Use of NEC-2 to Calculate Collecting Area

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1 Introduction

This report describes a simple method for determining the collecting area of an antenna using the Numerical Electromagnetics Code (NEC-2) code. This technique determines the collecting area of the antenna under test directly in "receive mode", as opposed to an alternative technique in which the collecting area is determined from the pattern gain in "transmit mode." The technique is used to estimate the collecting area of a thin straight half-wave dipole, for which theoretical results are also available. Without too much effort to optimize accuracy, it is found in this case that the predicted collecting area is about 6% larger than the theoretical value. Greater accuracy might be possible with additional effort. This approach is easily extended to facilitate modeling of complete LWA stations, assuming sufficient memory is available. The more familiar indirect (transmit mode) method yields accuracy of better than 1%, but relies on reciprocity arguments which may be difficult to apply for complex arrays in realistic scenarios.

2 Problem Statement

We wish to determine the collecting area – referred to here as the effective aperture, A_e – of a thin straight half-wave dipole. For simplicity, we consider only one frequency $f = 38$ MHz, for which the free space wavelength $\lambda = 7.895$ m, and the dipole is exactly $\lambda/2$ long. The dipole is constructed from perfectly-conducting material of circular cross section having a radius of 0.05 mm. Its terminals are located at the origin of the coordinate system and it is aligned along the z-axis, such that its pattern maximum is in the $x-y$ plane and it's nulls are along the z axis. For simplicity we shall consider only A_e in the direction of maximum gain; in this case, in the $x-y$ plane.

A simple technique by which to determine A_e for any antenna is simply to use the relationship [1]:

$$A_e = \frac{\lambda^2}{4\pi} G \ , \tag{1}$$

where G is the pattern gain. The theoretical value of G for a vanishingly-thin half-wave dipole is 2.15 dB, yielding $A_e = 8.137$ m^2 at 38 MHz. G can be obtained for any antenna by the principle of reciprocity [1]. For example, one might apply a voltage source to the antenna terminals, then use NEC or some other technique to calculate the transmit pattern gain, and exploit reciprocity to interpret this value as the receive pattern gain. This is quite straightforward for simple antennas in free space; however the reciprocity argument is more difficult to apply for arrays of complex elements in the vicinity of complex media, such as the interface between lossy ground and free space. Thus, it is desireable to have an independent method by which the calculation can be done "directly" with the antenna system operating in "receive mode."

3 Proposed Method

The alternative approach proposed here uses directly the definition of A_e as the ratio of P_r, the power successfully received by the antenna, to the incident power density S^i, typically having units of W/m^2. Of course, P_r depends on the impedance presented by the load attached to the terminals of the antenna under test. Typically the *maximum* value of A_e is of interest, which occurs when the load impedance Z_L is equal to the complex conjugate of the antenna's terminal impedance Z_A; i.e., $Z_L = Z_A^*$. Thus, it is necessary to know Z_A before proceeding.

Determination of Z_A using NEC-2 is a simple matter. A NEC "deck" (input file describing the problem) for this is shown in Figure 1. This deck models the antenna using 11 segments, with a voltage source with magnitude 1 V applied to the center segment, which corresponds to the terminals. When this deck is run, NEC computes the resulting currents including the current in the segment containing the voltage source. From this, Z_A is computed as the ratio of voltage to current in the segment corresponding to the terminals; NEC reports $Z_A = 77.41 + j45.09$ Ω. Thus, we choose

$Z_L = 77.41 - j45.09\ \Omega$.

At this point it is noted that a slight modification to NEC deck shown in Figure 1 – specifically, the addition of an "RP" ("radiation pattern") "card" (line) – can be used to obtain G, the pattern gain in transmit mode. In this case, I obtain G = 2.16 dB, which corresponds to $A_e = 8.156$ m^2, or just 0.2% higher than the theoretical value.

To compute A_e using the alternative method, it is proposed to illuminate the antenna under test with a plane wave having a known S^i and then to measure the power P_r delivered to the load. NEC-2 provides a mechanism for plane wave excitation of antennas, however I find this feature to be poorly documented and I am reluctant to trust it. An equally effective way to generate the desired incident wave is simply to use another antenna located in the far field of the antenna under test. As I would ultimately like to use this same method to study LWA stations, I wish the source dipole to be also in the far field of a station. Assuming a maximum dimension $D = 100$ m for a station, the far field criterion is $R > 2D^2/\lambda = 2.53$ km; and I choose $R = 25$ km to be sure.

For simplicity, the source dipole is chosen to be identical to the antenna under test, except the load is replaced by a voltage source $V_t = 1 + j0$ V applied to the center segment. Because we are interested in A_e in the $x - y$ plane, the terminals of the source dipole are located at $x = R$, $y = 0$, $z = 0$. The source dipole is oriented in the same direction so as to be co-polarized with the antenna under test.

For this problem, A_e can be determined as follows:

1. The electric field radiated by a vanishingly-thin half-wave dipole is known to be [1]:

$$\mathbf{E}(\theta_t, R) = \hat{\theta}_t \, \frac{\cos(\frac{\pi}{2}\cos\theta_t)}{\sin\theta_t} \, \frac{j\omega\mu}{2\pi k} \, I_t \, \frac{e^{-jkR}}{R} \ , \tag{2}$$

   where $I_t = V_t/Z_A$ is the current at the terminals, θ_t is the angle measured from the $+z$ direction with associated unit vector $\hat{\theta}_t$, $j = \sqrt{-1}$, $\omega = 2\pi f$, $k = 2\pi/\lambda$, and $\mu = 4\pi \times 10^{-7}$ H/m (the permeability of free space).

2. In the far field of the source, the radiated wave appears locally as a plane wave. Thus the power density of the plane wave incident on the antenna under test is [1]:

$$S^i = \frac{1}{2\eta} \left|\mathbf{E}\right|^2 \ , \tag{3}$$

   where $\eta = 376.7\ \Omega$, the impedance of free space.

3. The power delivered to the matched load of the antenna under test is determined by first calculating the current I_r generated at the terminals of the antenna under test using NEC. The NEC deck used in this case is shown in Figure 2.

4. The power delivered to the load is then:

$$P_r = \mathrm{Re}\left\{ \frac{1}{2} \left|I_r\right|^2 Z_L \right\} \tag{4}$$

5. Finally, we calculate $A_e = P_r/S^i$.

4 Results and Comparison to Theory

Using the procedure described in the preceding section, I obtained $I_r = 0.3340 - j0.3185\ \mu$A, leading to an estimate of $A_e = 8.64$ m^2.

One reason for selecting this particular antenna to evaluate is because theoretical results are available for comparison. The effective aperture of any vanishingly-thin co-polarized linear antenna attached to matched load is [1]:

$$A_e = \frac{\eta l_e^2}{4R_{rad}} \ , \tag{5}$$

where $R_{rad} = \mathrm{Re}\left\{Z_A\right\}$ for a lossless antenna and l_e is the "effective length", given in our case by:

$$l_e = \frac{1}{I_r}\int I(z)dz \ , \tag{6}$$

where $I(z)$ is the current at the indicated position on the dipole. For a half-wave dipole, $l_e = \lambda/\pi$, and if the dipole is vanishingly thin, $R_{rad} = 73\ \Omega$. Thus we obtain the well-known formula $A_e = 0.13\lambda^2$, which in our case is 8.16 m^2. Our computed result is 5.9% greater than this well-known theoretical result; perhaps reasonable agreement.

An important distinction between the actual problem and the theoretical problem is that the actual radius of the dipole is finite. This has a small but significant effect on R_{rad}, which we found to be 77.41 Ω as opposed 73 Ω. This changes the result obtained using Equation 5 to $A_e = 7.69$ m^2, making our computed result now 12.3% greater than theoretical. At first glance, this seems counterintuitive as one would expect that actions that make the theoretical model conform more closely to the actual situation would result in an improvement in agreement, which is clearly not the case here. However, if R_{rad} is modified then perhaps l_e should also be modified. Using Equation 6 with the dipole current reported by NEC, we find a significant increase in l_e, now yielding $A_e = 8.19$ m^2. Thus the computed results are just 5.5% greater than the theoretical result computed using Equation 5, if we take into account both the actual load impedance and the NEC-derived dipole currents. This is a slight improvement in agreement.

5 Discussion

The above results indicate that the method proposed yields an estimate of collecting area that is about 6% higher than that expected from theory. This level of uncertainty is probably acceptable for most applications, since the magnitudes of other errors are likely to have similar values. Increasing the number of segments in the NEC calculations might help; however, given that the intended use of this procedure is to analyze complete LWA stations, an increase in the number of segments required per dipole will greatly increase the computation time and thus is not desired. In any event, this method provides a useful independent check of A_e calculations based on reciprocity arguments.

```
CM thin half-wave dipole with unit voltage source
CE
GW 1 11 0.0000 0.0000 -1.9737 0.0000 0.0000 1.9737 0.00005
GE 0
EX 0 1 6 0 0
FR 0 1 0 0 38.0 0.0
EN
```

Figure 1: NEC deck used to determine Z_A for the antenna under test.

```
CM thin half-wave dipole with matched load illuminated by another dipole in far field
CE
GW 1 11 +25000.0000 0.0000 -1.9737 +25000.0000 0.0000 +1.9737 0.00005
GW 2 11      0.0000 0.0000 -1.9737      0.0000 0.0000 +1.9737 0.00005
GE 0
EX 0 1 6 0 0
LD 4 2 6 6 +77.4100 -45.0900
FR 0 1 0 0 38.0 0.0
EN
```

Figure 2: NEC deck used to determine I_r for the antenna under test (with matched load), using another dipole in the far field as the source.

References

[1] W.L. Stutzman and G.A. Thiele, *Antenna Theory and Design*, 2nd Ed., Wiley, 1997.