Announcements

- Homework #2 due today
- Homework #3 due next Thursday
- Seminar on binary black holes today 3205 PAIS – Greg Walsh (UVA)
- Sign up for the Society of Physics Students at UNM https://physics.unm.edu/pandaweb/undergraduate/sps.php

Gravitation Part II

Newton's Laws of Motion and Gravity

Newton (1642-1727)

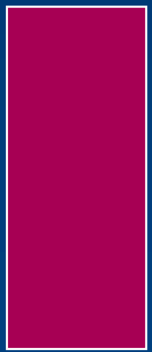
Kepler's laws were basically playing with mathematical shapes and equations and seeing what worked – empirical.

Newton's work based on experiments of how objects interact.

His three laws of motion and law of gravity described how all objects interact with each other.

Newton's First Law of Motion

Every object continues in a state of rest or a state of uniform motion in a straight line unless acted on by a force.



Newton's Second Law of Motion

When a force, F, acts on an object with a mass, m, it produces an acceleration, a, equal to the force divided by the mass.

$$a = \frac{F}{m}$$

$$\text{or} \quad F = ma$$

acceleration is a change in velocity or a change in direction of velocity.

Newton's Second Law of Motion

DEMO - Smash the HAND

Newton's Second Law of Motion

Demo - Measuring Force and Acceleration

Newton's Third Law of Motion

To every action there is an equal and opposite reaction.

Or, when one object exerts a force on a second object, the second exerts an equal and opposite force on first.

Newton's Third Law of Motion

DEMO: PUCKS

DEMO: WALLS

Newton's accomplishments (partial list)

- Replaced empirical descriptions with fundamental, physical explanations. This is really the foundation of modern physics.
- Laws of motion of objects on the Earth also pertains to objects in the heavens. Study of motion produced by applied forces is called *mechanics*.
- Unified all motions into three simple laws.
- (Also developed calculus (about the same time as Leibnitz), studied light, Newtonian reflecting telescope…)

First, some definitions

- Speed measures how fast something is moving, for instance, car moving 100 km/hr. A scalar quantity.
- Velocity measures speed in a specified direction, for instance, car moving 100 km/hr north. A vector quantity.
- Acceleration is the rate at which velocity changes. Can be change in speed, direction, or both. A vector quantity.
- Question: is there acceleration involved in uniform circular motion?

- Yes!
- Uniform circular motion is accelerated motion!
- Direction of velocity is changing => acceleration

DEMO

Question: How many accelerators does your car have?

Newton's Laws of Motion

Built on Galileo's ideas from his experiments

Newton's first law of motion (law of inertia):

A body remains at rest, or moves in a straight line at a constant speed, unless acted upon by a net outside force.

Implication: since planets do not have straight line motion, there must be a force acting on them!

Newton's second law of motion:

The acceleration of an object is proportional to the net outside force acting on the object, and inversely proportional to its mass.

$$a = \frac{F}{m}$$

or

$$F = ma$$

Acceleration in same direction as applied force. Force is also a vector.

Unit of force in mks system is the Newton (N). Must have same units as *ma*, so 1 N = 1 kg m s^{-2}.

One implication: planets move in elliptical orbits, so accelerating. Must be moving under an applied force!

Newton’s third law of motion:

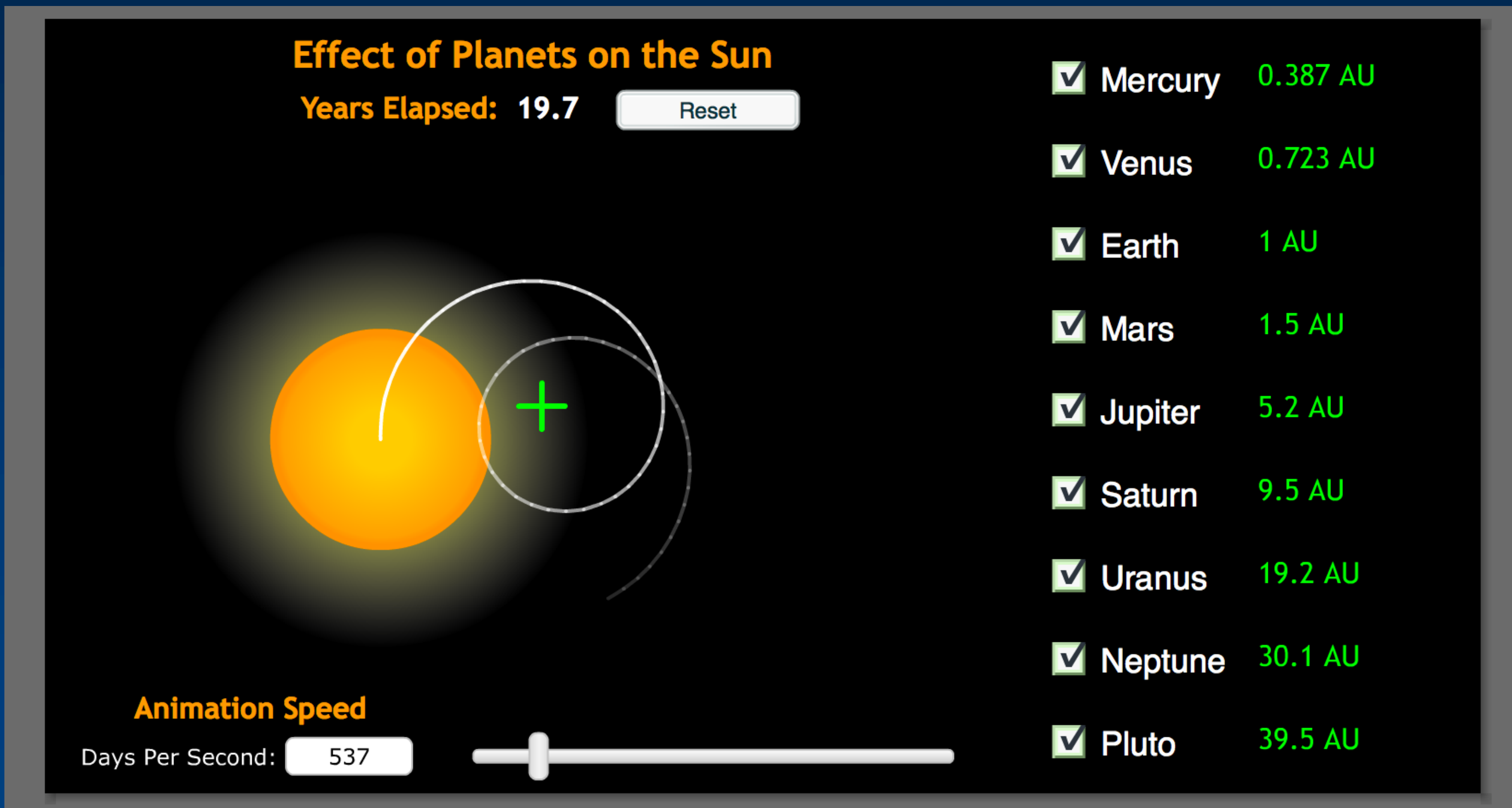
Whenever one body exerts a force on a second body, the second body exerts an equal and opposite force on the first body.

Forces are interactions and act in simultaneous pairs.

Implication: a planet and the Sun exert equal and opposite forces on each other.

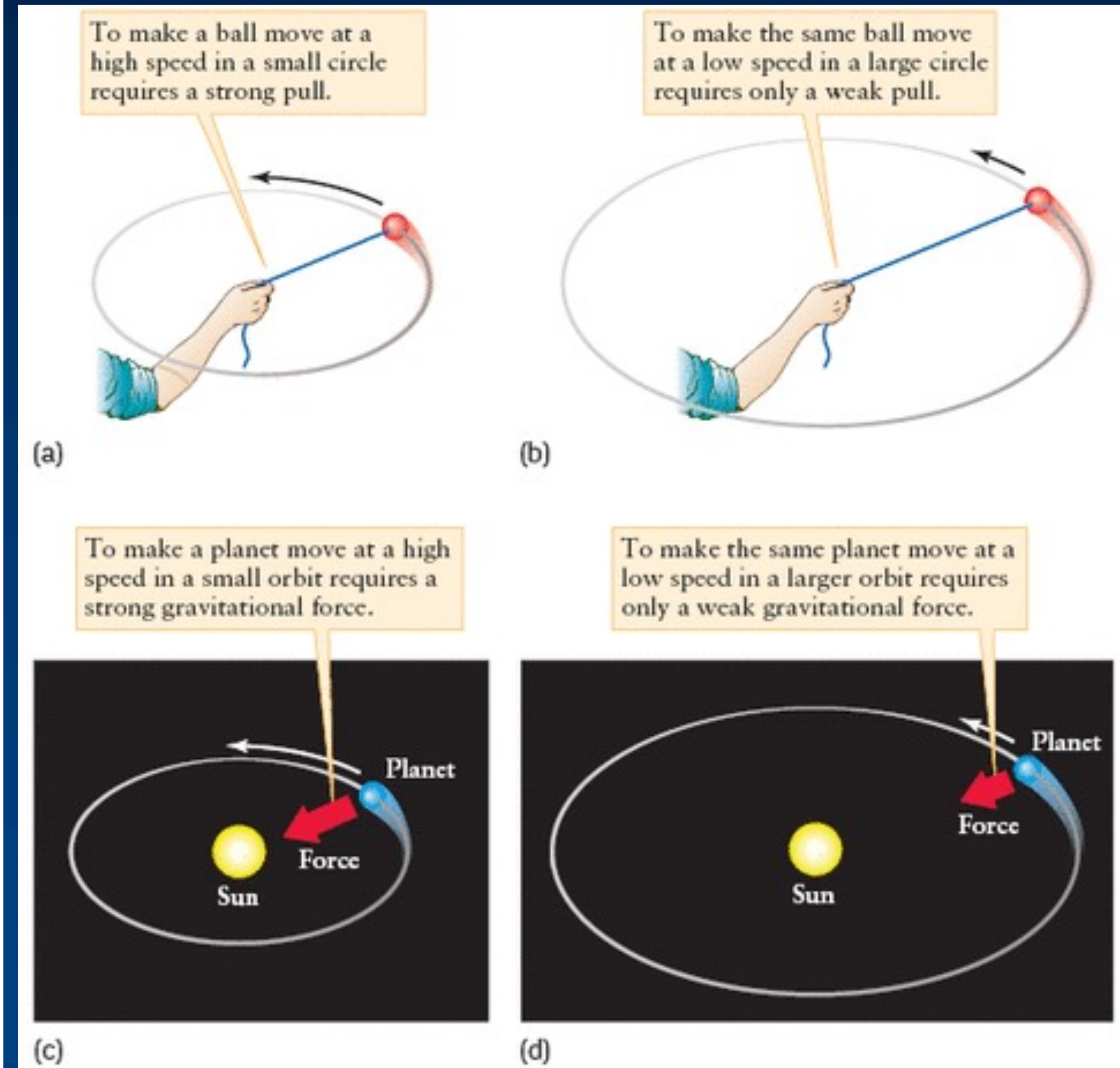
Question: Then why doesn't the Sun circle the planets?

Hence, Sun wobbles due to pull from all the planets. Each has different acceleration, period, eccentricity.



motion of sun animation

What’s the nature of the force of gravity?



Planets near the Sun move at high speed, and need strong force from the Sun. Planets in larger orbits need weaker pull from the Sun to stay in orbit.

Newton’s law of universal gravitation

Two bodies attract each other with a force that is directly proportional to the mass of each body and inversely proportional to the square of the distance between them.

- Gravity is an attractive, universal and mutual force
- Depends on masses and separations of objects – and NO OTHER characteristics, eg. not colors, shapes, compositions…
- Newton derived this from his three laws of motion and measurements of accelerations.

In equation form:

$$F = G\frac{m_1 m_2}{r^2}$$

where:

F = gravitational force between 2 objects,

m_1 = mass of first object,

m_2 = mass of second object,

r = distance between objects,

G = Newton's gravitational constant

= 6.67×10^{-11} Nm^2/kg^2

Your weight is the force of gravity between you and the Earth:

$$weight = F = G\frac{m_{you}m_{Earth}}{r_{Earth}^2}$$

Your mass does not change in different gravities, but your weight does.

So mks units of weight are N (in Imperial units, pounds)

The mass of the Earth

- Acceleration of gravity at surface of Earth measured to 9.8 m/s^2
- Radius of the Earth measured to 6378 km
- Since

$$F = G\frac{m_1 m_2}{r^2} = G\frac{m_{earth} m_2}{r_{earth}^2} = m_2 a$$

=>

$$m_{earth} = \frac{a r_{earth}^2}{G} = \frac{9.8\ \text{m/s}^2 \times (6.378\times 10^6\ \text{m})^2}{6.67\times 10^{-11}\ \text{m}^3/\text{kg/s}^2} = 5.98\times 10^{24}\ \text{kg}$$

Newton's law of gravity provides us with a way to estimate masses of objects, using their motions!

Newton used the motion of the Moon to test his law of gravitation.

Fact: Earth-Moon distance is about 60 Earth radii, so acceleration of the Moon toward Earth should be $1/60^2$, or 1/3600 as much as on object dropping on Earth's surface. Is it?

Worksheet #4

For motion in a circle, the centripetal ("directed toward the center") acceleration a is

$$a = \frac{V^2}{r}$$

What’s the Moon’s speed?

Distance = rate x time, so

Speed = distance traveled in an orbit ÷ time for one orbit:

$$V = \frac{2\pi r}{P}$$

where $2\pi r$ is circumference of orbit, and P is the sidereal period of the Moon.

P = 27.3 days x 24 hr/day x 60 min/hr x 60 s/min

= 2.36×10^6 s

The radius of the Moon's orbit is 3.84×10^8 m, so

$$V = \frac{2\pi(3.84 \times 10^8 \text{m})}{2.36 \times 10^6 \text{s}} = 1.02 \times 10^3 \text{ m/s}$$

The Moon's centripetal acceleration is then

$$a = \frac{V^2}{r} = \frac{(1.02 \times 10^3 \text{ m/s})^2}{3.84 \times 10^8 \text{ m}} = 2.7 \times 10^{-3} \text{ m/s}^2$$

which is indeed $(9.8 \text{ m/s}^2)/3600$.

Nice proof that it's the *same* force pulling on the Moon, that causes objects near Earth's surface to fall, and that it must vary as the inverse square of the separation!

Newton's form of Kepler's first law

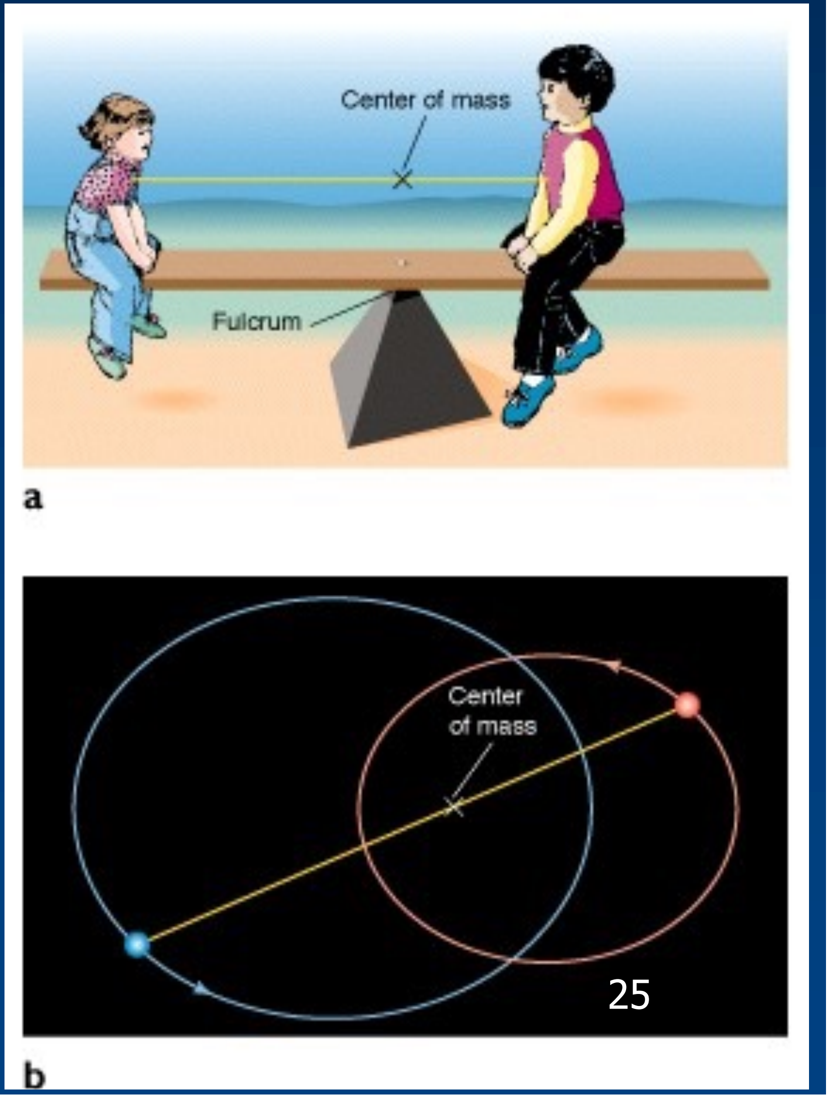
Since $F = G\frac{m_1 m_2}{r^2} = m_1 a_1 = m_2 a_2$

neither mass can be stationary.
Hence, Sun cannot sit at one focus.

Newton found <u>center of mass</u>
is at the focus.

If distances of m_1 and m_2 to C of M are r_1 and r_2, then $m_1 r_1 = m_2 r_2$

r_1 and r_2 change over orbit, but this relation always holds.



Newton's form of Kepler's third law

Newton found that Kepler's third law ($P^2 \propto a^3$) needed to be modified – into a general and *very* useful equation.

Newton's form of Kepler's third law:

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)} \right] a^3$$

with P in seconds, a in meters, masses in kg. a is the mean separation of the objects over their orbit.

Q: Why did Kepler miss the term with the masses?

Because mass of all planets = 0.1% mass of Sun

Example: By measuring period and semimajor axis for a satellite, you can calculate the sum of masses of satellite and its planet.

Imagine you've discovered a planet around another star. The planet has the same mass as the Earth, and the semimajor axis is 1 AU, but the star's mass is 4 times the Sun's mass. What's the planet's sidereal period?

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)}\right] a^3$$

For the Earth-Sun system , $m_1 + m_2 \approx M_{Sun}$ so this simplifies to

$$P^2 = \frac{4\pi^2}{GM_{Sun}} a^3$$

For star with mass four times that of Sun:

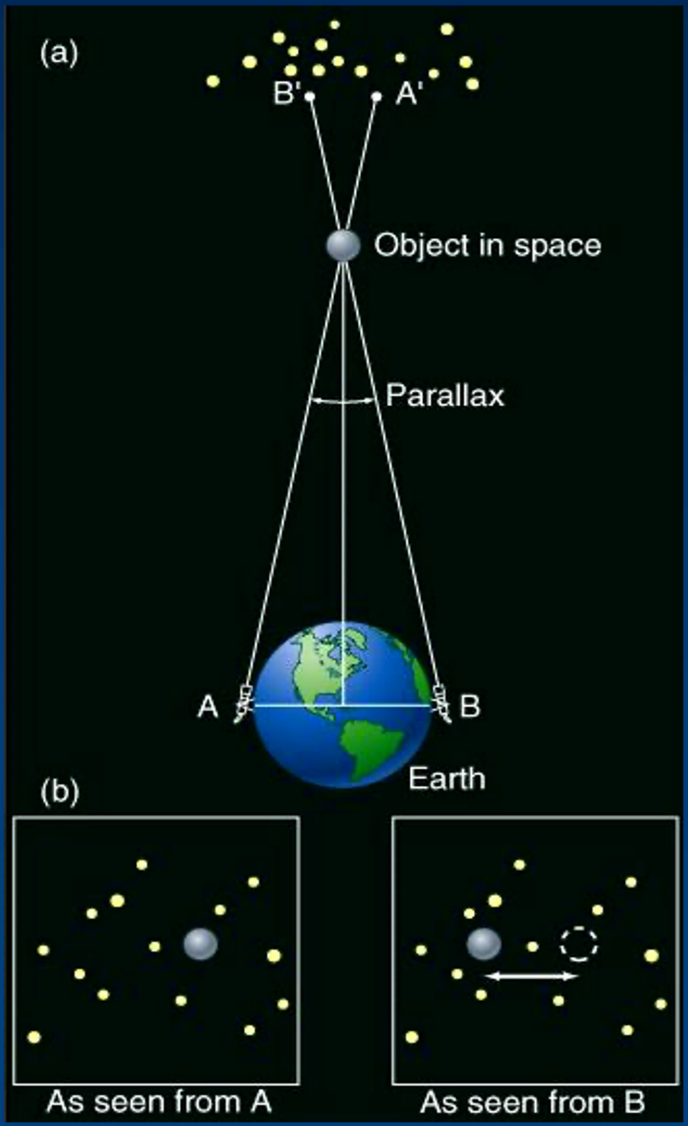
$$P^2 = \frac{4\pi^2}{4GM_{Sun}} a^3 = \frac{1}{4}\left[\frac{4\pi^2}{GM_{Sun}}\right] a^3$$

Semimajor axis is 1 AU in both cases, so P^2 is smaller by a factor of ¼ compared to solar system, so P must be ½ year.

Note: if P in years, a in AU, and masses in M⊙ , it is still the case that

$$P^2 = \frac{a^3}{m_1 + m_2}$$

In Newton's time, actual distances of planets from Sun were just starting to be measured. One technique is "Earth-baseline parallax".



"Earth-baseline" or "diurnal" parallax uses telescopes on either side of Earth to measure planet distances.

Use small angle formula:

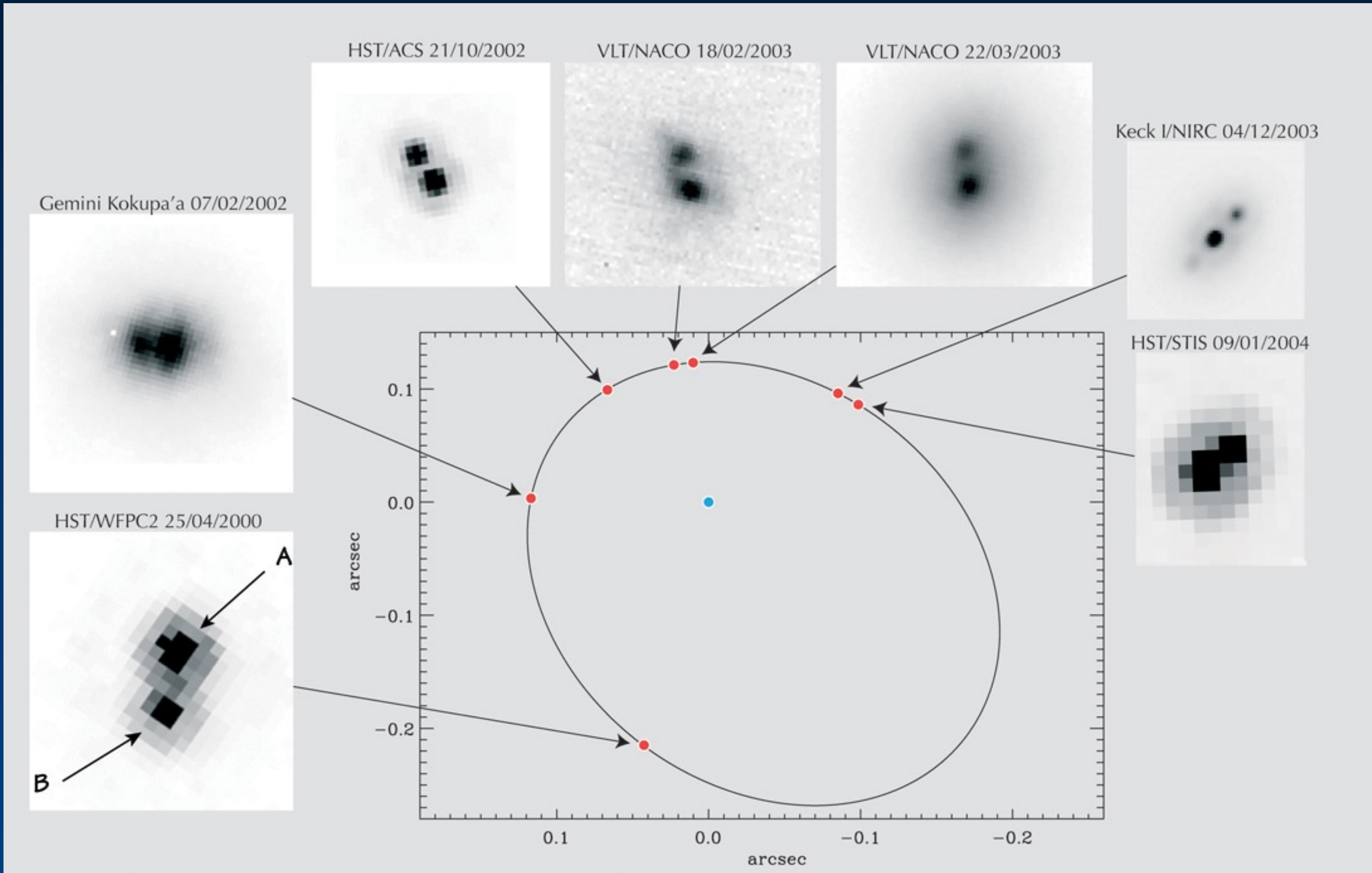
$$diam = (p / 206{,}265) \times dist$$

so

$$dist = diam / (p / 206{,}265)$$

First attempted by Cassini for Mars in 1672. Got answer 7% too small. Refined by Halley (1761) using parallax of Venus transit.

Newton said, *"I have not yet disclosed the cause of gravity, nor have I undertaken to explain it, since I could not understand it from phenomena"*



Orbit of the Ultra-cool Stars in 2MASSW J0746425+2000321
(NACO/VLT)

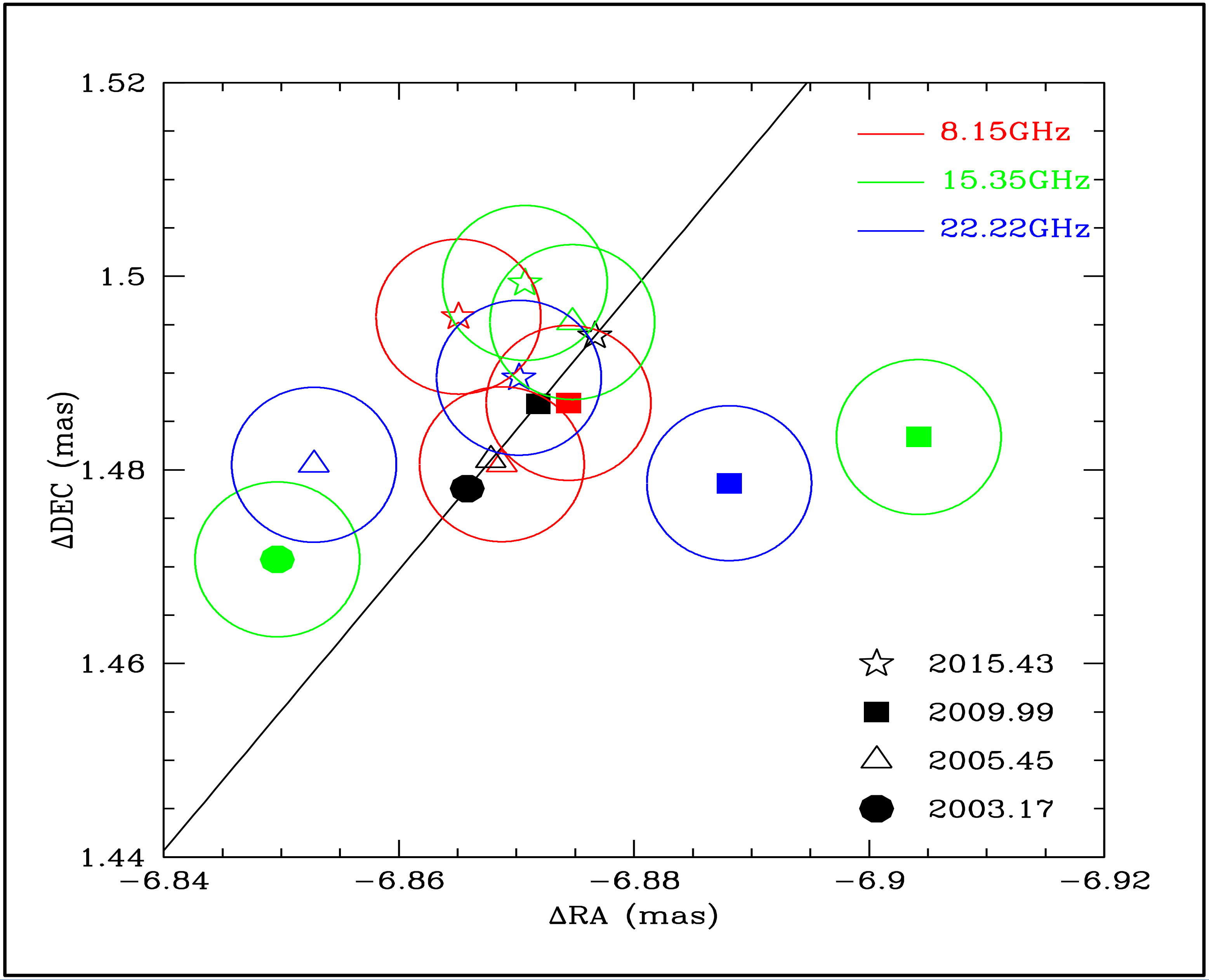
ESO PR Photo 19a/04 (15 June 2004)


Binary Black Holes

0402+379

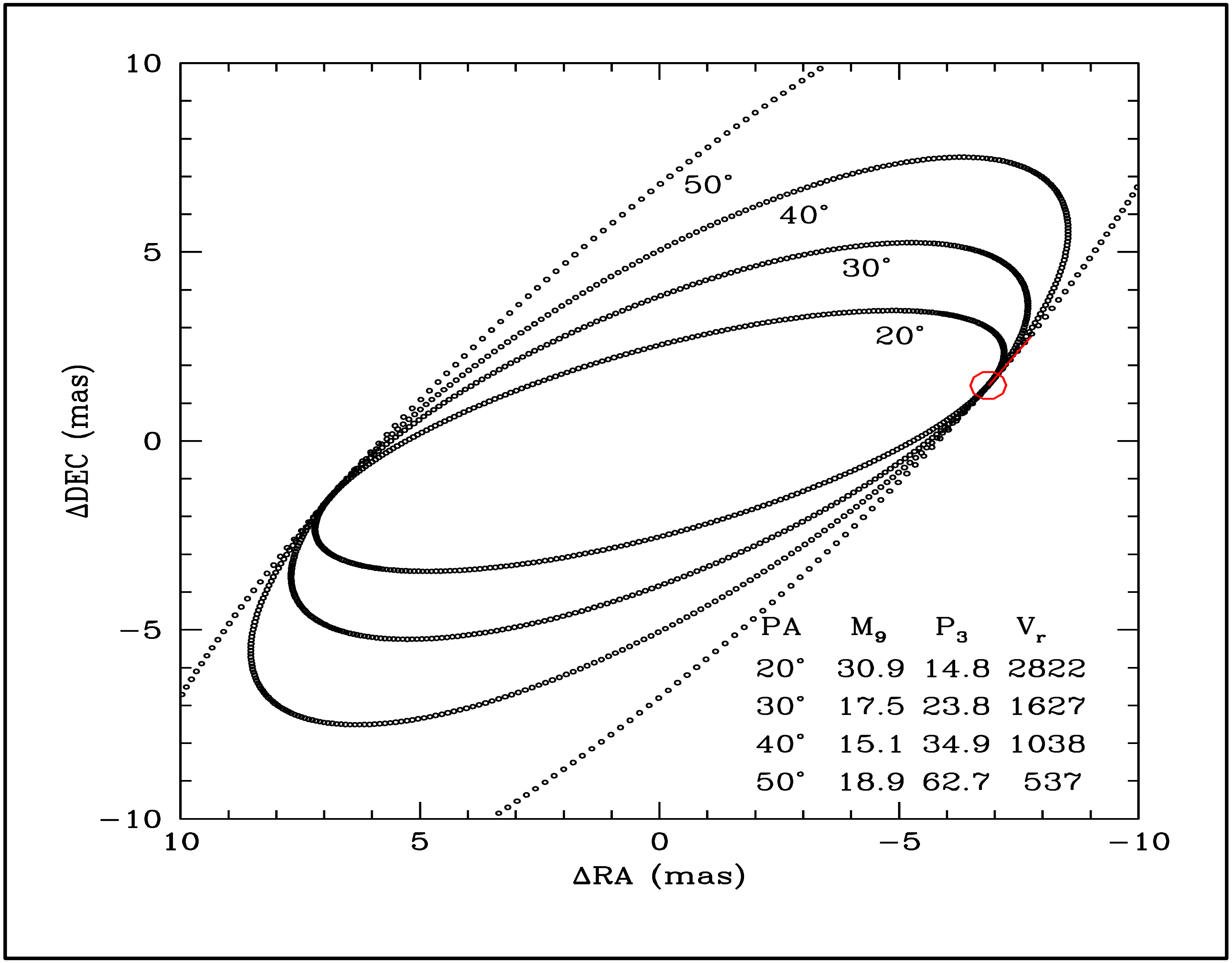
- Elliptical Galaxy
- Distance ~ 750 million light years
- Separation ~ 7 pc
- The most compact black hole binary system known

Results



1.52
1.5
1.48
1.46
1.44
−6.84
−6.86
−6.88
−6.9
−6.92
ΔDEC (mas)
ΔRA (mas)
8.15GHz
15.35GHz
22.22GHz
2015.43
2009.99
2005.45
2003.17

Results



50°
40°
30°
20°
PA M9 P3 Vr
20° 30.9 14.8 2822
30° 17.5 23.8 1627
40° 15.1 34.9 1038
50° 18.9 62.7 537
ΔDEC (mas)
ΔRA (mas)
10
5
0
−5
−10

Determine the total mass of a system

- The α Centauri system is a binary star system with a period of 79.92 years. The A and B components have a mean separation of 23.7 AU (although the orbits are highly elliptical). *What is the total mass of the system?*

- Use the generalized form of Kepler's third law

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)} \right] a^3$$

$$P = 79.92 \times 365 \times 24 \times 60 \times 60 = 2.52 \times 10^9 \mathrm{s}$$

$$a = 23.7 \times 1.496 \times 10^{11} = 3.55 \times 10^{12} \mathrm{m}$$

$$(m_1 + m_2) = \frac{4\pi^2 a^3}{GP^2} = \frac{4\pi^2 (3.55 \times 10^{12})^3}{6.67 \times 10^{-11} \times (2.52 \times 10^9)^2} = 4.2 \times 10^{30} \mathrm{kg}$$

Unit check: $m^3/ Nm^2kg^{-2}s^2 = m^3/ (kg\ m\ s^{-2})m^2\ kg^{-2}s^2$

$=m^3/(m^3kg^{-1}s^{-2}\ s^2)=kg$

Compare to mass of our Sun = 1.99×10^{30} kg

Newton's description of motion

A complete explanation of motions of objects:

- Easy to state in words or mathematics
- Universal physical laws that apply to all moving objects
- Unify phenomena by explaining everything with the same set of self-consistent rules