A Parametric Model for the Normalized Power Pattern of a Dipole Antenna over Ground

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1 Introduction

This memo describes a mathematical model for the normalized power pattern of a dipole of the type used in LWA and other low-frequency arrays. The model parameterizes the E- and H-planes using 4 real-valued parameters each (total 8 parameters). Under the assumption that the pattern is smoothly varying, patterns in other planes can be obtained by a simple trigonometric interpolation. Suitable model parameters are determined for the bowtie-type inverted-V dipole used in LWA-1 at 38 MHz and 74 MHz.

2 Model

The *normalized power pattern* of an antenna is defined the normalized directivity as a function of direction. *Normalized directivity* is defined as directivity in a given direction, divided by the directivity in the direction in which it is maximum, so that the maximum value is 1.

For the purposes of this memo, the ground is assumed to lie in the $x-y$ plane, with $+z$ pointing to zenith, and θ measured from the $+z$ axis. The dipole is assumed to lie in the $x-z$ plane; we then refer to this as the E-plane. The H-Plane is the $y-z$ plane. The normalized power pattern in *any* plane containing the z-axis, according to the model, is:

$$p(\theta) = \left[1 - \left(\frac{\theta}{\pi/2}\right)^{\alpha}\right] \cos^{\beta}\theta \;+\; \gamma\left(\frac{\theta}{\pi/2}\right)\cos^{\delta}\theta \quad , \quad 0 \leq \theta \leq \pi/2 \quad . \tag{1}$$

It should be noted that this model is not guaranteed to yield a reasonable and properly-normalized power pattern for all possible choices of the parameters; the intent is to have a model which is capable of accurately describing realistic patterns using only a small number of parameters. The parameters would normally be determined by some form of best-fit method using measured or simulation/calculated data.

Normally, dipole patterns are not *balanced*, meaning that the patterns in the E- and H-plane patterns are different. In this case, different sets of parameters are used to define the pattern in the E- and H-planes. Assuming the dipole is not electrically large, the pattern should vary slowly and smoothly in ϕ, so that the pattern in any other plane can be obtained by interpolation from the E- and H-plane patterns. Given E- and H-plane patterns $p_E(\theta)$ and $p_H(\theta)$, the recommended interpolation is:

$$p(\theta,\phi) = \sqrt{\left[p_E(\theta)\cos\phi\right]^2 + \left[p_H(\theta)\sin\phi\right]^2} \quad . \tag{2}$$

3 Application to LWA-1

The LWA inverted-V bowtie antenna is described in [1]. Also in this reference, the effective length of a single "standalone" LWA dipole is calculated at various frequencies and in various planes, including the E- and H-planes, using the method of moments. Since effective length is proportional to directivity, the model parameters α, β, γ, and δ for the E- and H-plane normalized power patterns can be extracted from this data. Using a trial-and-error best-fit procedure, I obtained the following model parameters:

38 MHz, E-Plane: $\alpha = 1$, $\beta = 3$, $\gamma = 1$, $\delta = 1.7$
38 MHz, H-Plane: $\alpha = 9$, $\beta = 1.3$, $\gamma = \delta = 0$

74 MHz, E-Plane: $\alpha = 1$, $\beta = 4$, $\gamma = 1$, $\delta = 1.4$
74 MHz, H-Plane: $\alpha = 9$, $\beta = 1.2$, $\gamma = \delta = 0$

A comparison of the model result compared to the original data is shown in Figures 1 and 2.

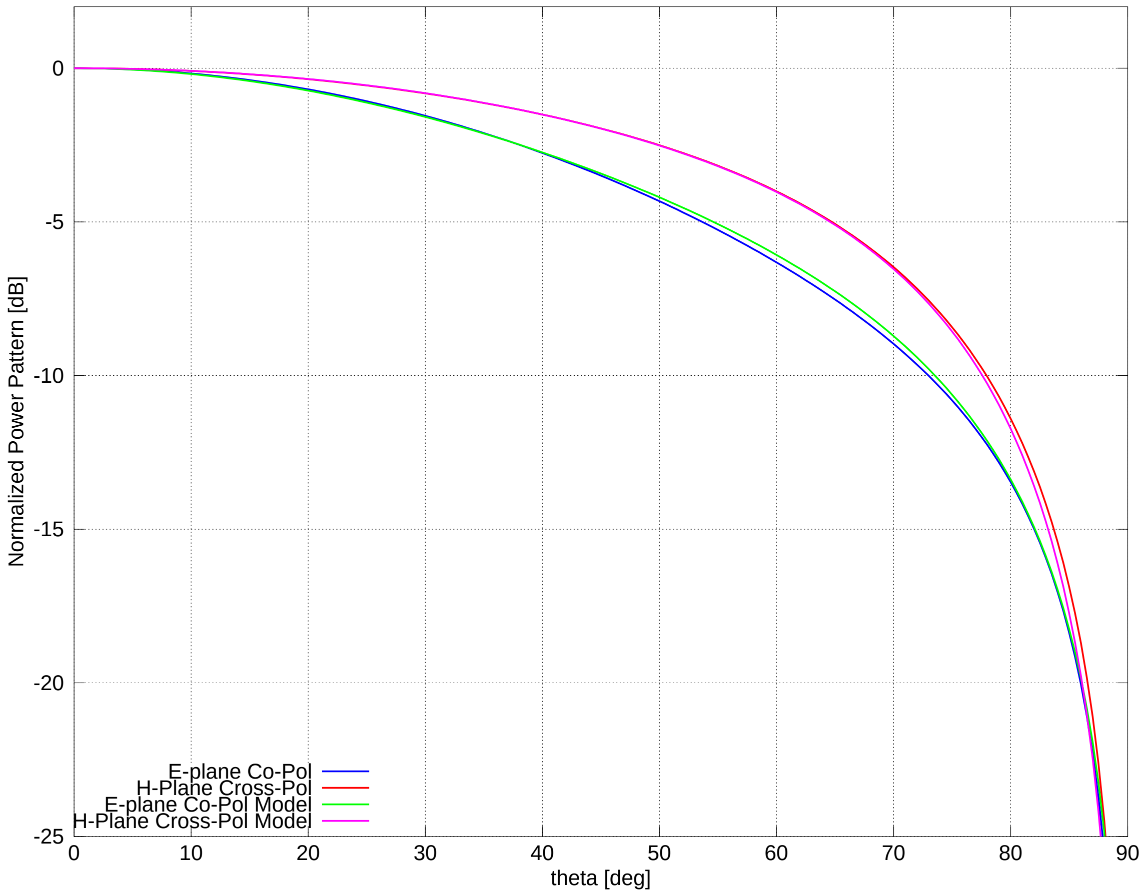

Figure 1: LWA-1 model results compared to moment method data from [1], 38 MHz. Note that this is for a single "standalone" dipole.

Note that the agreement between the model and the moment method data is very good (in fact, difficult to see because the curves are so close), with the largest difference emerging near $\theta = \pi/2$, where the directivity is smallest.

References

[1] S. Ellingson *et al.*, "The Long Wavelength Array," *Proc. IEEE*, Vol. 97, No. 8, pp. 1421-1430, Aug 2009. Also available as LWA Memo 157.

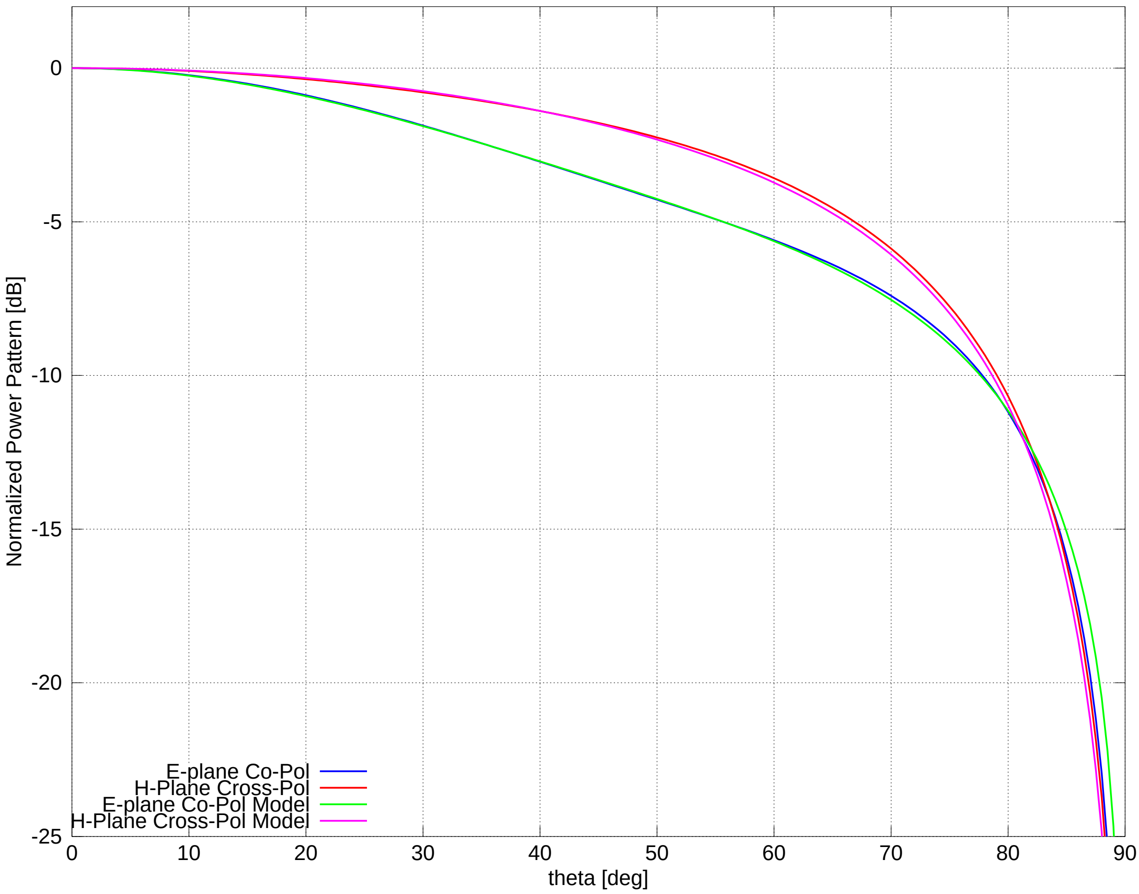

Figure 2: LWA-1 model results compared to moment method data from [1], 74 MHz. Note that this is for a single "standalone" dipole.