Newton's form of Kepler's third law

Newton found that Kepler's third law ($P^2 \propto a^3$) needed to be modified – into a general and *very* useful equation.

Newton's form of Kepler's third law:

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)}\right] a^3$$

with P in seconds, a in meters, masses in kg. a is the mean separation of the objects over their orbit.

Q: Why did Kepler miss the term with the masses?

Because mass of all planets = 0.1% mass of Sun

Note: <u>if</u> P in years, a in AU, and masses in M⊙ ,
it is still the case that

$$P^2 = \frac{a^3}{m_1 + m_2}$$

Determine the total mass of a system

- The α Centauri system is a binary star system with a period of 79.92 years. The A and B components have a mean separation of 23.7 AU (although the orbits are highly elliptical). *What is the total mass of the system?*

- Use the generalized form of Kepler's third law

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)}\right] a^3 \quad \text{or} \quad P^2 = \frac{a^3}{m_1 + m_2}$$

s, m, kg

years, AU, M_sun

$m1+m2=(23.7)^3 / (79.92)^2 = 2.08\ M_{sun}$

Compare to mass of our Sun = 1 M_{sun}

or

$$P = 79.92 \times 365 \times 24 \times 60 \times 60 = 2.52 \times 10^{9}\,\mathrm{s}$$

$$a = 23.7 \times 1.496 \times 10^{11} = 3.55 \times 10^{12}\,\mathrm{m}$$

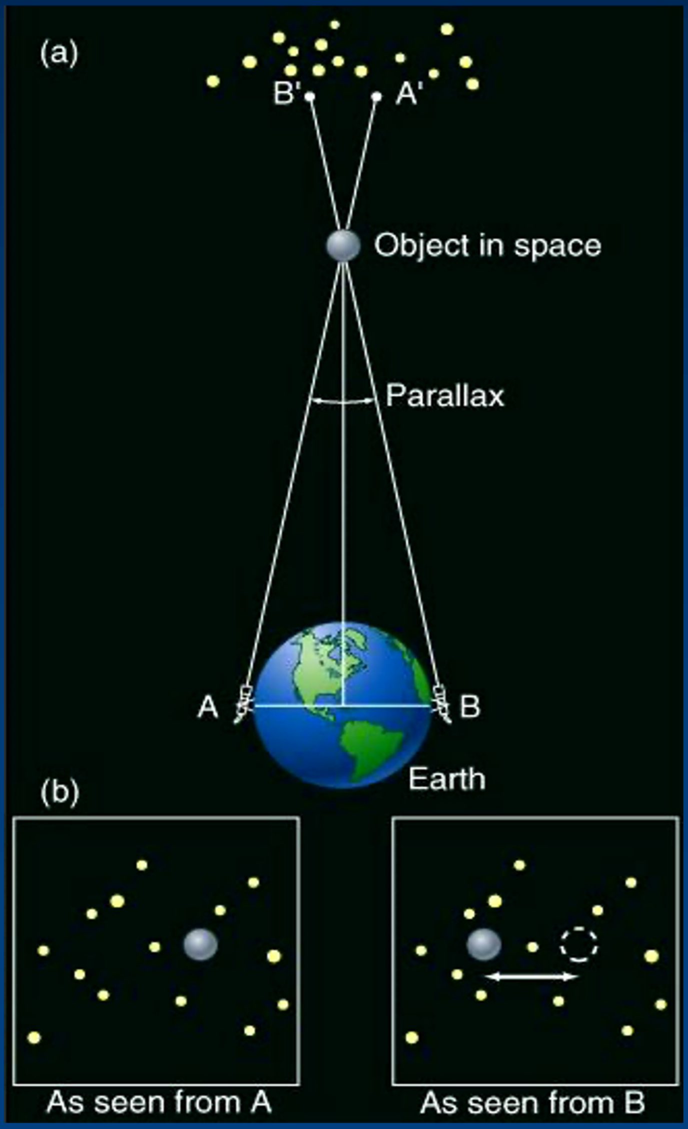
$$(m_1 + m_2) = \frac{4\pi^2 a^3}{GP^2} = \frac{4\pi^2 (3.55 \times 10^{12})^3}{6.67 \times 10^{-11} \times (2.52 \times 10^{9})^2} = 4.2 \times 10^{30}\,\mathrm{kg}$$

Unit check: $m^3/\, Nm^2kg^{-2}s^2 = m^3/\,(kg\ m\ s^{-2})m^2\ kg^{-2}s^2$

$= m^3/(m^3kg^{-1}s^{-2}\ s^2) = kg$

Compare to mass of our Sun = 1.99×10^{30} kg

In Newton's time, actual distances of planets from Sun were just starting to be measured. One technique is "Earth-baseline parallax".



"Earth-baseline" or "diurnal" parallax uses telescopes on either side of Earth to measure planet distances.

Use small angle formula:

$$diam = (p / 206{,}265) \times dist$$

so

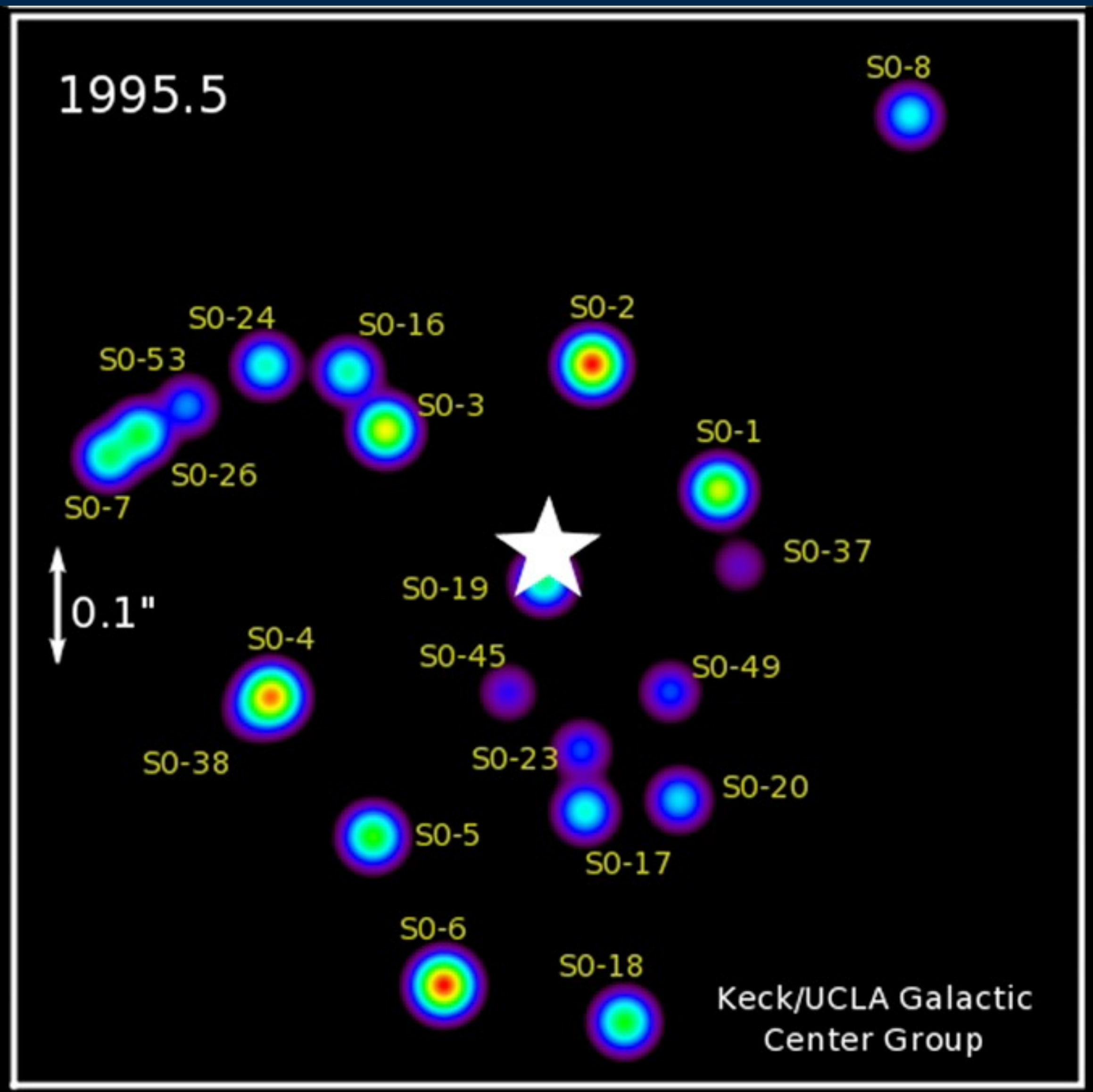
$$dist = diam / (p / 206{,}265)$$

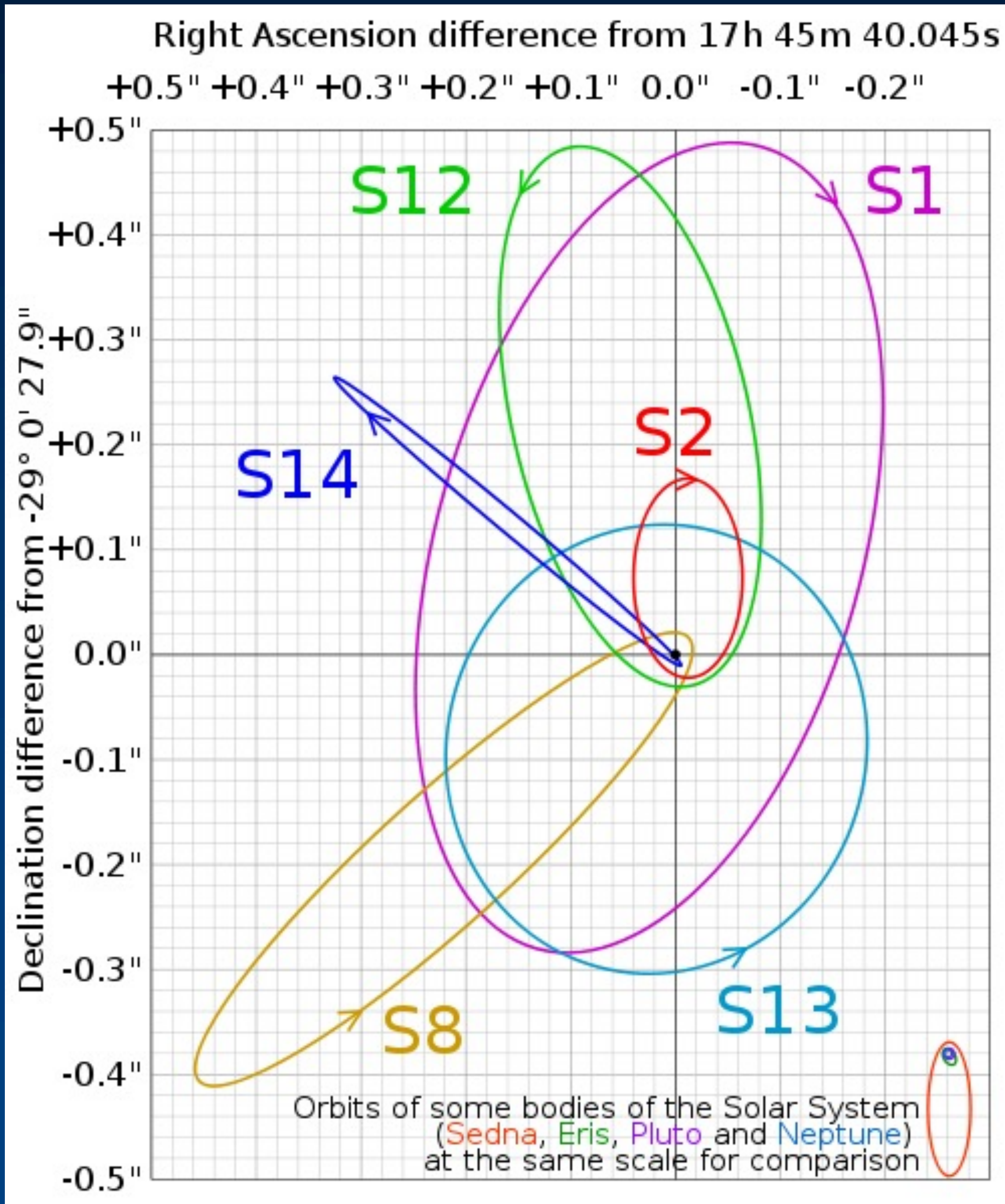
First attempted by Cassini for Mars in 1672. Got answer 7% too small. Refined by Halley (1761) using parallax of Venus transit.

Gravitation Part III

Orbits

Supermassive (3 million solar mass) Black Hole at the Galactic Center





Orbit of S2

P =15.6 yrs

a = 955 AU

e = 0.881

i = -48.1

From Kepler's 3rd law

$M = a^3/P^2$

$= 3.6 \times 10^6\ M_{sun}$

Satellites

Remote Sensing:

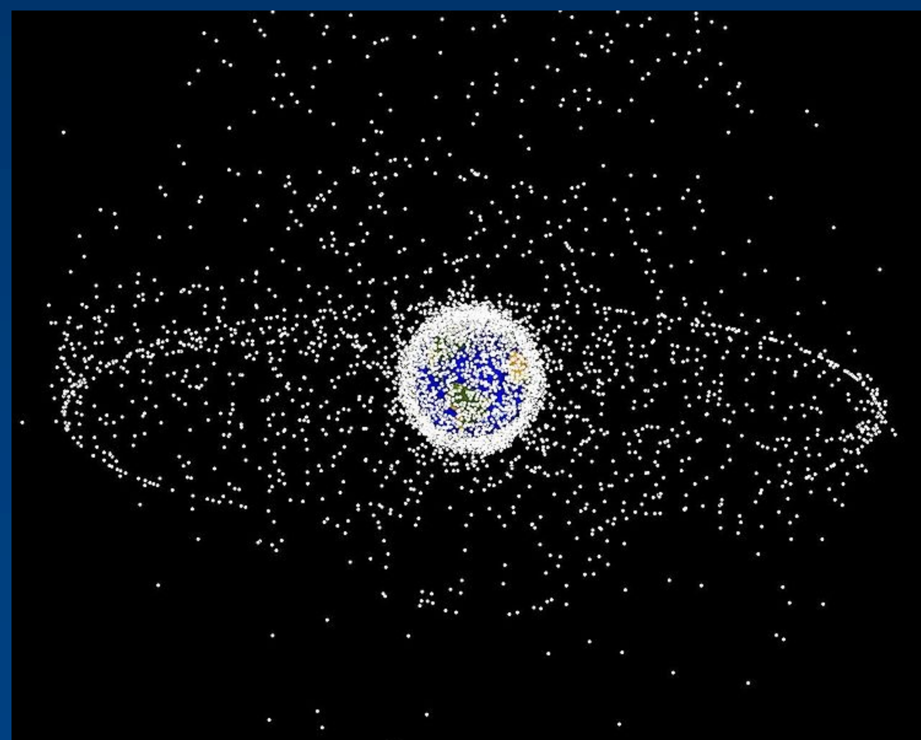
- Weather forecasting
- Tracking hurricanes
- Soil moisture and other climate data
- Astronomy (Chandra, Swift, Fermi, Hubble, ...)
- Communications
- Spy satellites

Space Debris

- Launching satellites and probes

At last count US strategic command says there are more than 30,000 man-made objects 10 cm and larger, and NASA estimates could be few x 10^5 or even 10^6 smaller, non-trackable pieces of debris.

Right: Region of space within 2000 km of Earth surface to show most concentrated area for orbital debris.



Tragedy of the commons

Mega Constellations

Starlink: 4755 launched by SpaceX as of today
Expect to deploy 30,000 and considering 42,000

Oneweb: Launched 75, went bankrupt, restructured, launching today now 600+
Others to follow?

Space Debris

Telkom-1 in August 2017 explodes

Need to have plan to de-orbit (or move to “graveyard orbit”) all vehicles at end of life

Need to clean up LEO and GEO

The Importance of Orbits

More examples:

- Can use to measure masses (most fundamental way of measuring masses of stars, their most fundamental property)

- Clues to origin and history of whatever is orbiting (e.g. comet orbits imply huge reservoirs of unseen comets, orbits of stars and gas in galaxies imply existence of Dark Matter).

Gravity's predictive power

- Halley's comet: Edmund Halley found orbit of 1682 comet appeared similar to other comets spotted 1607 and 1537 => predicted it would return in 1758/59. It did!

- The discovery of Neptune: 1781 Herschel discovered Uranus, but by 1840 the predicted positions were much offset from measured => gravitational influence from other body. The 8th planet found 1845!

Keplar's 3rd Law Application

- Example: At what height above the surface of the Earth are geostationary satellites located?

  Geostationary orbits are defined to keep the satellite above the same place on Earth.

  Worksheet #5

Universal - valid everywhere

- Example: At what height above the surface of the Earth are geostationary satellites located?

  Geostationary orbits are defined to keep the satellite above the same place on Earth, which means they have a period P=23.93 hrs (why not 24 hrs?).

  a is mean separation of the objects. For circular orbits where one is much more massive, a is approximately the orbit radius of the less massive one.

$$P = 23.93\ \text{hr/day} \times 3600\ \text{s/hr} = 8.61 \times 10^4\ \text{s}$$

$$m_1 + m_2 \approx m_{Earth} = 5.97 \times 10^{24}\ \text{kg}$$

$$a^3 = \frac{GP^2 m_{Earth}}{4\pi^2} = \frac{6.67 \times 10^{-11}\ \text{m}^3/\text{kg/s}^2 (8.61 \times 10^4\ \text{s})^2 (5.97 \times 10^{24}\ \text{kg})}{4\pi^2} = 7.48 \times 10^{22}\ \text{m}^3$$

$$a = 4.21 \times 10^7\ \text{m} = 42{,}100\ \text{km}$$

Now subtract off Earth's radius to get the altitude: 35,750 km.

Can you have a geostationary satellite anywhere else than above the equator?

Weightlessness

Astronauts on the space shuttle feel weightless because:

a) there is no gravity in space and they do not weigh anything.

…but then what would make them orbit in the first place?

b) space is a vacuum and there is no gravity in a vacuum.

…not true

c) they are in free-fall where the net force on them is zero.

right!

d) the astronauts are far from Earth's surface at a location where gravitation has a minimal affect.

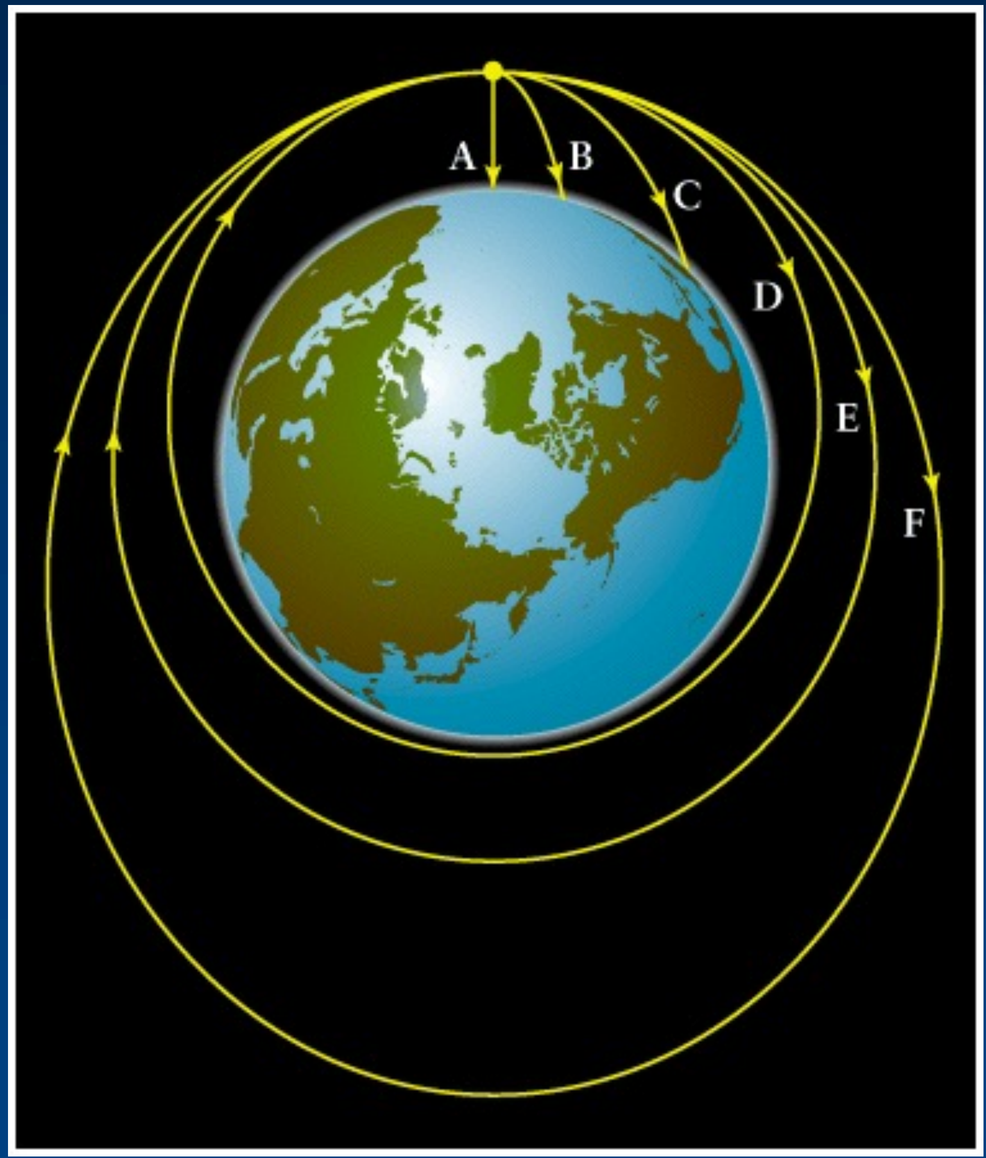
Hm, at 400 km orbit, g is reduced from 9.8 to 8.7 m/s^2 - not enough

Weightlessness

- Weightlessness is a sensation
- In orbit no force counteracts gravity - you are constantly accelerating. In "free-fall".
- So is the shuttle and everything in it. So it does not push up on you like the Earth does => simulates gravity-free environment

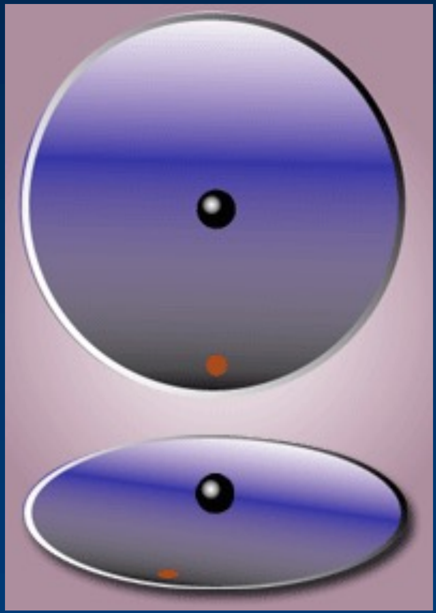
Paths A-F correspond to harder and harder throws. If throws are hard enough, rock “misses” the Earth completely!

Cases D-F are orbits of the rock around the Earth. The type of orbit depends on the injection velocity.



DEMO
Projectile Motion

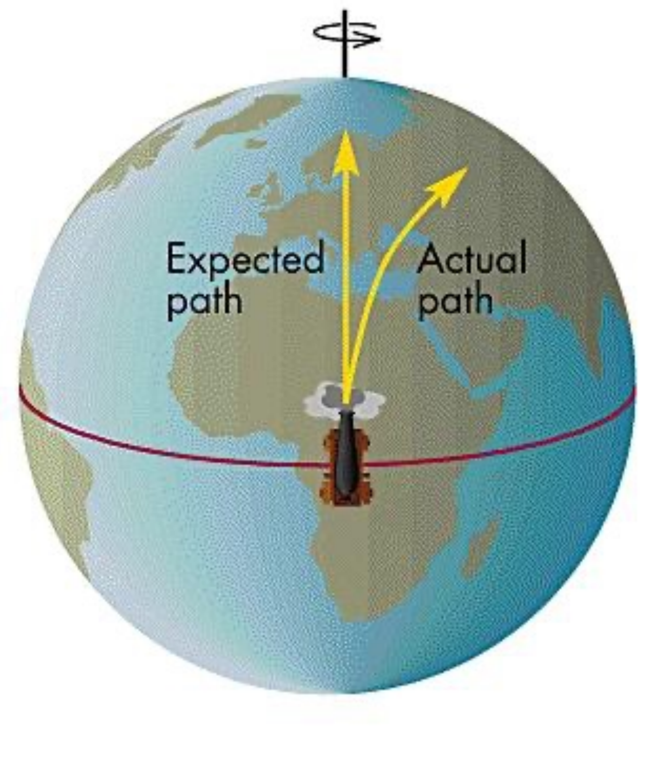
The Coriolis Effect



- First described by the French physicist Gustave Coriolis 1835
- An effect of viewing straight-line motion from a rotating frame of reference.
- On Earth, it results in an apparent deflection of a projectile
- Example: fire a cannonball due north from the equator.

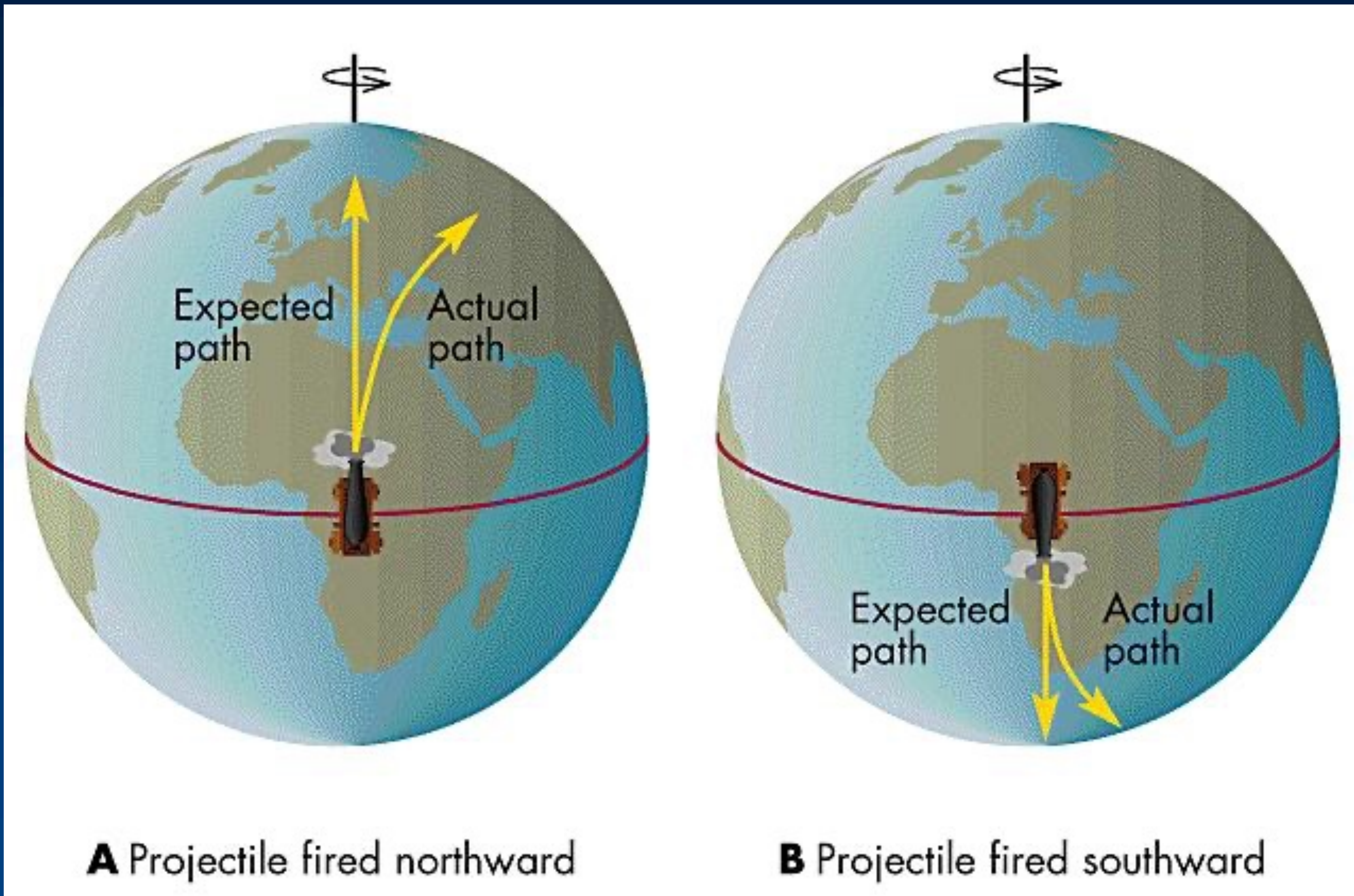
- The cannon is also moving east with the rotation of the Earth at 1670 km/h.
- The cannonball retains this initial eastward speed as it travels north (Newton's first law).
- But the further north it gets, the slower is the eastward motion of the Earth's surface beneath.

⇒ Result is an eastward deflection of the cannonball with respect to its initial trajectory northward.



A Projectile fired northward

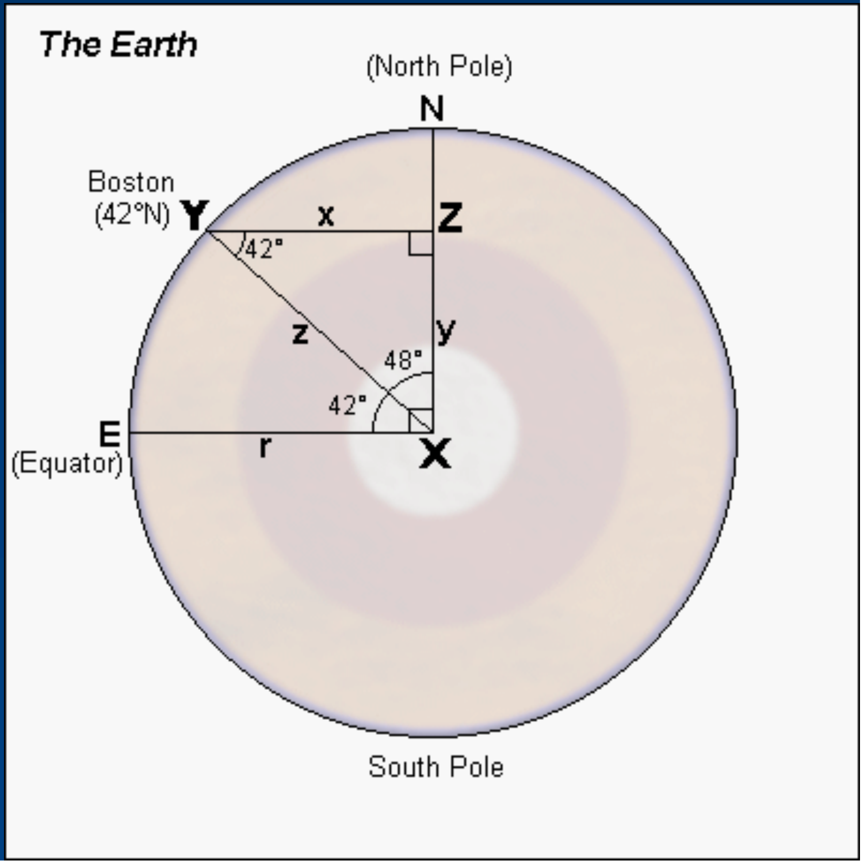
The same happens if you fire your cannonball to the south.



What about projectile fired south from north pole?

Earth's rotation speed vs. latitude

- Equator rotation speed: 1670 km/h
- Rotation speed greatest at the equator, and decreases with increasing latitude, as the distance covered in 1 day is less ($2\pi x$)



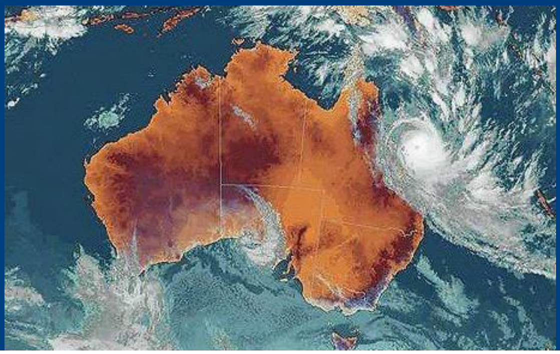
- Figure shows rotation speed decreases as cos(latitude)
- In Albuquerque (35º N), the rotation speed is (1670 km/h) x (cos(35º)) =1370 km/h.

Consequences of the Coriolis effect

- Affects storms and low/high pressure systems:

Flow around a low pressure system in N hemisphere

  - Hurricanes in the northern hemisphere rotate *counter-clockwise*
  - In the southern hemisphere they rotate *clockwise*



Hurricane Florence

Example: How fast does a satellite have to move in order for it to be in a circular orbit around the Earth?

Example: How fast does a satellite have to move in order for it to be in a circular orbit around the Earth?

For circular motion, the acceleration is the centripetal acceleration:

$$a_{centripetal} = \frac{V_{circular}^{\;2}}{r}$$

So

$$F_{gravity} = m a_{centripetal}$$

$$\frac{GM_{Earth} m_{rock}}{r^2} = m_{rock} \frac{V_{circular}^{\;2}}{r}$$

Canceling m_{rock} and one factor of 1/r,

$$\frac{GM_{Earth}}{r} = V_{circular}^{\;2}$$

or

$$V_{circular} = \sqrt{\frac{GM_{Earth}}{r}}$$

This circular speed is the magnitude of the injection velocity, perpendicular to direction to center of Earth, needed for a circular orbit at distance r from the Earth.

This is general!

Example: what is the circular speed of an object located at 1 AU from the Sun?

M = M_{Sun} = 2 x 10^{30} kg; r is 1.5 x 10^{11} m, and the constant G is 6.7 x 10^{-11} in SI units, so

$$V_{circular} = \sqrt{\frac{(6.7 \times 10^{-11})(2 \times 10^{30})}{1.5 \times 10^{11}}}$$

$$V_{circular} = 3.0 \times 10^{4} \text{ m/s}$$

$$V_{circular} = 30 \text{ km/s}$$

Accurate average speed of the Earth around the Sun: 29.79 km/s

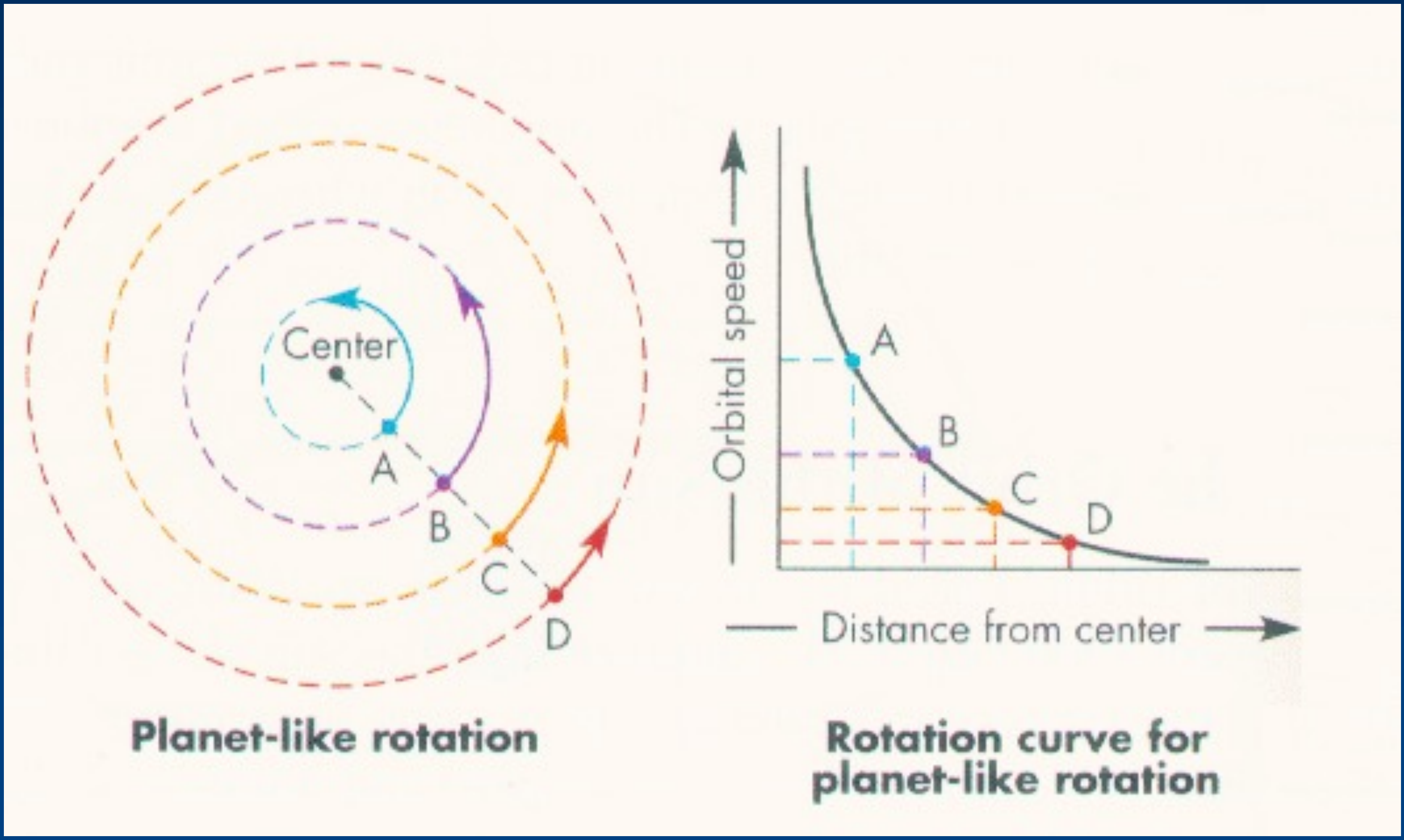
How about if the Earth had twice the mass?

Question: How would the speed of the planets around the Sun vary with distance from the Sun? Eg., what is functional relation between v and r?

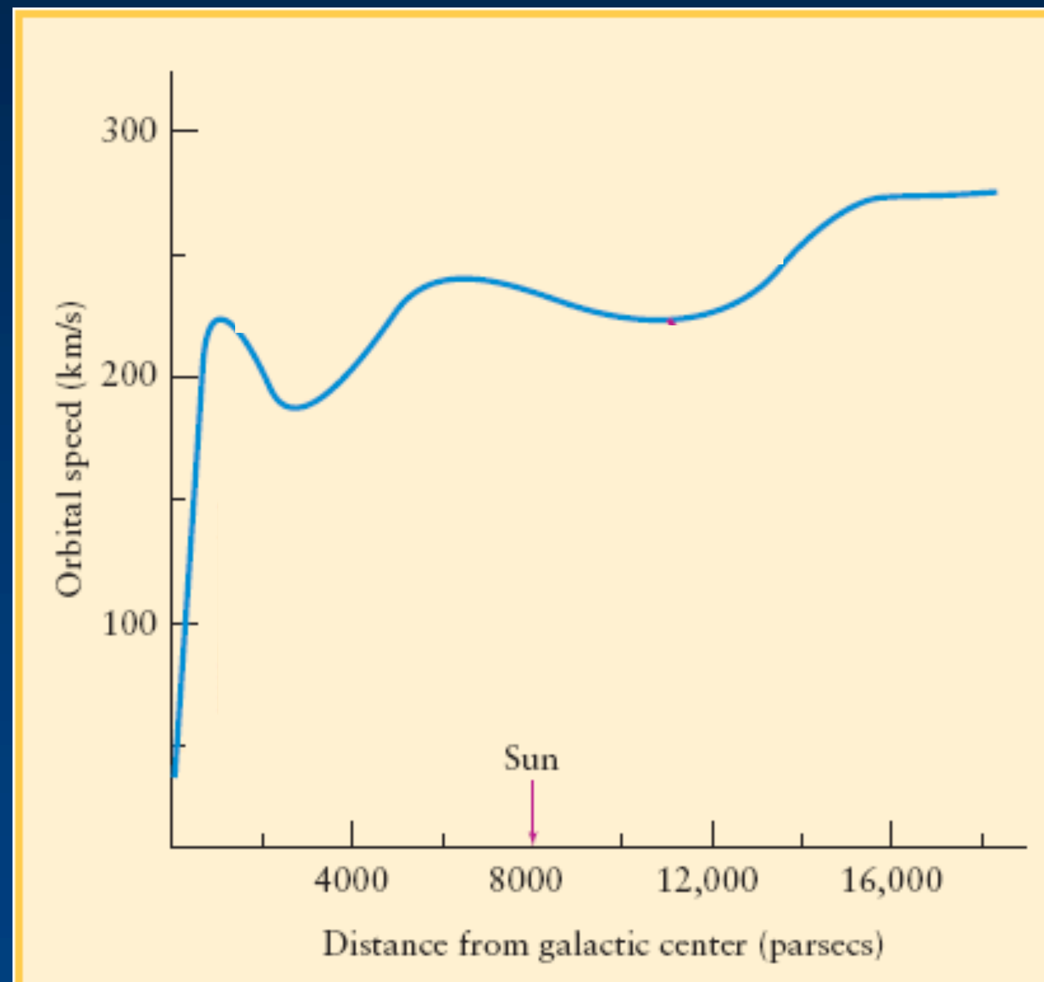
This is what Kepler saw!

Keplerian rotation curves

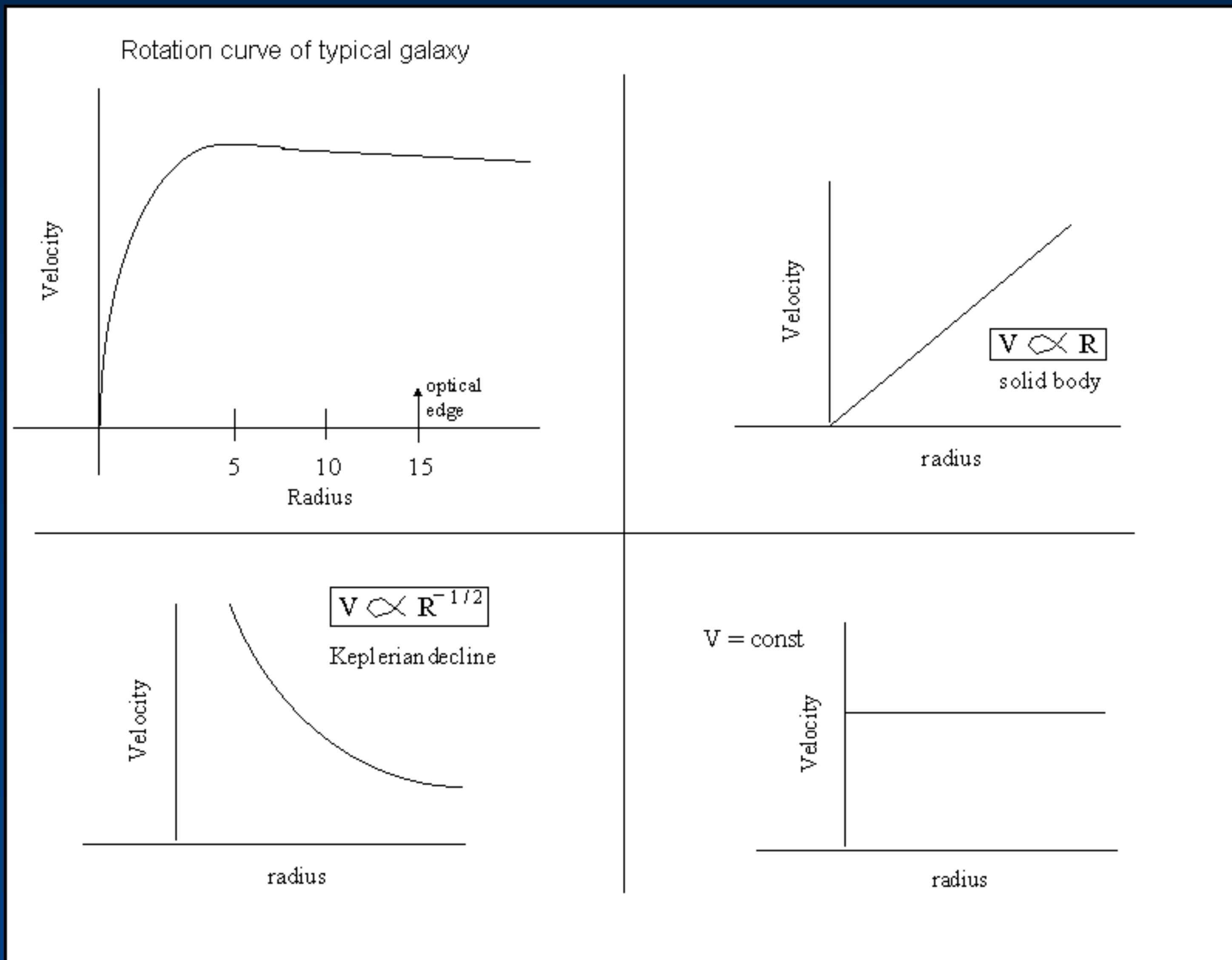
- When the system is dominated by the central mass: $v \propto r^{-1/2}$



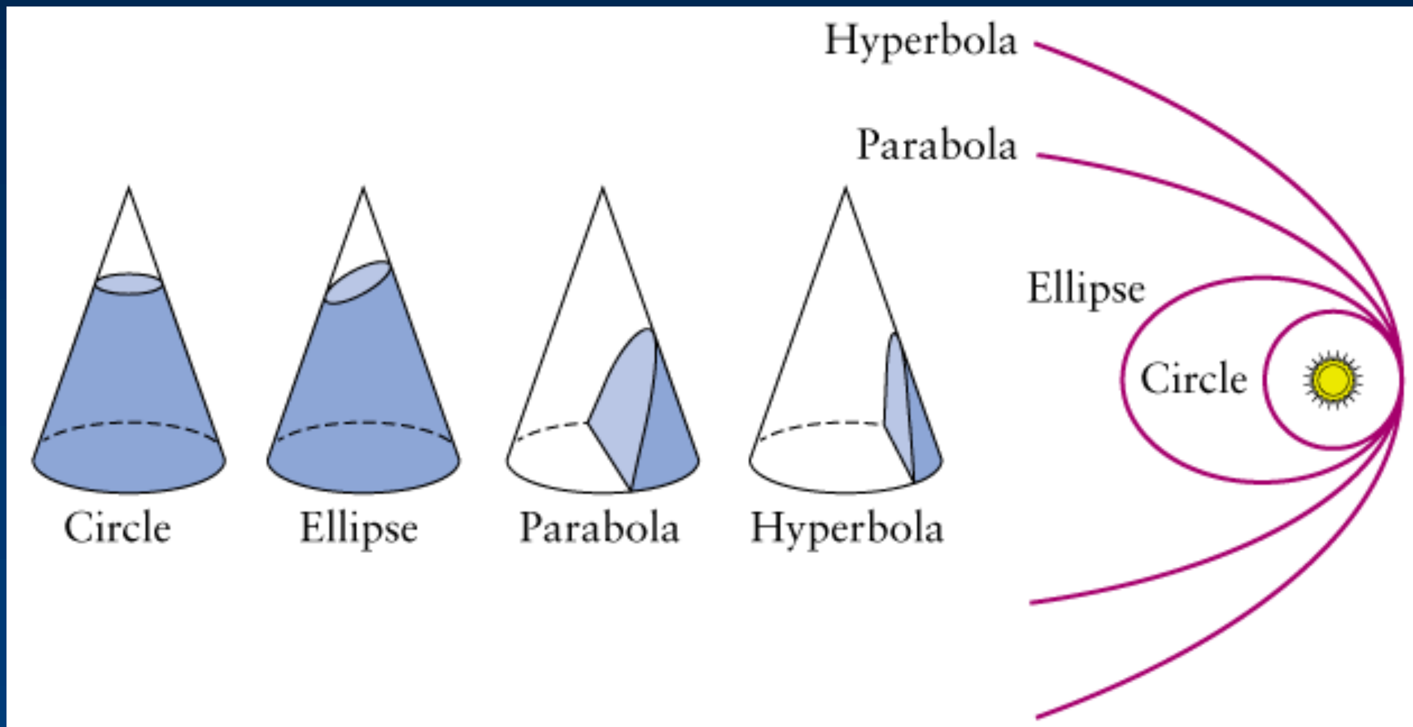
Milky Way also rotates. But rotation curve not Keplerian, but nearly “flat”. Milky Way mass is not dominated by a central mass



Rotation Curves



What if injection velocity isn’t the circular velocity? In general, orbits are conic sections (Newton).

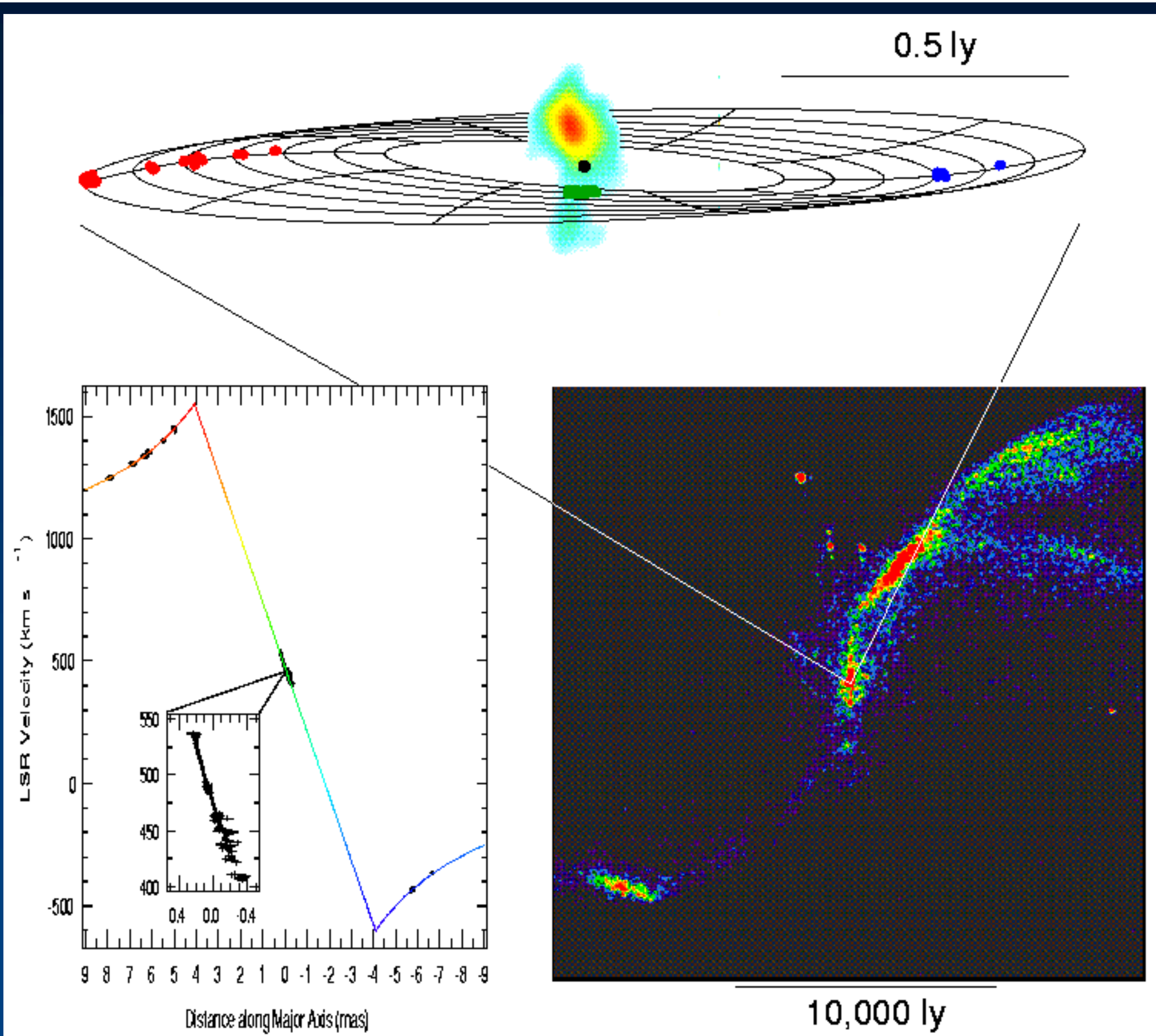


Ellipses and circles are *closed* orbits, hyperbolic and parabolic orbits are *open*. Objects on these orbits do not return – some comets do this!

General rules of orbits

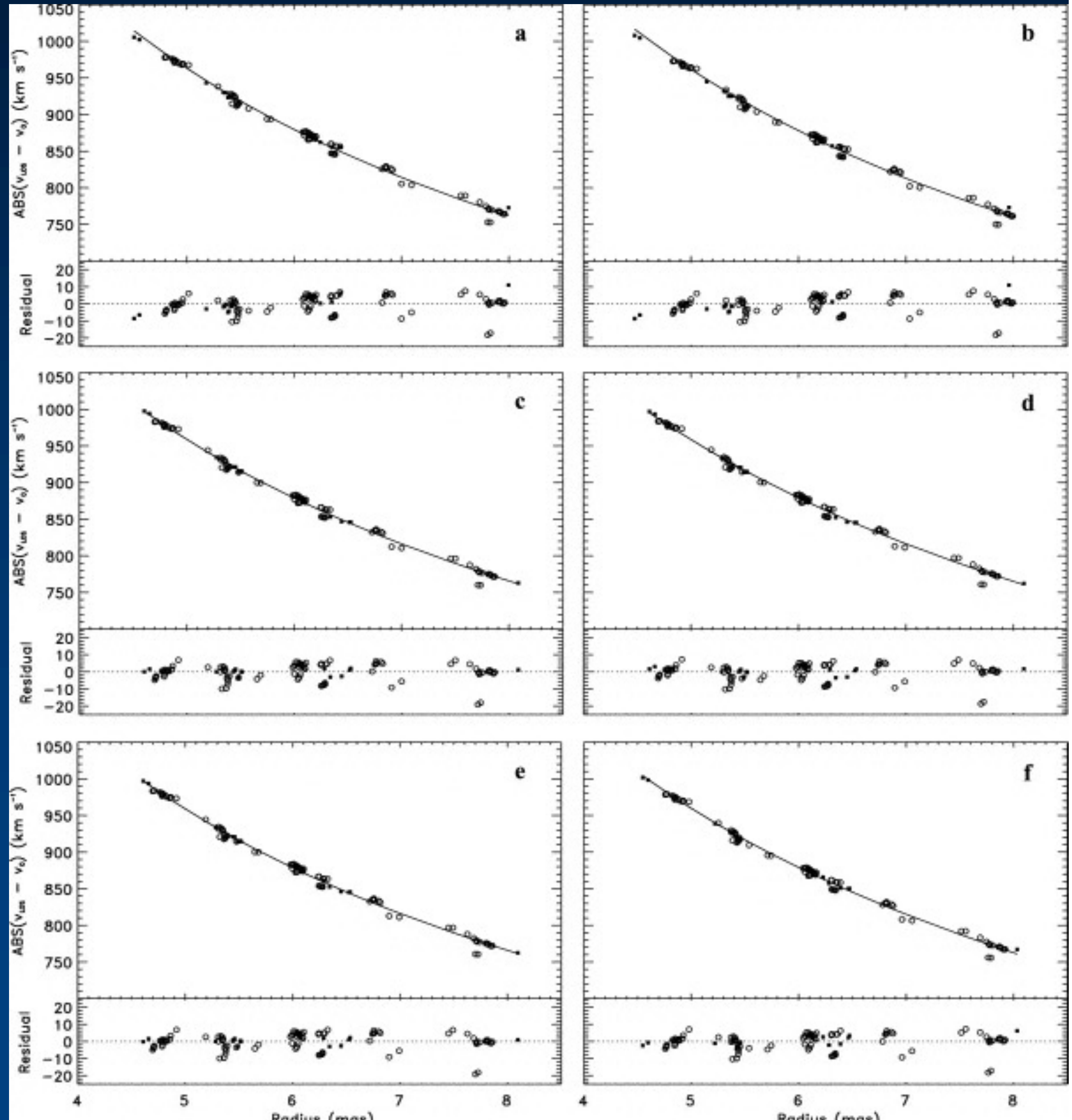
Orbit shape	System energy
Parabolic	Zero
Elliptical/circular	Negative
Hyperbolic	Positive

- Negative energy orbits are "bound", positive energy orbits are "unbound".
- What orbit is the most bound (minimum energy)?



0.5 ly
1500
1000
500
0
-500
LSR Velocity (km s^{-1})
550
500
450
400
0.4
0.0
-0.4
9 8 7 6 5 4 3 2 1 0 -1 -2 -3 -4 -5 -6 -7 -8 -9
Distance along Major Axis (mas)
10,000 ly

NGC4258 fits



Lagrange Points