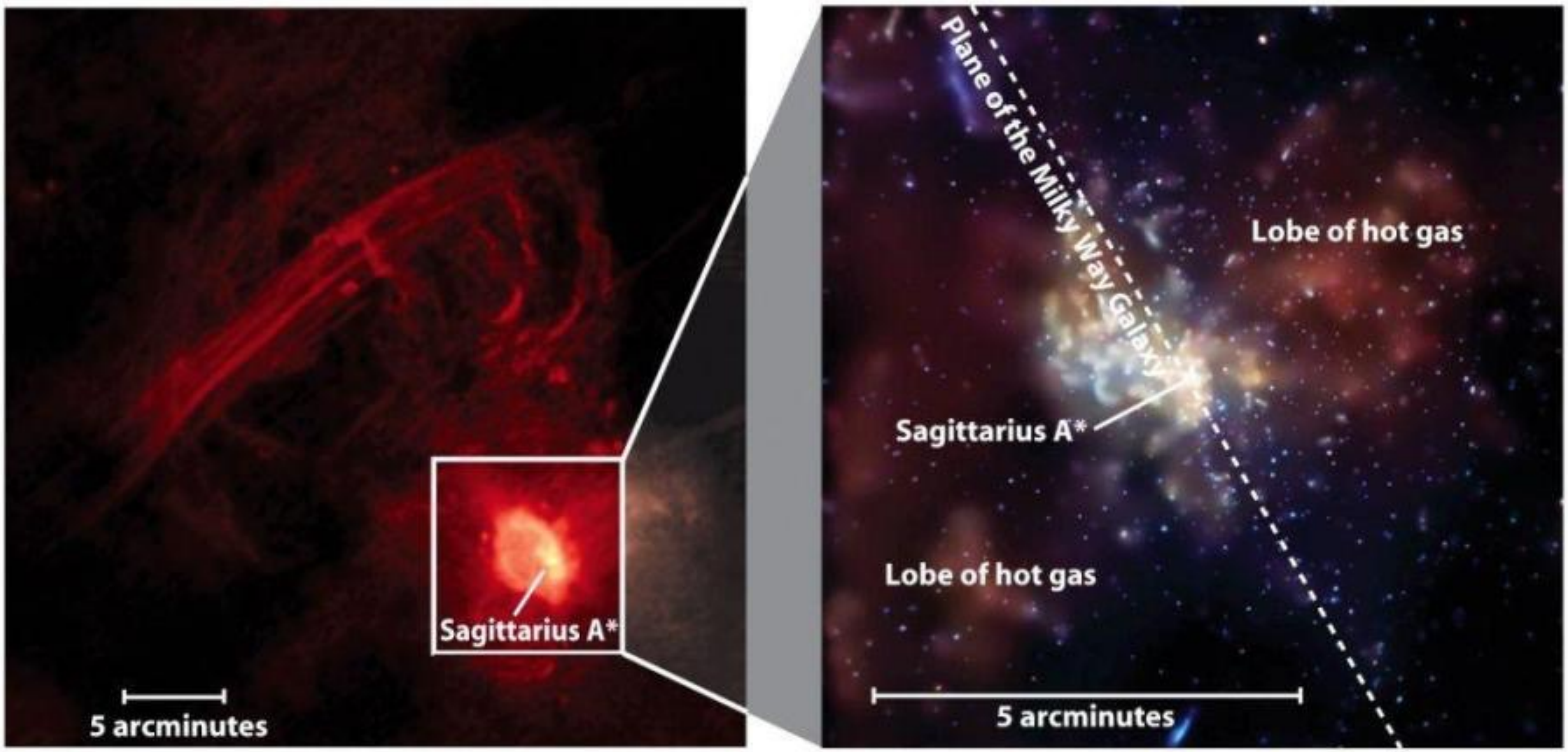


Best evidence: NGC4258



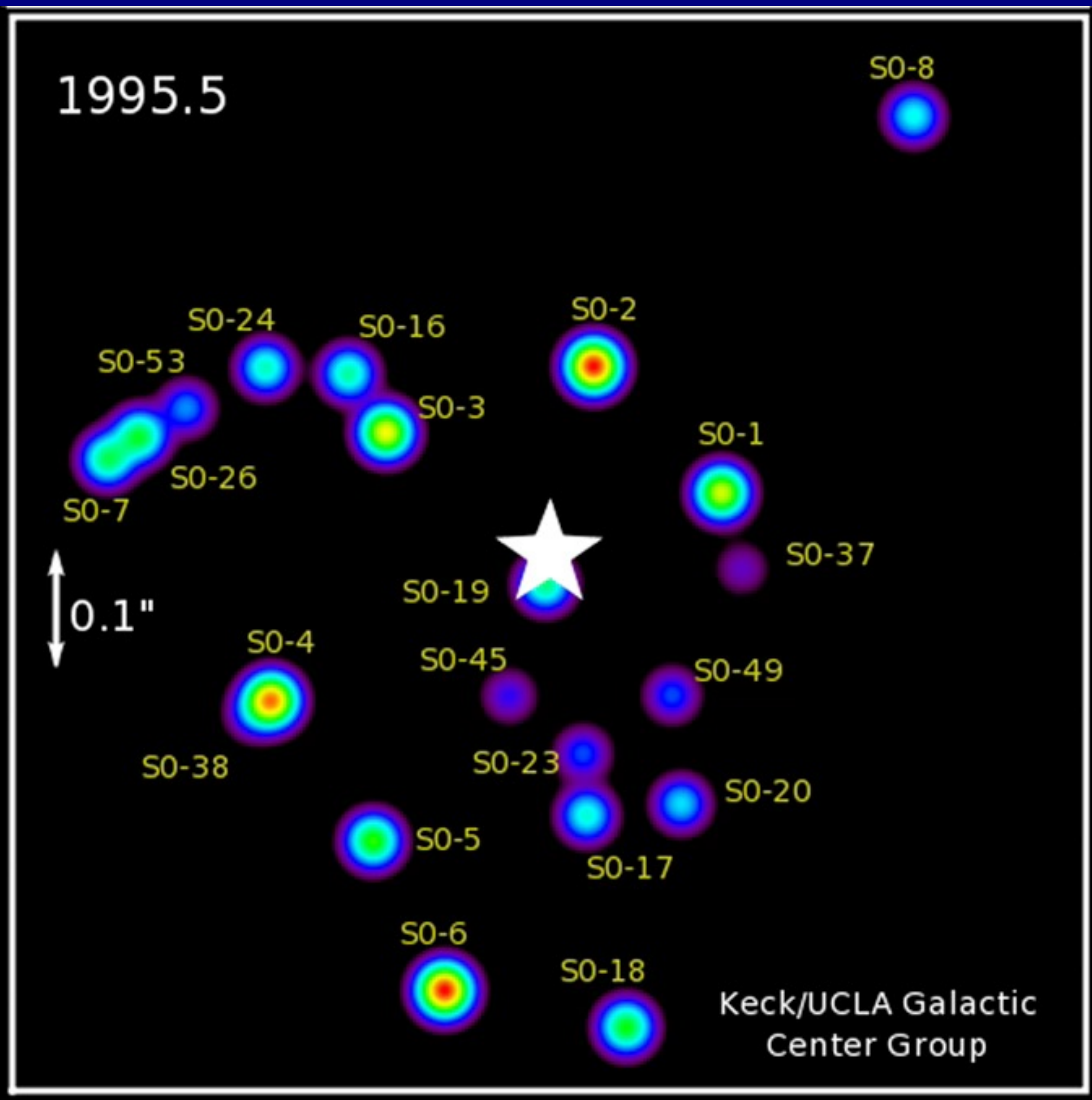
SgrA*

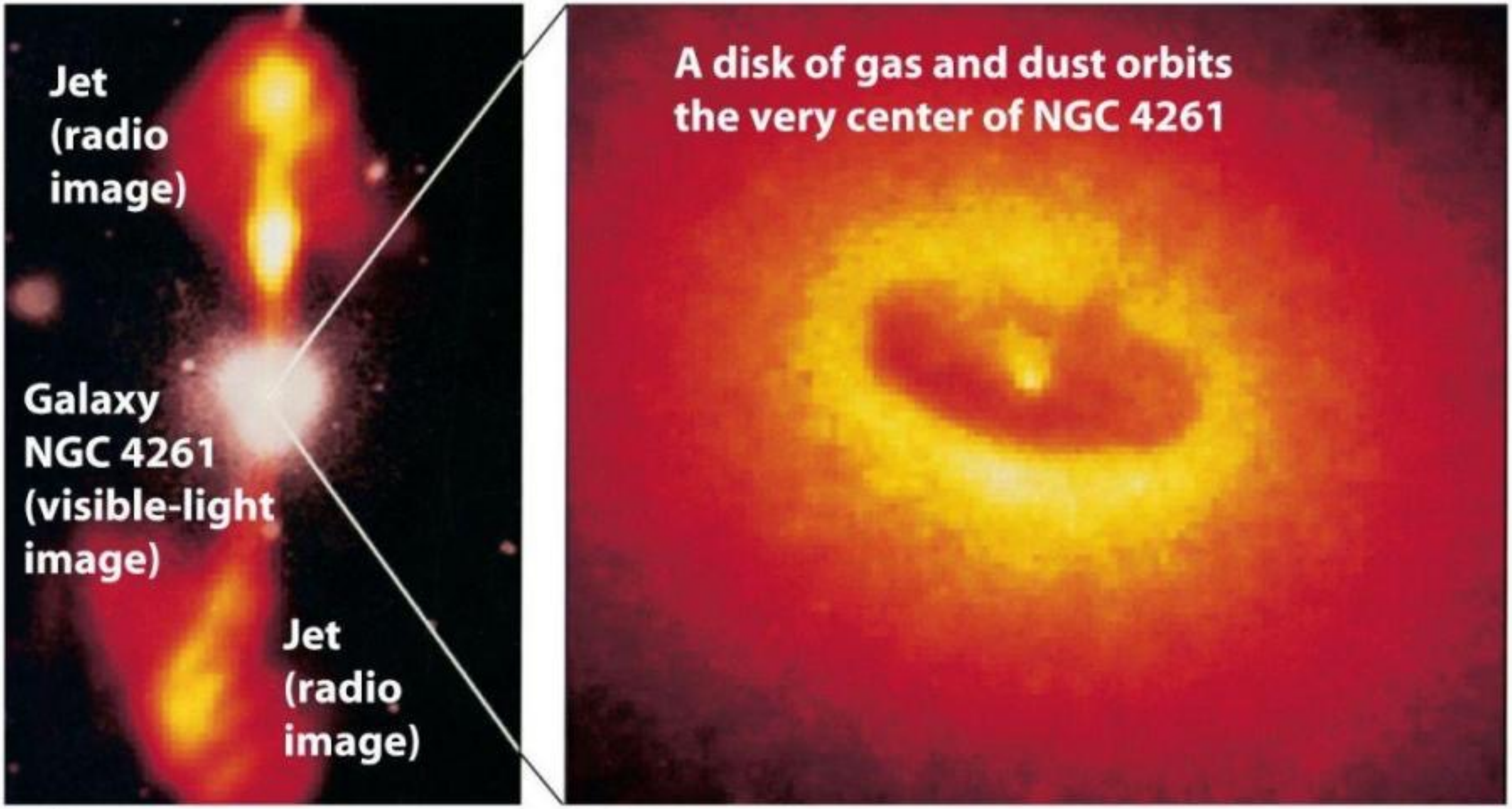
- The dynamical center of the Milky Way is called SgrA*, and contains a supermassive black hole.



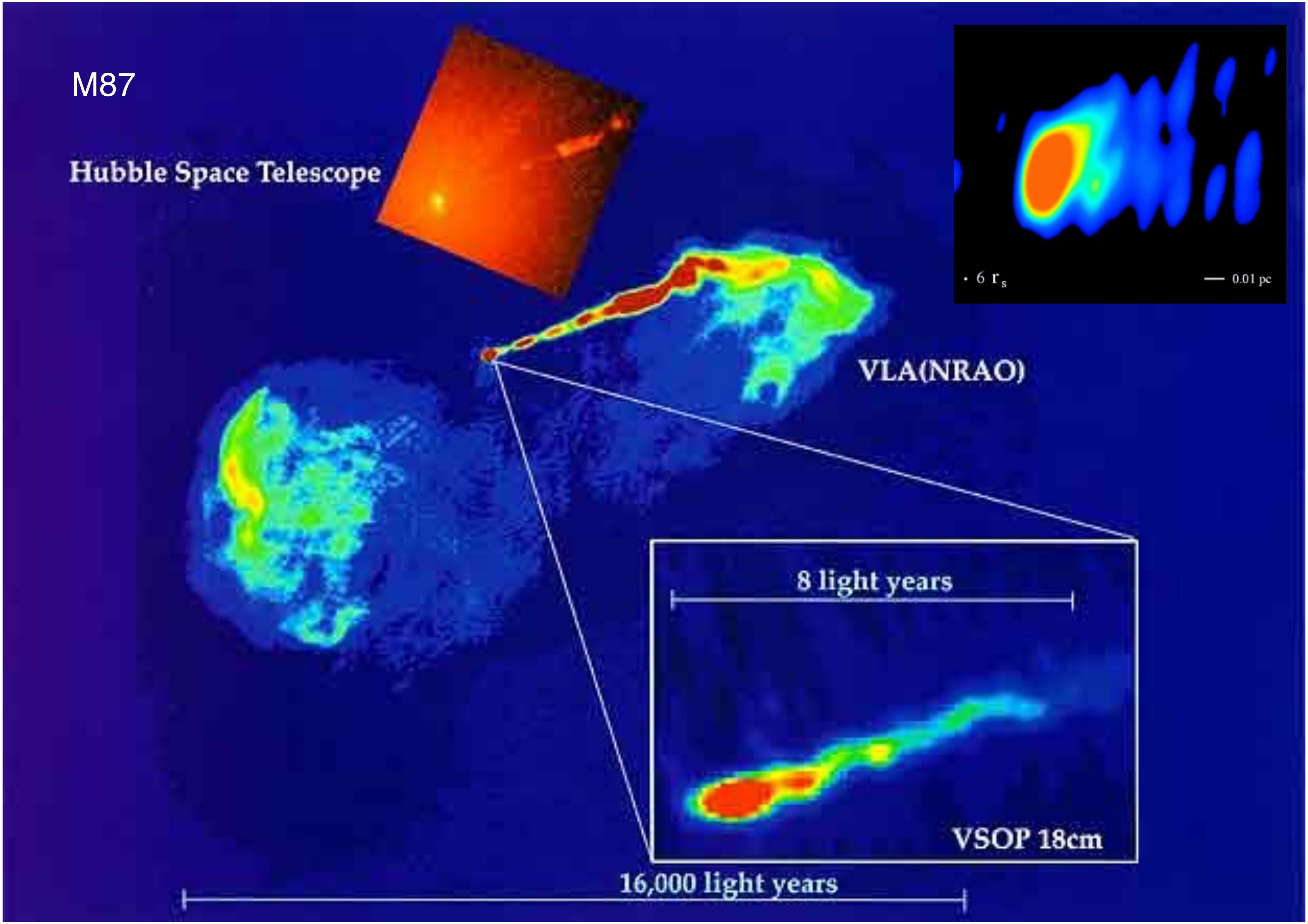
(a) A radio view of the galactic center (b) An X-ray view of the galactic center

Supermassive (3 million solar mass) Black Hole at the Galactic Center

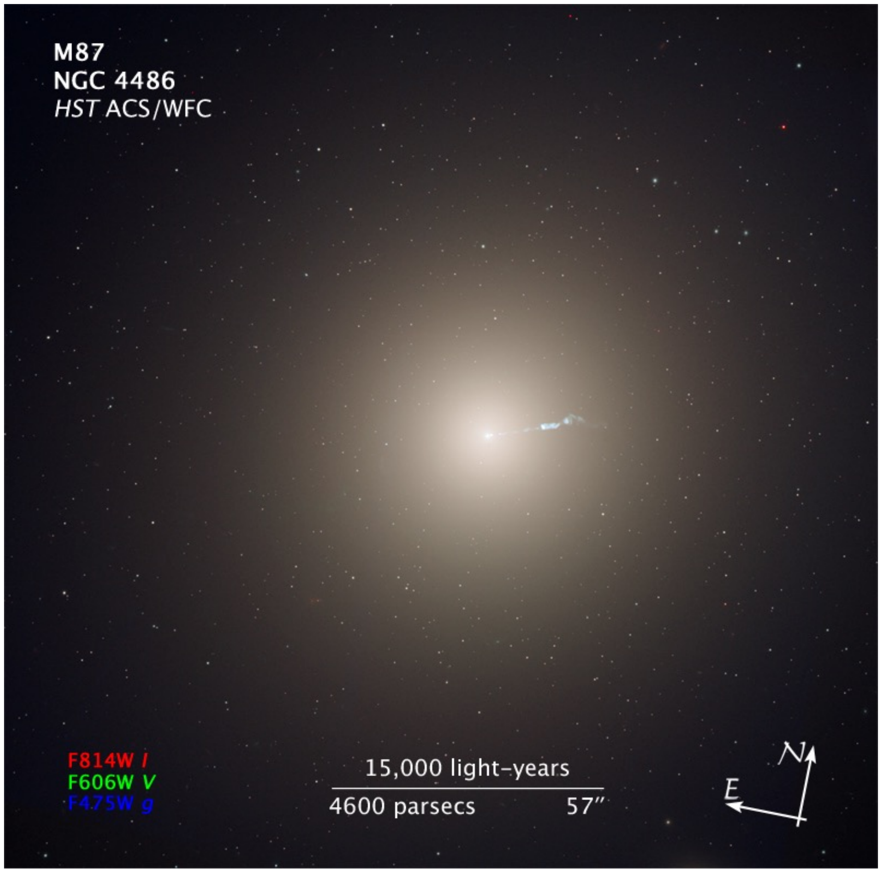




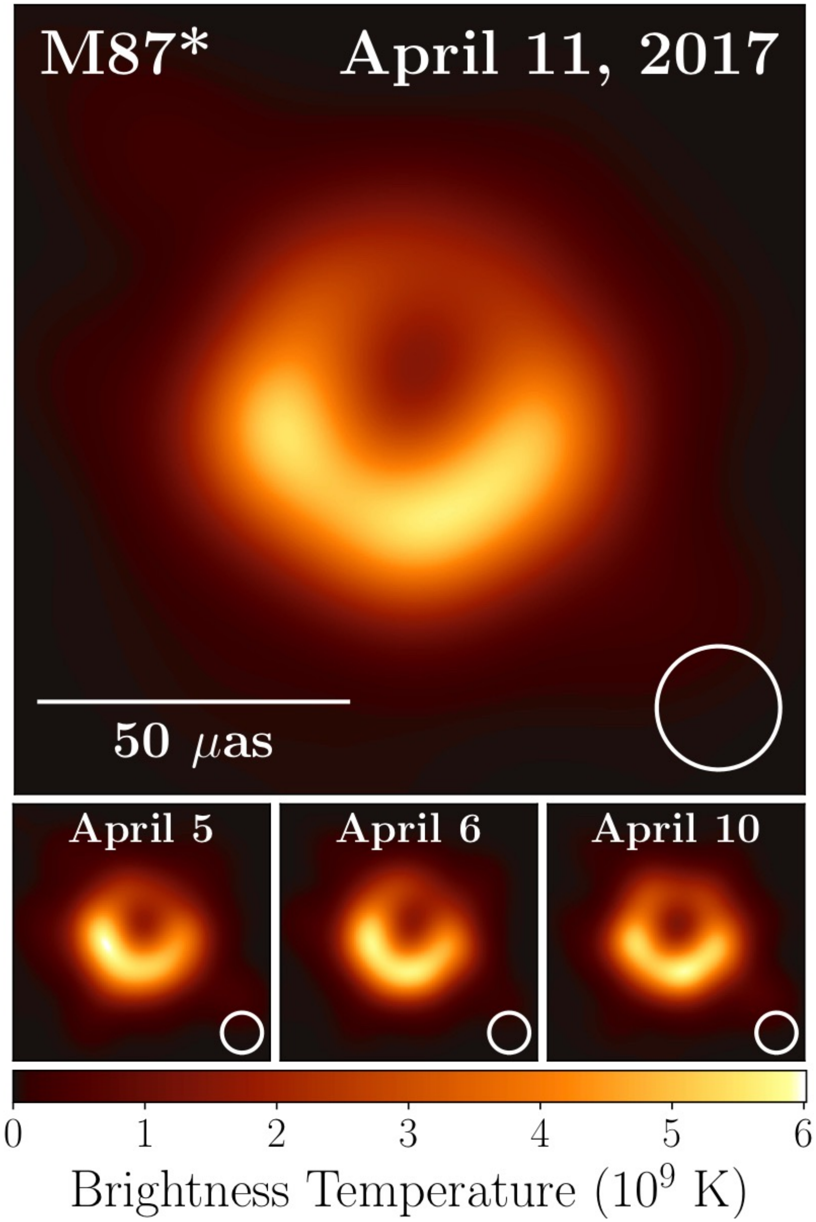
(a) Galaxy NGC 4261 **(b)** Evidence for a supermassive black hole in NGC 4261


M87
Hubble Space Telescope
VLA(NRAO)
6 r_s
0.01 pc
8 light years
VSOP 18cm
16,000 light years

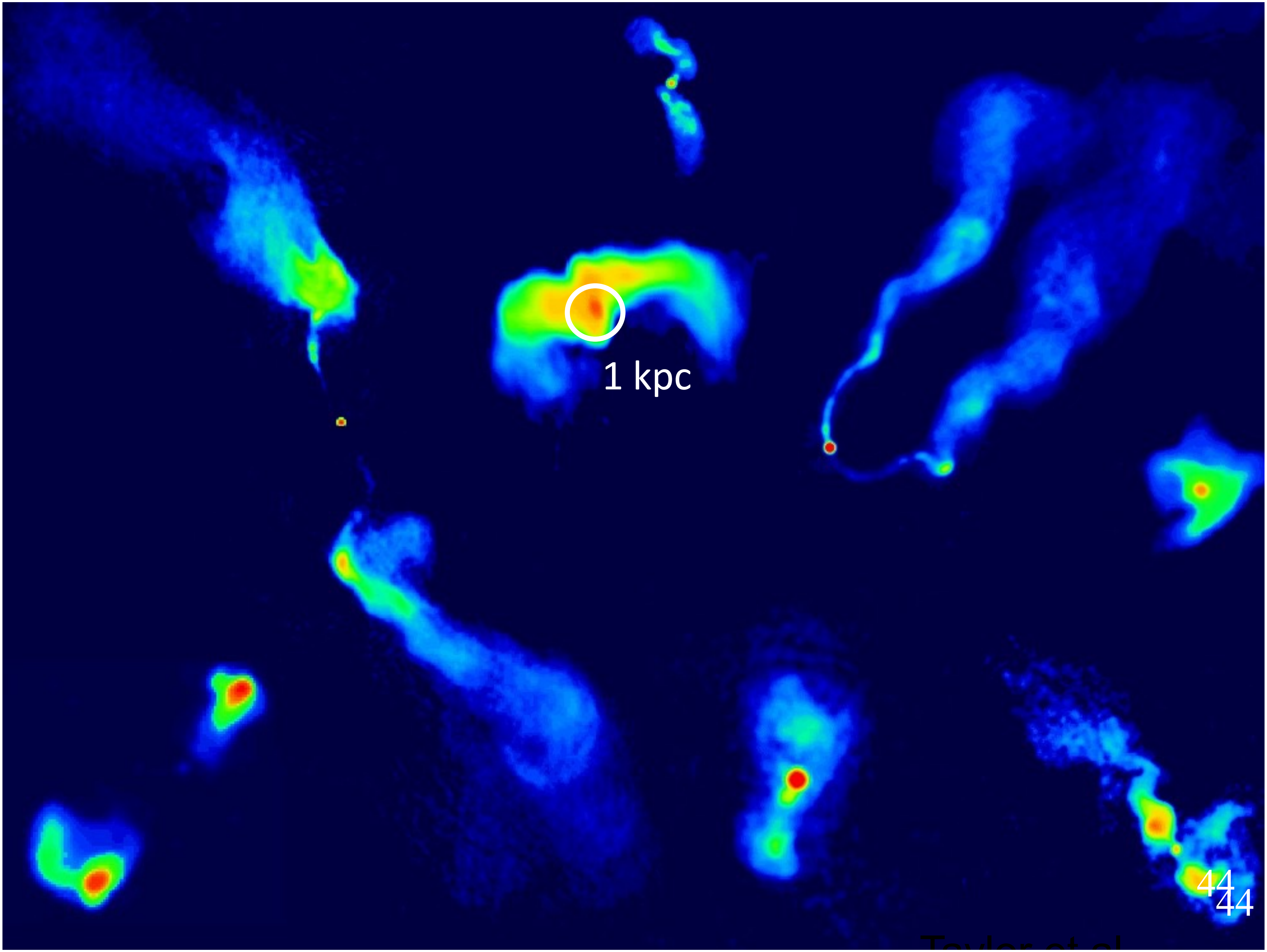
Shadow of a Black Hole



Optical image of the host galaxy



Radio image from Event Horizon Telescope



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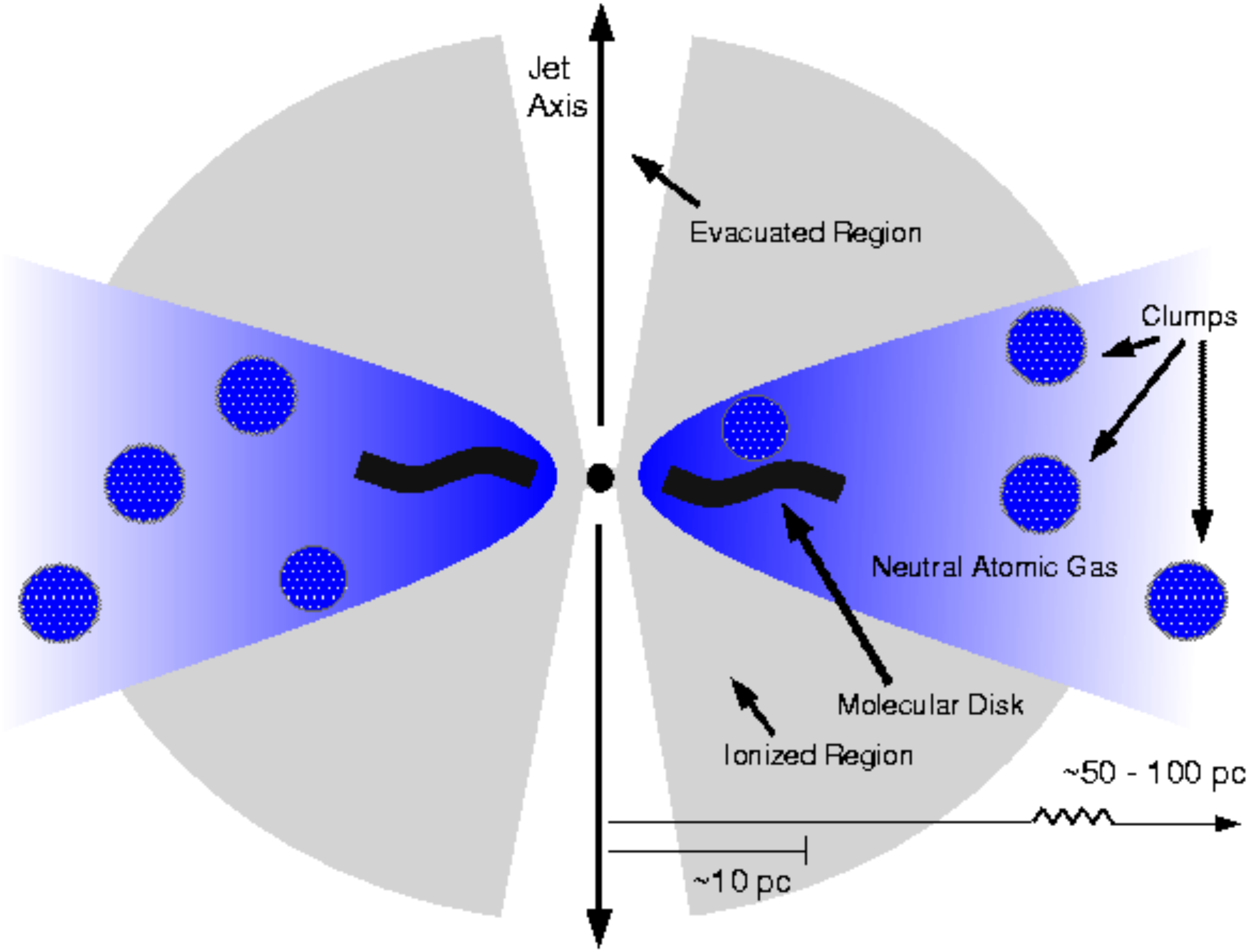
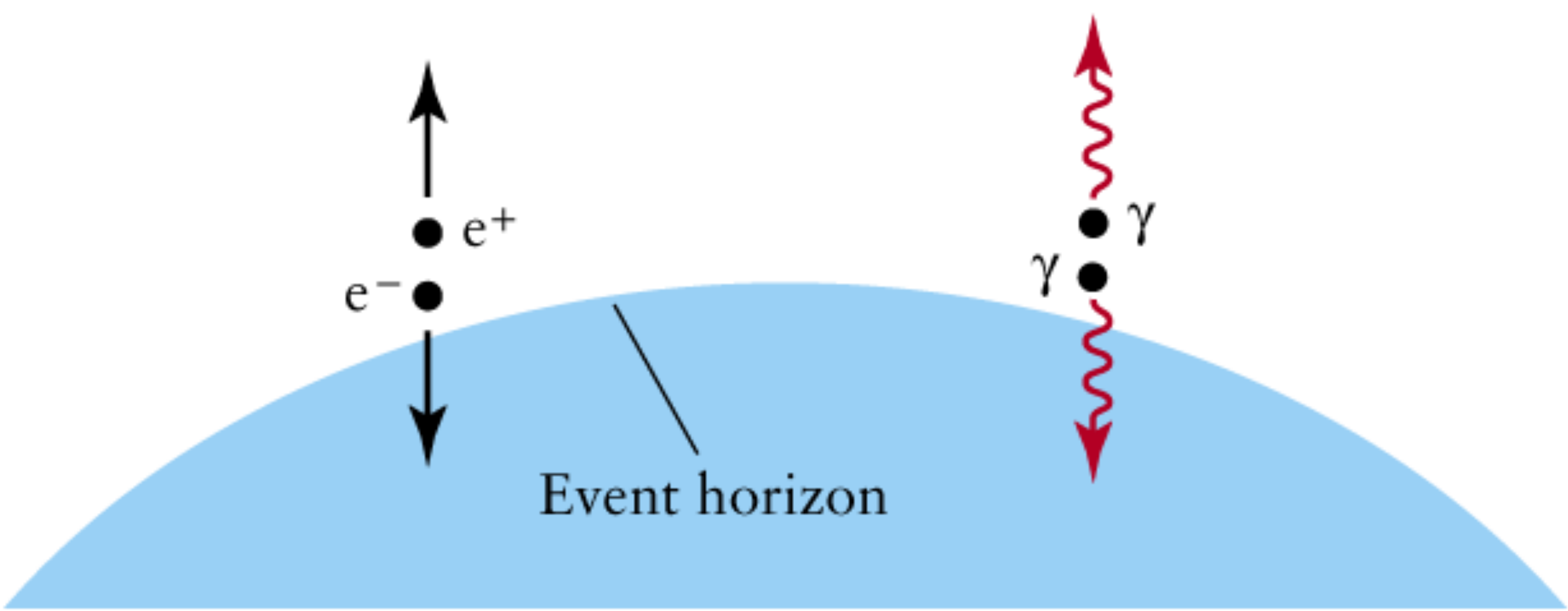

Jet
Axis
Evacuated Region
Clumps
Neutral Atomic Gas
Molecular Disk
Ionized Region
~50 - 100 pc
~10 pc

Do black holes exist forever?

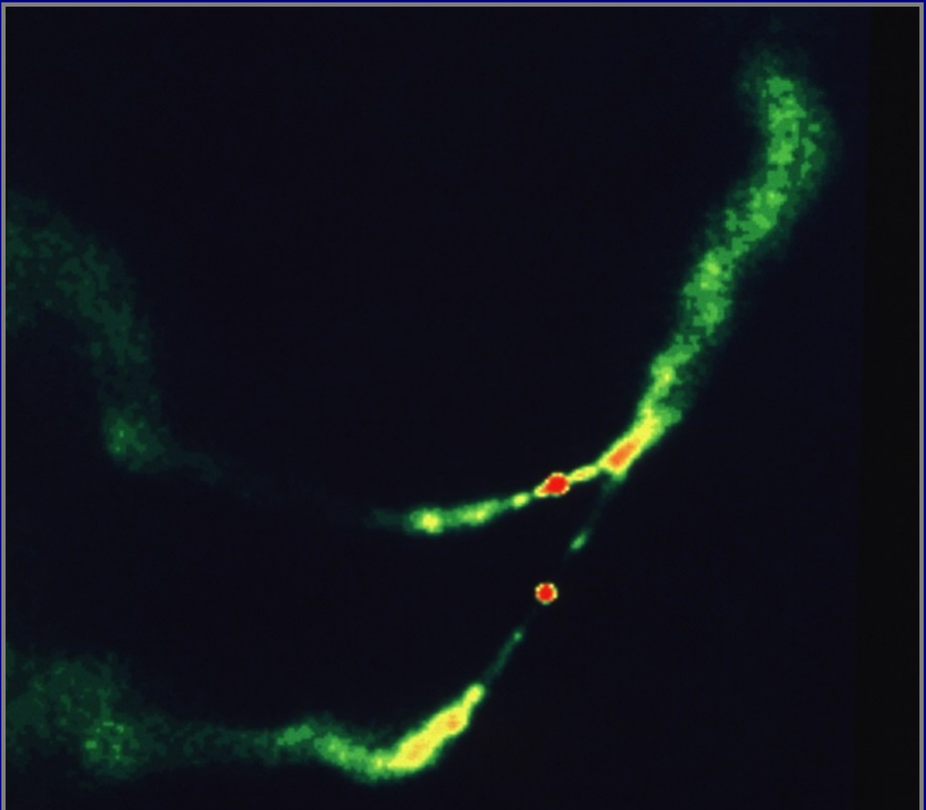
- Maybe not: there may be something called *Hawking radiation.*
- Virtual pairs of particles pop in and out of existence. If they are near the event horizon, one might get caught, the other escape, cause the black hole to evaporate. Relevant only for mini-black holes.



Supermassive Binary Black Holes

3C 75

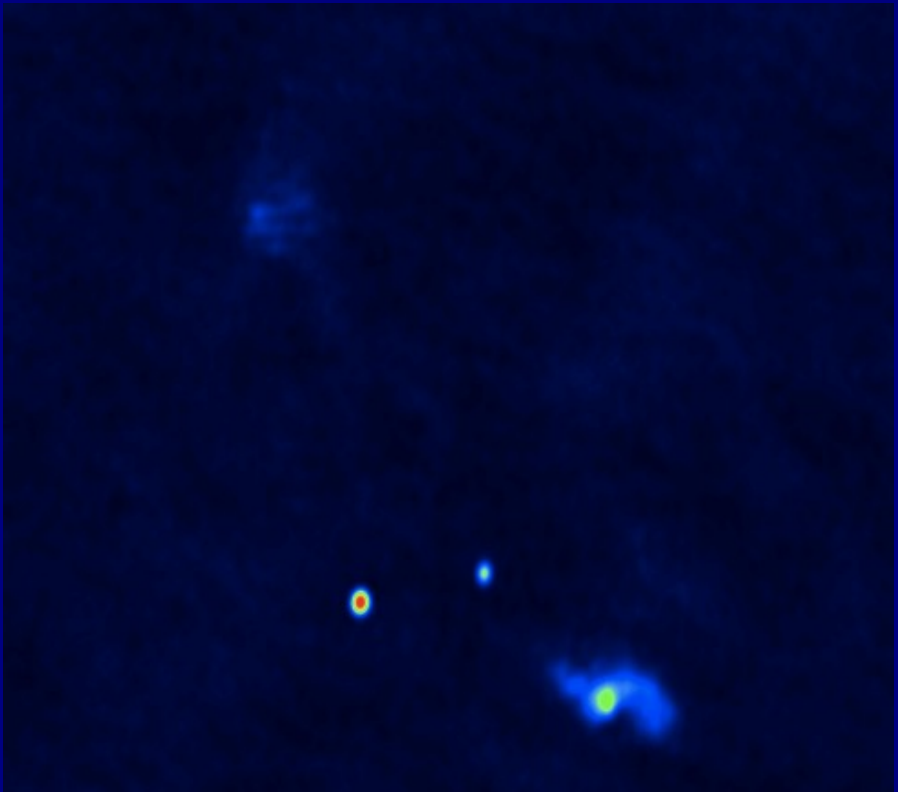
⇨ 7 kpc separation



VLA image of 3C 75 at 6 cm (Owen *et al.* 1985)

0402+379

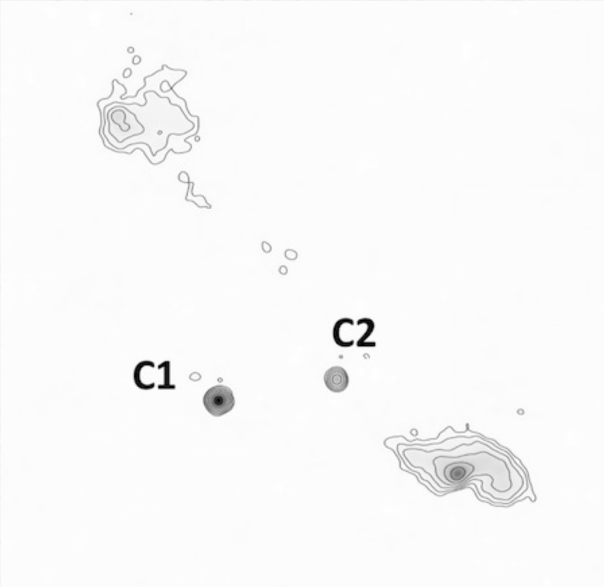
7 pc separation



VLBA image at 2 cm (Rodriguez *et al.* 2006)

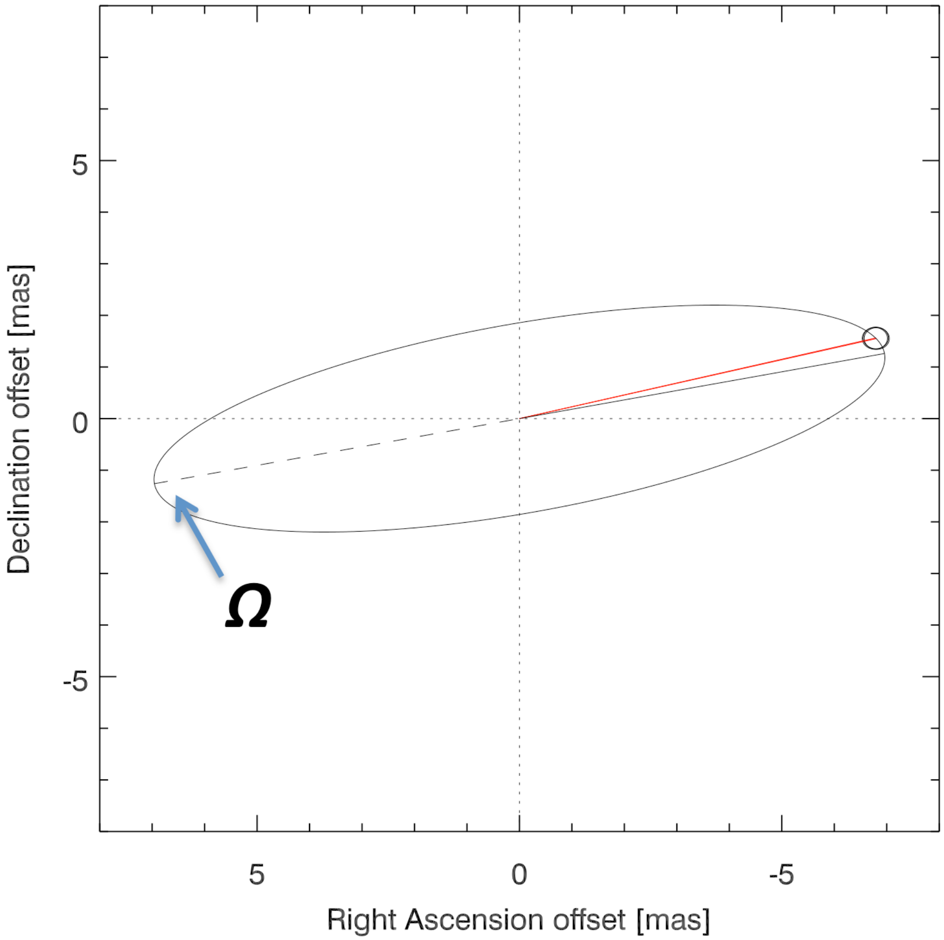
Constraining the Orbit in 0402+379

Supermassive binary black hole system 0402+379

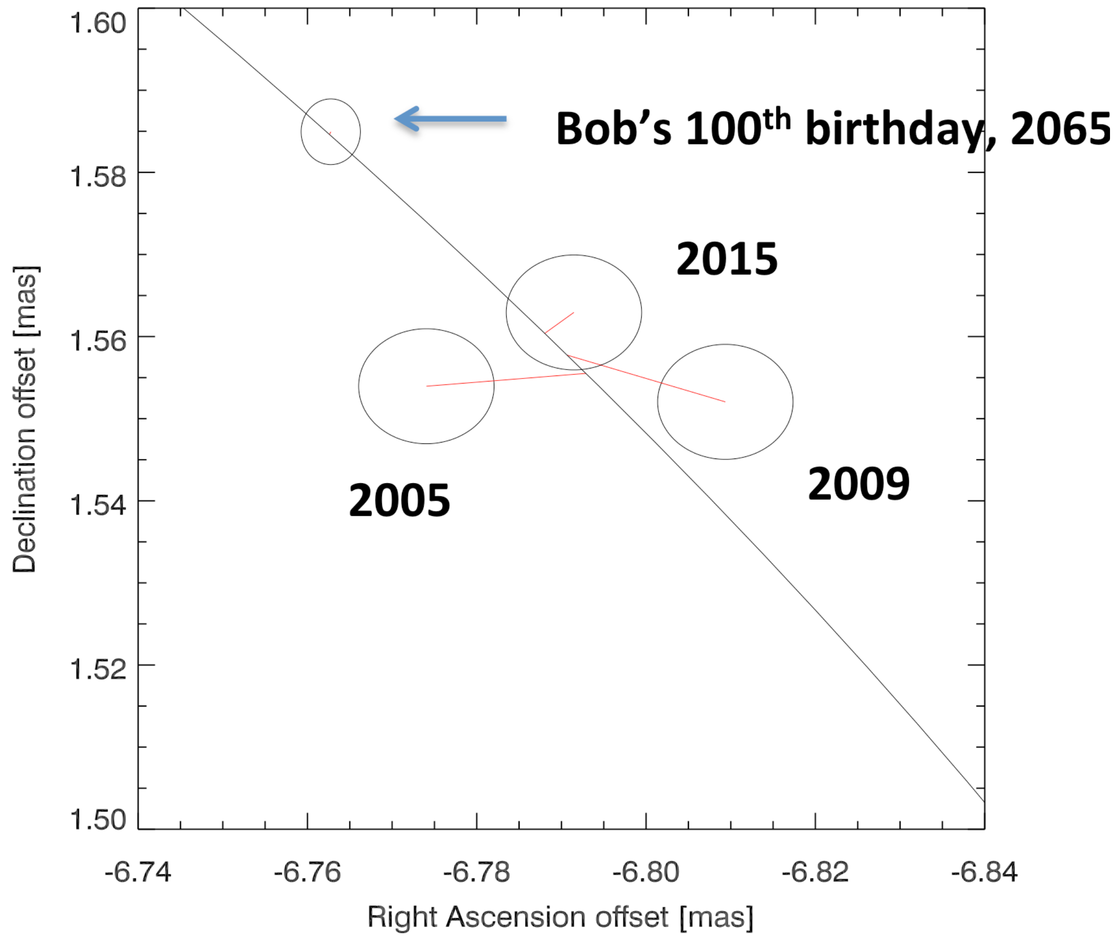


Bansal et al. 2017

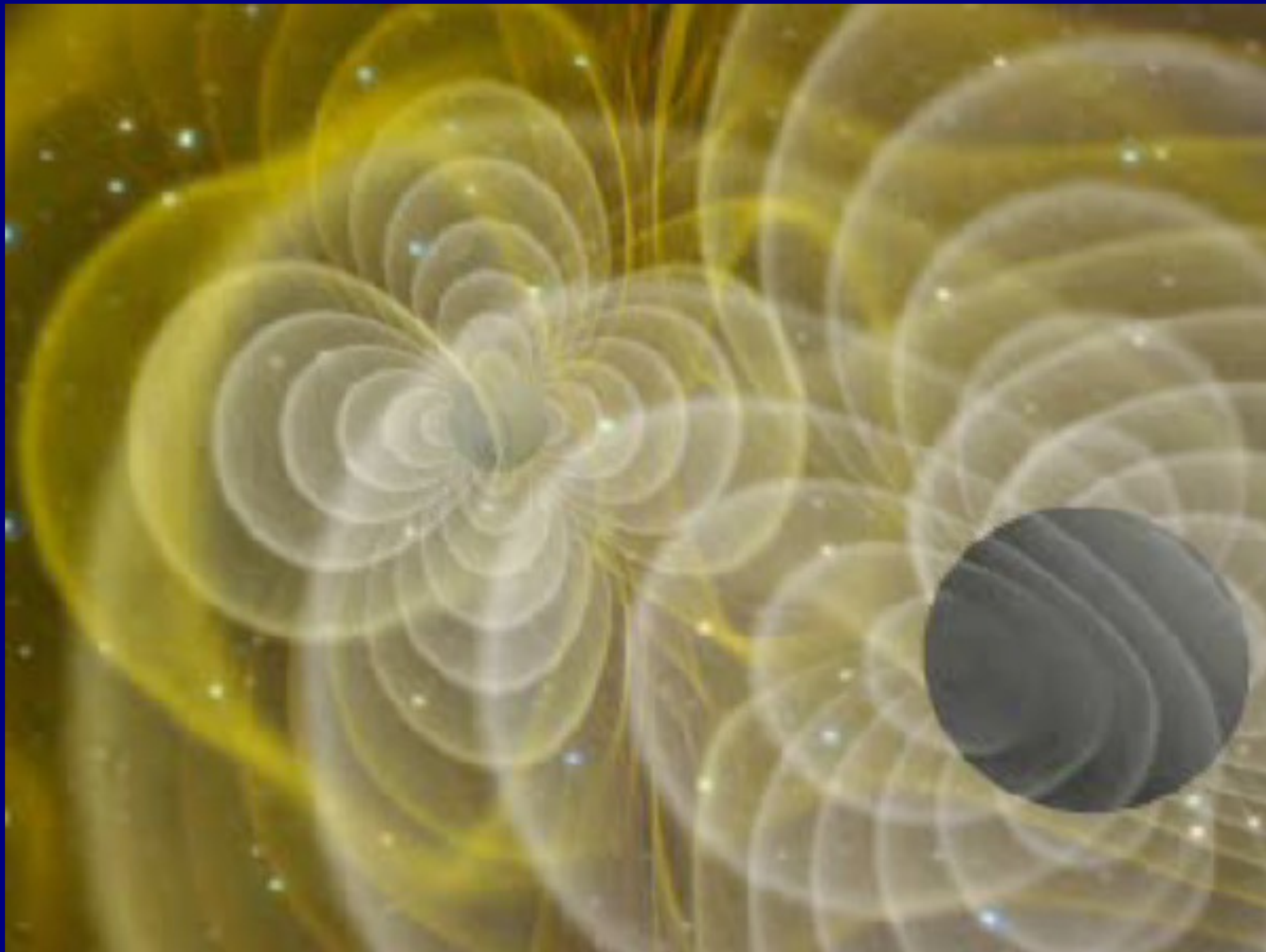
Orbital Fitting



Circular orbit at an inclination



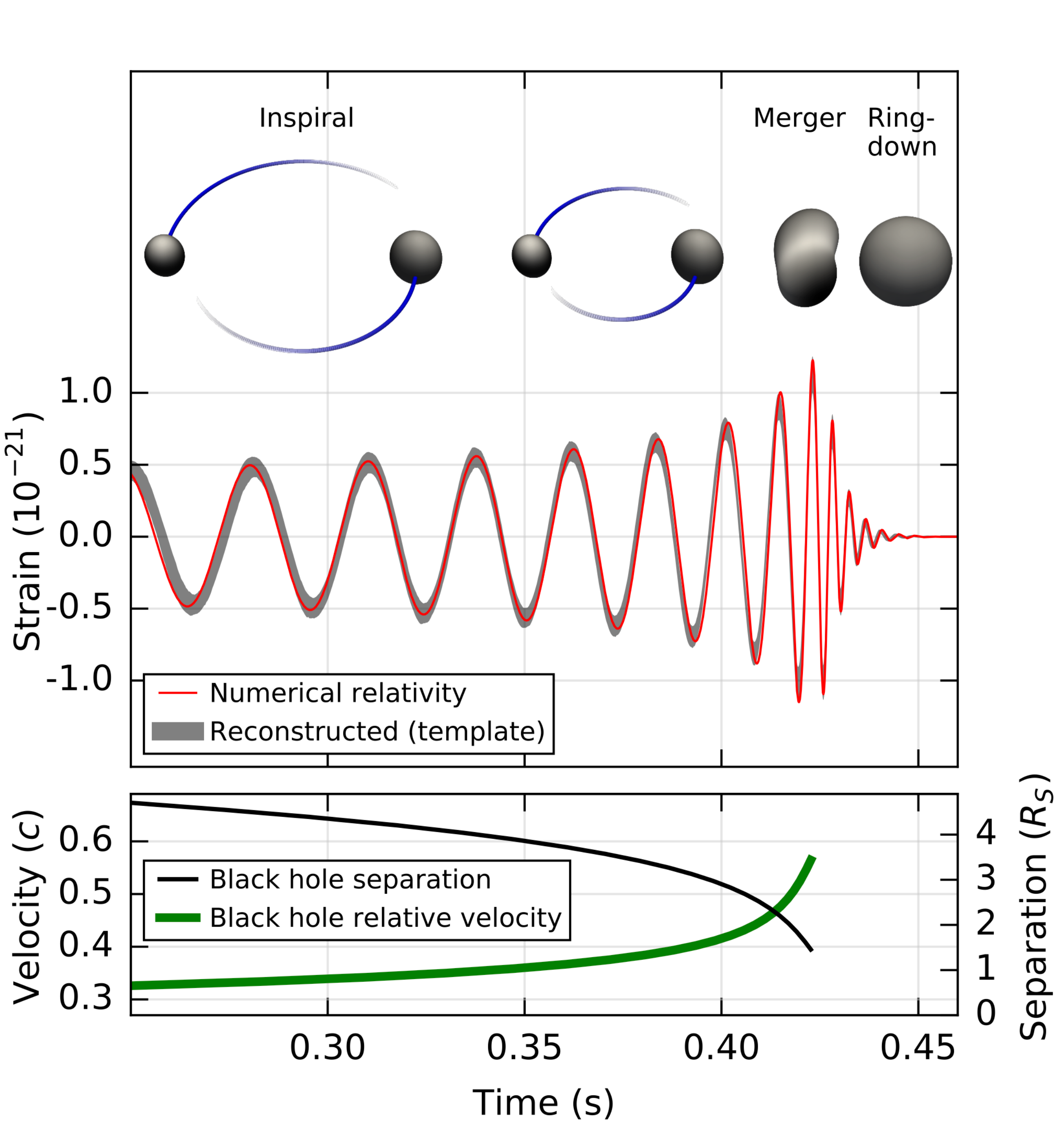
Close-up view



Gravitational Waves

LIGO (Laser Interferometric Gravity-Wave Observatory)

Gravitational Waves



Gravitational Waves